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ESTIMATION OF ECONOMIC GROWTH USING GREY COBB-DOUGLAS PRODUCTION FUNCTION: AN APPLICATION FOR US ECONOMY

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ABSTRACT

Purpose- In this paper, we apply the Grey Cobb-Douglas production model to predict the GDP, examine the effects of the variation rate of capital, and labor inputs to economic growth. Many factors contribute to economic growth, such that technological progress, labor force, capital accumulation, the optimal using of sources, energy, institutional innovation ext. In reality, a variate of economic factors often intertwine with each other.

Methodology- The capital and labor are main elements of economic growth. Improving the capital and labor performance plays important role in increase the wealth of a country. Traditionally, Cobb-Douglas (C-D) production model use only capital stock and labor to describe the economic growth. In this study, firstly C-D production function is established and confirmed that the capital and labor has a positive impact on economic growth (GDP). Then GM(1,1) prediction model is used to predict the future values of capital stock and labor force inputs.

Findings- The future GDP values are predicted by the estimated capital and labor values putting into the Cobb-Douglas model. We also obtained the production elasticities of capital and labor inputs. Findings suggest that the contribution rate of capital is 0.403 and labor is 1.094 to economic growth. The sum of the contributions of factors is 1.497 and greater than one.

Conclusion- Findings of this empirical studies shows that percentage of the increase in GDP is greater than that of the increase in capital stock and labor.

Keywords: Cobb-Douglas production function, economic growth, Grey-prediction model.
JEL Codes: E23, E37, C53

1. INTRODUCTION

The production level in general economy or firm environment is described by production function. One of the main problems for the economic authorities is to choice the functional relationship between the economic inputs and production value. In the literature generally four different production functions is used. In literature, very often used production functions are linear production function, Cobb-Douglas production function(C-D), Constant Elasticity of Substitution production function (CES), Variable Elasticity Substitution production function (VES), Leontief Production function and Translog production function (Cheng, M. and Han, Y., 2017). In this study, we used Cobb-Douglas production function to establish model impact of capital investments and labor on economic growth.

Christensen, Jorgenson and Lau (1973) consider an extension of Cobb-Douglas function that is called *translog production* function. Important contributes made by Romer (1986), Lucas (1988), Mankiw, Romer and Weil (1992) and Benhabib and Spiegel (1994).

The error term in Cobb-Douglas production function is modelled by either additive or multiplicative. In this study, we will use additive form. Bahatti (1993), Hossain et al. (2010), Prajneshu (2008), Golfeld and Quandt (1970) are used classic regression model estimate the Cobb-Douglas function.

There are a lot of factor have contributed to production level such that capital, technology optimal allocation of sources, innovations etc. In this study, the future economic production level is forecasted based on GM (1, 1) Grey Cobb-Douglas production model. In addition, we make an empirical analysis on the elasticity of substitution, direction of technical change and the contribution rate of US economic growth factors to total factor productivity.

2. PRODUCTION FUNCTIONS

Cobb-Douglas function has a constant cost share of capital and strong co-movement in labor productivity and capital productivity. Cobb-Douglas function is every time Hicks neutral. Another words, the marginal productivities of capital and labor do not effect from allocation of factors. The growth of economics generally has measured by Gross Domestic Production (GDP) rate in current price. In this study, we assume that economic growth is only based on asset and employment. In substance, economics production is effected from various environmental factors such as capital, labor, agricultural activities, technology, industry, energy, raw materials etc.

In economic theory, the production function is described as the empirical relationship between given the quantity of economic inputs and specified outputs. One of the basic problems for economics governance in production process is determining the functional relationship between the production output and input factors. The general form of production function is described by $f: D \to R_+$, $D \in R_+^n$ and

$$Q = f(X_1, X_2, \dots, X_n) \tag{1}$$

where $X_1, X_2, \dots, X_n$ are inputs and Q is production level. A production function with n input factors is called $h-$ homogeneous, $h > 0$, if

$$f(kX_1, kX_2, \dots, kX_n) = k^h f(X_1, X_2, \dots, X_n) \tag{2}$$

where $k \epsilon R$ and,

- If $h > 1$, per percent increase in input levels would result greater than per percent increase in the output level,
- If $h < 1$, per percent increase in input levels would result less than per percent increase in output,
- $h = 1$ represent the constant return to scale.

Traditionally assumed that the most important factors that affect the production are capital and labor inputs (Cobb, C. W. and Douglas, P. H.,1928). We can write the production function with two input factor as follows,

$$Q = f(L, K) \tag{3}$$

where Q is the quantity of total output (production), L is the quantity of labor and K is the capital used.

The output Q is usually measured by physical units produced or by their values, Labor is typically measured in man - hours or number of employees. Capital represents aggregations of different components. Determination of its value is difficult.

There are different functional forms for$f$, Table 1 shows some of widely accepted functions (Godin, A. and Kinsela, S., 2013).

Table 1: Different Functional Forms of Production Function

Cobb-Douglas production function	$Q = AK^{\alpha}L^{\beta}$, $\alpha + \beta = 1$
Constant Elasticity of Substitution (CES) production function	$Q = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\vartheta/\rho}$
Translog production function	$\ln Q = \alpha_1 \ln K + \alpha_2 \ln L + \beta_{11}(\ln K)^2 + \beta_{12} \ln K . \ln L + \beta_{22}(\ln L)^2$
Polynomial (cubic) production function	$Q = \alpha_1 K + \alpha_2 K^2 + \alpha_3 K^3 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3$
Leontief production function	$Q = min[\alpha K, \beta L]$

