

## Effect of Void Ratio and Water Contents on the Small-Strain Shear Modulus for Sands

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### Abstract

The dynamic behavior of sand samples in earthquake-prone regions must be thoroughly characterized and incorporated into building design processes. Seismic events induce significant changes in the stress-strain behavior and strength characteristics of soils, which in turn lead to ground deformations and potential structural damage. Therefore, it is essential to determine key dynamic soil properties such as the maximum shear modulus, the normalized shear modulus reduction curve, and the damping ratio curve, which are representative of the site conditions. However, the laboratory-based determination of the maximum shear modulus is both time-consuming and costly. To address this limitation, empirical models have been developed to provide practical alternatives for researchers. In this study, a series of resonant column tests were conducted on silty sand samples collected from a site located along the North Anatolian Fault Zone (NAFZ), one of the most active fault systems in Turkey. The tests were performed under varying confining pressures, void ratios, and water contents. Based on the experimental results, an empirical model with high predictive capability is proposed for engineering applications. The  $R^2$  (0.6537, 0.7683, 0.8312) and RMSE (1.88, 1.76, 1.91) values of these models have been obtained.

**Keywords:** Shear modulus, Damping ratio, Resonant column, Empirical model

## Boşluk Oranı ve Su Muhtevasının Kumların Düşük Kayma Modülüne Etkisi

### Özet

Deprem riski taşıyan bölgelerdeki kum numunelerinin dinamik davranışı ayrıntılı şekilde belirlenmeli ve yapı tasarım süreçlerinde dikkate alınmalıdır. Depremler, zeminlerin gerilme-şekil değiştirme ilişkisi ile dayanım özelliklerinde önemli değişikliklere yol açmakta, bu durum ise zemin deformasyonlarına ve yapısal hasarlara neden olmaktadır. Bu nedenle bölgeye özgü zemin özelliklerini yansıtan maksimum kayma modülü, normalize kayma modülü azalma eğrisi ve sönüm oranı eğrisi gibi dinamik zemin parametrelerinin belirlenmesi gerekmektedir. Ancak, maksimum kayma modülünün laboratuvar ortamında belirlenmesi hem zaman alıcı hem de maliyetlidir. Bu nedenle araştırmacıların kullanımına yönelik pratik çözümler sunan ampirik modeller geliştirilmiştir. Bu çalışmada, Türkiye'nin en aktif fay sistemlerinden biri olan Kuzey Anadolu Fay Zonu (KAFZ) üzerinde yer alan bir bölgeden alınan siltli kum numuneleri üzerinde farklı çevre basıncı, boşluk oranı ve su içeriği değerleri altında rezonant kolon deneyleri gerçekleştirilmiştir. Elde edilen deneysel veriler doğrultusunda mühendislik uygulamalarında kullanılabilecek yüksek öngörü gücüne sahip bir ampirik model önerilmiştir. Bu modellerin  $R^2$  (0.6537, 0.7683, 0.8312) ve RMSE (1.88, 1.76, 1.91) değerleri elde edilmiştir.

**Anahtar Kelimeler:** Kayma modülü, Sönüm oranı, Rezonant kolon, Ampirik model

## 1. INTRODUCTION

The seismic response of various soil types is governed by a multitude of interrelated factors, including soil composition, stratification, intrinsic dynamic properties, and soil-structure interaction mechanisms. A comprehensive understanding of these parameters is essential for developing of reliable seismic design methodologies and effective risk mitigation strategies, particularly in seismically active regions [1-3]. Under dynamic loading conditions, soils undergo stress-induced deformations that can significantly influence the seismic performance of overlying structures. Consequently, the characterization of soil behavior under cyclic or repeated loading becomes critical to accurately estimate the seismic demands imposed on structural systems [4-5].

As seismic waves propagate from the earthquake source toward the ground surface, their amplitude and frequency content are modulated by the intervening soil layers. This interaction can result in either amplification or attenuation of the wave energy, directly affecting the extent of surface-level damage. Therefore, detailed investigations into the dynamic behavior of subsurface soils are imperative to predict and mitigate potential seismic hazards [6-7]. In this regard, both laboratory and in-situ testing play a pivotal role in determining the stress-strain response of soils subjected to cyclic loading. The dynamic parameters derived from such testing particularly shear wave velocity, shear modulus, and damping ratio are essential inputs for numerical modeling of seismic wave propagation and for the rational design of earthquake-resistant infrastructure [8-9].

Among the various laboratory techniques developed for dynamic soil characterization, the Resonant Column (RC) test remains one of the most extensively employed methods for evaluating small-strain behavior. This test, standardized under ASTM D4015, enables the determination of shear modulus ( $G$ ) and damping ratio within a controlled shear strain range typically between 0.001% and 0.1%. Numerous studies have employed RC tests to investigate the small-strain stiffness of sands, clays, and silty soils, contributing valuable insights into their deformation characteristics under seismic loading [10-11]. However, the accurate determination of dynamic soil properties often requires high-precision instrumentation, significant laboratory effort, and considerable financial and time investments. Moreover, the limited availability of advanced testing facilities can restrict the routine implementation of such tests, particularly in regions lacking the required infrastructure.

