

## CHILDREN'S CLASSIFICATION OF GEOMETRIC SHAPES

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### Özet

Geometrik şekiller erken çocukluk döneminde matematik eğitiminin temel konularından biridir. Çocuklar günlük yaşamlarındaki deneyimler aracılığıyla, formal eğitime başlamadan önce de geometrik şekillerle ilgili bir takım informal bilgilere sahip olabilmektedirler. Bu dönemde edinilen bilgilerin bir kısmı yanlış olabilmekte ve bu yanlış bilgiler çocukların ileriki geometri eğitimlerini olumsuz yönde etkileyebilmektedir. Bu nedenle de, çocukların geometrik şekilleri ne kadar tanıyıp sınıflandırabildiklerinin ve sınıflandırmada kullandıkları kriterlerin neler olduğunun incelenmesi, okul öncesi dönemde verilecek matematik eğitimin içeriğinin belirlenmesi açısından son derece önemlidir. Geometrik düşüncenin gelişimi ile ilgili olan Van Hiele teorisi, çocukların geometrik şekilleri sınıflandırması ile ilgili temel teorilerden biri olarak kabul edilmektedir. Öte yandan, van Hiele teorisinin erken çocukluk döneminde geometrik düşüncenin gelişimini açıklamada yetersiz kaldığına son dönemlerde bir takım iddialar bulunmaktadır. Bu nedenle, bu araştırma okul öncesi dönem 3- 6 yaş çocuklarının ve ilköğretim 1. ve 4.sınıf öğrencilerinin geometrik şekilleri tanıması ve şekilleri bir birinden ayırt ederken kullandıkları kriterleri belirlemek amacıyla gerçekleştirilmiştir. Bu amaçla 150 çocukla bireysel görüşmeler gerçekleştirilmiştir. Veri toplama aracı olarak dört tane sınıflama testi kullanılmıştır (üçgen, dikdörtgen, kare ve daire sınıflama testi). Veri toplama aracı, önceki araştırmalar temel alınarak araştırmacılar tarafından geliştirilmiştir. Her bir sınıflama testi, A4 kâğıdı üzerine yerleştirilmiş 12 tane şekilden meydana gelmektedir. Araştırmanın sonuçları, küçük çocukların geometrik şekillerin tipik örneklerini tanımada başarılıyken, tipik olmayan örneklerini (örneğin, farklı boyut, konum ve basıklıktaki şekiller) tanımada yeterince başarılı olmadıklarını ortaya koymaktadır. Aynı zamanda, küçük çocukların sınıflama işlemi sırasında çoğunlukla şekillerin görsel özelliklerine dikkat ettikleri belirlenmiştir. Bunun aksine, daha büyük yaşta çocukların ya hem görsel hem de niteliksel özelliklere ya da sadece niteliksel özelliklere dikkat ettikleri saptanmıştır. Böylece, bu araştırmanın sonuçları van Hiele teorisine yönelik son dönemdeki iddiaları desteklemektedir.

**Anahtar Kelimeler:** Erken çocukluk, matematik, geometri, geometrik şekilleri sınıflama

### **Abstract**

Geometric shapes are one of the primary subjects in mathematics education during early childhood. Even before entering formal schooling, children will have some basic information about geometric shapes through their everyday experiences. Some of this early information about geometric shapes might be erroneous, which might negatively impact children's further understanding of geometric shapes. For that reason, exploring how children recognize and classify geometric shapes and the criteria they use for classification is critical in determining the content of early mathematics education. Van Hiele's theory about the development of geometrical thinking is regarded as one of the basis theories concerning children's classification of geometric shapes. However, recently there seems to be some arguments put forward claiming that van Hiele's theory is insufficient in explaining development of geometric thinking in early childhood. Thus, this study was conducted to determine 3 to 6 year-old-preschoolers' and primary 1<sup>st</sup> grade and 4<sup>th</sup> grade students' recognition levels of geometric shapes and the criteria they use to distinguish one group of shape from the other. To this end, individual interviews were conducted with 150 children. Four classification tasks (triangle, rectangle, square and circle) were used in this study as data collection tool. The data collection tool used in this study was developed by the researchers based on the previous studies. Each classification task consisted of classifying 12 shapes, which were outlined on an A4 size paper. Results of the study revealed that while young children were successful in recognizing typical examples of the geometric shapes, they were not successful enough in recognizing the atypical examples (such as shapes with different size, orientation, aspect ratio skewness). It was also determined that younger children were paying attention mostly to visual attributes of the shapes in the process of classification. In contrast, older children were found to pay attention both to visual and property attributes or only to property attributes. Thus, results of the present study hand support to recent criticism of van Hiele's theory

**Key Words:** Early childhood, mathematics, geometry, classification of geometric shapes.

### **Introduction**

Teaching geometric shapes is one of the basic issues of early childhood mathematics and it is far more important in the early childhood period than most people realize. For example, geometric examples are used to teach and learn arithmetical concepts, and knowledge of informal geometry is required to teach and learn reading and writing as well. Furthermore, for children with special needs, geometry offers rich opportunities to strengthen perceptual and motor skills as well as visual discrimination abilities (Schultz, Colarusso & Strawderman, 1989).

