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Probabilistic Slope Stability Analysis: A Case Study

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ABSTRACT

Probabilistic slope stability analyses are becoming more and more popular to evaluate the safety level of slopes and associated risk and reliability, especially in the recent years. The probabilistic approach can take into account the uncertainties and natural variability in material properties, as well as in environmental factors, by using various statistical distribution functions (such as normal, lognormal etc.) for random variables. It is already noted by various researchers that, a slope with a deterministic factor of safety larger than 1.00 using average values of soil parameters may have a significant level of probability of failure, if the material properties are unknown, or contain significant uncertainty/variability. In this study, a well-documented landslide case study is used to demonstrate the importance of probabilistic approach in slope stability; to investigate the effects of considering variability in material properties; and to compare deterministic and probabilistic slope stability analyses results. Deterministic limit equilibrium, probabilistic limit equilibrium, and probabilistic finite element analyses are conducted for Lodalen landslide in Oslo, Norway and the results are compared with each other. The factor of safety, the probability of failure and the most critical failure surface are investigated with and without statistical cross-correlation of soil's shear strength parameters. The results of this study can provide further insights into the comparison of deterministic and probabilistic approaches in slope safety.

Keywords: slope stability, probabilistic analysis, limit equilibrium, probability of failure

1. INTRODUCTION

Soil is an inherently heterogeneous, three-phase material which has natural variability and uncertainty in its properties. Geotechnical design should incorporate the variability of the soil to assess the reliability and risk associated with the projects. Accounting for soil variability and carrying out probabilistic analyses also result in significant savings in the cost of the project, and can lead to possible prediction of failure events.

Determination of the safety level of slopes is important for proper geotechnical risk assessment, especially in urban areas. The safety level can be determined via probabilistic slope stability analyses with the consideration of uncertainties and variability in material properties, as well as in environmental factors, such as the level of the groundwater table. In this study, a landslide case study is used to demonstrate the importance of probabilistic approach in slope stability, and to investigate the effects of considering variability in material properties. Deterministic limit equilibrium, probabilistic limit equilibrium, and probabilistic finite element analyses are conducted for Lodalen landslide in Oslo, Norway and the results are compared with each other. Probability of failure (PF) and the most critical failure surface are investigated with and without statistical cross-correlation of soil's shear strength parameters. Lodalen landslide in Oslo, Norway, is chosen as the case study since it has been well-documented in the literature [1, 2].

1.1. Background information about probabilistic slope stability

In recent years, probabilistic approaches are more and more widely used for the safety assessment of slopes [3-9]. It is noted by various researchers that a slope with a deterministic factor of safety (FS) larger than 1.00

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may have a probability of failure larger than 0% [7, 9]. For example, results of [9] show that there is an inverse relation between FS and PF which is demonstrated to be nonlinear and that coefficient of variation (COV=standard deviation / mean) level, i.e. the level of variability in material properties, has a significant effect on this relationship (Figure 1).

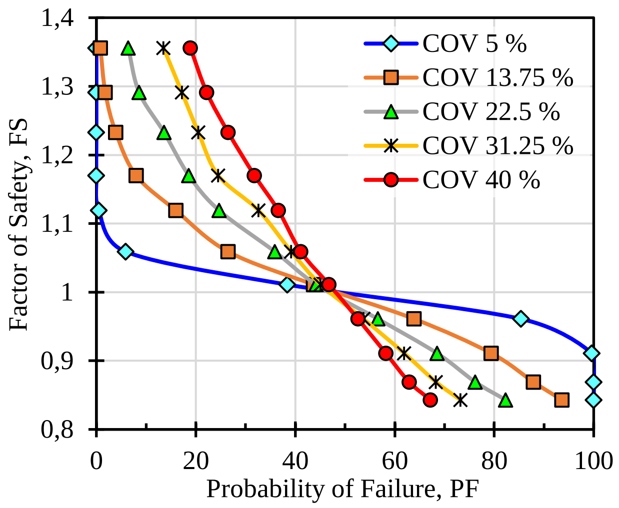


Figure 1. The nonlinear relationship between FS and PF and the effect of COV% [9]

Increase in the COV% level causes an increase in PF value for slopes with $FS_{deterministic}$ value greater than 1.00 (Figure 1). The PF and the location of the critical failure surface are significantly influenced by: (i) the COV level, (ii) the consideration of cross correlation of shear strength parameters, and (iii) by the traditional deterministic FS level of the slope [9]. Furthermore, the deterministic critical failure surface with minimum FS value is not always the most critical slip surface [9]. Similar results have been reported by [6] where they stated that generally probabilistic analyses are carried out on the critical deterministic surface, but that surface may not have the highest PF value.