In response to these constraints, recent research has increasingly focused on developing of empirical models capable of predicting key dynamic parameters such as maximum shear modulus ( $G_{\max}$ ), based on more accessible geotechnical input data. These models offer a cost-effective and time-efficient alternative for large-scale seismic site characterization and are particularly valuable in situations where conventional laboratory testing is impractical or infeasible [12-15].

Within this research context, resonant column tests were conducted on silty sand specimens retrieved from a seismically active region to evaluate the influence of key variables such as void ratio, confining pressure, and water content on  $G_{\max}$ . Based on the experimental findings and subsequent statistical analysis, a practical and robust empirical model has been proposed for estimating  $G_{\max}$ , offering a high degree of applicability for engineering assessments in similar soil conditions. The primary aim of this study is to determine the shear modulus and damping ratio of sands at low strain levels. Considering certain limitations, a model has been proposed to serve as a practical tool for researchers in their studies or under laboratory conditions. This model is particularly recommended because the experimental setup is challenging to implement and time-consuming compared to other test series.

The model developed by Darendeli (2001) constitutes a significant reference in the field of soil dynamics, particularly for examining the behavior of sandy soils at small to medium shear strain levels. Compared to other models in the literature, the Darendeli model successfully captures the general trends, although it tends to slightly underestimate  $G/G_{\max}$  values for soils with high plasticity indices; therefore, the limitations and uncertainties of the model should be carefully considered in engineering applications. Within this

context, the Darendeli model is regarded as a critical tool for analyzing the dynamic properties of sandy soils and providing reliable predictions in geotechnical design.

## 2. MATERIALS AND METHODS

### 2.1 Resonant Column Test System

One of dynamic parameters of soils, the maximum shear modulus ( $G_{max}$ ), can be determined through both field and laboratory testing methods. In laboratory settings, the Resonant Column Test System (RCTS) is commonly used to calculate  $G_{max}$  by applying shear waves to the soil specimen. In this system, longitudinal or torsional vibrations are induced in the specimen, and the shear wave velocity is measured accordingly (Hardin and Richart, 1963). The connection mechanism and vertical cross-section of the Resonant Column Test System are illustrated in Figure 1.

The testing apparatus operates by fixing the specimen at the base while exciting it with vertical or torsional vibrations. Once the fundamental resonant frequency is identified, the resonance frequency and vibration amplitude are measured. Using the principles of elasticity theory, shear strain and wave propagation velocity are calculated. The shear modulus is then derived from the measured wave velocity and the density of the specimen. Based on the determined resonant frequency, amplitude values and half-power bandwidth points are obtained, from which damping values are subsequently calculated. Through this testing procedure, both shear modulus ( $G$ ) and damping ratio ( $D$ ) values can be accurately determined within a strain range of approximately 0.001% to 0.1% [16].

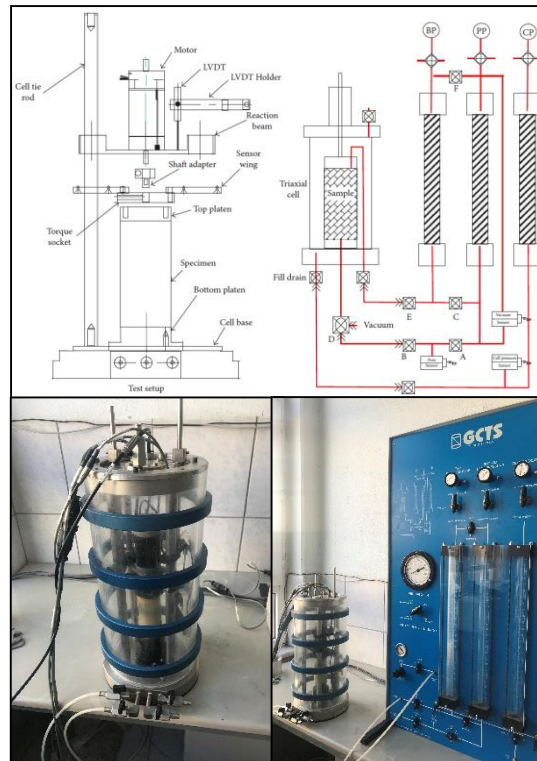


Figure 1. Resonant column test system

Initially, an experimental program was developed for the study, as presented in Table 1. Subsequently, Resonant Column (RC) tests were conducted in accordance with this program. The test setup, as illustrated in Figure 2, includes the soil specimen, motor, proximity sensor (proximeter) and pressure valves. This configuration represents the general layout of the experimental system.