Most of the studies on children's recognition of the geometric shapes are demonstrated with one of the following two traditional approaches: Piaget's approach about the development of geometrical thinking in children and van Hiele's approach. Piaget indicated that the development of geometrical thinking in children takes place in two phases. This approach explains the recognition of environment and shapes in

children by topological geometry (Copeland, 1974; Calikoglu Bali & Boz, 2003; Dodwell, 1963; Kellough, Carin, Seefeldt, Barbour & Souviney, 1996; Piaget & Inhelder, 1967). According to Piaget, during the first phase children are able to recognize the familiar shapes, however this recognition excludes Euclidean shapes. According to Piaget, during this phase, children acquire topological knowledge such as whether the shapes are open or close through sensorimotor activities and can distinguish shapes by their topological attributes. According to Piaget (Piaget & Inhelder, 1967), in the second phase children can recognize the Euclidean shapes such as circle, square, triangle, rectangle and can distinguish them from one another.

On the other hand van Hiele (1986) asserts that the development of geometrical thinking takes place not in two phases as suggested by Piaget, but in five distinct levels. Like Piaget's developmental steps, van Hiele's levels are also sequential and success in one level depends on the geometrical thinking attribute of the previous level (Altun, 1997; Aktas, 2002). However, according to van Hiele (1986), Piaget's theory of geometrical thinking is a developmental theory and not an educational theory. Piaget was not concerned on how children can be supported to move from one geometrical thinking level to the other. In addition, Piaget describes the development of geometrical thinking only with two levels. Van Hiele points out we need more than this to explain the development of geometrical thinking in children.

For van Hiele (1986), the first level of geometrical thinking is the visual level. In this level, children perceive the shapes as a whole and classify them by comparing the shapes with a prototype (Bennie, 1998; Burger & Shaughnessy, 1986; Clements, 1999; Clements, Swaminathan, Hannibal, & Sarama, 1999; De Villers, 1996; Hannibal & Clements, 2000; Jami & Gutierrez, 1994, van Hiele, 1999). This level includes the first two years of the primary education. In this level children do not pay attention to the defining (property) attributes of the shapes such as the side or corner. According to the van Hiele theory, when the child starts to define a geometrical shape depending on its property attributes such as number of side or corner, the child is at second level of the geometric thinking, which is the analysis level (Hannibal & Clements, 2000). Children achieve this level in the 3<sup>rd</sup> and 4<sup>th</sup> grades of the primary education (Altun, 1997).

When the recent studies about the development of geometrical thinking in younger children are examined (Clements & Battista, 1992; Clements, et al. 1999; Hannibal & Clements, 2000), several criticisms against the van Hiele theory are observed. According to Clements and colleagues (1999), van Hiele emphasized the development of geometrical thinking in elder children and was not concerned about the development in younger children. They argued that the first level of the van Hiele theory falls short in explaining preschoolers' understanding of geometric shapes. Some preschool children are not able to distinguish the shapes from the other shapes, which are not in the same category. Therefore, these children should be considered to be in the transition phase towards the visual level rather than at the visual level. These children should be classified as "Prerecognitive Level".

Clements and colleagues (Clements et al., 1999; Hannibal & Clements, 2000) also claim that the transition from the visual level to the analysis level is not as clear as it was described in the van Hiele theory. According to the van Hiele theory (van Hiele, 1986), if the child is classifying a shape by looking at it as a whole and comparing it to

a prototype, then this shows that the child is in the visual level. On the other hand, if the child is classifying the shape by considering the property attributes such as the sides and the corners, then the child is in the analysis level. However, according to Clements (Clements, et al. 1999), there is no such clear transition from the visual level to the analysis level. Clements argues that there is a syncretic level in which the children classify the shapes both by comparing them to a visual prototype and by paying attention to the property attributes (i.e., “This also has three sides, but it doesn’t look like that one. For this reason it is not a triangle”). Thus, Clements (Clements, et al. 1999) claims that the visual level should be renamed as the “Syncretic Level”.