The historical evolution of production function see also refer to Misra, S. K.(2010). We can state the marginal productivity of factors as follows,

$$MP_L = \frac{\Delta Q}{\Delta L}\Big|_{K\ is\ constant} = \beta\ AK^{\alpha}L^{\beta-1} = \beta\left(\frac{Q}{L}\right) \quad , \text{(marginal productivity of labor)}$$

$$MP_K = \frac{\Delta Q}{\Delta K}\Big|_{L\ is\ constant} = \alpha A\ K^{\alpha-1}L^{\beta} = \alpha\left(\frac{Q}{K}\right) \quad , \text{(marginal productivity of capital)}$$

In Cobb-Douglas production function, capital receives the constant share α total product. For a Cobb-Douglass production function with two inputs, *elasticity of substitution*;

$$\sigma = \frac{d\ln(K/L)}{d\ln(MP_L/MP_K)} = 1\,. \tag{4}$$

This specification creates isoquants that are convex. The distribution of national income between capital and labor determine the elasticities of substitution. If $\sigma = 1$, any change in K/L is matched by a proportional change in w/r and te relative income shares of capital and labor stay constant, where w is wage rate and r is rental rate of capital. As a result, constant shares of output are allocated to capital and labor even though the capital – labor ratio may change over time. An elasticity of substitution equal to unit implies that these factor shares will remain constant for any capital – labor ratio because any changes in factor proportional will be exactly offset by changes in the marginal productivities of the factor input (Miller, E., 2008).

For a production function that has more than two inputs, *Hicks elasticity of substitution measure* is described as (Stern, D. I., 2011),

$$\sigma_{ij} = -\frac{\partial\ln\left(X_i/X_j\right)}{\partial\ln\left((\partial Q/\partial X_i)/\left(\partial Q/\partial X_j\right)\right)}\,. \tag{5}$$

Cobb-Douglas function is based on the assumption that the elasticity of substitution between factors is one.

2.1 Cobb-Douglas Production Function

The studies on production function were made firstly by Knut Wicksell (economist) in 1906. Then, Cobb-Douglas production function was developed by Charles W. Cobb (mathematician) and Paul H. Douglas (economist) in 1928. The Cobb-Douglas production function is widely used in economic studies. This function is describes the economic output as a function of two factors, capital and labor. Cobb-Douglas production function is used the modeling the substitution between capital input, labor services and technical change. This model implies the elasticity of substitution equals one. This function is describes the economic output as a function of two factors, capital and labor. The Cobb-Douglas production function is given by

$$Q = f(K, L) = AK^{\alpha}L^{\beta} \tag{6}$$

where Q is total production (the monetary value of all goods produced in a year), (usually use GDP), A is productivity of existing technology (total factor productivity) (technical process, economic system etc.), K is investment capital input which is represent by the total investment in fixed assets (the monetary worth of all machinery, equipment and buildings) and L is the quantity of the labor input (the total number of person - hours worked in a year) (Cobb, C. W. and Douglas, P. H., 1928). Parameter α and β are the output elasticities to capital and labor, respectively.

Output elasticity measures the responsiveness of output to a change in levels of either labor or capital used in production and given by,

$$\begin{aligned} \alpha &= \frac{\partial Q/Q}{\partial K/K} \quad , \text{(Output elasticity coefficient of capital)} \\ \beta &= \frac{\partial Q/Q}{\partial L/L} \quad , \text{(Output elasticity coefficient of labor)} \end{aligned} \tag{7}$$

Cobb-Douglas production function allow us to change the magnitude of inputs response to factor price changes. One of the limitation of model is that use two factor input to explain the production (Liao, Q., Wu, Z. and Xu, J., 2010).

2.2. Properties of Cobb-Douglas Production Function

$a)$ Cobb-Douglas production function is a homogeneous function. The degree of homogeneity of function is described with $\alpha + \beta$. Let $(c > 1) \in R$ and $Q_1 = AK_1^{\alpha}L_1^{\beta}$ then,

$$Q_2 = A(cK_1)^{\alpha}(cL_1)^{\beta} = c^{\alpha+\beta}AK_1^{\alpha}L_1^{\beta} = c^{\alpha+\beta}Q_1\,.$$

If each factor is increased by a factor c, total output Q will increase by $c^{\alpha+\beta}$. The production function shows *constant returns to scale* if changing in input factors by positive proportion has changing output by the same proportion.

$i)$ $\alpha+\beta=1$, function denotes the constant return the scale,

$ii)$ $\alpha+\beta>1$, it shows increasing returns to scale,

$iii)$ $\alpha+\beta<1$, diminishing returns to scale (Besanko, D. A. and Braeutigam, R. R., 2010).

Here, return to scale is determined by,

$$\text{Return to scale} = \frac{\%\Delta\ (\text{quantity of output})}{\%\Delta\ (\text{quantity of all inputs})}.$$

b) Cobb-Douglas production function is linear.

c) The marginal productivity of capital, $MP_K=\frac{\partial Q}{\partial K}$ and the marginal product of labor, $MP_L=\frac{\partial Q}{\partial L}$

d) The marginal rate of the technical substitution of labor (L) for capital (K) is given by

$$MRTS=\frac{MP_L}{MP_K}=\frac{\partial Q}{\partial L}\Big/\frac{\partial Q}{\partial K}=\left(\frac{\beta}{\alpha}\right)\left(\frac{K}{L}\right). \tag{8}$$

The generalized form of Cobb-Douglas function is given as,

$$Q(\boldsymbol{X})=AX_1^{\beta_1}X_2^{\beta_2}\dots X_n^{\beta_n}. \tag{9}$$

If the logarithms of both sides are taken, we obtained that

$$\ln Q=\beta_0+\beta_1\ln X_1+\cdots+\beta_n\ln X_n \tag{10}$$

where $\beta_0=lnA$, $\boldsymbol{X}=(X_1,X_2,\dots,X_n)\in R_+^n$, $A>0$ is a constant, Q is total production level, X_i are input factors, $i=1,2,\dots,n$ and $\beta_1, \beta_2,\dots,\beta_n$ are parameters, n is the number of factors which are used in the production function (Vîlcu, G. E. ,2018; Wang, X., 2016).