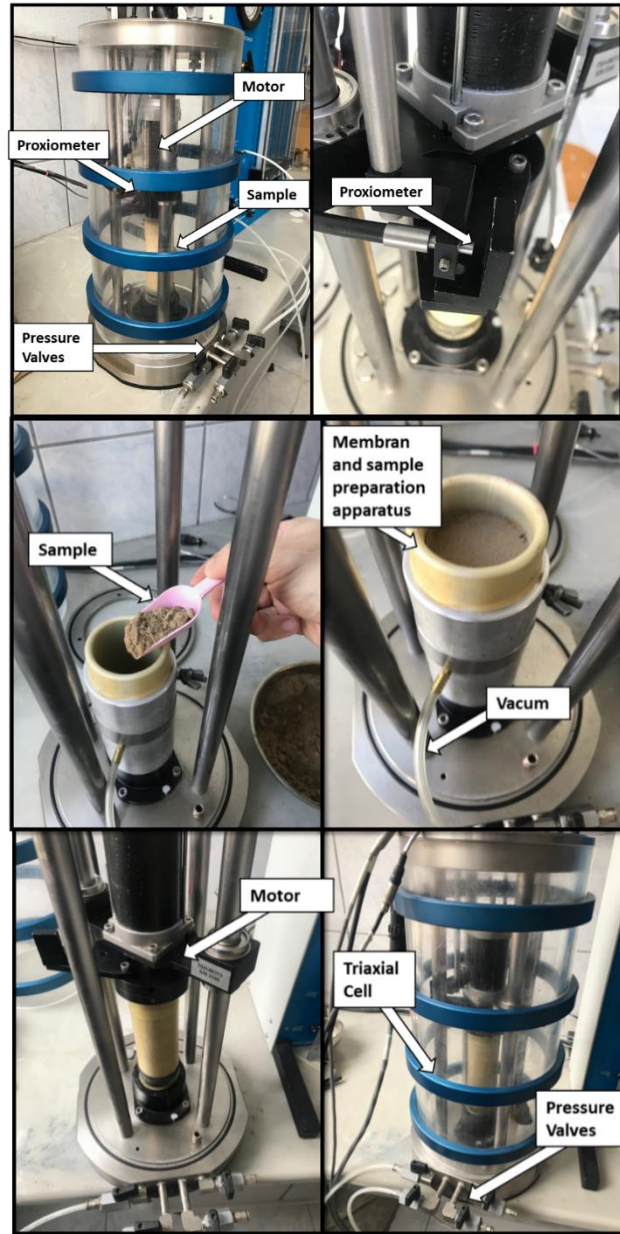


Figure 2. Experimental equipment and sample placement

The confining pressure required during testing is supplied by a compressor unit. In the test system, a specimen placement apparatus was first used to position the membrane, and the sand sample was then placed under vacuum. Specimens were prepared at different void ratios ( $e = 0.70, 0.80$ ) and varying water contents ( $w = 0.05, 0.10, 0.15$ ), as shown in Figure 8. After sample preparation, the cell was assembled around the specimen, and the valve settings were adjusted accordingly. Confining pressures of 50, 100, and 150 kPa were applied through the system during testing. Using the force generated by the motor, the required dynamic parameters were obtained. A total of 18 tests were performed throughout the study. As a result of these experiments, the maximum shear modulus ( $G_{\max}$ ) and damping ratio ( $D$ ) values were determined [17]. The properties of the specimen used in the experiment are as follows: Soil index: SC, ( $G_s$ ) = 2.60, LL: 49%, PL: 37%.



Table 1. Test programme

Test No	Sample No	Test type	Void ratio (e)	Water content (w%)	Confining Pressure (kPa)
1	S-1	RC	0.70	0.05	50
2	S-1	RC	0.70	0.05	100
3	S-1	RC	0.70	0.05	150
4	S-2	RC	0.70	0.1	50
5	S-2	RC	0.70	0.1	100
6	S-2	RC	0.70	0.1	150
7	S-3	RC	0.70	0.15	50
8	S-3	RC	0.70	0.15	100
9	S-3	RC	0.70	0.15	150
10	S-4	RC	0.80	0.05	50
11	S-4	RC	0.80	0.05	100
12	S-4	RC	0.80	0.05	150
13	S-5	RC	0.80	0.1	50
14	S-5	RC	0.80	0.1	100
15	S-5	RC	0.80	0.1	150
16	S-6	RC	0.80	0.15	50
17	S-6	RC	0.80	0.15	100
18	S-6	RC	0.80	0.15	150

### 3. RESULTS AND DISCUSSION

The determination of the dynamic parameters of sand specimens specifically the shear modulus and its variation with shear strain provides critical insight into the soil's behavior under dynamic loading conditions. These parameters are particularly important for understanding how the soil responds during seismic events. Moreover, identifying the influence of various factors on dynamic parameters allows for a better assessment of how these factors alter soil response under such loading conditions.

In the present study, a total of 18 different test sets were prepared. The experimental program can be evaluated in two stages. In the first stage, the effects of increasing water content and confining pressure were investigated. Water content values were systematically increased as  $w = 0.05, 0.10$ , and  $0.15$ , while confining pressures were applied at  $50, 100$ , and  $150$  kPa.