In line with the research on young children's understanding of geometric shapes, the objective of this study is to examine the development of geometrical thinking of children by determining the 3 to 6-year-old preschoolers’ and primary 1<sup>st</sup> grade and 4<sup>th</sup> grade students' recognition levels of the basic geometric shapes and the criteria they use to distinguish one group of shape from the other. The reason of inclusion of the 1st and 4th graders in the present study is to observe the transition of children's geometric thinking from 1st level to 2nd level in van Hiele's theory of development of geometric thinking. To achieve this end, we have tested Clements et al.'s theory of development of geometrical thinking in young children using different tasks with a different age group of children (primary 1st and 4th graders) coming from a culturally different society.

## **Method**

### *Participants*

The study was conducted by the participation of 3 -6 years old preschool children who were attending the kindergarten of a university and primary 1<sup>st</sup> grade and 4<sup>th</sup> grade students were attending a public primary school. The children were selected by random sampling from the children population of the schools subject to this study. Total of 150 children, 25 children from each age group and class level were selected for the sampling. Seventy of the selected children were girls and eighty were boys.

### *Materials*

Four classification tasks (triangle, rectangle, square and circle, See Appendix) were used in this study as data collection tool. The data collection tool used in this study was developed by the researchers based on the previous studies (Clements et al., 1999; Hannibal & Clements, 2000; Satlow & Newcombe, 1998). Each classification task consisted of classifying 12 shapes, which were outlined on an A4 size paper. Previous studies (Clements et al., 1999; Hannibal & Clements, 2000) pointed out that some non-defining attributes (aspect ratio, skewness, orientation and size) caused children to make mistakes in their classification decisions. Due to this, in each task, atypical examples containing non-defining attributes (for example, T2, T3, T4, T5, T6 and T7) were placed in addition to the typical examples of the shapes (T1 in triangle task, R3 in rectangle task, S1 in square task and C1 in circle task). Besides, in order to test the children’s ability to distinguish the shapes from the ones that do not belong to the same shape group, palpable distractors (for example, TD3 and TD4 in triangle task) and

impalpable distractors (TD1, TD2 and TD5 in the triangle task) were placed to each task.

Previous studies (Clements, 1999; Clements et al., 1999; Hannibal & Clements, 2000) pointed out that the equilateral triangle was perceived by young children as a typical example of the triangle. For this reason, first, an equilateral triangle (T1) was placed while designing the triangle tasks. Then, in order to test the effect of the aspect ratio, which is the ratio of height to base in triangle, on children's triangle classification decision T3 and T6 were designed (in T3 the ratio of height to base is 1/8, in T6 the ratio of height to base is 8/1).

Skewness indicates the distance of the top to the centreline in triangles. In order to test the effect of skewness on children's classification decision, T7 and T2 were designed (in T7 the distance from the top to the centreline is half the length of the base while in T2 the distance from the top to the centreline is the same length as the length of the base). The orientation of the shape as a non-defining attribute was tested in the triangle classification test. In order to test the effect of orientation on the children's classification decisions, T4 and T5 were designed (T4 was placed at a 45° angle and T5 was placed at a 90° angle).

The studies conducted by Clements and colleagues (Clements et al., 1999, Clements, 1999; Hannibal & Clements, 2000) show that the prototype of rectangle for young children is a shape with four sides which has two sides almost parallel to one another and long. Based on these findings a typical example of rectangle as R3 was placed to the rectangle task. In order to test the effect of aspect ratio on the children's square classification decision, R5 (the ration of height to base is 1/8), to test the effect of orientation R1 and R2 (R1 was placed at 90° angle, and R2 was placed at 45° angle), to test the effect of size, R4 (a rectangle at 1 x 2 cm size) were designed. Besides, in order to test the children's distinguishing the rectangle from other shapes, palpable distractors (RD3, RD5, RD6 and RD7) and impalpable distractors (RD1, distractor with concave sides; RD4, distractor with convex sides and RD2, distractor with broken side) were designed.

In the square task, S1 was placed as a typical example. In order to test the orientation on the children's classification decision S2 (atypical example placed at 45° angle), in order to test the effect of size S3 (1 x 1 cm), to test the effect of both the orientation and size S4 (atypical example placed at 45° angle, 1 x 1 cm in size) were designed. Besides, in order to test the effect of palpable distractor SD2 (circle), SD3 (parallelogram), SD4 (rectangle), SD5 (triangle), SD6 (rhombus) and SD8 (trapezoid), to test the effect of impalpable distractors SD1 (distractor with concave sides) and SD4 (distractor with convex sides) were placed.