Frequently, to carry out a probabilistic slope stability analyses, Monte Carlo (MC) method is used [7, 9]. In this approach, firstly, the input parameters that are to be treated as random variables in the analyses are determined (for example, unit weight of soil, cohesion, friction angle, pore water pressure coefficient, r_u, etc.). For each of these random variables, a representative statistical distribution (such as normal, lognormal etc.) is selected. Statistical distributions are typically represented by a mean value and a standard deviation, or a COV%. If sufficient number of field tests and laboratory test results are not available to determine site-specific values for these, typical COV% values from the literature can be utilized for that soil parameter. Then the required/sufficient number (N) of Monte Carlo analyses (runs) are determined and slope stability analyses are carried out N times. By the end of these analyses, (i) a statistically distributed, N-times factor of safeties, (ii) probability of failure, (iii) critical probabilistic failure surface and (iv) reliability index depending on the FS distribution will be obtained. PF is defined as the ratio of the number of slope stability analyses that end up with FS smaller than 1.00 to the number of total analyses.

In order to carry out a MC analysis, random numbers have to be generated from a given statistical distribution of the input variables. However, most typically, soil parameters are not independent random variables; they are dependent on each other. For example, an increase in unit weight will indicate a denser soil and slightly higher shear strength; or as cohesion increases, friction angle can be expected to decrease, which means a negative cross correlation between cohesion and friction angle. The cohesion and internal friction angle of the soil generally have an inversely correlated relationship [10, 11].

[9] noted that considering the cross-correlation between cohesion and friction angle significantly decreases PF for slopes having $FS_{deterministic}$>1.00, but increases for others. [8] also mentioned that cross-correlation between random values of soil properties can reduce the PF for simple slope cases. In their study, [8] investigated cohesive slopes with cross-correlation between cohesion and unit weight and compared the results with the cases without cross-correlation. Negative cross-correlation between cohesion and friction angle and positive cross correlation between cohesion and unit weight, and friction angle and unit weight were considered [8]. Results for cohesive soil slopes indicated that PF decreased for increasing negative cross-correlation between cohesion and friction angle, and increasing positive correlation between cohesion and unit weight, and friction angle and unit weight.

1.2. Background information about Lodalen landslide

Lodalen landslide took place on October 6, 1954, with a circular failure plane. The details of the slide are summarized from [1]. The width of the slide was about 50 m, slope height was 17 m and a 5-m-deep, almost-vertical head scarp at the top of the slope was observed. The slope was originally produced by cutting/excavating a natural slope 30 years ago and its slope angle was 1V:2H before the time of failure. 9 boreholes for in-situ vane tests and 7 boreholes (3 of which were in landslide mass) were done. Thin-walled 54-mm-diameter tubes were used for undisturbed

sampling and block samples having 10 cm height and 6 cm length were also taken. In three boreholes, a direct indication of the depth of the sliding surface was found at depths of 7.2-8.0 m, 9.5 m, and 8.2-9.0 m.
A relatively homogeneous marine clay with thin silt layers existed at the site. Liquid limit was in the range of 31-42% with an average value of 35.5%, plasticity index was in the range of 12-22% with an average value of 17%, average in-situ moisture content was about 30%, clay-size fraction was about 40% and unit weight of the marine clay was 18.7 kN/m^3. The sensitivity of the clay ranges from 3 to 15. The activity (plasticity index / clay size fraction) was 0.35. The salt content of the clay was 3-15 g/l, however, it was originally 20-30 g/l (similar to sea water), and therefore there has been significant leaching [1].

Unconfined compression tests were conducted on undisturbed samples, and in-situ vane shear tests were conducted. Measured values of undrained shear strength varied between 29-88 kPa with an average of 49 kPa. 10 consolidated undrained triaxial tests with pore water pressure measurements were conducted on undisturbed samples taken from 3 m to 19 m depths. Mean effective cohesion, c′, was 10 kPa and the standard deviation was 2.2 kPa (range 6.9-14.3 kPa). The mean friction angle was 27.1° and the standard deviation was 1.7° (range 24.0° and 29.4°) [2] as can be seen in Figure 2.