Hicks J.R and Allen, R.G (1934) proposed a generalization of Hicks elasticity coefficient as,

$$H_{i,j}(\boldsymbol{X})=\frac{\dfrac{1}{X_i\left(\frac{\partial f}{\partial X_i}\right)}+\dfrac{1}{x_j\left(\frac{\partial f}{\partial X_j}\right)}}{-\dfrac{\frac{\partial^2 f}{\partial X_i\partial X_i}}{\left(\frac{\partial f}{\partial X_i}\right)^2}+\dfrac{2\left(\frac{\partial^2 f}{\partial X_i\partial X_j}\right)}{\left(\frac{\partial f}{\partial X_i}\right)\left(\frac{\partial f}{\partial X_j}\right)}-\dfrac{\frac{\partial^2 f}{\partial x_j\partial X_j}}{\left(\frac{\partial f}{\partial X_j}\right)^2}} \tag{11}$$

where $=(X_1,X_2,\dots,X_n)\,\epsilon\,R_+^n$, $1\le i\ne j\le n$ and the $H_{i,j}$ denotes the Hicks elasticity of substitution of i th production factor respect to the *j* th production factor (Bang-Yen Chen, 2012).

2.3.Parameter Estimations of Generalized Cobb-Douglas Production Function

Cobb-Douglas Production function defined as in Eq. (9) is used to estimate the following regression equation,

$$Q_i=AX_{1i}^{\beta_1}X_{2i}^{\beta_2}\dots X_{ni}^{\beta_n}+\varepsilon_i$$

Shifting the ε_i left side of equation then taking the logarithm on both side of equation,

$$log\,(Q_i-\varepsilon_i)=logA+\beta_1 logX_{1i}+\beta_2 logX_{2i}+\cdots+\beta_n logX_{ni} \tag{12}$$

Let $m_i=\varepsilon_i/Q_i$ (the relative error of the observations of Q_i) (Mahaboob, B. et al., 2017). We consider m_i is small, i.e. ε_i are small. In this case, we can write,

$$log(Q_i-\varepsilon_i)=logQ_i+log(1-m_i)\cong logQ_i-m_i\,. \tag{13}$$

Hence,

$$logQ_i=logA+\beta_1 logX_{1i}+\beta_2 logX_{2i}+\cdots+\beta_n logX_{ni}+m_i \tag{14}$$

where, $m_i\sim N\left(0\,,\sigma^2/Q_i^2\right)$ so it is not satisfy the assumption of linear regression. Now we multiply by Q_i on both of side equation $logQ_i$ then, we obtain new equation as,

$$Q_i\, logQ_i = Q_i logA + Q_i\beta_1 logX_{1i} + Q_i\beta_2 logX_{2i} + \cdots + Q_i\beta_n logX_{ni} + Q_i m_i$$

$$Q_i logQ_i = Q_i logA + Q_i\beta_1 logX_{1i} + Q_i\beta_2 logX_{2i} + \cdots + Q_i\beta_n logX_{ni} + \varepsilon_i \tag{15}$$

where $\varepsilon_i \sim N(0\,, \sigma^2)$.

In the matrix notation,

$$\boldsymbol{Q} = \begin{pmatrix} Q_1 logQ_1 \\ Q_2 logQ_2 \\ \vdots \\ Q_n logQ_n \end{pmatrix} \quad , \quad \widehat{\boldsymbol{\beta}} = \begin{pmatrix} log\hat{A} \\ \widehat{\beta_1} \\ \vdots \\ \widehat{\beta_n} \end{pmatrix} \quad , \quad A = \begin{pmatrix} Q_1 & Q_1 logX_{11} & \cdots & Q_1 logX_{n1} \\ Q_2 & Q_2 logX_{12} & \cdots & Q_1 logX_{n1} \\ \vdots & \vdots & \vdots & \vdots \\ Q_n & Q_n logX_{1n} & \cdots & Q_1 logX_{nn} \end{pmatrix} \tag{16}$$

The least squares estimation of the parameters obtains as follows (Qi, W., Yingsheng, S. and Pengfei, J., 2010),

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{A^T A})^{-\boldsymbol{1}} \boldsymbol{A^T Q}. \tag{17}$$

3. GREY PREDICTION MODEL GM(1,1)

In the analysis of empirical data is widely used the multiple regression models, prediction of the future value of response variable based on multiple regression models need to know the future value of the independent random variables. Grey system theory initiated in J. L. Deng (1982) can provide a more flexible approximation to fitting a model to observed data. Grey model was building on the differential equation. Solution of differential equation has an exponential function.

In the grey system theory, the basic model is GM(1,1). It is predict a single variable. This model eliminates the randomness of data by accumulating the original time series (Honghong Liu, 2010). We can summarize the model calculation as following,

Step 1: Original data:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\right) \tag{18}$$

We assume that the original data non-negative and generally assumed that, $n \geq 4$.

Step 2: Accumulate generating data (1-AGO):

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\right) \tag{19}$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)\,, \qquad k = 1,2, \dots, n \tag{20}$$

this sequence is monotone increasing.

Step 3: The generated mean sequence $Z^{(1)}$ is described as,

$$Z^{(1)} = \left(z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n)\right) \tag{21}$$

where

$$z^{(1)}(k) = 0.5\left(x^{(1)}(k) + x^{(1)}(k-1)\right), \quad k = 2,3, \dots, n \tag{22}$$

$X^{(1)}$ represent the monotonic increasing sequence, so it is similar to the solution of first order linear differential equation. This differential equation is called whitening equation for $X^{(1)}$ and is given by

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \tag{23}$$

where a is called developing parameter and parameter b is called grey input coefficient.

