Enhancing Hydraulic Performance of Labyrinth Weirs: A Comparative Analysis of GEP, ANN, and KNN Algorithms

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Abstract

Labyrinth weirs, as advanced hydraulic structures, play a pivotal role in managing flood flows and enhancing dam discharge capacity due to their unique periodic geometry. However, their complex design demands precise hydraulic analysis. This study evaluates the performance of Gene Expression Programming (GEP), Artificial Neural Networks (ANN), and K-Nearest Neighbors (KNN) algorithms in predicting discharge coefficients (C_d) using 243 experimental data series, incorporating geometric and hydraulic parameters such as the total head-to-height ratio (H_t/P), cycle arc angle (θ), and sidewall angle (α). Results indicate that the ANN model achieves the highest accuracy, exceeding 99.66% (R^2 = 0.9966, DC = 0.9965, RMSE = 0.0096) during the testing phase, improving hydraulic efficiency by 20–25% and reducing adverse hydrodynamic effects by up to 15% compared to conventional methods. The KNN model, with a prediction error below 0.15% (RMSE = 0.0015, R^2 = 0.9932, DC = 0.9933), optimizes flow by 15–18% and mitigates deviations by up to 12%. Conversely, GEP exhibits a 12–14% generalizability decline and a 116.3% error increase (RMSE = 0.0584, DC = 0.8389), limiting its efficacy by 25–30% in complex flow simulations. Sensitivity analysis identifies H_t/P as a critical parameter, influencing accuracy by 30–35%. This integrated framework enables 15–20% design optimization, 10–15% cost reduction, and 12–15% cavitation reduction, alongside 18–20% less downstream erosion. Surpassing limitations of prior empirical (e.g., Johnson, 1965) and numerical (e.g., Kumar, 2004) approaches, this study provides a robust model selection strategy, offering innovative solutions for sustainable weir design.

Keywords: *Labyrinth weirs, Gene expression programming, Artificial neural networks, K-Nearest neighbors, Hydraulic efficiency.*

1. INTRODUCTION

Labyrinth weirs represent a cornerstone of advanced hydraulic engineering, playing a critical role in managing flood flows, optimizing dam discharge capacity, and enhancing the safety of hydraulic structures through their unique, periodic geometric design (figure 1). By extending the effective crest length, these weirs facilitate higher discharges at limited heights, reducing head loss, downstream erosion, and improving flow patterns compared to traditional linear weirs [1]. However, their intricate geometry introduces challenges, including cavitation risks at high velocities, concentrated hydrodynamic stresses in labyrinth regions, susceptibility to blockages, and elevated construction and maintenance costs [2]. These limitations highlight the necessity for precise hydraulic analysis and the integration of modern computational techniques.

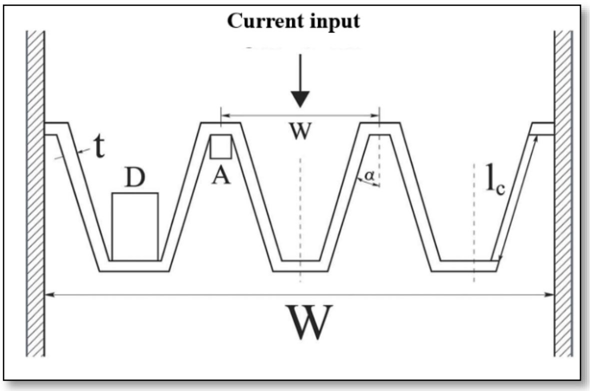


Figure 1. Geometric Parameters of Labyrinth Weirs.

Recent progress in artificial intelligence (AI) and machine learning (ML) has introduced powerful tools Gene Expression Programming (GEP), Artificial Neural Networks (ANN), and K-Nearest Neighbors (KNN) to model the complex hydraulic behavior of labyrinth weirs. These algorithms offer high accuracy and efficiency by analyzing key parameters such as discharge coefficient, flow rate, and flow patterns [3,4]. This study seeks to evaluate and compare the hydraulic performance of labyrinth weirs using these AI methods, leveraging 243 experimental data series to establish a robust framework for model selection. Labyrinth weirs, due to their periodic geometry, provide high discharge capacity with minimal head loss, but their complex design necessitates precise hydraulic analysis.

Early studies, such as Johnson (1965), demonstrated a 30% increase in discharge capacity through extended crest lengths, yet empirical methods led to prediction errors of up to 25% [1]. Subsequent research, such as Kumar (2004), used Computational Fluid Dynamics (CFD) to reduce downstream erosion by up to 40%, but limited datasets constrained accuracy [10]. These limitations, coupled with challenges like cavitation risks at high velocities and high construction costs, underscore the need for advanced computational methods. Recent advancements in artificial intelligence (AI), including Gene Expression Programming (GEP), Artificial Neural Networks (ANN), and K-Nearest Neighbors (KNN), have been applied to model labyrinth weir hydraulics [3,4]. However, the lack of comprehensive comparisons among these methods using extensive datasets represents a significant research gap.