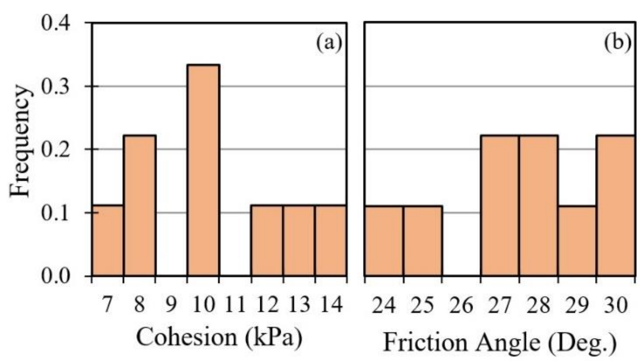


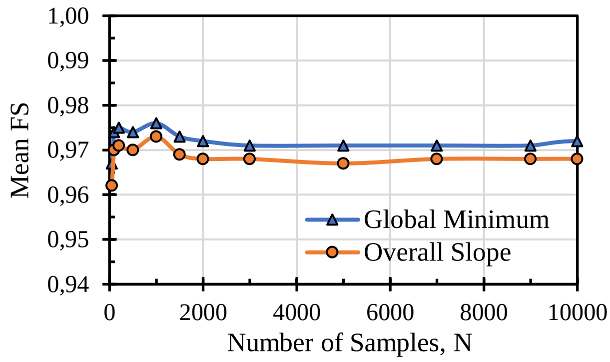
Figure 2. Distribution of (a) cohesion and (b) friction angle using measured data [1]

2. METHODS OF ANALYSES

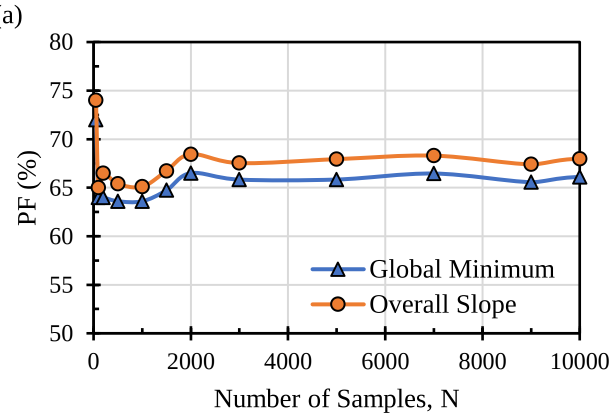
Slope stability analyses are conducted using limit equilibrium method (LEM) and method of slices (using Rocscience Slide software), as well as by finite element method (FEM) using Rocscience RS2 software. In the FEM analyses, a failure surface does not need to be assumed since the method uses stress-strain compatibility and finds the location of the failure surface by checking the maximum shear strains. In the LEM, for an assumed failure surface, the soil mass is divided into a number of vertical slices and force and moment equilibrium is checked. Spencer method [12] is used in the analyses since it satisfies all force and moment equilibrium conditions and is known to give an accurate factor of safety. Since the slope failure occurred 30 years after the slope is formed, it is considered a "drained, long term" failure event controlled by drained shear strength parameters of the marine clay [1, 2].

For probabilistic LEM slope stability calculations, two types of analyses are carried out: global minimum (GM) and overall slope (OS). In GM type analysis, using the mean values of all soil parameters the most critical deterministic failure surface having the lowest FS value is obtained; and after that, probabilistic analysis is carried out only for this failure surface, using the statistical distribution of material properties. OS type analysis, however, carries out a search for the most critical failure surface by N times, where N is the number of randomly selected soil properties and carries out probabilistic analysis for each found critical failure surface. As expected, OS analysis takes significantly longer run time as compared to GM analysis.

