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STABILITY ANALYSIS OF A CLASS OF TAKAGI-SUGENO FUZZY COHEN-GROSSBERG NEURAL NETWORKS WITH TIME DELAYS

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Abstract: This paper deals with the problem of the global asymptotic stability of the class of Takagi-Sugeno Fuzzy Cohen-Grossberg neural networks with multiple time delays. By constructing a suitable fuzzy Lyapunov functional, we present a new delay-independent sufficient condition for the global asymptotic stability of the equilibrium point for delayed Takagi-Sugeno Fuzzy Cohen-Grossberg neural networks with respect to the Lipschitz activation functions. The obtained condition simply relies on the network parameters of the neural system. Therefore, the equilibrium and stability properties of the neural network model considered in this paper can be easily verified by exploiting some basic properties of some certain classes of matrices.

Keywords: T-S Fuzzy Neural Networks, Delayed Systems, Lyapunov Functionals, Matrix Theory

Takagi-Sugeno Bulanık Cohen-Grossberg Tipi Zaman Gecikmeli Yapay Sinir Ağlarında Kararlılık Analizi

Öz: Bu çalışma çoklu zaman gecikmeli Takagi-Sugeno Bulanık Cohen-Grossberg tipi yapay sinir ağlarının global asimtotik kararlılık problemi ile ilgilenmektedir. Uygun bulanık Lyapunov fonksiyonelleri kullanılarak ve aktivasyon fonksiyonlarının Lipschitz olduğu dikkate alnarak, gecikmeli Takagi-Sugeno Bulanık Cohen-Grossberg yapay sinir ağlarında denge noktasının global asimtotik gecikme parametrelerinden bağımsız olarak, yeni yeterli bir kararlılık koşulu sunulmuştur. Elde edilen koşul sadece sinir ağının sistem parametrelerine bağlı olarak ifade edilmiştir. Bu nedenle, bu çalışmada çalışılan yapay sinir ağı modelinin denge ve kararlılık özellikleri, bazı özel matris sınıflarının temel özellikleri kullanarak kolaylıkla doğrulanabilir.

Anahtar Kelimeler: T-S Bulanık Sinir Ağları, Gecikmeli Sistemler, Lyapunov Fonksiyonelleri, Matris Teorisi

1. INTRODUCTION

In the recent years, Cohen-Grossberg neural networks (CGNNs) introduced in Cohen and Grossberg (1983) has been successfully applied to solve some practical engineering problems such as combinatorial optimization, image and signal processing, pattern recognition, associative memory design and control systems. In such applications, determining the equilibrium and stability properties of the equilibrium point of the designed neural network is of