C1 and C4 were placed as the typical example in the circle task. In order to test the effect of size on children's classification decision C5, to test the effect of the thickness of the sidelines C2 and C3 were designed. Besides, in order to test the effect of palpable distractors on the children's circle classification decision CD1 (rectangle), CD4 (ellipse), CD5 (square) and CD7 (triangle) were placed. CD1, CD3 and CD6 (distractors with broken sides) were designed to test the effect of impalpable distractors.

To test the validity and reliability of the data collection tool, item and test analysis were conducted and strength and distinction indices were calculated. In the

measuring tool it was found that none of the items had an item distinction below .15 and that the item strength varied between .32 and .99.

The KR 20 alpha value was calculated to determine the reliability of the geometric shape recognition test. The results of the analysis showed that the KR 20 alpha value was .80 for triangle recognition test, .88 for rectangle recognition test, .81 for square recognition test and .77 of circle recognition test.

#### *Data Collection*

The data were collected by interviews held by one of the researchers with each child individually in one of the empty rooms of the school the children attended. First, the researcher gave brief information about the purpose of the study to the teacher of the class that will be interviewed on that day. In order not to affect the result of the study, the teacher was requested not to perform any activities with children about geometric concepts. Then, with the assistance of the teacher the children were brought to the interview room one by one and a brief introduction conversation was made with each child. After that the test was conducted.

First, the triangle recognition task and two colored pencil were handed to the child and it was requested from the child by asking “there are some shapes here and are mixed. Can you put a cross mark (X) on the triangles with the blue pencil?” to mark the triangles. After the child completed marking, the child was asked “Can you now mark the shapes which are not triangle with the red pencil?” After all the marking process was completed if there were any unmarked shape left, the child was asked “I can see that you did not mark this shape, do you think that this is not a triangle?” At the end of the marking process all the shapes were shown to the child and the child was asked the following “You marked this as a triangle (or non triangle), why do you think that this is (or is not) a triangle?” The responses from the child were recorded to the comments section on the interview registration form. The same procedure was followed for the other tasks.

#### *Data Analysis*

The responses from children regarding their shape classification decision were categorized under three main categories: “visual response”, “property response” and “I don’t know”. If the child referred to the visual attribute of the shape (e.g. fat, thin, big), indicating that it looks like another object or like one of the shapes on the paper or anything similar to these which do not include the property attribute (e.g. side, corner) of the shape it was coded as “Visual Response.” If the responses from the child included the property attributes of the shape it was coded as “Property Response.” If the child responded as “I don’t know” it was coded as “I don’t know”. In addition, after the interview was held individually with the child, the researcher gave “1” point for the correct responses and “0” point for the wrong responses.

After this process, percentage and frequency distribution was calculated for the collected data. Variance analysis was used to test whether there is a significant difference between the recognition of each classification task with respect to ages. In the interpretation of the results .05 was accepted as the significance level.

## Results

The reliability of the geometric shape-classification task used in this study was measured by calculating the KR 20 alpha value. The KR 20 alpha value for the triangle-classification task is .78, for the rectangle-classification task is .86, for the square-classification task is .80, and for the circle-classification task is .73. Based on these findings indicates that this task is reliable enough to be used in this study.

Table 1. Age Group Based Percentage Distribution of the Correct Classification the Children Achieved in Each classification Task

| Task      | Percentage of Achievement<br>(N=150)<br>% | Age Groups                   |                             |                              |                            |                                      |                                      |
|-----------|-------------------------------------------|------------------------------|-----------------------------|------------------------------|----------------------------|--------------------------------------|--------------------------------------|
|           |                                           | 3 year - olds<br>(N=25)<br>% | 4 year -olds<br>(N=25)<br>% | 5 year - olds<br>(N=25)<br>% | 6 Year-olds<br>(N=25)<br>% | 1 <sup>st</sup> grade<br>(N=25)<br>% | 4 <sup>th</sup> grade<br>(N=25)<br>% |
| Triangle  | 70                                        | 62                           | 70                          | 67                           | 72                         | 67                                   | 80                                   |
| Rectangle | 76                                        | 53                           | 58                          | 81                           | 89                         | 82                                   | 91                                   |
| Square    | 77                                        | 49                           | 81                          | 78                           | 84                         | 79                                   | 91                                   |
| Circle    | 90                                        | 77                           | 85                          | 92                           | 95                         | 96                                   | 95                                   |

Table 1 shows that the children achieved 70% from the triangle task, 76% from the rectangle task, 77% from the square task and 90% from the circle task. These findings show that the highest achievement levels were obtained from the circle-classification task and lowest from the triangle and rectangle-classification tasks. These findings overlap with the findings of the other studies which used similar tasks (Clements et al., 1999; Aktas Arnas & Aslan, 2004). In addition, these findings show that there is an increase in recognizing the shapes as the age increases in the preschool period. However, when primary first grade students were compared to the six year-olds group, a decrease was observed. That is, the 6 year-olds group was more successful than the primary first grade students.