For a random sampling of soil parameters from a given statistical distribution, MC method is used. This method has been commonly used by other researchers for probabilistic slope stability analyses [3, 4, 7, 13, 14]. Since MC method requires long computational time, a series of MC LEM slope stability runs are conducted first, to determine how many samples (number of runs) are sufficient. The effect of number of simulations, N, on the results can be seen in Figure 3. In the analyses, shear strength parameters, cohesion and friction angle, are considered to be "not correlated" to each other, for simplicity in the analyses. Slope geometry, ground water table level, soil properties and their statistical mean and standard deviation, together with a lognormal distribution, as reported in [1], are used in the MC probabilistic slope stability analyses. The results in Figure 3 indicate that, after an optimum value of N, the results are not significantly influenced by the number of simulations. Therefore, in this study, 3000 MC samples are found to be sufficient for the purposes of this study.



(a)

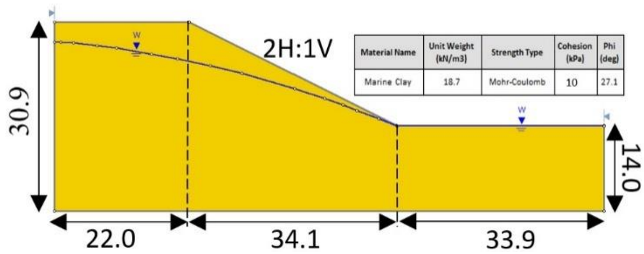


(b)

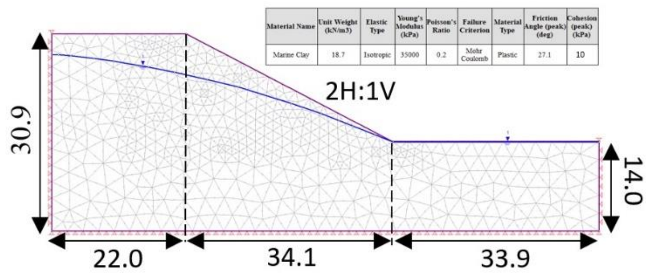
Figure 3. The effect of the number of Monte Carlo samples, N, on the calculated (a) mean FS and (b) PF

For soil shear strength, Mohr-Coulomb model is used with cohesion (c) and internal friction angle (□) as the random variables, each of which can be defined as having normal or lognormal statistical distributions. For soil properties (which cannot have a negative value) lognormal distribution is widely used and has been shown to perform well in the geotechnical literature [6, 13, 15, 16, 17]. The cross-correlation between cohesion and friction angle of soil is reported to be in the range of -0.37 and -0.7 [9, 10, 13]. In this study, -0.5 is used and both “cross-correlated” and “not-correlated” analyses are carried out.

The geometry of the model [1] is shown in Figure 4a and the finite element mesh of the model can be seen in Figure 4b.



(a)



(b)

Figure 4. (a) Cross section of the Lodalen slope [1], (b) finite element mesh of the slope (all dimensions are in meters)

Mohr-Coulomb soil constitutive model is used in this study in finite element analyses. There exists more sophisticated soil models, however, the goal of this study is not capturing the slope displacements with the highest accuracy, but to investigate the effects of variability in soil properties in a probabilistic manner. Therefore, the Mohr-Coulomb model was found to be sufficient for the purposes of this study. Soil parameters used in limit equilibrium and finite element analyses can be seen in Table 1.

Table 1. Soil parameters used in the study for the marine clay [1, 2]

Soil Parameter	Mean Value	Std. Dev.	COV %
Unit Weight (kN/m^3)	18.7	-	-
Young's Modulus (kPa)	35000	5000	14
Poisson's Ratio	0.20	-	-
Friction Angle (degrees)	27.1	1.7	6
Cohesion (kPa)	10	2.2	22

Cohesion and friction angle are defined as random variables having a mean, a standard deviation and lognormal distribution in LEM analyses. In addition to these parameters, Young’s Modulus is also considered as a random variable in FEM analyses. Other soil parameters are considered deterministic, i.e. they have only one value as given in Table 1.

In this study, the effect of changing water level on PF is also studied by carrying out probabilistic slope stability analyses using different ground water tables as shown in Figure 5. In Figure 5, WT1 to WT4 indicates the ground water table that is increased gradually, until the highest ground water table (which is the original ground water table) is reached. In these analyses, cohesion and friction angle are not cross-correlated, and the values in Table 1 are used.