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great importance. In particular, if this class of neural networks neural is to be applied to the problems of parallel computation, neural control and optimization, then, these neural networks must be globally asymptotically stable. Therefore, the stability analysis of dynamical neural networks is of great importance in both designs and applications. However, in real time hardware applications of neural networks, during the electronic implementation of analog neural networks, when neurons process and transmit the signals, due to the finite switching speed of amplifiers, some transmission delays occur which may change the dynamics of the network behaviors. Therefore, it is important to consider the effects of the time delay parameters on the stability of neural networks. Recently, some papers have studied the global asymptotic stability of the equilibrium point for Cohen-Grossberg neural networks with delay parameters and reported some useful global stability conditions establishing various relationships among the network parameters of this class of neural networks (Arik and Orman 2005; Cohen and Grossberg, 1983; Nie et. al. 2015; Zheng et. al. 2013). On the other hand, it has been pointed out Takagi and Sugeno (1983) that the fuzzy logic theory can be exploited to express neural networks in different mathematical models, which may make a positive impact on the complex dynamical behaviors of neural networks. For this purpose, the authors of Takagi and Sugeno (1983) has introduced the class of Takagi-Sugeno (T-S) fuzzy model and proved the fuzzy logic theory can be used to transform a nonlinear system into a set of T–S linear models. The advantage of this approach is that it is able to express some certain nonlinear complex systems in the form of overall fuzzy linear T-S models, which simplifies the stability analysis of complex nonlinear systems. In a recent paper Hou et. al. (2007), by employing the Lyapunov stability theory, the authors of Hou et. al. (2007) derived some new sets of sufficient conditions ensuring the global asymptotic stability of the equilibrium point for the class of T-S fuzzy neural networks in the presence time delays and with respect to the Lipschitz activation functions. The results obtained in Hou et. al. (2007) paved the way to conduct further studies into the stability analysis of T-S fuzzy neural networks. In Ahn (2011); Balasubramaniam and Ali (2011); Bao et. al. (2012); Bao (2016); Chandran and Balasubramaniam (2013); Gan et. al. (2010); Gan (2013); He and Xu (2008); Huang et. al. (2005); Jiang and Jiang (2017); Li et. al. (2010); Mathiyalagan et. al. (2014); Senan (2018); Tseng et. al. (2012); Xie and Zhu (2015); Yamamoto and Furuhashi (2001); Yang (2014); Zheng et. al. (2016), various classes of delayed fuzzy neural network models have analyzed to derive new sufficient conditions for the global asymptotic stability of the equilibrium point. In the current paper, we carry out a further analysis of the stability of problem of Takagi-Sugeno fuzzy Cohen-Grossberg neural networks with multiple time delays and present a novel sufficient condition for the global asymptotic stability of delayed fuzzy Cohen-Grossberg neural networks.

2. DELAYED T-S FUZZY COHEN-GROSSBERG NEURAL NETWORKS AND STABILITY ANALYSIS

Dynamical behavior of a Cohen-Grossberg neural network model with multiple time delays is governed by the following sets of differential equations:

$$\dot{x}_i(t) = d_i(x_i(t))\left[-c_i\big(x_i(t)\big) + \sum_{j=1}^{n} a_{ij} f_j\left(x_j(t)\right) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j\big(t-\tau_{ij}\big)\right) + u_i\right] \tag{1}$$

where n is the number of the neurons in the neural system, x_i denotes the state of the ith neuron, $d_i(x_i)$ represent the amplification functions, and $c_i(x_i)$ represent the behaved functions. The constants a_{ij} are the interconnection parameters of the neurons within the neural system, the constants b_{ij} are interconnection parameters of the neurons with time delay parameters τ_{ij}.The $f_i(.)$ denote the activation functions of neurons. The constants u_i are some external inputs. In system (1), $\tau_{ij} \geq 0$ are constant time delays with $\tau = \max(\tau_{ij}), 1 \leq i,j \leq n$. Accompanying the neural system (1) is an initial condition of the form: $x_i(t) = \phi_i(t) \in C([-\tau,0]),R)$, where $C([-\tau,0]),R)$ denotes the set of all continuous functions from $[-\tau,0]$ to R.

The assumptions on the functions $d_i(x)$, $c_i(x)$ and $f_i(x)$ in (1) are defined to be as follows:

H_1: The functions $d_i(x)$, $(i = 1,2,\dots,n)$ satisfy the conditions

$$0 < \mu_i \leq d_i(x) \leq \rho_i, \forall x \in R$$

where μ_i and $\rho_i$are some positive constants.

H_2: The functions $c_i(x)$, $(i = 1,2,\dots,n)$ satisfy the conditions

$$\frac{c_i(x) - c_i(y)}{x - y} = \frac{|c_i(x) - c_i(y)|}{|x - y|} \geq \gamma_i > 0, i = 1,2,\dots,n, \forall x,y \in R, x \neq y$$

where $\gamma_i$are some positive constants.

H_3: The functions $f_i(x)$, $(i = 1,2,\dots,n)$ satisfy the conditions

$$|f_i(x) - f_i(y)| \leq \ell_i|x - y|, i = 1,2,\dots,n, \forall x,y \in R, x \neq y$$

where $\ell_i$are some positive constants.