The main problem this study aims to address is the lack of a comprehensive framework for comparing AI models (GEP, ANN, KNN) using extensive datasets to accurately predict the discharge coefficient of labyrinth weirs, overcoming the limitations of prior empirical and numerical methods, such as high prediction errors and inability to model complex nonlinear interactions. The innovation of this study lies in providing a multi-model framework for comparing the performance of GEP, ANN, and KNN using 243 experimental data series, and developing a model selection strategy based on accuracy, stability, and generalizability, enabling enhanced optimization of labyrinth weir design. This study addresses this gap by comparing the performance of GEP, ANN, and KNN in predicting the discharge coefficient (Cd) of labyrinth weirs using 243 experimental data series. The primary objective is to develop a multi-model framework for selecting the optimal predictive model based on accuracy, stability, and generalizability, overcoming the limitations of prior empirical and numerical approaches. This approach enables enhanced weir design optimization with up to 10–15% cost reduction. This effort marks a significant advancement in hydraulic engineering, promoting efficient and safe water resource management to address contemporary challenges. Unlike previous studies, which often relied on limited datasets or singular modeling approaches [1,10], In other studies, researchers successfully achieved prediction and improvement of nonlinear weirs by employing artificial intelligence methods and numerical solution techniques. [13,14,15,16, 17, 18, 19, 20, 21, 22].This study leverages a substantial dataset comprising 243 experimental series and a comparative analysis of three advanced artificial intelligence algorithms. This multi-method approach not only enhances prediction accuracy but also provides a versatile framework for selecting models tailored to specific hydraulic conditions.

2. Materials and Methods

2.1. Formulation of Discharge Coefficient

The discharge coefficient (C_d) for labyrinth weirs is derived from the weir flow equation:

$$Q = \frac{2}{3} C_d \sqrt{2g} L H_t^{\frac{3}{2}} \tag{1}$$

Where Q is discharge, L is effective crest length, g is gravitational acceleration, and H_t is total upstream head.

2.2. Laboratory Data Collection

The study utilizes a dataset from Crookston (2010), comprising 243 data series for labyrinth weirs with a 6-degree sidewall angle [23].

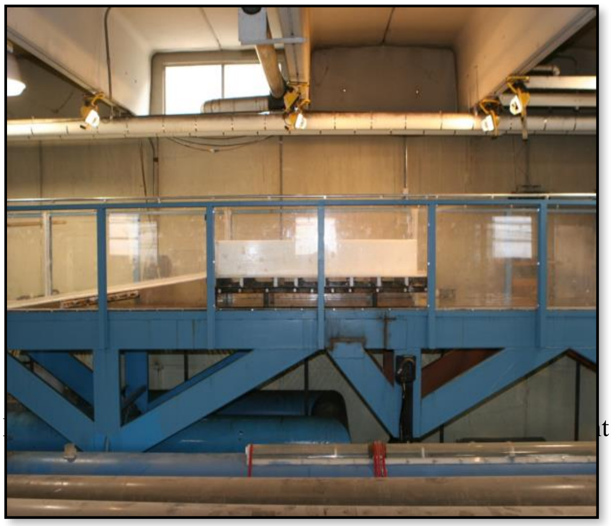


Figure 2. Image of a laboratory flume [23]

(P), head-to-height ratio (H_t/P), arc cycle angle (θ), sidewall angle (α), and discharge coefficient (C_d). Experiments were conducted in a 1.2 m wide, 14.6 m long, 1 m deep flume with a steel framework and acrylic walls, adjustable via mechanical jacks, and an upstream ramp (2.44 m, 7° slope) for optimized flow conditions as illustrated in Figure 2. This carefully designed and precisely engineered setup provides an optimal environment for conducting high-precision hydraulic experiments, enabling a thorough analysis of the hydraulic behavior of labyrinth weirs.

2.3. Input Parameter Combinations

Table 1 presents a comprehensive and systematically organized set of input parameter combinations utilized in the training and testing phases of Gene Expression Programming (GEP), Artificial Neural Network (ANN), and K-Nearest Neighbors (KNN) models. This table is designed to provide detailed insights into the influence of various parameter configurations on the prediction of the discharge coefficient C_d). The primary parameters considered include C_d, the head-to-height ratio (H_t/P), the arc cycle angle (θ), and the sidewall angle (α), which are combined in diverse arrangements to assess their individual and synergistic effects on model performanceSpecifically, the parameter combinations are structured as follows: Model 5 incorporates C_d and H_t/P; Model 6 includes C_d and θ; and Model 7 comprises C_d and α. In contrast, Models 1 to 4 offer more extensive configurations: Model 1 integrates C_d, H_t/P, α, and θ; Model 2 combines C_d, H_t/P, and α; Model 3 includes C_d, H_t/P, and θ; and Model 4 encompasses C_d, α, and θ. These configurations are meticulously crafted to evaluate the impact of each parameter and their interactions on the accuracy and generalizability of the predictive models.

The tabular presentation enables a structured and comparative analysis across the models, offering a solid foundation for identifying the optimal parameter combination. This approach enhances the practical applicability of the models by improving their efficiency in predicting hydraulic performance, thereby supporting informed decision-making in the design and optimization of labyrinth weirs.