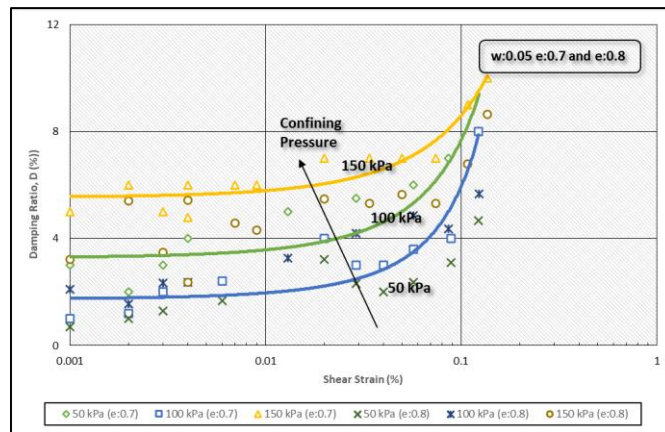


Figure 3. Effects on damping ratio

In the analysis of the test results, the influence of water content variation was first examined. The increase in water content exhibited a noticeable impact at all confining pressure levels. Additionally, it was found that void ratio also significantly influenced the dynamic parameters. As shown in Figure 3, damping ratio

values for two different void ratios were compared. For each of the three confining pressures (50, 100, and 150 kPa), changes in damping ratio reached approximately 40%. This highlights the void ratio as a critical factor affecting dynamic soil behavior.

According to the results, an increase in water content led to a reduction in shear modulus values. This behavior can be attributed to the fact that, although the sand samples were adequately compacted, the presence of water interfered with interparticle contacts, reducing the overall stiffness and strength of the specimens. This trend was consistently observed across all confining pressure levels.

The results for Tests 1-9, focusing on shear modulus–shear strain behavior and damping ratio values, are presented in Figures 4 and 5.