Step 4: a and b parameters are determined by discrete form of above differential equation. The difference equation is written as,

$$X^{(0)}(k) + aZ^{(1)}(k) = b, \qquad \mathrm{k} = 1, 2, 3, \dots \tag{24}$$

The model parameters a and b are estimated as follows,

$$\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix} = (B^T B)^{-1} B^T Y \tag{25}$$

where

$$B = \begin{pmatrix} -\frac{1}{2}\left(x^{(1)}(1) + x^{(1)}(2)\right) & 1 \\ -\frac{1}{2}\left(x^{(1)}(2) + x^{(1)}(3)\right) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left(x^{(1)}(n-1) + x^{(1)}(n)\right) & 1 \end{pmatrix}, \qquad Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix} \tag{26}$$

Pa rameter α and b also estimated as follows by using the time series form by the least square method:

$$\hat{a} = \frac{\sum_{k=2}^{n} z^{(1)}(k) \ \sum_{k=2}^{n} x^{(0)}(k) - (n-1)\sum_{k=2}^{n} x^{(0)}(k) \ \ z^{(1)}(k)}{(n-1)\sum_{k=2}^{n}[z^{(1)}(k)]^2 - (\sum_{k=2}^{n} z^{(1)}(k))^2}, \tag{27}$$

$$\hat{b} = \frac{\sum_{k=2}^{n} x^{(0)}(k) \ \sum_{k=2}^{n}\left[z^{(1)}(k)\right]^2 - \sum_{k=2}^{n} x^{(0)}(k) z^{(1)}(k) \sum_{k=2}^{n} z^{(1)}(k)}{(n-1)\sum_{k=2}^{n}[z^{(1)}(k)]^2 - (\sum_{k=2}^{n} z^{(1)}(k))^2}. \tag{28}$$

Step 5: Solving the differential equation with an initial condition $x^{(1)}(1) = x^{(0)}(1)$, we obtain a prediction model as follows,

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}\right] e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} \quad , \quad k = 1,2,3,\dots \tag{29}$$

We estimates the original data by inverse accumulated generating operation (IAGO) which is defined as

$$\hat{x}^{(0)}(k+1) = \left(1 - e^{-\hat{a}}\right) \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-\hat{a}k} \ , \qquad k = 1,2,3,\dots \tag{30}$$

4. EMPIRICAL ANALYSIS

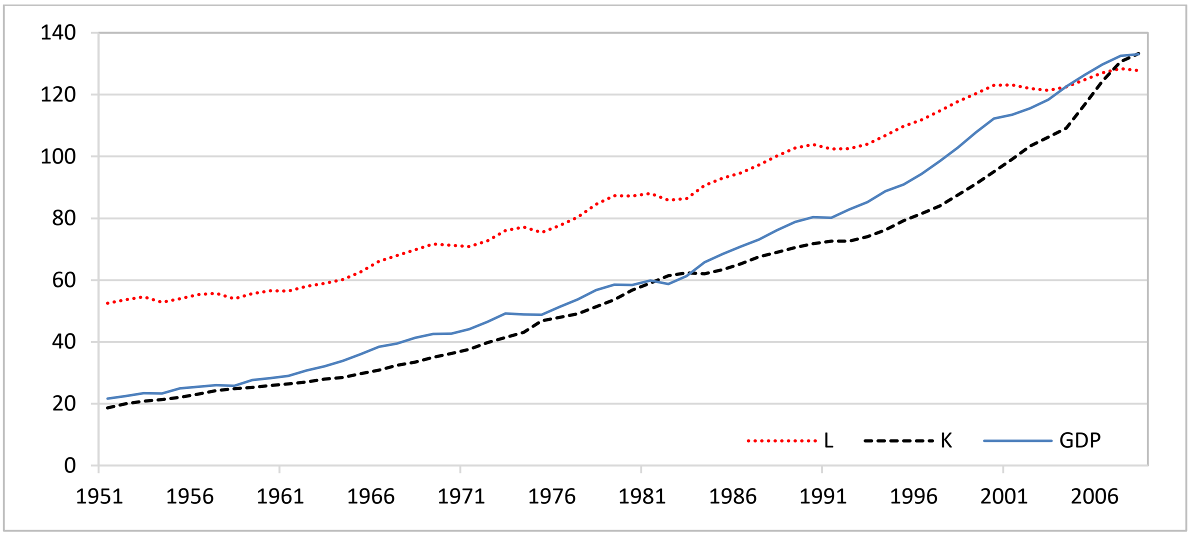
In this study, we analysis the economic growth of US, first we describe the behavior pattern of economic factors and then we measure the input factor's contribution rate to economic growth.

We are used the annual data Gross Domestic Product (GDP-Billion US dollar) (Q_t) as a output and Labor (the number of employees (10.000 people)) (L_t) and Capital (fixed asset investment)(Billion US dollar) (K_t) as a input factors series for United State of American economy for 1951 to 2008. Data obtained from the United States Department of Commerce and the NIPA National Income Product Accounts (Thompson, H., 2016). The present series in Fig. 1 include labor and capital inputs and GDP. The capital stock time series used in models are not directly observable, then we obtain them by using capital formation index and GDP. Generally, the deflate values of inputs and outputs are used as a proxy of physical quantities (De Loecker, 2011).

Capital : Capital was taken as the total of the stock of machines, equipment and structures used in production. The time series of capital is usually not found in economics databases. We can built it using two index as Gross capital formation (GKF) and GDP as follows (Godin, A. and Kinsella, S., 2013),

$$K_t = \frac{GKF_t}{\overline{GDP} + 0.05}. \tag{31}$$

Figure 1: Trends in Labor, Capital and GDP



Labor: Labor is computed as total man-hours worked during the time period as

$$L_t = pop_t . part_t (1 - nairi_t) hours_t \tag{32}$$

where pop_t: total population, $part_t$: participation rate, $nairi_t$: non-accelerating inflation rate of unemployment, $hours_t$: average hours worked per capita. Or, if we don't have data, we use the number of people working (labor force) or the total population of working age (15-64).