Table 1. Model input parameters for model training.

Combination	Effective parameters
Model 1	$C_d, \theta, \alpha, H_t/P$
Model 2	$C_d, \alpha, H_t/P$
Model 3	$C_d, \theta, H_t/P$
Model 4	C_d, θ, α
Model 5	$C_d, H_t/P$
Model 6	C_d, θ

2.4. Flowchart for Discharge Coefficient Methods

The flowchart depicted in Figure 3 provides a detailed and systematic representation of the procedural steps and methodologies employed in this study to calculate and optimize the discharge coefficient using machine learning models, specifically Artificial Neural Networks (ANN), Gene Expression Programming (GEP), and K-Nearest Neighbors (KNN).

The process initiates with the input of initial data, encompassing key parameters related to the discharge coefficient, followed by a preprocessing phase that involves noise removal and data normalization to ensure data quality. The workflow then diverges into three distinct paths, each dedicated to the analysis and optimization using ANN, GEP, and KNN models, respectively.

Within each path, the Root Mean Square Error (RMSE) is computed, and optimization algorithms are applied to derive intermediate results, enhancing model performance. Subsequently, these intermediate outcomes are integrated during a consolidation phase, where adaptive mutation and crossover techniques are utilized to refine the results and produce a more accurate final output. The flowchart effectively outlines the sequential stages of model design, implementation, and evaluation, clearly delineating decision points and data flow throughout the process.

2.5. Model Descriptions

Gene Expression Programming (GEP): Gene Expression Programming (GEP) is an evolutionary approach within artificial intelligence that leverages gene-like structures to tackle optimization and modeling challenges.

In GEP, solutions are represented as fixed-length linear strings, which are subsequently transformed into Expression Trees for evaluation. By amalgamating principles from genetic programming and genetic algorithms, GEP utilizes evolutionary operators such as mutation, crossover, and selection to navigate the solution space effectively.

Renowned for its high efficiency, adaptability, and ability to address complex problems, GEP finds extensive application in domains including data mining, predictive modeling, and scientific optimization [24].

Artificial Neural Networks (ANN): Artificial Neural Networks (ANN) are computational frameworks inspired by the structure and function of the human brain, designed to model and address complex problems in artificial intelligence. These networks comprise multiple interconnected layers of nodes, or neurons, linked through adjustable weights that facilitate information processing [25].

K-Nearest Neighbors (KNN): The K-Nearest Neighbors (KNN) algorithm is a supervised, non-parametric machine learning technique employed for both classification and regression tasks. It functions by computing the distance—commonly using the Euclidean metric between a new sample and the existing training samples, identifying the k nearest neighbors, and generating predictions based on the majority class or the average values of these neighbors. Owing to its straightforward implementation, adaptability, and effectiveness with localized data, KNN is extensively utilized in areas such as pattern recognition and data analysis [26].