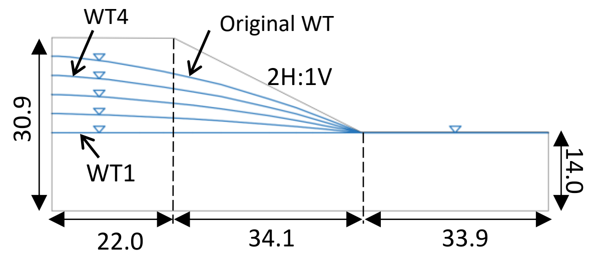


Figure 5. Different water table conditions analyzed in this study

3. RESULTS OF ANALYSES

The results of the analyses are presented in Table 2 and in Figure 6 through Figure 11. Table 2 presents the $FS_{deterministic}$ values, probabilistic FS_{mean} values and PF for different water table conditions given in Figure 5. In the probabilistic slope stability analyses, both GM and OS type analyses are conducted in Roscience Slide software. In these analyses, soil parameters in Table 1 are utilized together with lognormal distribution, and c-□ are not cross-correlated. It is observed that deterministic and probabilistic FS values are very similar to each other. Furthermore, the mean FS values do not differ much from each other when the GM and OS type probabilistic analyses are compared, however, their PF values are slightly different. Table 2 and Figure 6 shows that as the ground water table is increased, the FS value decreases nonlinearly.

Table 2. Deterministic and probabilistic limit equilibrium method analyses results

Water level in Fig. 5	$FS_{deterministic}$	Probabilistic Analysis Type	FS_{mean}	PF (%)
Original WT	0.968	GM	0.971	65.833
		OS	0.968	67.533
WT4	1.139	GM	1.143	3.533
		OS	1.142	4.000
WT3	1.279	GM	1.284	0.033
		OS	1.282	0.033
WT2	1.399	GM	1.399	0.000
		OS	1.398	0.000
WT1	1.488	GM	1.488	0.000
		OS	1.488	0.000
No WT (WT0)	1.530	GM	1.531	0.000
		OS	1.532	0.000

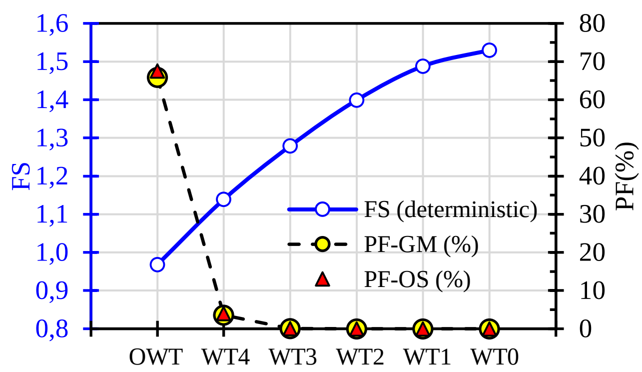


Figure 6. Effect of different water levels on FS and PF (data from Table 2)

FEM results are demonstrated in Figure 7, where the deformed mesh of the model, horizontal displacement values and failure mode of the soil body can be seen. Figure 8 shows the histograms of the computed FS by LEM-OS and FEM.

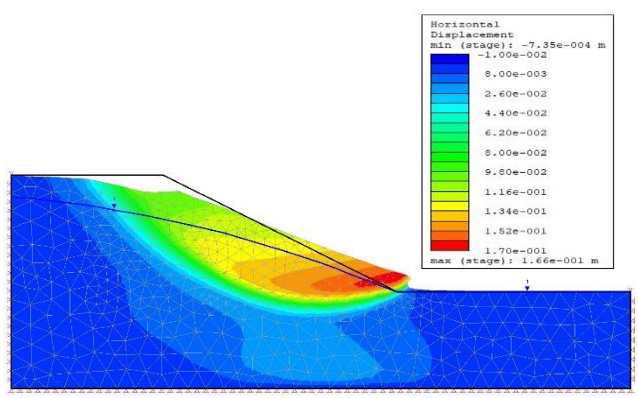


Figure 7. Deformed mesh and horizontal displacements in the slope by FEM results

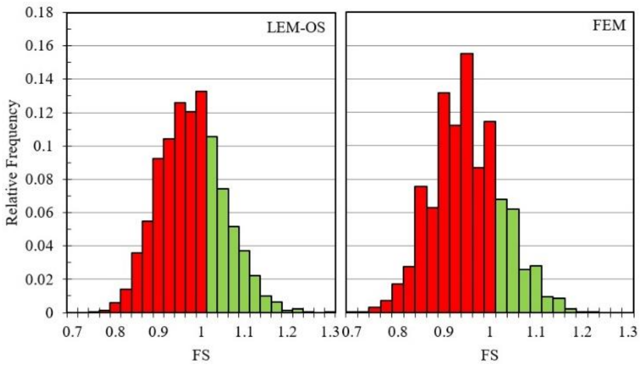


Figure 8. Histograms of the FS for the Lodalen landslide by two different probabilistic methods

