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Perceptions of Several Sets in Ideal Nano Topological Spaces

Ilangovan Rajasekaran[1,*] <sekarmelakkal@gmail.com>
Ochanan Nethaji[2] <jionetha@yahoo.com>
Rajendran Prem Kumar[3] <prem.rpk27@gmail.com>

[1]Department of Mathematics, Tirunelveli Dakshina Mara Nadar Sangam College, T. Kallikulam-627 113, Tirunelveli District, Tamil Nadu, India
[2]School of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu, India
[3]Department of Mathematics, Senthamarai College of Arts and Science, Vadapalanji-21, Madurai District, Tamil Nadu, India

Abstaract − In this paper, we introduce the concepts of t-nI-set and $\mathcal{R}$-nI-set are investigate and deal with an ideal nano topological spaces.

Keywords − *t-nI-open set, $\mathcal{R}$-nI-open set, and t_α-nI-open set α-nI-open set, pre-nI-open set and $\mathcal{R}_\alpha$-nI-open set.*

1 Introduction

An ideal I [8] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

1. $A \in I$ and $B \subset A$ imply $B \in I$ and
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a topological space (X, τ) with an ideal I on X. If $\wp(X)$ is the family of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [1]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ [7] is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the $\star$-topology finer than τ. The topological space together with an ideal on X is called an ideal topological space or an ideal space denoted by (X, τ, I). We will simply write $A^\star$ for $A^\star(I, \tau)$ and $\tau^\star$ for $\tau^\star(I, \tau)$.

*Corresponding Author.

Some new notions in the concept of ideal nano topological spaces were introduced by Parimala et al. [3, 4].

In this paper, we introduce the notions of t-nI-set, $\mathcal{R}$-nI-set, t_α-nI-set and $\mathcal{R}_\alpha$-nI-set are investigate and deal with an ideal nano topological spaces.

2 Preliminaries

Definition 2.1. *[5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. *[2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then R(X) satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*

2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by n-int(A) and n-cl(A), respectively.

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [3] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [3] the family of nano open sets containing x.

In future an ideal nano topological spaces $(U, \mathcal{N}, I)$ is referred as a space.

Definition 2.3. *[3] Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $A \subseteq U$, $A^\star_n(I, \mathcal{N}) = \{x \in U : G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $A^\star_n$ for $A^\star_n(I, \mathcal{N})$.*

Theorem 2.4. *[3] Let $(U, \mathcal{N}, I)$ be a space and A and B be subsets of U. Then*

1. $A \subseteq B \Rightarrow A^\star_n \subseteq B^\star_n$,
2. $A^\star_n = n\text{-}cl(A^\star_n) \subseteq n\text{-}cl(A)$ *($A^\star_n$ is a n-closed subset of $n\text{-}cl(A)$),*
3. $(A^\star_n)^\star_n \subseteq A^\star_n$,
4. $(A \cup B)^\star_n = A^\star_n \cup B^\star_n$,
5. $V \in \mathcal{N} \Rightarrow V \cap A^\star_n = V \cap (V \cap A)^\star_n \subseteq (V \cap A)^\star_n$,
6. $J \in I \Rightarrow (A \cup J)^\star_n = A^\star_n = (A - J)^\star_n$.

Theorem 2.5. *[3] Let $(U, \mathcal{N}, I)$ be a space with an ideal I and $A \subseteq A^\star_n$, then $A^\star_n = n\text{-}cl(A^\star_n) = n\text{-}cl(A)$.*

Definition 2.6. *[3] Let $(U, \mathcal{N}, I)$ be a space. The set operator $n\text{-}cl^\star$ called a nano $\star$-closure is defined by $n\text{-}cl^\star(A) = A \cup A^\star_n$ for $A \subseteq X$.*

It can be easily observed that $n\text{-}cl^\star(A) \subseteq n\text{-}cl(A)$.

Theorem 2.7. *[4] In a space $(U, \mathcal{N}, I)$, if A and B are subsets of U, then the following results are true for the set operator $n\text{-}cl^\star$.*

1. $A \subseteq n\text{-}cl^\star(A)$,
2. $n\text{-}cl^\star(\phi) = \phi$ *and* $n\text{-}cl^\star(U) = U$,
3. *If* $A \subset B$, *then* $n\text{-}cl^\star(A) \subseteq n\text{-}cl^\star(B)$,
4. $n\text{-}cl^\star(A) \cup n\text{-}cl^\star(B) = n\text{-}cl^\star(A \cup B)$.
5. $n\text{-}cl^\star(n\text{-}cl^\star(A)) = n\text{-}cl^\star(A)$.