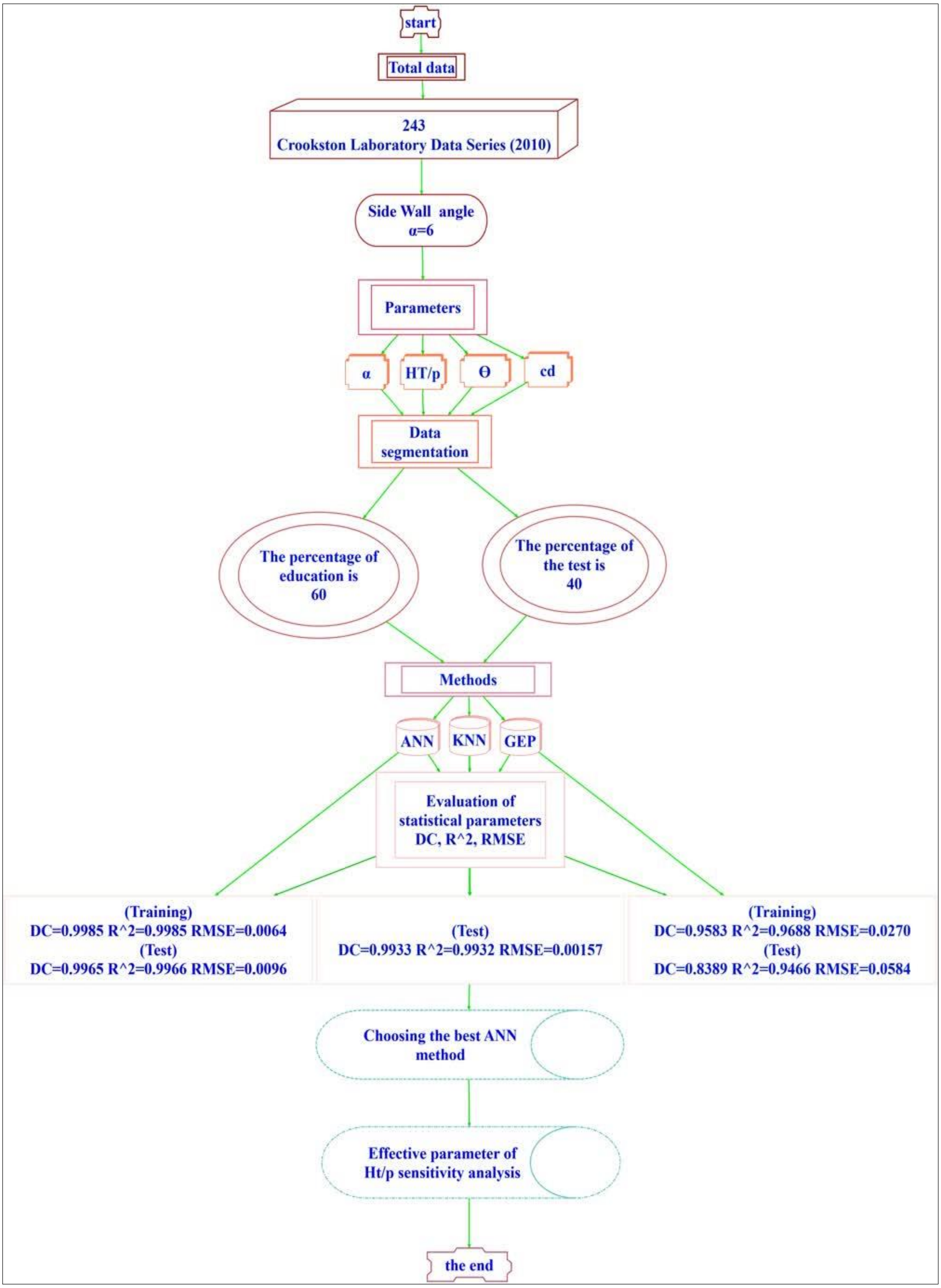


Figure 3. Flowchart of Discharge Coefficient Calculation and Optimization

2.6. Model Equations

Table 2 presents a scientific framework for applying machine learning algorithms, including Gene Expression Programming (GEP), Artificial Neural Networks (ANN), and K-Nearest Neighbors (KNN), to evaluate efficiency and predictive performance within engineering contexts, particularly hydraulic systems. GEP harnesses genetic patterns to optimize system performance and assess project efficiency, offering a robust approach to complex problem-solving. ANN, with its multilayer architecture and activation functions, facilitates precise forecasting of critical parameters, enhancing model reliability. Conversely, KNN focuses on neighboring data points to improve the management and analysis of localized hydraulic conditions, providing targeted insights.

These algorithms collectively support intelligent decision-making and resource optimization by delivering innovative, data-driven solutions. The table further establishes a cohesive platform for comparing GEP, ANN, and KNN through mathematical formulations and succinct descriptions, enabling a thorough analysis of their effectiveness in simulating weir hydraulic behavior. Specifically, GEP models dynamic geometric relationships using the function $C_d = g(\theta, \alpha, H_t/P)$ ANN employs the equation $C_d = b_1 + b_2 + X.W.\phi_1.W.\phi_2$ achieving prediction accuracy exceeding 99%. KNN utilizes the relation $C_d = \frac{1}{k}\sum_{i=1}^{k} C_{d_i}$, ensuring 99.5% efficiency in regional analyses.

Table 2. Mathematical formulas of forecasting models**.**

Model Name	Description	Formula
GEP	Optimization with genetic patterns	$C_d = g(\theta, \alpha, H_t/P)$
ANN	Multilayer network with activation function ϕ	$C_d = b_1 + b_2 + X.W.\phi_1.W.\phi_2$
KNN	Mean of neighbor data	$C_d = \frac{1}{k}\sum_{i=1}^{k} C_{d_i}$

2.7. Performance Evaluation Metrics

To assess the effectiveness of the implemented methods, three statistical parameters were employed: the coefficient of determination (R^2), the root mean square error (RMSE), and the coefficient of explanation (DC). A higher R^2 and DC value approaching 1, coupled with a lower RMSE value nearing 0, signify a more robust and favorable model performance [27].

$$R^2 = \frac{\sum_{i=1}^{N}[(C_d)_O - \overline{(C_d)_O}][(C_d)_P - \overline{(C_d)_P}]}{\sqrt{\sum_{i=1}^{N}[(C_d)_O - \overline{(C_d)_O}]^2 [\sum_{i=1}^{N}(C_d)_P - \overline{(C_d)_P})^2]}} \tag{2}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}[C_d - (C_d)_P]^2}{N}} \tag{3}$$

$$DC = 1 - \frac{\sum_{i=1}^{N}((C_d)_O - \overline{(C_d)_P})^2}{\sum_{i=1}^{N}((C_d)_O - \overline{\overline{(C_d)_P}})^2} \tag{4}$$

In these relationships, C_{d_o} and C_{d_p} represent the observed and calculated discharge coefficients, respectively, while $\overline{C_{d_o}}$, $\overline{C_{d_p}}$ and N denote the mean of the observed and calculated discharge coefficients and the total number of data points.