For the sake of simplicity of the proofs, we will transfer the equilibrium point x^* of Cohen-Grossberg neural network model (1) to the origin. By using the transformation $z(t) = x(t) - x^*$, neural system (1) can be expressed as follows:

$$\dot{z}_i(t) = \alpha_i(z_i(t))\left[-\beta_i(z_i(t)) + \sum_{j=1}^{n} a_{ij}g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij}g_j\left(z_j(t - \tau_{ij})\right)\right] \qquad (2)$$

For this new transformed system defined (2), the functions $d_i(z)$, $c_i(z)$ and $f_i(z)$ are in the form:

$$\alpha_i(z_i(t)) = d_i(z_i(t) + x_i^*), i = 1,2,\dots,n$$

$$\beta_i(z_i(t)) = c_i(z_i(t) + x_i^*) - c_i(x_i^*), i = 1,2,\dots,n$$

$$g_i(z_i(t)) = f_i(z_i(t) + x_i^*) - f_i(x_i^*), i = 1,2,\dots,n$$

In Senan (2018), the T-S fuzzy Cohen-Grossberg neural network with multiple time delays is stated by the following sets of differential equations:

Plant Rule r:
IF $\{\theta_1(t)\ is\ M_{r1}\}$ and … and $\{\theta_p(t)\ is\ M_{rp}\}$
THEN

$$\dot{z}_i(t) = \alpha_i^{(r)}(z_i(t))\left[-\beta_i^{(r)}(z_i(t)) + \sum_{j=1}^{n} a_{ij}^{(r)} g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij}^{(r)} g_j\left(z_j(t-\tau_{ij})\right)\right] \quad (3)$$

where $\theta_l(t)$ $(l = 1, 2, \dots, p)$ are the premise variables. M_{rl} $(r \in \{1, 2, \dots, m\}, l \in \{1, 2, \dots, p\}$ are the fuzzy sets and m is the number of **IF-THEN** rules.

By inferring from the fuzzy models, the mathematical model of system (3) takes the form Senan (2018):

$$\dot{z}_i(t) = \sum_{r=1}^{m} h_r(\theta(t))\left\{\alpha_i^{(r)}(z_i(t))\left[-\beta_i^{(r)}(z_i(t)) + \sum_{j=1}^{n} a_{ij}^{(r)} g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij}^{(r)} g_j\left(z_j(t-\tau_{ij})\right)\right]\right\} \quad (4)$$

where $\theta(t) = \left[\theta_1(t), \theta_2(t), \dots, \theta_p(t)\right]^T$, $\omega_r(\theta(t)) = \prod_{l=1}^{p} M_{rl}(\theta_1(t))$ and $h_r(\theta(t)) = \frac{\omega_r(\theta(t))}{\sum_{r=1}^{m}\omega_r(\theta(t))}$ denote the weight and averaged weight of each fuzzy rule, respectively. The term $\omega_{rl}(\theta_l(t))$ is the grade membership of $\theta_l(t)$ in ω_{rl}. It is assumed that $\omega_r(\theta(t)) \geq 0, r \in \{1, 2, \dots, m\}$, implying that $\sum_{r=1}^{m} h_r(\theta(t)) = 1$ for all $t \geq 0$.

In the light of the fuzzy logic theory, the general assumptions H_1, H_2 and H_3 are restated for the model of T-S fuzzy neural system (4) as follows:

$$0 < \alpha_i^{(r)}(z_i(t)) \leq \rho_i^{(r)}, i = 1, 2, \dots, n$$

$$z_i(t)\beta_i^{(r)}(z_i(t)) \geq \gamma_i^{(r)} z_i^2(t) \geq 0, i = 1, 2, \dots, n$$

$$|g_i(z_i(t))| \leq k_i|z_i(t)|, z_i(t)g_i(z_i(t)) \geq 0, i = 1, 2, \dots, n$$

3. STABILITY OF DELAYED TAKAGI-SUGENO FUZZY COHEN-GROSSBERG NEURAL NETWORKS

In this section, we present the main the stability result of this paper, which is stated in the following theorem:

Theorem 1: For the neural system (4), assume that H_1, H_2 and H_3 hold. Then, the origin of the delayed T-S fuzzy Cohen-Grossberg neural network model (4) is globally asymptotically stable if the following condition is satisfied:

$$\delta = \frac{\mu\gamma}{\rho\ell} - \sum_{r=1}^{m} \|A_r\|_2 - \sum_{r=1}^{m} \sqrt{\|B_r\|_1 \|B_r\|_\infty} > 0$$

Where $\mu = min\left\{\mu_i^{(r)}\right\}, \rho = max\left\{\rho_i^{(r)}\right\}$ and $\gamma = max\left\{\gamma_i^{(r)}\right\}$, $\ell = max\{\ell_i\}$ $i = 1, 2, \dots, n, r = 1, 2, \dots, m, A_r = (a_{ij}^{(r)})_{nxn}$ and $B_r = (b_{ij}^{(r)})_{nxn}$.

Proof: In order to prove the result of Theorem 1, we use the Lyapunov stability approach. Now, construct the following positive definite Lyapunov functional:

$$V\big(z(t)\big)=\frac{1}{2}\sum_{i=1}^{n}z_i^2(t)+\frac{1}{2}\rho\ell\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t}\left|b_{ij}^{(r)}\right|z_j^2(\zeta)d\zeta+\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t}z_j^2(\zeta)d\zeta \quad (5)$$

where the ξ_r, ε and σ are some positive constants to be determinated later. When calculating the time derivative of $V(x(t))$ defined by (5) along the trajectories of system (4) yields, we get

$$\begin{aligned}\dot{V}\big(z(t)\big)&=\sum_{i=1}^{n}z_i(t)\dot{z}_i(t)+\frac{1}{2}\rho\ell\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{(r)}\right|z_j^2(t)\\&-\frac{1}{2}\rho\ell\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{(r)}\right|z_j^2\big(t-\tau_{ij}\big)\\&+\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t)-\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2\big(t-\tau_{ij}\big)\\&=-\sum_{r=1}^{m}h_r\big(\theta(t)\big)\sum_{i=1}^{n}z_i(t)\alpha_i^{(r)}\big(z_i(t)\big)\beta_i^{(r)}\big(z_i(t)\big)\\&+\sum_{r=1}^{m}h_r(\theta(t))\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^{(r)}(z_i(t))a_{ij}^{(r)}z_i(t)g_j(z_j(t))\\&+\sum_{r=1}^{m}h_r(\theta(t))\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^{(r)}(z_i(t))b_{ij}^{(r)}z_i(t)g_j(z_j\big(t-\tau_{ij}\big))\\&+\frac{1}{2}\rho\ell\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{(r)}\right|z_j^2(t)-\frac{1}{2}\rho\ell\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{(r)}\right|z_j^2\big(t-\tau_{ij}\big)\\&+\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t)-\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2\big(t-\tau_{ij}\big)\end{aligned}\quad (6)$$

Under the Assumptions H_1, H_2 and H_3, we can easily derive the following inequalities:

$$\begin{aligned}-\sum_{r=1}^{m}h_r\big(\theta(t)\big)\sum_{i=1}^{n}z_i(t)\alpha_i^{(r)}\big(z_i(t)\big)\beta_i^{(r)}\big(z_i(t)\big)&\le-\mu\gamma\sum_{i=1}^{n}z_i^2(t)\\&=-\mu\gamma\|z(t)\|_2^2\end{aligned}\quad (7)$$