Although the findings suggest that children's recognition of geometrical shapes increase with the age, the difference between 6 year-olds group and first graders indicate an inconsistency. In this study the shapes presented in the tasks were two dimensional. However, in Turkish Educational system, shapes with two dimensions are thought in preschools, but three dimensional shapes are thought in the first grade. Furthermore, since preschool education is not compulsory in Turkey, only some of the participants among first graders had previous preschool experience. Therefore, only some of the first graders were exposed to the geometrical shapes as two-dimensional presentation. Thus, we think, this situation explains why some of the first graders were less successful than six year-old children.

Preschool and primary first grade students were successful in recognizing the typical examples of the triangle, square, rectangle and circle, while they were less successful in recognizing the atypical examples (such as, different size, orientation, aspect ratio, skewness) of these shapes. Also, they were less successful in recognizing the shapes which belong to different groups and in recognizing the distractors. This was more explicitly observed especially in 3 and 4 year-olds groups. Similar studies

(Clements, et al. 1999; Hannibal & Clements, 2000; Aktas Arnas & Aslan, 2004) also show the same findings. When compared to the other age groups the primary fourth grade students were more successful in recognizing the typical and atypical examples of the shapes and in recognizing the shape that does not belong to the specified group.

The results of the variance analysis concerning whether there is a difference between ages in the classification task achievement showed that there is no statistically significant difference among the groups in the triangle-classification task ( $F=1,32$ ;  $p>.05$ ), while statistically significant differences were found between the groups in rectangle ( $F=16,40$ ;  $p<.01$ ), square ( $F=17,03$ ;  $p<0.1$ ) and circle ( $F=11,52$ ;  $p<.01$ ) classification tasks. The results of the Scheffe-f test, which was applied to determine the groups between which the differences occurred, showed that statistically significant differences were found between the 3 year – olds group and 5, 6 year – olds group, 1<sup>st</sup> and 4<sup>th</sup> grade students against the 3 year-olds group in the rectangle- classification task. Also, statistically significant differences were found between 4 years – old group and the 5, 6 year-olds group and 1<sup>st</sup> and 4<sup>th</sup> grade students in the rectangle-classification task against the 4 year-olds group. When the children's achievement in the square-classification task is examined, it was observed that compared to the other groups statistically significant differences were found against the 3 years old group. Finally, when the children's achievement on the circle-classification task was examined statistically significant differences were found between the 3 year – olds group and 5 – 6 year – olds group, 1<sup>st</sup> and 4<sup>th</sup> grade students against the 3 years old group. In addition, statistically significant differences were found between 4 year – olds group and the 4<sup>th</sup> grade students against the 4 year-olds group. The results of this study show that there are differences between the first years of the preschool period and the later age groups in classification the basic geometric shapes and that as the age increases the achievement levels in classification tasks increased. This can be explained by the biological development and the formal geometry education received at schools.

Table 2. The Verbal Responses of Children’s Selection Reasons on the Classification Tasks