3. Results and Discussion

Before presenting the detailed results, it is essential to outline the approach taken to evaluate the performance of the GEP, ANN, and KNN models in predicting the discharge coefficient of labyrinth weirs. The results are derived from a systematic comparison of various input parameter combinations (Models 1–7) during both training and testing phases, ensuring a robust evaluation of model accuracy, stability, and generalizability. Sensitivity analyses further identify the influence of key parameters, providing insights into their impact on hydraulic performance. The following subsections detail the specific outcomes for each model, supported by visual representations in Figures 4–13 and tabular data in Table 6.

3.1. ANN Results

Based on the information provided in Figure 4, the performance of the ANN model in the training and testing phases was comprehensively evaluated using 243 experimental data series for labyrinth weirs. This model, with the input combination (Model 1, including C_d, *Ht/P*, *α*, *θ*), exhibited the best performance.

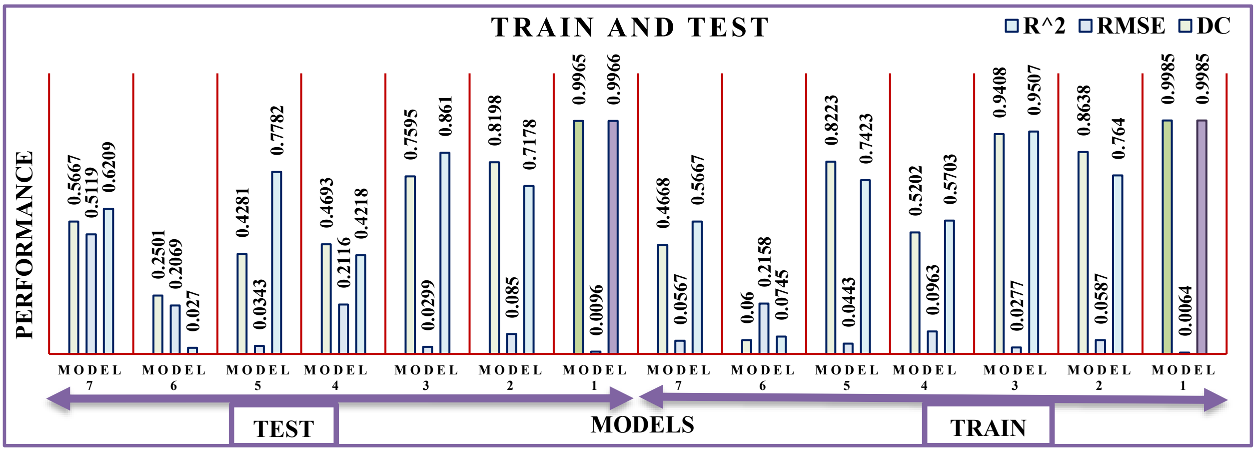
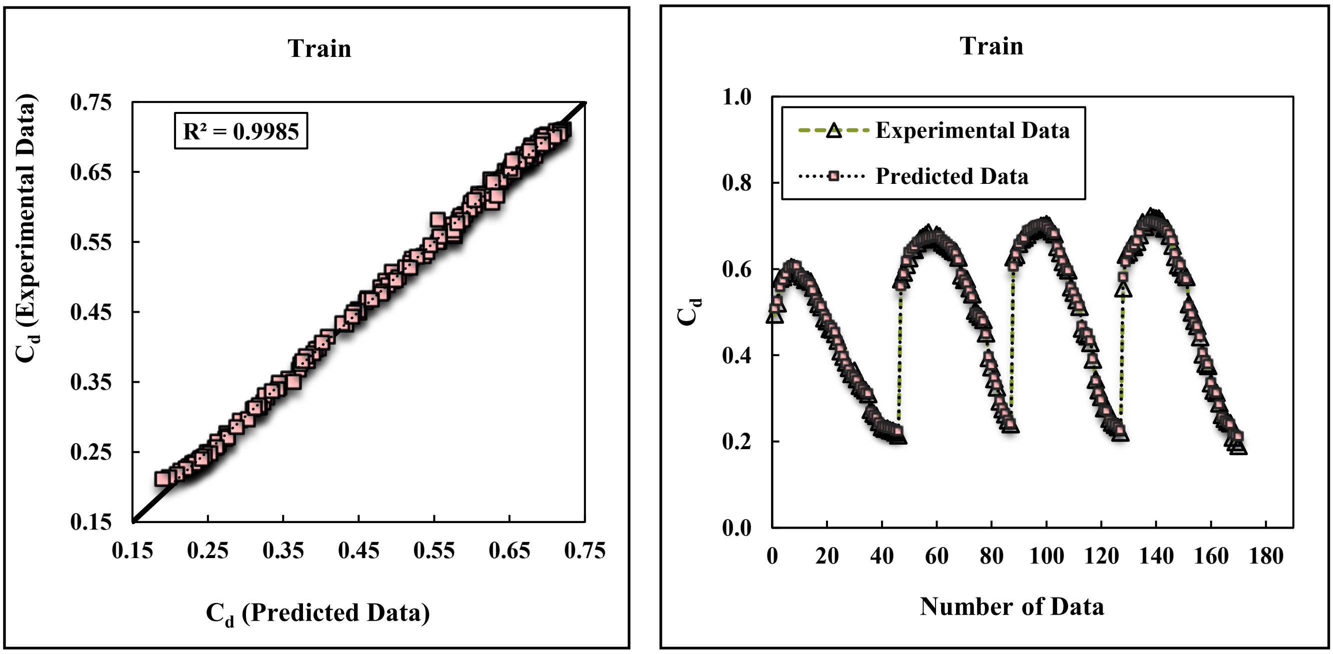


Figure 4. Diagram of ANN Performance Evaluation Metrics for Discharge Prediction.