Definition 2.8. *[6] A subset A of space $(U, \mathcal{N}, I)$ is said to be*

1. *nano α-I-open (briefly, α-nI-open) if $A \subset n\text{-}int(n\text{-}cl^\star(n\text{-}int(A)))$,*
2. *nano pre-I-open (briefly, pre-nI-open) if $A \subset n\text{-}int(n\text{-}cl^\star(A))$.*

3 On nano t-I-set, nano t_α-I-set, nano $\mathcal{R}$-I-set and nano $\mathcal{R}_\alpha$-I-sets

Definition 3.1. *A subset A of a space $(U, \mathcal{N}, I)$ is called*

1. *nano t-I-set (briefly, t-nI-set) if n-$int(A) = n$-$int(n$-$cl^\star(A))$,*
2. *nano t_α-I-set (briefly, t_α-nI-set) if n-$int(A) = n$-$int(n$-$cl^\star(n$-$int(A)))$,*
3. *nano $\mathcal{R}$-I-set (briefly, $\mathcal{R}$-nI-set) if $A = P \cap Q$, where P is n-open and Q is t-nI-set,*
4. *nano $\mathcal{R}_\alpha$-I-set (briefly, $\mathcal{R}_\alpha$-nI-set) if $A = P \cap Q$, where P is n-open and Q is t_α-nI-set.*

Example 3.2. *Let $U = \{a_1, a_2, a_3, a_4\}$ with $U/R = \{\{a_2\}, \{a_4\}, \{a_1, a_3\}\}$ and $X = \{a_3, a_4\}$. Then $\mathcal{N} = \{\phi, \{a_4\}, \{a_1, a_3\}, \{a_1, a_3, a_4\}, U\}$. Let the ideal be $I = \{\phi, \{a_3\}\}$.*

1. *$A = \{a_2\}$ is t-nI-set.*
2. *$B = \{a_4\}$ is t_α-nI-set.*
3. *$C = \{a_2, a_3\}$ is $\mathcal{R}$-nI-set*
4. *$D = \{a_1, a_3\}$ is $\mathcal{R}_\alpha$-nI-set*

Remark 3.3. *In a space $(U, \mathcal{N}, I)$,*

1. *each n-open set is $\mathcal{R}$-nI-set.*
2. *each t-nI-set is $\mathcal{R}$-nI-set.*

Proposition 3.4. *Let A and B be subsets of a space $(U, \mathcal{N}, I)$. If A and B are t-nI-sets, then $A \cap B$ is t-nI-set.*

Proof. Let A and B be t-nI-sets. Then we have

$$\begin{aligned} n\text{-}int(A \cap B) &\subset n\text{-}int(n\text{-}cl^\star(A \cap B)) \\ &\subset n\text{-}int(n\text{-}cl^\star(A) \cap n\text{-}cl^\star(B)) \\ &= n\text{-}int(n\text{-}cl^\star(A)) \cap n\text{-}int(n\text{-}cl^\star(B)) \\ &= n\text{-}int(A) \cap n\text{-}int(B) \\ &= n\text{-}int(A \cap B). \end{aligned}$$

Then n-$int(A \cap B) = n$-$int(n$-$cl^\star(A \cap B))$ and hence $A \cap B$ is a t-nI-set.

Example 3.5. *In Example 3.2, $H = \{a_1, a_3\}$ and $K = \{a_3, a_4\}$ is t-nI-set. But $H \cap K = \{a_3\}$ is t-nI-set.*

Proposition 3.6. *For a subset A of a space $(U, \mathcal{N}, I)$, the following properties are equivalent:*

1. *A is n-open,*

2. *A is pre-nI-open and $\mathcal{R}$-nI-set.*

Proof. (1) $\Rightarrow$ (2): Let A be n-open. Then

$$A = n\text{-}int(A) \subset n\text{-}int(n\text{-}cl^\star(A))$$

and A is pre-nI-open. Also by Remark 3.3 A is $\mathcal{R}$-nI-set.

(2) $\Rightarrow$ (1): Given A is $\mathcal{R}$-nI-set. So $A = P \cap Q$ where P is n-open and

$$n\text{-}int(Q) = n\text{-}int(n\text{-}cl(Q))$$

Then $A \subset P = n\text{-}int(P)$. Also, A is pre-nI-open implies

$$A \subset n\text{-}int(n\text{-}cl(A)) \subset n\text{-}int(n\text{-}cl^\star(Q)) = n\text{-}int(Q)$$

by assumption. Thus

$$A \subset n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(A)$$

and hence A is n-open.