4.1. Statistical Analysis of Parameter Estimation

Cobb-Douglas production function in Eq. (6) can be converted to linear form as follows,

$$\ln Q = \ln A + \alpha \ln K + \beta \ln L.$$

The regression results obtained as Table 2.

Table 2: The Regression Results

Adjusted R Square	0.9955
F statistics	6306.6823

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-5.7761	1.1151	-5.1799	3.252E-06
ln K	0.4030	0.0677	5.9545	1.905E-07
Ln L	1.0936	0.1281	8.5377	1.170E-11

Obtained Cobb-Douglas model parameters are given in Table 3.

Table 3: Cobb-Douglas Model Parameters

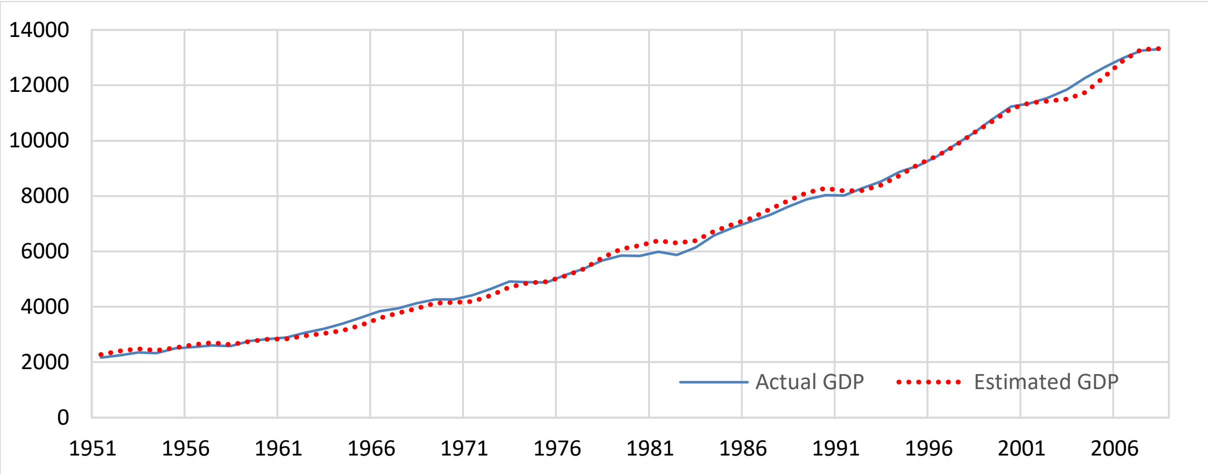
$\hat{A}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha} + \hat{\beta}$
0.0031	0.403	1.094	1.497

The model coefficient of determination is $R^2 = 1 - \frac{\sum_{t=1}^{58}(Q_t - \hat{Q}_t)^2}{\sum_{t=1}^{58}(Q_t - \bar{Q}_t)^2} = 0.9957$. It shows that the model has a very high fitting precision. Finally, Cobb-Douglas production function regression equation can be obtained as follows,

$$\hat{Q}_{regression} = 0.0031\, K^{0.403}\, L^{1.094}\,. \tag{33}$$

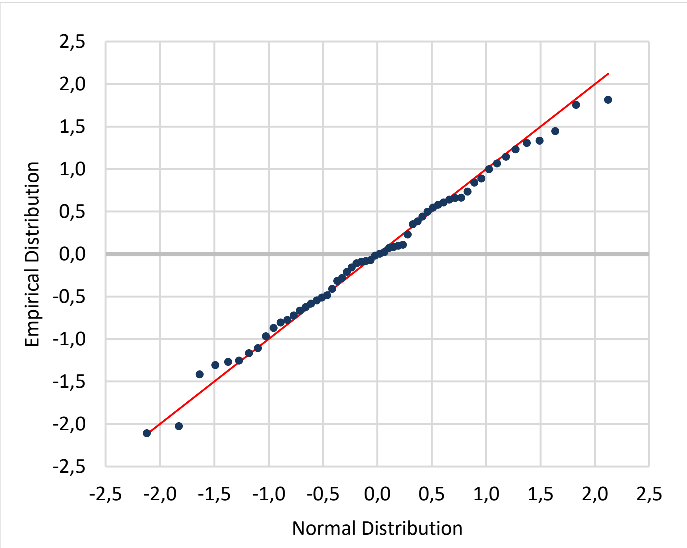
Result of the regression show that regression equation is significant. Then, the explained ratio of GDP by changes in capital and labor is 99.55%.

Figure 2: Actual GDP and Estimated GDP According to Cobb - Douglas Production Function for $\hat{A} = 0.0031$, $\hat{\alpha} = 0.403$ and $\hat{\beta} = 1.097$.



Errors between the observed real GDP values and the values which estimated by Cobb-Douglas production function are distributed as a normal. This situation is shown by using the Q-Q plot in Fig. 3.

Figure 3: Q-Q Plot of Residuals between Real GDP and Estimated GDP



The points in the Q-Q plot lie on straightforward line. This shows that the residuals are based on normal distribution.

Production function in Eq. (33) show that level of production technology is 0.0031. The elasticity of capital α is 0.403. This value shows that 1% increase in capital lead to 0.403% increase in GDP. The elasticity of labor β is also 1.094. This value shows that a 1% increase in labor lead to a 1.094% increase in GDP. The sum of elasticities of input factors is

$$\alpha + \beta = 0.403 + 1.094 = 1.497 > 1.$$

This shows that per percent of increase in GDP is greater than that of the increase in capital and number of employees, i.e. it shows increasing return to scale.