| Response Category | Responses                                                                  | Classification Tasks |    |           |    |        |    |        |    |
|-------------------|----------------------------------------------------------------------------|----------------------|----|-----------|----|--------|----|--------|----|
|                   |                                                                            | Triangle             |    | Rectangle |    | Square |    | Circle |    |
|                   |                                                                            | f                    | C  | f         | C  | f      | C  | f      | C  |
| Visual Response   | Drawing by hand and saying: “Because it is like this”                      | 87                   | 14 | 118       | 16 | 104    | 13 | 49     | 5  |
|                   | Looks like a (shape)                                                       | 242                  | 48 | 298       | 54 | 296    | 66 | 305    | 59 |
|                   | Doesn’t look like a (shape)                                                | 43                   | 31 | 47        | 11 | 34     | 9  | 31     | 6  |
|                   | Showing another shape on the paper and saying: “It looks like this”        | 50                   | 11 | 38        | 7  | 33     | 8  | 19     | 4  |
|                   | Showing another shape on the paper and saying: “It doesn’t look like this” | 9                    | 5  | 55        | 19 | 6      | 2  | 6      | 1  |
|                   | Compare to an object “It looks like a (object)”                            | 61                   | 14 | 8         | 1  | 64     | 15 | 98     | 28 |
|                   | Compare to an object “It doesn’t look like a (object)”                     | -                    | -  | -         | -  | 2      | 1  | 30     | 7  |
|                   | Fat/ skinny                                                                | 2                    | 1  | 3         | 2  | 1      | 1  | -      | -  |
|                   | “That’s the way it is”                                                     | 180                  | 33 | 147       | 28 | 178    | 33 | 215    | 28 |
|                   | Orientation                                                                | 10                   | 6  | 6         | 3  | 17     | 7  | -      | -  |
|                   | Length                                                                     | 12                   | 7  | 10        | 5  | 2      | 1  | 1      | 1  |
|                   | Size                                                                       | 2                    | 1  | 13        | 6  | 4      | 3  | 3      | 3  |
|                   | “They made it like this”, “God made it like this”                          | 27                   | 5  | 11        | 2  | 19     | 2  | 21     | 4  |
|                   | Total of visual responses                                                  | 725                  |    | 754       |    | 760    |    | 778    |    |
| Property Response | Number of corners                                                          | 129                  | 34 | 99        | 25 | 70     | 17 | 185    | 39 |
|                   | Number of sides                                                            | 236                  | 45 | 160       | 36 | 147    | 29 | 64     | 17 |
|                   | Type of line                                                               | 156                  | 48 | 182       | 54 | 213    | 62 | 549    | 75 |
|                   | Length of Sides                                                            | -                    | -  | 164       | 31 | 183    | 31 | 15     | 6  |
|                   | Total of property responses                                                | 521                  |    | 605       |    | 613    |    | 813    |    |
| IDK               | I don’t know                                                               | 3                    | 1  | 10        | 3  | 26     | 5  | 23     | 5  |
|                   | Grand Total                                                                | 1249                 |    | 1369      |    | 1399   |    | 1614   |    |

“C” indicates the number of children in the age groups who gave the mentioned responses at least once.

Responses given by children about the reasons of correct classification they made in the triangle task can be seen in the table 2. When the table is examined it can be seen that the most frequent response for the reasons of classification was “Because it looks like a [shape]”. When the distribution of the responses according to the age groups are examined it can be determined that the most frequent response given by the 3 year-olds group was “That’s the way it is”, the most frequent response from the 4 year-olds group was “Because it looks like a [shape]”, the students in the 5 year-olds group, in 1<sup>st</sup> grade and 4<sup>th</sup> grade gave responses related to the side numbers of the shape and the 6 year-olds group’s responses were based on the corner numbers of the shape. Only one of the responses from the 3 year-olds group was based on the property, while this number was 34 in the 4 years-old group, 70 in the 5 year-olds group, and 86 in the 6 year-olds group. The number of property responses exceeded the number of visual responses in primary 1<sup>st</sup> grade and 4<sup>th</sup> grade (106 and 224, respectively). A continuous decrease in visual

responses was observed with the increase in age. In the light of all the findings it can be indicated that with the increase in age, children use property attributes more than the visual attributes in distinguishing the triangle from the other shapes. The reason for the high number of visual responses during the preschool period can be due to comparing the shape to objects, such as “the roof of the house,” or showing only one example (equilateral triangle) while teaching triangles in stead of using the properties of the shape. The reason for the increase in property based responses in later years can be due to pointing out the properties of the geometric shapes while teaching them in the primary school.

The children's responses regarding the reasons of correct classification they made in the rectangle task can be seen in the table 2. The most frequent response given was “Because it looks like a [shape]”. When the distribution of the responses according to the age groups are examined, it can be determined that the most frequent response given by the 3, 4 and 6 year-olds group was “Because it looks like a [shape]”, 5 year-olds group’s most frequent response was “Because it looks like this [another shape on the paper]”, 1st grade students gave responses related to the corner numbers of the shape and 4th grade students’ responses were related to the side length of the shape. These results show that there is an increase in the property responses with the increase in age. These results indicate that in distinguishing the rectangle from other shapes preschool children gave responses that are basically related to the visual attributes of the shapes while 1st and 4th grade students paid attention to the property attributes of the shapes. The reason for this can be due to introducing the shapes visually in the preschool period and not pointing out the property attribute of the shape.

Responses given by children about the reasons of correct classification they made in the square task can be seen in the table 2. When the table is examined it can be seen that the most frequent response for the reasons of classification was “Because it looks like a [shape]”. When the distribution of the responses according to the age groups are examined it can be determined that the most frequent response given by the 3 and 4 year-olds group was “Because it looks like a [shape]”, 5 year-olds group gave responses based on the type of line (-e.g., “because its lines are not straight), 6 year-olds group’s most frequent response was related to the side numbers of the shape, “That’s the way it is” was the most frequent response from the 1st grade students and “side length” related responses were for the 4th grade students. The results show that 3 year-olds group gave responses according to the visual attributes of the shape when deciding on whether a shape is a square or not, 4-5-and 6 year-olds children and 1st grade students used the property attribute besides the visual attributes and finally 4th grade students mostly used the property attribute in deciding on the shape. The reason for the high number of visual responses in early years can be due to using the visual prototypes of the shapes in teaching the squares and not pointing out the property attribute of the shape.