Remark 3.7. *In a space the family of pre-nI-open sets and the family of $\mathcal{R}$-nI-sets are independent.*

Example 3.8. *In Example 3.2,*

1. *$A = \{a_1, a_4\}$ is pre-nI-open but not $\mathcal{R}$-nI-set.*
2. *$B = \{a_2\}$ is $\mathcal{R}$-nI-set but not pre-nI-open.*

Remark 3.9. *In a space $(U, \mathcal{N}, I)$,*

1. *each n-open set is $\mathcal{R}_\alpha$-nI-set.*
2. *each t_α-nI-set is $\mathcal{R}_\alpha$-nI-set.*

These relations are shown in the diagram.

$$\begin{array}{ccccc} t\text{-}nI\textbf{-set} & & & & t_\alpha\text{-}nI\textbf{-set} \\ \downarrow & & & & \downarrow \\ \mathcal{R}\text{-}nI\textbf{-set} & \longleftarrow & n\textbf{-open} & \longrightarrow & \mathcal{R}_\alpha\text{-}nI\textbf{-set} \end{array}$$

The converses of diagram is not true as shown in the following Example.

Example 3.10. *In Example 3.2,*

1. *$A = \{a_2\}$ is $\mathcal{R}$-nI-set but not n-open set.*
2. *$B = \{a_1, a_3, a_4\}$ is $\mathcal{R}$-nI-set but not t-nI-set.*
3. *$A = \{a_1\}$ is $\mathcal{R}_\alpha$-nI-set but not n-open set.*
4. *$B = \{a_1, a_3, a_4\}$ is $\mathcal{R}_\alpha$-nI-set but not t_α-nI-set.*

Proposition 3.11. *If A and B are t_α-nI-sets of a space $(U, \mathcal{N}, I)$, then $A \cap B$ is a t_α-nI-set.*

Proof. Let A and B be t_α-nI-sets. Then we have

$$\begin{aligned} n\text{-}int(A \cap B) &\subset n\text{-}int(n\text{-}cl^\star(n\text{-}int(A \cap B))) \\ &\subset n\text{-}int[n\text{-}cl^\star(n\text{-}int(A)) \cap n\text{-}cl^\star(n\text{-}int(B))] \\ &= n\text{-}int(n\text{-}cl^\star(n\text{-}int(A))) \cap n\text{-}int(n\text{-}cl^\star(n\text{-}int(B))) \\ &= n\text{-}int(A) \cap n\text{-}int(B) \\ &= n\text{-}int(A \cap B). \end{aligned}$$

Then $n\text{-}int(A \cap B) = n\text{-}int(n\text{-}cl^\star(n\text{-}int(A \cap B)))$ and hence $A \cap B$ is a t_α-nI-set.

Example 3.12. *In Example 3.2, $H = \{a_2, a_3\}$ and $K = \{a_1, a_2\}$ is t_α-nI-set. But $H \cap K = \{a_2\}$ is t_α-nI-set.*

Proposition 3.13. *For a subset A of a space $(U, \mathcal{N}, I)$, the following properties are equivalent:*

1. *A is n-open;*
2. *A is α-nI-open and a $\mathcal{R}_\alpha$-nI-set.*

Proof. (1) $\Rightarrow$ (2): Let A be n-open. Then

$$A = n\text{-}int(A) \subset n\text{-}cl^\star(n\text{-}int(A))$$

and

$$A = n\text{-}int(A) \subset n\text{-}int(n\text{-}cl^\star(n\text{-}int(A)))$$

Therefore A is α-nI-open. Also by (1) of Remark 3.9, A is a $\mathcal{R}_\alpha$-nI-set.

(2) $\Rightarrow$ (1): Given A is a $\mathcal{R}_\alpha$-nI-set. So $A = P \cap Q$ where P is n-open and

$$n\text{-}int(Q) = n\text{-}int(n\text{-}cl^\star(n\text{-}int(Q)))$$

Then $A \subset P = n\text{-}int(P)$. Also A is α-nI-open implies

$$A \subset n\text{-}int(n\text{-}cl^\star(n\text{-}int(H))) \subset n\text{-}int(n\text{-}cl^\star(n\text{-}int(Q))) = n\text{-}int(Q)$$

by assumption. Thus

$$A \subset n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(A)$$

and A is n-open.

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