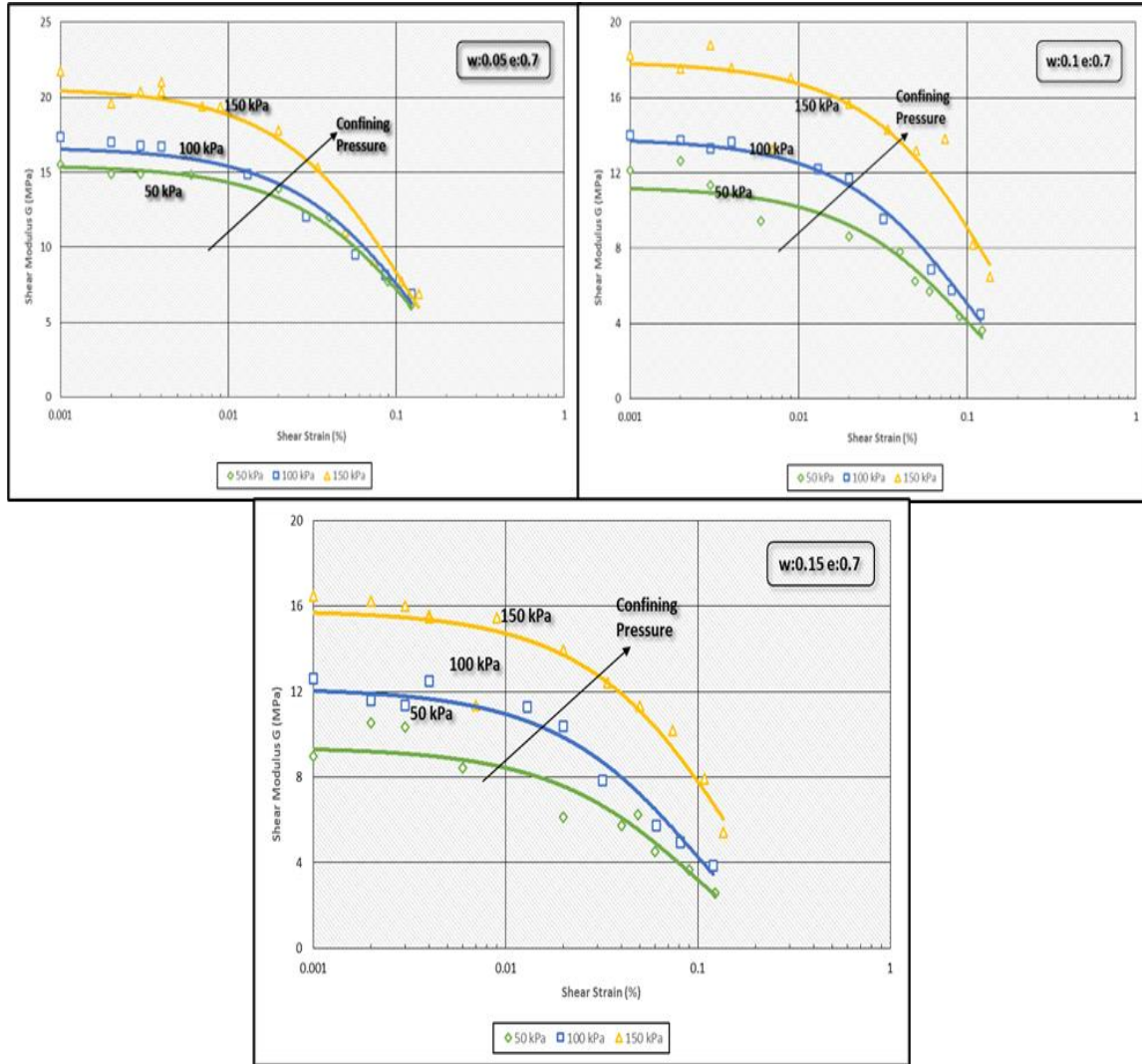


Figure 4. Effects of water content

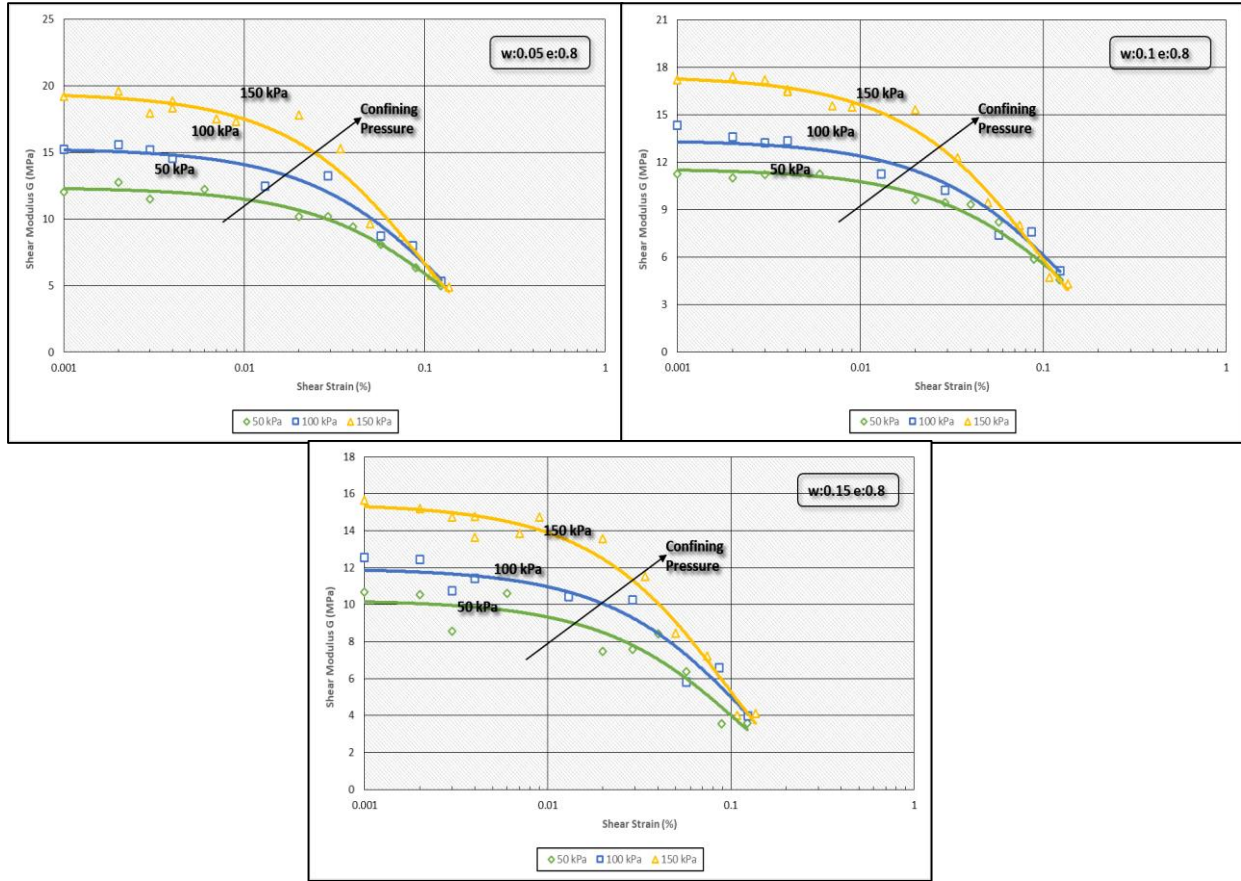


Figure 5. Effects of water content

Regarding the effect of water content, Figure 4 presents results obtained at a constant void ratio ( $e = 0.70$ ). The initial tests conducted at  $w = 0.05$  revealed a direct relationship between confining pressure and shear modulus. As water content increased to  $w = 0.10$  and  $0.15$ , a corresponding decrease in shear modulus was observed. Furthermore, additional experiments at a higher void ratio demonstrated that both water content and void ratio contributed to the reduction in shear modulus values. Specifically, increasing the void ratio resulted in a further decline in shear stiffness.