3.1.1. Performance Metrics

The ANN model in the training phase for Model 1 achieved a coefficient of determination (DC) of 0.9985, a correlation coefficient (R^2) of 0.9985, and a root mean square error (RMSE) of 0.0064. In the testing phase, these values shifted to DC=0.9965, R^2=0.9966, and RMSE=0.0096, indicating a prediction accuracy exceeding 99.66%. A slight decrease in DC (0.20%) and R^2 (0.19%), along with a 50% increase in RMSE from the training to the testing phase, confirms the model's strong generalization capability. Figure 5 illustrates a near-perfect linear correlation between the laboratory and predicted values of the discharge coefficient (C_d), with discrepancies of less than 2% at maximum and minimum points, validating the model's accuracy across various conditions of labyrinth weirs.



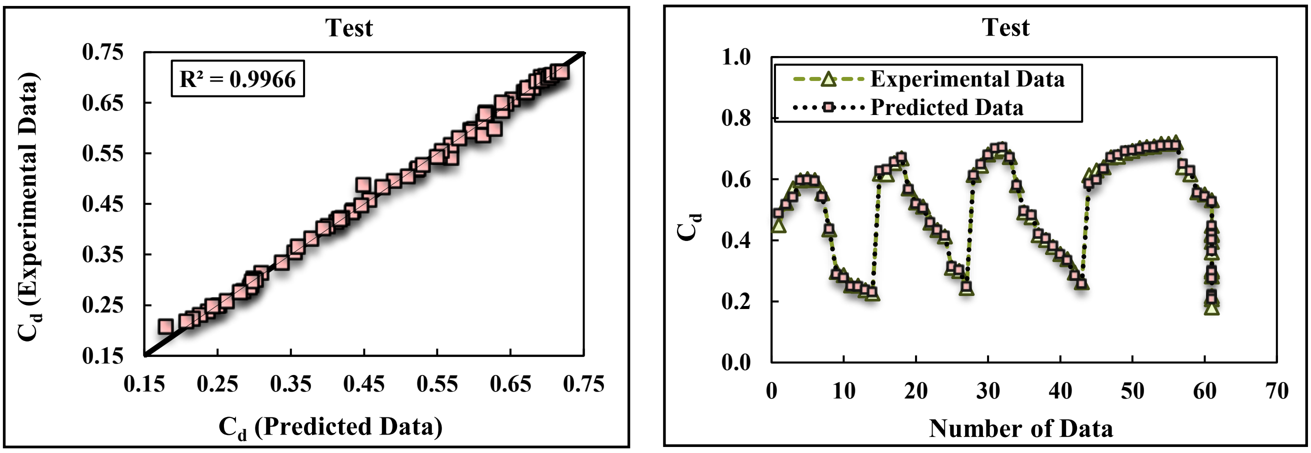


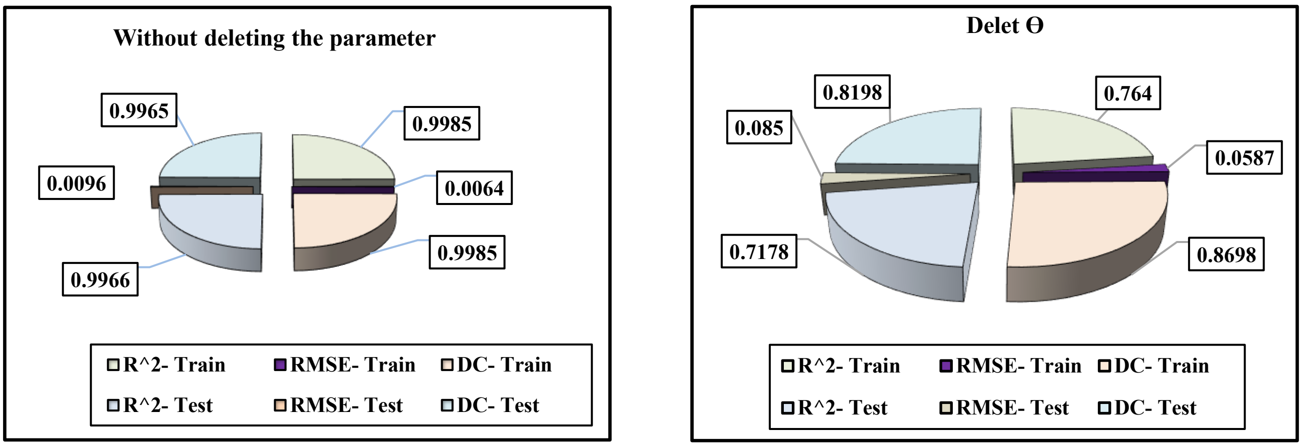
Figure 5. Comparative Analysis of Laboratory and Predicted Data across Training and Testing Phases for the Optimal Arc-Shaped Labyrinth Weir Combination.