Figure 9 compares the failure surfaces obtained by various methods with the original observed failure surface reported by [1]. The probabilistic FEM results are also shown in Figure 9 by the maximum shear strain values (as the shades of red color). It is observed that the FEM gives slightly different failure zone, which extends slightly more backward into the slope, without developing a vertical scarp. Deterministic and probabilistic LEM gives very similar failure surfaces, which seems to agree with the actual slip surface.

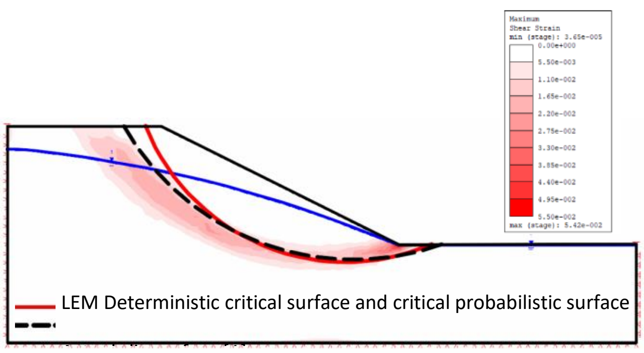


Figure 9. Comparison of failure surfaces obtained by deterministic and probabilistic limit equilibrium and finite element methods with the observed failure surface of [1]

Probabilistic FEM and LEM results are compared in Table 3. Probabilistic FEM gives very slightly lower FS_{mean} value as compared to probabilistic LEM.

Table 3. Comparison of probabilistic finite element and limit equilibrium method results

	FEM	LEM	
		GM	OS
FS_{mean}	0.94	0.971	0.968
Std. Dev. of FS	0.077	0.0771	0.0756
PF (%)	77.85	65.83	67.53

The effects of different COV levels and the effects of whether or not using cross-correlation of c-φ on the PF can be seen in Figure 10. The analyses are conducted for lognormally distributed c and φ, and for the original water table condition in Figure 5, which had a $FS_{deterministic}$ value of 0.968 (i.e. "deterministically not safe" slope). It can be seen in Figure 10 that, as the COV level increases from 5% to 40%, the PF decreases from about 73%-81% to 61%-66%. When the c-φ are cross-correlated (by cross-correlation coefficient of -0.5), the PF value is larger as compared to the case where c-φ are not cross-correlated. Furthermore, GM and OS analyses are observed to give slightly different PF results, especially for the low COV% levels.

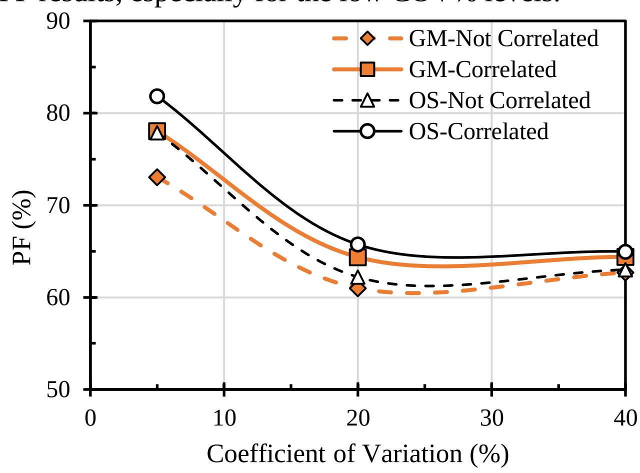


Figure 10. Effect of COV level and the effects of whether or not using cross-correlation of c-φ on the probability of failure of slope ($FS_{deterministic}$ = 0.968)

Figure 11 shows the effects of different COV levels and the effects of whether or not using cross-correlation of c-φ on the PF, for water table at WT3 level in Figure 5, and soil properties from Table 1 and lognormally distributed. $FS_{deterministic}$ value is 1.279 (i.e. "deterministically safe" slope). It can be seen that as the COV level increases from 5% to 40%, the PF increases from 0% to 40%. When the c-φ are cross-correlated, the PF value is less, as compared to the case where c-φ are not cross-correlated. Furthermore, GM and OS analyses are observed to give almost identical PF results.