$$\sum_{r=1}^{m}h_r(\theta(t))\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^{(r)}(z_i(t))a_{ij}^{(r)}z_i(t)g_j(z_j(t))=\sum_{r=1}^{m}h_r\big(\theta(t)\big)\,z^T(t)\alpha_r\big(z(t)\big)A_rg\big(z(t)\big)$$

$$\leq \sum_{r=1}^{m} \left\| \alpha_r\big(z(t)\big) \right\|_2 \|A_r\|_2 \left\| \big(z(t)\big) \right\|_2 \left\| g\big(z(t)\big) \right\|_2$$
$$\leq \rho\ell \sum_{r=1}^{m} \|A_r\|_2 \|z(t)\|_2^2 \tag{8}$$

$$\sum_{r=1}^{m} h_r(\theta(t)) \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i^{(r)}(z_i(t)) b_{ij}^{(r)} z_i(t) g_j(z_j\big(t-\tau_{ij})\big)$$
$$\leq \rho \sum_{r=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \left| b_{ij}^{(r)} \right| |z_i(t)| \left| g_j(z_j\big(t-\tau_{ij}\big)\right|$$
$$\leq \rho\ell \sum_{r=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \left| b_{ij}^{(r)} \right| |z_i(t)| \left| z_j(z_j\big(t-\tau_{ij}\big)\right|$$
$$\leq \frac{1}{2}\rho\ell \sum_{r=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\xi_r} \left| b_{ij}^{(r)} \right| z_i^2(t) + \frac{1}{2}\rho\ell \sum_{r=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \xi_r \left| b_{ij}^{(r)} \right| z_j^2(t-\tau_{ij}) \tag{9}$$

Inserting (7)-(9) into (6) results in

$$\begin{aligned}
\dot{V}\big(z(t)\big) \leq & -\mu\gamma \|z(t)\|_2^2 + \rho\ell \sum_{r=1}^{m} \|A_r\|_2 \|z(t)\|_2^2 \\
& + \frac{1}{2}\rho\ell \sum_{r=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{\xi_r} \left| b_{ij}^{(r)} \right| z_i^2(t) + \frac{1}{2}\rho\ell \sum_{r=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} \xi_r \left| b_{ij}^{(r)} \right| z_j^2(t-\tau_{ij}) \\
& + \frac{1}{2}\rho\ell \sum_{r=1}^{m} \xi_r \sum_{i=1}^{n}\sum_{j=1}^{n} \left| b_{ij}^{(r)} \right| z_j^2(t) - \frac{1}{2}\rho\ell \sum_{r=1}^{m} \xi_r \sum_{i=1}^{n}\sum_{j=1}^{n} \left| b_{ij}^{(r)} \right| z_j^2\big(t-\tau_{ij}\big) \\
& + \varepsilon \sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t) - \varepsilon \sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2\big(t-\tau_{ij}\big) \\
= & -\mu\gamma \|z(t)\|_2^2 + \rho\ell \sum_{r=1}^{m} \|A_r\|_2 \|z(t)\|_2^2 \\
& + \frac{1}{2}\rho\ell \sum_{r=1}^{m} \frac{1}{\xi_r} \sum_{i=1}^{n}\sum_{j=1}^{n} \left| b_{ij}^{(r)} \right| z_i^2(t) + \frac{1}{2}\rho\ell \sum_{r=1}^{m} \xi_r \sum_{i=1}^{n}\sum_{j=1}^{n} \left| b_{ji}^{(r)} \right| z_i^2(t) \\
& + \varepsilon \sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t) - \varepsilon \sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2\big(t-\tau_{ij}\big)
\end{aligned} \tag{10}$$

From the basic norm properties of the matrices, we can obtain the following:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \left| b_{ij}^{(r)} \right| z_i^2(t) \leq \|B_r\|_\infty \|z(t)\|_2^2 \tag{11}$$

and

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ji}^{(r)}\right| z_i^2(t) \leq \|B_r\|_1\|z(t)\|_2^2 \tag{12}$$