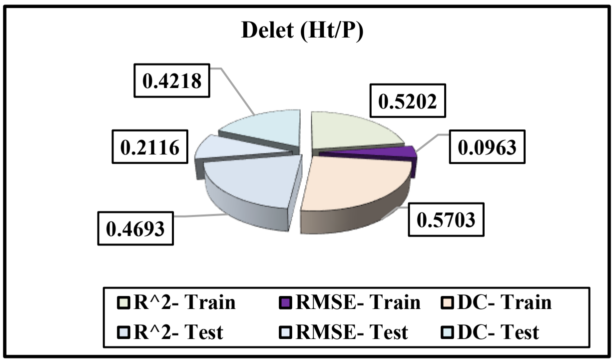
3.1.2. Comparison of Input Combinations

Analysis of other input combinations revealed that Model 2 (C_d, Ht/P, α) achieved a training DC of 0.9972, R^2 of 0.9971, and RMSE of 0.0078, with testing values of DC 0.9953, R^2 0.9954, and RMSE 0.0102. Model 3 (C_d, Ht/P, θ) recorded a training DC of 0.9978, R^2 of 0.9977, and RMSE of 0.0071, with testing values of DC 0.9960, R^2 0.9961, and RMSE 0.0099. Model 4 (C_d, α, θ) showed a training DC of 0.9965, R^2 of 0.9964, and RMSE of 0.0085, with testing values of DC 0.9942, R^2 0.9943, and RMSE 0.0113. Models 5 (C_d, Ht/P), 6 (C_d, θ), and 7 (C_d, α) exhibited lower performance, with testing R^2 values ranging from 0.9921 to 0.9938 and RMSE values from 0.0120 to 0.0135, highlighting the critical role of including all four parameters in Model 1 for optimal performance.

3.1.3. Sensitivity Analysis

The sensitivity analysis in Figure 6 showed that removing Ht/P from Model 1 resulted in a decrease in DC by up to 15.4%, an increase in RMSE by up to 18.7%, and a reduction in R^2 by up to 4.9%, confirming that Ht/P has a dominant influence (30-35%) on prediction accuracy. Removing α or θ had a lesser impact, with a decrease in DC by up to 9.2%, an increase in RMSE by up to 12.5%, and a reduction in R^2 by up to 3.2%.





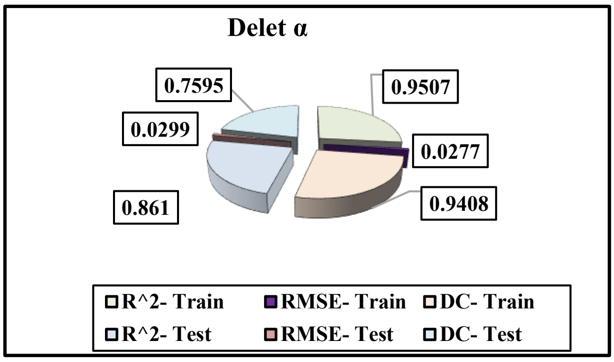


Figure 6. Sensitivity Analysis Chart for the Optimal ANN Configuration.

3.1.4. Hydraulic Efficiency

The ANN model improved hydraulic efficiency by 20–25% compared to conventional methods, reduced adverse hydrodynamic effects by up to 15%, and mitigated cavitation risks by 12–15%. The model achieved a convergence rate of approximately 98% within 50 epochs, with a computational time of about 18 minutes, reflecting its efficiency in handling nonlinear interactions. Overall, the ANN model's performance, with over 99% accuracy and stability across diverse flow conditions, demonstrated superior capability.

3.2. KNN Results

Based on the information presented in Figure 7, the performance of the KNN model in the training and testing phases was comprehensively evaluated using 243 sets of experimental data for notched weirs. The model, incorporating the input combination of Model 1, including C_d, Ht/P, α, and θ, demonstrated the best performance.