To evaluate the experimental results, the widely adopted Darendeli (2001) [18] model was employed for comparison. This model was selected as it provides more accurate results at low strain levels. As illustrated in Figure 6, the experimental data obtained at confining pressures of 50, 100, and 150 kPa were assessed against this model. While the experimental results exhibited slightly lower values than those predicted by the model, they remained within the overall data range. Furthermore, the agreement between the model and the experimental results improved with increasing confining pressure.

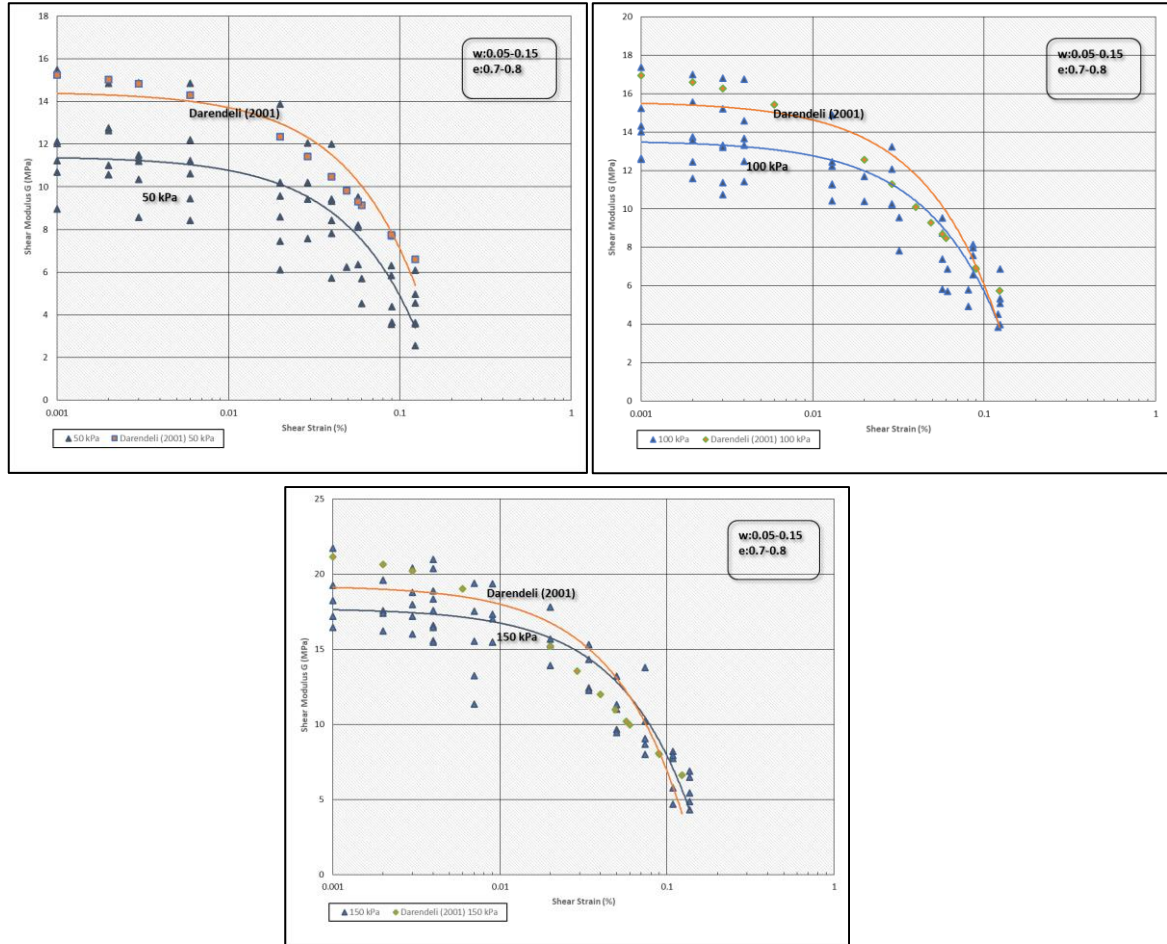


Figure 6. Validation of experimental results

When evaluating the experimental data, it was determined that both increasing water content and void ratio adversely affect the shear modulus, indicating an inverse relationship (Figure 7). Based on these findings, empirical models were proposed to estimate the shear modulus for sand samples within the tested range of water content and void ratio. These models aim to assist researchers in preliminary evaluations and to reduce time and cost in future studies. The proposed values are presented in Table 2.

Due to the time-consuming nature and high costs associated with determining the dynamic parameters of soils through laboratory testing, this study proposes new empirical models to support and facilitate future research in this field. Increasing the amount of experimental data is expected to enhance the reliability of such models. In this study, the empirical models presented in Table 2 were developed based on the influence of confining pressure, water content, and void ratio. For all confining pressure levels, the water content range was defined as  $w = 0.05-0.15$ , and the void ratio range as  $e = 0.70-0.80$ .