We obtain the picture shown in Fig. 4 of Cobb-Douglas production function in Eq. (33) having increasing return to scale.

Figure 4: Cobb-Douglas Production Function for $\hat{A} = 0.0031$, $\hat{\alpha} = 0.403$ and $\hat{\beta} = 1.094$.

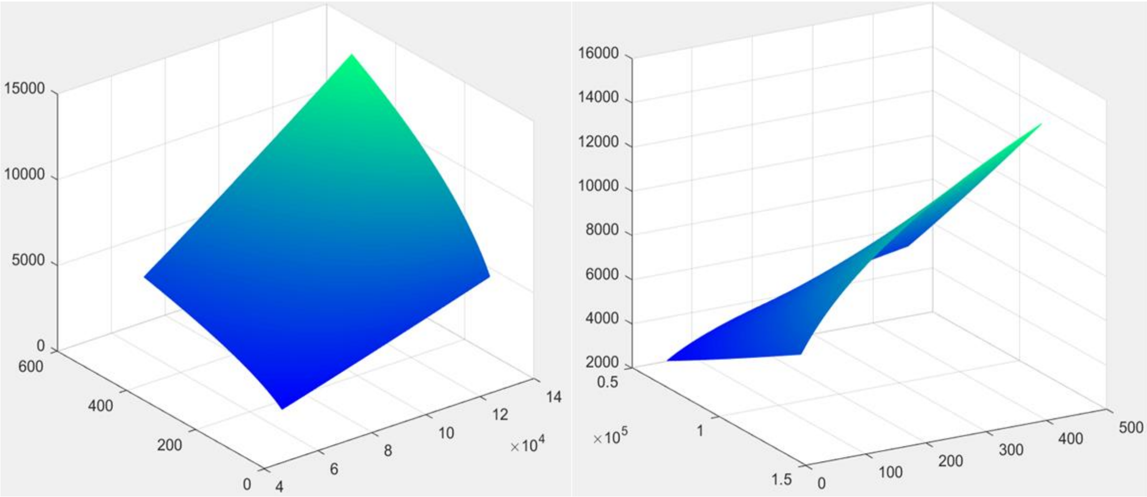


Figure 5: Labor Productivity and Capital Productivity

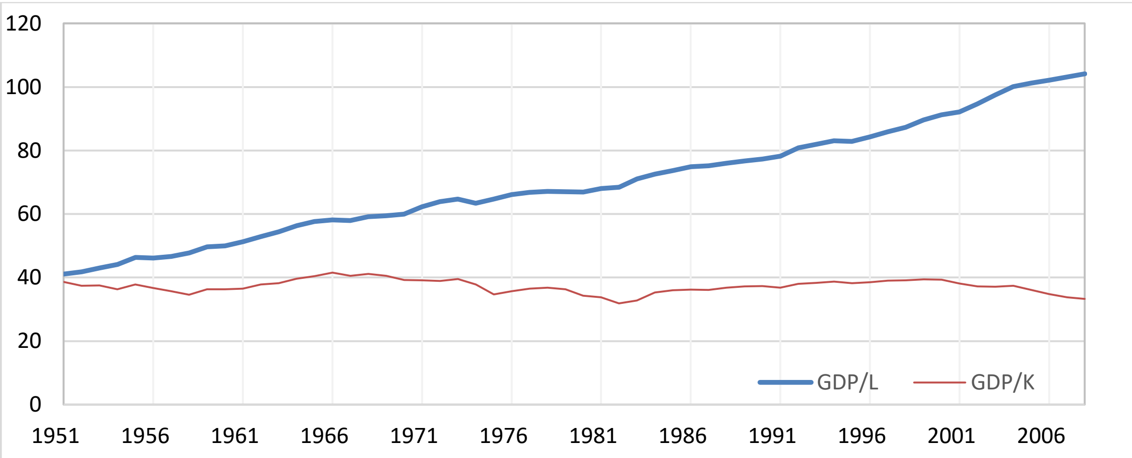
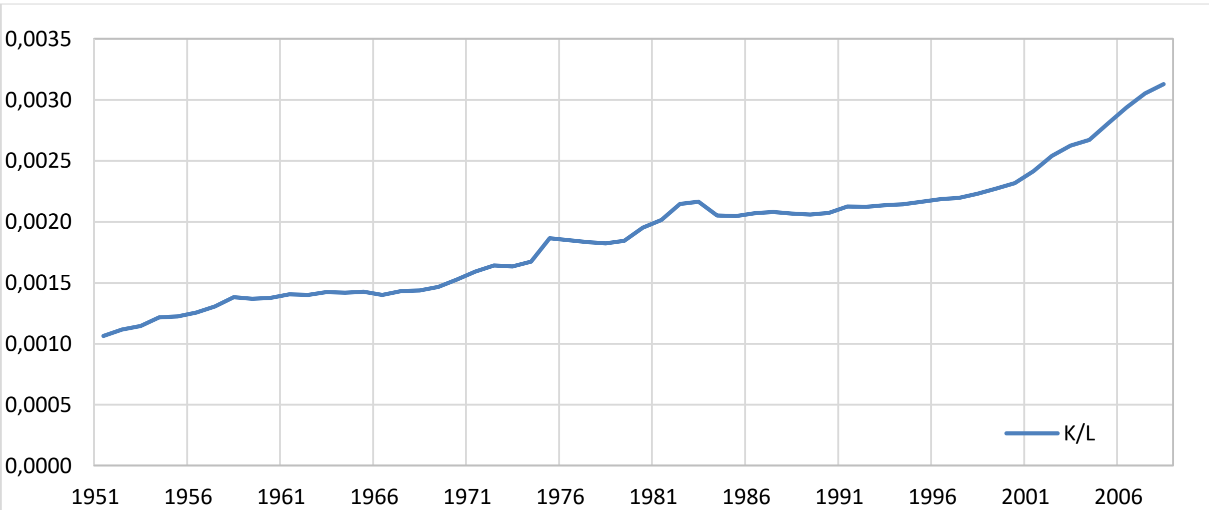


Figure 6: Capital – Labor Ratio



4.2. Grey Cobb-Douglas Production Function

Predictions for the values of K and L input factors obtained by using GM(1,1) model shown in Table 4.