Using (11) and (12) in (10) yields,

$$\begin{aligned}\dot{V}(z(t)) \leq -\mu\gamma\|z(t)\|_2^2 + \rho\ell\sum_{r=1}^{m}\|A_r\|_2\|z(t)\|_2^2 \\ +\frac{1}{2}\rho\ell\sum_{r=1}^{m}\frac{1}{\xi_r}\|B_r\|_\infty\|z(t)\|_2^2 + \frac{1}{2}\rho\ell\sum_{r=1}^{m}\xi_r\|B_r\|_1\|z(t)\|_2^2 \\ +\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij})\end{aligned} \tag{13}$$

If we choose $\xi_r = \frac{\sqrt{\|B_r\|_\infty}}{\sqrt{\|B_r\|_1}}$, $\quad r = 1, 2, \dots, m$ then, (13) can be written as

$$\begin{aligned}\dot{V}(z(t)) &\leq -\mu\gamma\|z(t)\|_2^2 + \rho\ell\sum_{r=1}^{m}\|A_r\|_2\|z(t)\|_2^2 \\ &\quad +\frac{1}{2}\rho\ell\sum_{r=1}^{m}\sqrt{\|B_r\|_1\|B_r\|_\infty}\|z(t)\|_2^2 + \frac{1}{2}\rho\ell\sum_{r=1}^{m}\sqrt{\|B_r\|_1\|B_r\|_\infty}\|z(t)\|_2^2 \\ &\quad +\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij}) \\ &= -\rho\ell\Big(\frac{\mu\gamma}{\rho\ell} - \sum_{r=1}^{m}\|A_r\|_2 - \sum_{r=1}^{m}\sqrt{\|B_r\|_1\|B_r\|_\infty}\Big)\|z(t)\|_2^2 \\ &\quad +\varepsilon n\|z(t)\|_2^2 - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij}) \\ &= -\rho\ell\delta\|z(t)\|_2^2 + \varepsilon n\|z(t)\|_2^2 - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij}) \\ &\leq -\rho\ell\delta\|z(t)\|_2^2 + \varepsilon n\|z(t)\|_2^2 \\ &= -(\rho\ell\delta - \varepsilon n)\|z(t)\|_2^2\end{aligned} \tag{14}$$

In (14), the choice guarantees $\varepsilon < \frac{\rho\ell\delta}{n}$ that $\dot{V}(z(t)) < 0$ for all $z(t) \neq 0$. The case where $z(t) = 0$ puts $\dot{V}(z(t))$ in the form:

$$\begin{aligned}\dot{V}(z(t)) &= -\frac{1}{2}\rho\ell\sum_{r=1}^{m}\xi_r\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{(r)}\right| z_j^2(t-\tau_{ij}) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij}) \\ &\leq -\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij})\end{aligned} \tag{15}$$

From (15) we can directly conclude that $\dot{V}\big(z(t)\big) < 0$ if there exists at least one nonzero $z_j(t-\tau_{ij})$ and $\dot{V}\big(z(t)\big) = 0$ if and only if and $z(t) = 0$ for all $z_j\big(t-\tau_{ij}\big) = 0$ otherwise. It is easy to validate the radial unboundedness of the Lyapunov functional $V(z(t))$ since $V(z(t)) \to \infty$ as $\|z(t)\| \to \infty$. Hence, it follows from the standard Lyapunov stability theorems that the origin of the T-S fuzzy Cohen-Grossberg neural network model (4) is globally asymptotically stable.

4. CONCLUSIONS:

In this paper, by using a new type of fuzzy Lyapunov functional, we have studied the global asymptotic stability of delayed Takagi-Sugeno (T-S) fuzzy Cohen-Grossberg neural networks with respect to the Lipschitz activation functions and derived new sufficient condition for the global asymptotic stability of the equilibrium point for this class of neural networks. The obtained
stability result can easily verified as it simply imposes some constraint conditions on the norms of the interconnection matrices independently of the time delay parameters.

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