When the children were asked the reasons of correct classification they made in the circle task, the most frequent response given was the “type of line”. When the distribution of the responses according to the age groups are examined it can be determined that the most frequent response given by the 3 year-olds group was “Because it looks like a [shape]”, 4 year-olds group was “that’s the way it is”, 5 and 6

year-olds group's and 1st and 4th grade students' most frequent response was "the type of the shape lines". The results show that in deciding on whether a shape is a circle or not, 3, 4 and 5 year-olds groups use the visual attributes of the shape while 6 year-olds group and 1<sup>st</sup> and 4<sup>th</sup> grade students use the property attributes.

It can be seen that the number of correct answers were the highest for the circle-classification task when compared to the others. The reason for this might be because the circle does not have many atypical examples (only the size can be differed) and can be easily distinguished from the angular shapes.

### **Discussion and Conclusion**

Various findings have been obtained about children's recognition of the basic geometric shapes and the criteria they use to classify these shapes. First, preschoolers were found to pay attention both to the visual and property attributes of the shapes when classifying them. However, they use visual attributes more often when thinking about and deciding to name a shape. Results of similar studies conducted by Hannibal and Clements (Hannibal, 1999) and Aktas Arnas and Aslan (2004) demonstrated parallel findings. Majority of the 1<sup>st</sup> grade students also classify the shapes by considering the visual attributes. Although the majority of the 4<sup>th</sup> grade students classify the shapes by taking into account the property attributes, visual responses are still observed in this group.

Second, when the visual attributes (such as, size, orientation, aspect ratio, skewness) of the triangle, rectangle, square and circle are changed, some children, especially 3 to 4-year-olds, failed to recognize and sort the shapes. Our findings support Clements and Battista's (1992) findings claiming that there is a "precognitive level" in the geometrical thinking level before the visual level for children who could not reliably distinguish the shapes that do not belong to the same group. Furthermore, these findings also support Piaget's claims regarding the failure of children in this period experience in distinguishing the shapes from one another (Piaget & Inhelder, 1967).

Third, a number of the children in the 5-6 year-old-group and in 1<sup>st</sup> grade group used only visual attributes when classifying the shapes (e.g. "It looks like this") and some of them used only the property attributes (e.g. "It has four corner") while some children in this age group used both the visual and property attributes (i.e., "this also has three sides but it doesn't look like this, so it is not a triangle"). Similarly, in some previous studies (Clements et al., 1999; Hannibal & Clements, 2000; Aktas Arnas & Aslan, 2004) it was determined that some students used both the visual and property attributes at the same time. However, all these findings oppose the van Hiele theory. According to the van Hiele theory (van Hiele, 1986), depending on their geometrical thinking level children use either the visual attributes (visual level) or the property attribute (analysis level) when working on a classification of a shape. Hannibal and Clements (2000) oppose this classification and claims that there is not a clear transition from the visual level to the analysis level. Clements also claims that there is a period in which children use both the visual and property attributes simultaneously. For this reason, Clements indicates that the visual level should be renamed as the "Syncretic Level". Our findings also support Clements' above claim of renaming the visual level as

“Syncretic Level”. As a result, when this study and the other studies conducted by Clements his colleagues (Clements et al., 1999; Hannibal & Clements, 2000) are taken into account, levels 1 and 2 in the van Hiele’s theory should be revised.

In addition, when one takes educational point of view into account, it seems obvious that preschoolers and 1<sup>st</sup> grade students encounter problems in recognizing even basic geometric shapes and distinguishing one shape from the other. Probably, the most important reason for this is using activities that help the students to recognize the shapes only visually by using a prototype instead of providing activities that would help the students explore the property attributes of the shapes. The negative effects of this kind of education can be explicitly seen in the responses of the 4<sup>th</sup> grade students who still give visual responses. Thus, instead of teaching geometry to young children through a rote learning method by using the visual prototypes, educators should develop activities that would provide children with opportunities to explore the property attributes of the shapes being studied. These activities should include atypical examples (such as shapes with different size, orientation, aspect ratio and skewness) as well as typical examples.