Table 4: Prediction of Production Factors by Using GM(1,1) Model

Year	GM(1,1) prediction for L	GM(1,1) prediction for K	Year	GM(1,1) prediction for L	GM(1,1) prediction for K
2009	141277.17	467.76	2014	147525.08	452.55
2010	137815.06	397.71	2015	150057.65	467.40
2011	140180.94	410.77	2016	152633.70	482.74
2012	142587.44	424.25	2017	155253.97	498.58
2013	145035.25	438.17	2018	157919.23	514.94

Figure 7: Actual Capital and Estimated Capital Obtained by Using GM(1,1) Prediction for K

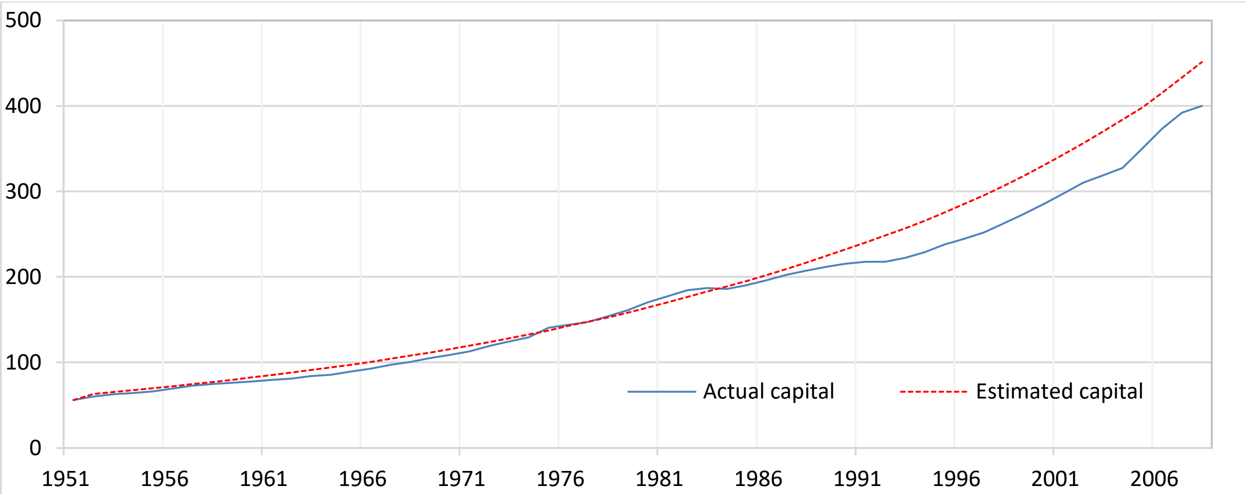


Figure 8: Actual Labor and Estimated Labor Obtained by Using GM(1,1) Prediction for L

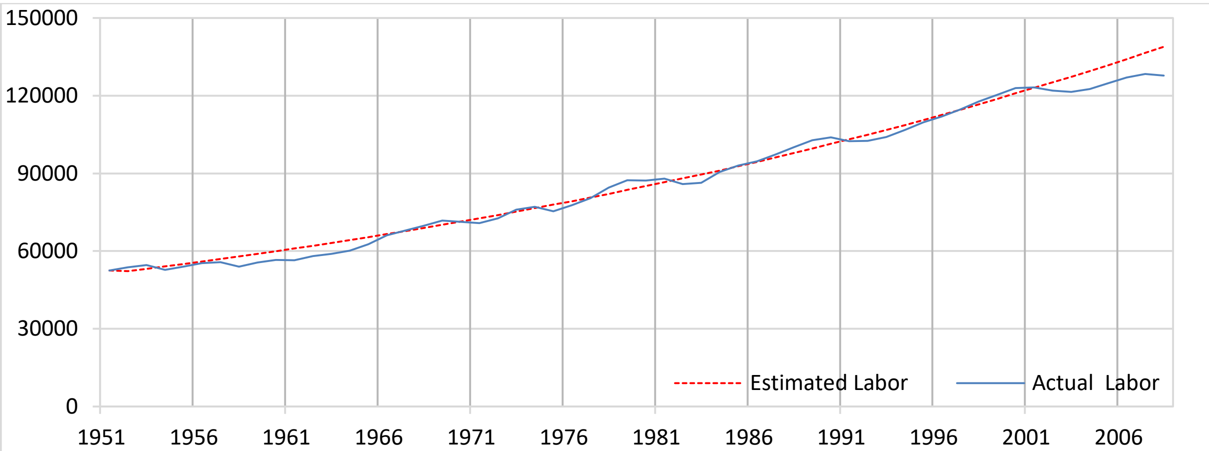
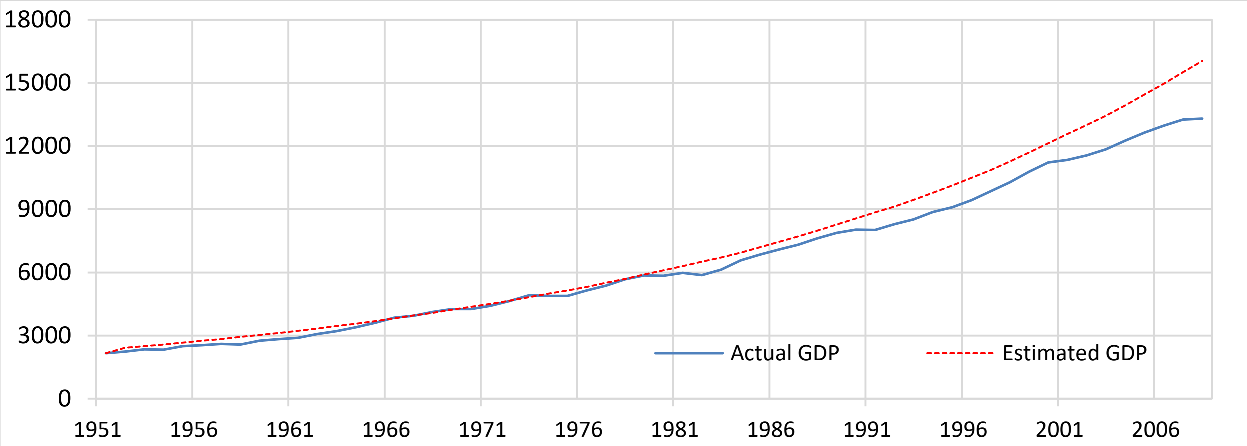