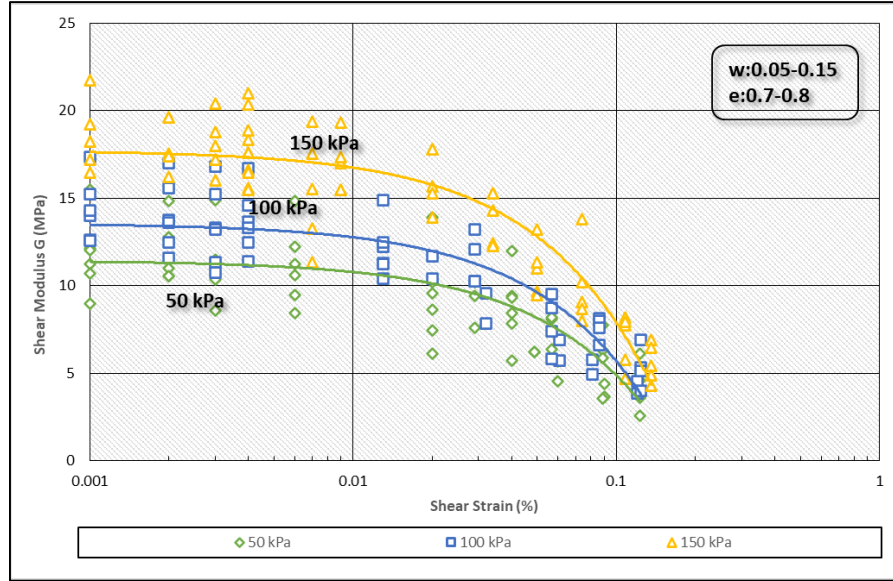


Figure 7. General shear modulus assessment

For structures where the first 15 meters of soil depth are especially critical, the coefficient of determination ( $R^2$ ) values for the proposed models were calculated as follows:

$R^2 = 0.6537$  at a confining pressure of 50 kPa

$R^2 = 0.7683$  at 100 kPa

$R^2 = 0.8312$  at 150 kPa

These values indicate an acceptable level of model accuracy for practical applications. The RMSE values obtained (1.88, 1.76, and 1.91) indicate a good agreement between the model predictions and the experimental results, with errors remaining below 2 across all confining pressures.

Table 2. General normalized shear modulus model

Shear Modulus (MPa)	50 kPa	$G = (-65.463) * \gamma + 11.429$ $R^2 = 0.6537$	w:0.05-0.15, e:0.7-0.8	RMSE:1.88
	100 kPa	$G = (-78.622) * \gamma + 13.549$ $R^2 = 0.7683$	w:0.05-0.15, e:0.7-0.8	RMSE:1.76
	150 kPa	$G = (-98.371) * \gamma + 17.729$ $R^2 = 0.8312$	w:0.05-0.15, e:0.7-0.8	RMSE:1.91

#### 4.CONCLUSION

Accurate identification of soil layers and the evaluation of their behavior under dynamic loads are essential for the safe design and construction of civil engineering structures. Laboratory-based experimental methods are widely used to assess the dynamic response of soils. Among the factors influencing the dynamic parameters of sandy soils, water content and void ratio are particularly significant. Given the high time and financial costs of traditional testing, many researchers have focused on models under various conditions to provide time-saving and cost-effective solutions.

The main findings of this study are summarized below:

- Determining the dynamic behavior of sandy soils and evaluating their impact on structures is of critical importance in geotechnical engineering.
- The Resonant Column Test system was effectively used to evaluate both shear modulus and damping ratio at small strain levels in sandy soils.
- An increase in void ratio was found to adversely affect dynamic parameters, resulting in lower shear modulus values.
- Increasing water content also led to a reduction in shear modulus values.
- The effect of confining pressure was observed in all specimens, with both water content and void ratio influencing the damping ratio.
- Based on these influences, empirical models were proposed specifically for sandy soils within defined boundary conditions to estimate shear modulus.
- The proposed models showed reliable coefficients of determination ( $R^2$ ), indicating their potential applicability in future studies.

In conclusion, since determining the dynamic parameters of sandy soils at small strain levels is both time-consuming and costly, the development of empirical models within defined ranges of influencing factors is important. Such models can assist researchers in preliminary assessments and contribute to more efficient and cost-effective geotechnical analysis.

The accurate determination of the behavior of sandy soils under dynamic loading is of critical importance for geotechnical engineering applications. Consistent with previous research (Hardin and Richart, 1963; Vucetic and Dobry, 1991 [19–20]), this study has demonstrated that void ratio and water content are key factors affecting the shear modulus and damping ratio at small strain levels. Resonant Column Tests ensured reliable measurement of these parameters, revealing that higher void ratios and increased water content reduce the shear modulus, while the damping ratio is influenced by void ratio, water content, and effective confining pressure. Within the defined boundary conditions of this study, the empirical models developed for sandy soils exhibited strong predictive capability, as indicated by high coefficients of determination ( $R^2$ ). The findings are consistent with existing literature and support the use of empirical models as a time- and cost-efficient approach for determining the dynamic parameters of sandy soils, providing valuable contributions for preliminary assessments in geotechnical design and analysis.