Furthermore, during the preschool period only two dimensional shapes are taught to children in Turkey, three dimensional shapes are taught in the 1st grade and then again two dimensioned shapes are taught in 2<sup>nd</sup> grade. This disunity between the education programs has great negative impacts on the geometry achievement. Therefore, starting from the preschool period the geometry programs must be modified and parallelism should be ensured in different educational levels.

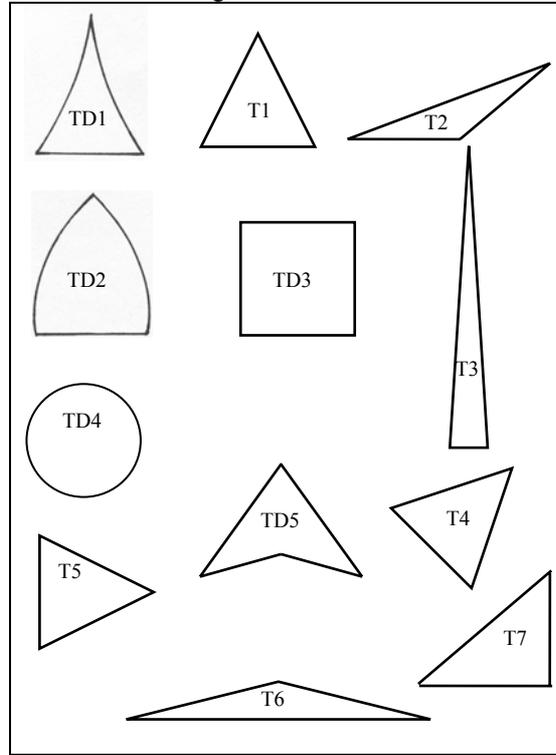
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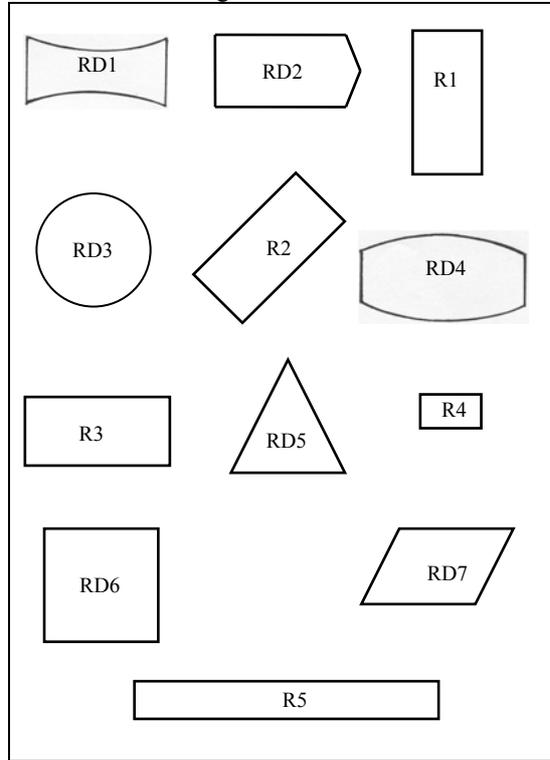
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**Appendix**

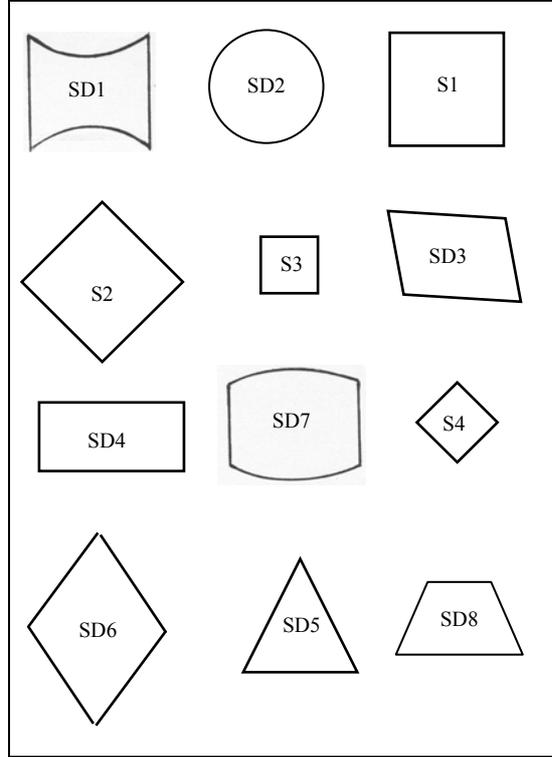
1- Triangle- classification task



2- Rectangle- classification task



3- Square- classification task



4- Circle- classification task