Figure 9: Actual GDP and Estimated GDP Obtained GM(1,1) Prediction for GDP



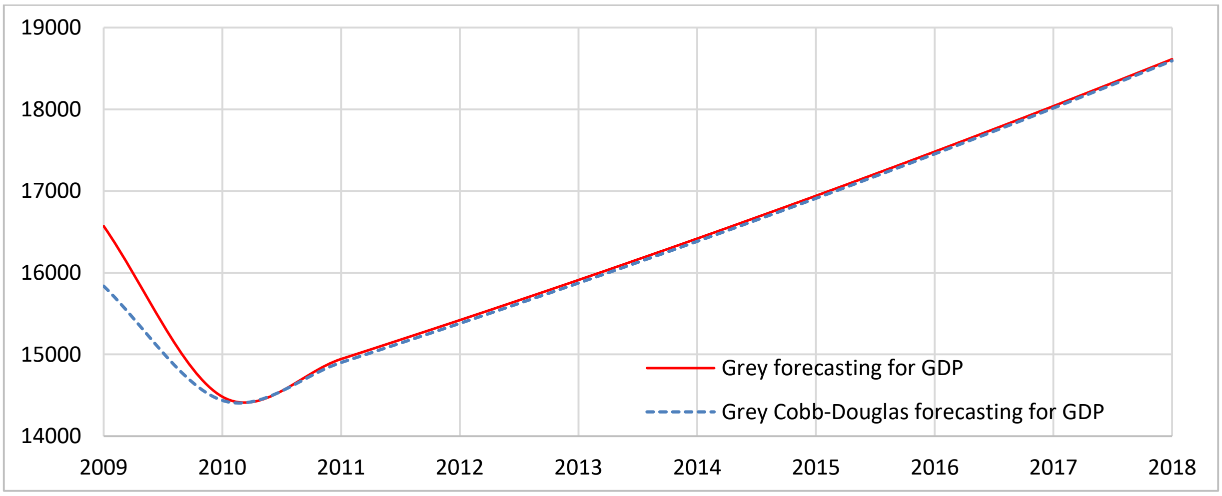
We determine the Grey Cobb-Douglas production function as (Zhu. S., Wu. Q. J. and Wang. Y., 2011).

$$\hat{Q}_t = \hat{A}\left(\hat{K}_t^{Grey}\right)^{\hat{\alpha}}\left(\hat{L}_t^{Grey}\right)^{\hat{\beta}}. \tag{42}$$

Table 5: The GM(1,1) Forecasting and Grey Cobb-Douglas Forecasting for GDP

Year	GM(1,1) forecasting for GDP	Grey Cobb-Douglas forecasting for GDP
2009	16570.35	15836.51
2010	14481.48	14437.28
2011	14942.97	14901.18
2012	15419.17	15379.99
2013	15910.55	15874.18
2014	16417.59	16384.25
2015	16940.78	16910.72
2016	17480.64	17454.09
2017	18037.72	18014.93
2018	18612.54	18593.79

Figure 10: Plot of the GM(1,1) Forecasting and Grey Cobb-Douglas Forecasting for GDP



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4.3. Error Analysis

Error Analysis is needed for examining the precision of forecasted results. The Mean Absolute Percentage Error (MAPE) is one of the most widely used methods that is evaluation of forecasting error. The MAPE is calculated as,

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Q_t - \hat{Q}_t}{Q_t}\right|. \tag{43}$$

Where Q_t is actual value. $\hat{Q}_t$ is also forecasting value at time t. n is the number of periods forecasted (Makridakis. S.. Wheelwright. S. C. and Hyndman. R. J., 2008).

Table 6: MAPE reference values for forecasting accuracy

MAPE	Forecasting Accuracy
Less than 10%	Highly accurate
11% to 20%	Good forecast
21% to 50%	Reasonable forecast
51% or more	Inaccurate forecast

MAPE value for Cobb-Douglas production function estimation is calculated as

$$MAPE = 0.0282.$$

This value is less than 10%, so we can say that Cobb-Douglas production function is suitable model for forecasting of GDP values.

5. CONCLUSION

The percent change growth of production is proportional to percentage change growth in the quantities of input factors without changing factor usage shares. That is the constant return to scale form of the production function. Cobb-Douglas production function model is applied to capital, labor, Gross Domestic Product (GDP) time series for United States of American economy for 1951 to 2008 and is obtained that marginal contribution to GDP of capital input is 0.403 and marginal contribution to GDP on labor input is 1.094. Findings show that United States economic (GDP) is labor intensive. In addition, it is confirmed that the labor force and capital has a positive effect on economic growth. Besides, GM(1,1) model is used to prediction of future labor and capital values. Predicted values for K and L putting into grey Cobb-Douglas production model is forecasted GDP values. Finally, improvement of the labor quality can help to increase of the GDP values.

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