## REFERENCES

- [1] C. O. Molua and J. O. Ataman, “Dynamic analysis of soil-structure interaction in earthquake-prone areas,” *International Journal of Applied and Structural Mechanics*, no. 12, pp. 19–29, Nov. 2024.
- [2] S. Janous, M. A. Abid, A. Afras, and A. E. Ghoulboursi, “Soil-structure interaction influence on the seismic performance of buildings,” *Civil Engineering and Architecture*, Mar. 2024.
- [3] D. Lal, B. Regmi, H. F. Bhat, and S. A. Kumar, “Analysis of seismic behavior of buildings with and without shear walls in various seismic zones and soil types,” 2021.
- [4] J. Permalatha, “Influence of soil structure interaction on seismic performance of steel structure interaction with different types of foundations and soil,” *International Journal for Multidisciplinary Research*, May 2024.
- [5] B. Bapir, L. Abrahamczyk, and A. Afroz, “Evaluation of soil-structure interaction for different RC structural systems and foundation sizes,” *Journal of Physics: Conference Series*, vol. 2647, no. 8, p. 082007, Jun. 2024.
- [6] C. Vrettos, “Soil-structure interaction,” in *Encyclopedia of Earthquake Engineering*. Berlin, Heidelberg: Springer, 2014, pp. 1–16.

- [7] F. Göktepe, “Effect of tunnel depth on the amplification pattern of environmental vibrations considering the seismic interactions between the tunnel and the surrounding soil: A numerical simulation,” *Revista de la Construcción*, vol. 19, no. 2, pp. 255–270, Sep. 2020.
- [8] S. Sica, A. D. Russo, F. Rotili, and A. L. Simonelli, “Ground motion amplification due to shallow cavities in nonlinear soils,” *Natural Hazards*, vol. 71, no. 3, pp. 1913–1935, Apr. 2014.
- [9] L. Álamo, A. Padrón, J. J. Aznárez, and O. Maeso, “Structure-soil-structure interaction effects on the dynamic response of piled structures under obliquely incident seismic shear waves,” *Soil Dynamics and Earthquake Engineering*, vol. 78, pp. 142–153, 2015.
- [10] R. Davoodi-Bilesavar and L. R. Hoyos, “Response of cohesive-frictional soils at small to medium shear strain levels from thermo-controlled resonant column testing,” *Canadian Geotechnical Journal*, Jun. 2023.
- [11] T. Wichtmann and T. Triantafyllidis, “Dynamische Steifigkeit und Dämpfung von Sand bei kleinen Dehnungen,” *Bautechnik*, vol. 82, no. 4, pp. 236–246, Apr. 2005.
- [12] G. Du, “Evaluation of maximum shear modulus of soft clay from seismic piezocone tests (SCPTU),” *Rock and Soil Mechanics*, Jan. 2008.
- [13] B. P. Rocha, B. C. R. da Silva, and H. L. Giacheti, “Maximum shear modulus estimative from SPT for some Brazilian tropical soils,” *Soils and Rocks*, vol. 46, no. 1, p. e2023005222, Feb. 2023.
- [14] M. Cruz, J. M. Santos, and N. Cruz, “Maximum shear modulus prediction by Marchetti dilatometer test using neural networks,” in *Proc. 5th Int. Symp. on Deformation Characteristics of Geomaterials*, Berlin, Heidelberg: Springer, 2011, pp. 335–344.
- [15] T. Lu, W. R. Bryant, and N. C. Slowey, “Empirical model of dynamic shear modulus for surface marine sediments,” *Marine Georesources & Geotechnology*, vol. 16, no. 2, pp. 95–109, Apr. 1998.
- [16] Utest, “Tam otomatik resonant kolon ve burgusal kesme sistemi,” [Online]. Available: <http://www.utest.com.tr/tr/20365/Tam-Otomatik-Resonant-Kolon-ve-Burgusal-Kesme-Sistemi>. [Accessed: Jul. 2, 2025].
- [17] ASTM D4015-21, *Standard Test Methods for Modulus and Damping of Soils by Fixed-Base Resonant Column Devices*, 2021.
- [18] M. B. Darendeli, “Development of a new family of normalized modulus reduction and material damping curves,” Ph.D. dissertation, Univ. of Texas, Austin, TX, 2001.
- [19] R. W. Hardin and W. C. Richart, “Elastic wave velocities in granular soils,” *Journal of Soil Mechanics and Foundations Division*, vol. 89, no. 1, pp. 33–65, 1963.
- [20] M. Vucetic and R. Dobry, “Effect of soil plasticity on cyclic response,” *Journal of Geotechnical Engineering*, vol. 117, no. 1, pp. 89–107, 1991.