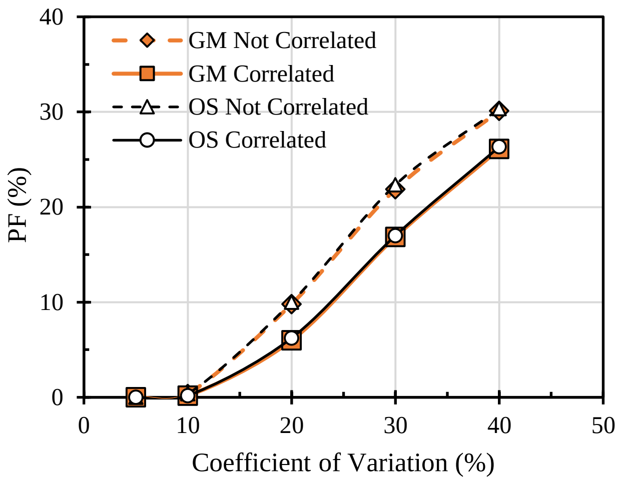


Figure 11. Effects of COV levels and the effects of whether or not using cross-correlation of c-□ on the PF ($FS_{deterministic}$ = 1.279)

It can be observed from Figure 11 that, eventhough a slope may be considered to be “safe enough” with a $FS_{deterministic}$ value close to 1.3, the probability of failure could be quite significant such as PF=30%, if the uncertainty is large, such as COV=40%.

4. DISCUSSION AND CONCLUSIONS

This study emphasizes the limitations of the deterministic factor of safety approach in evaluating the safety level of slopes. It is demonstrated that the uncertainty/variability in material properties can lead to unsafe or uneconomical slope design, depending on the level of COV. Therefore, a thorough site investigation and identification of soil properties are of significant importance in the correct assessment of the safety level of slopes.

As reported by [9], among others, depending on whether the slope is “deterministically safe” ($FS_{deterministic}$ >1.00) or “deterministically unsafe” ($FS_{deterministic}$<1.00), the effects of COV level on PF of the slope will be different (Figure 12). Similar behaviour is observed in this study, for the long term, drained analyses of a real slope failure in a marine clay (Figures 10 and 11).

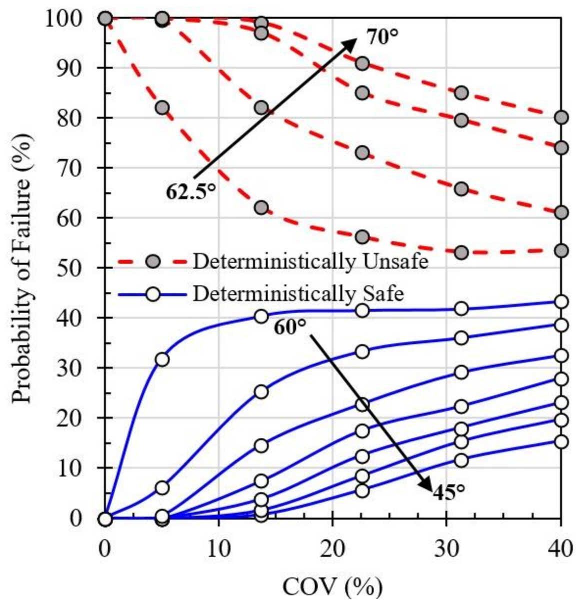


Figure 12. Effect of COV level on the PF of slope for deterministically safe and unsafe slopes represented by different slope angles [9]

The critical failure surface in a slope can be different in deterministic LEM, probabilistic LEM, and probabilistic FEM analyses. It would be best to identify the most critical failure surface by making use of all available methods, including probabilistic approaches.
The cross-correlation between shear strength parameters can have a significant influence on the probabilistic safety assessment of slopes. For example, when the correlation between c and □ are not taken into account, the probability of failure may be underestimated for a critical slope.
The results of this study highlight the importance of probabilistic slope stability concepts with the aim of better geotechnical risk management and communication.

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