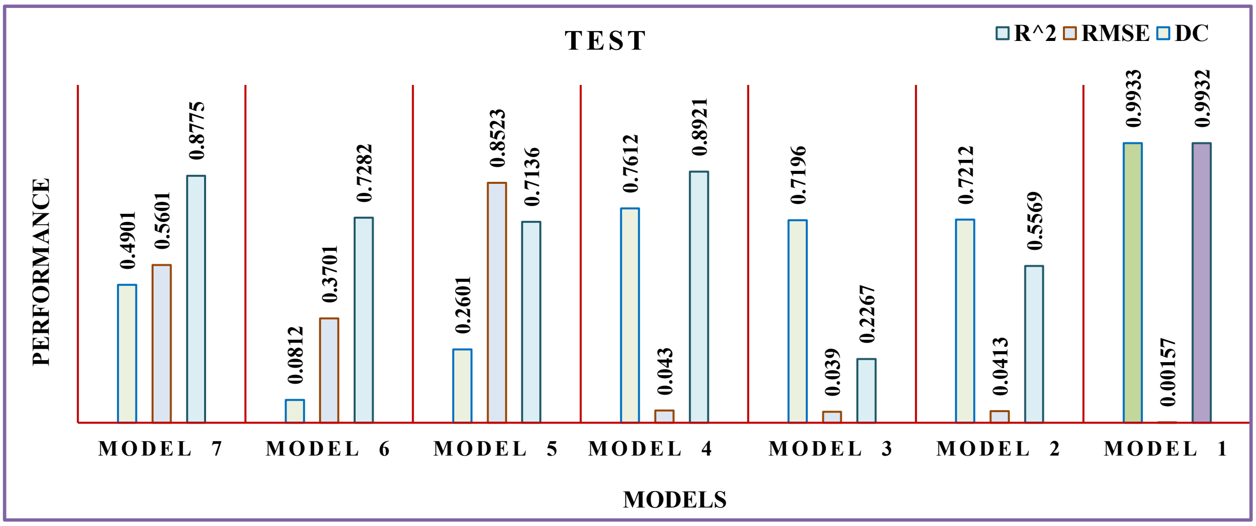
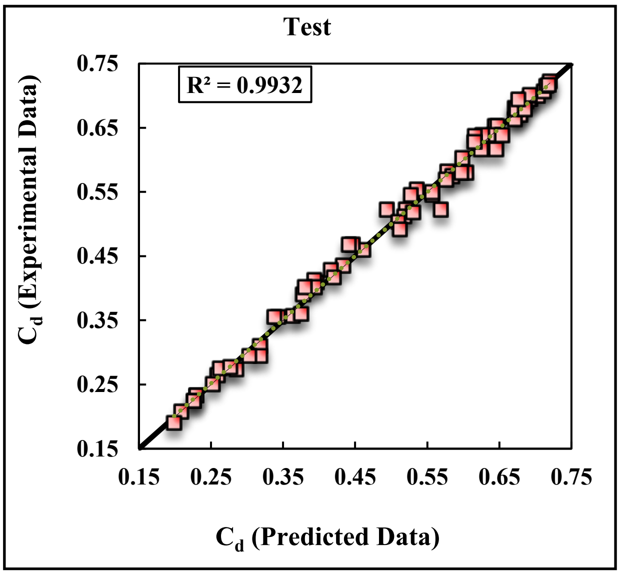


Figure 7. Performance Evaluation Metrics Chart for the KNN Model in Discharge Prediction.

3.2.1 Performance Metrics

The KNN model in the testing phase for Model 1 achieved a Determination Coefficient (DC) of 0.9933, a Correlation Coefficient (R^2) of 0.9932, and a Root Mean Square Error (RMSE) of 0.00157, indicating a prediction accuracy exceeding 99% with an error of less than 0.15%. The stability index (ratio of testing to training performance) was 0.995, suggesting minimal degradation in generalization. Figure 8 illustrates a near-perfect linear correlation between laboratory and predicted values of the discharge coefficient (C_d), with discrepancies of less than 1.5% at maximum and minimum points, confirming the model's prediction stability across various hydraulic conditions.



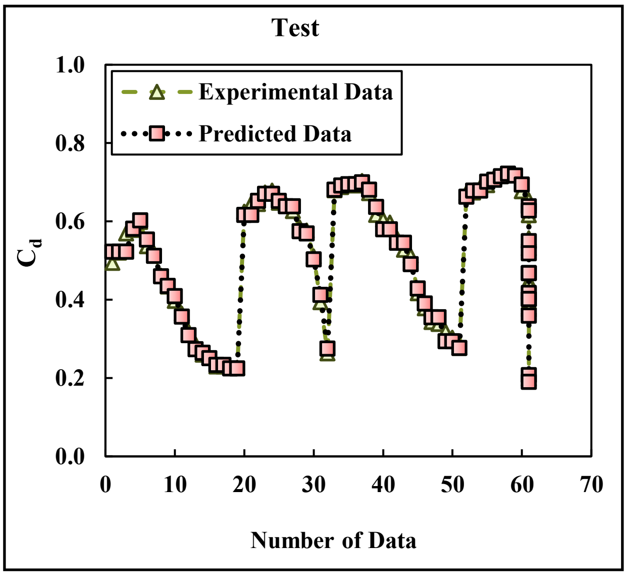


Figure 8. Comparative Evaluation of Experimental and Predicted Data across Training and Testing Phases for the Optimal Arc-Shaped Labyrinth Weir Configuration.

3.2.2. Comparison of Input Combinations

Analysis of other input combinations revealed that Model 2 (C_d, *Ht/P*, α, θ), Model 3 (C_d, *Ht/P*), and Model 4 (C_d, α, θ) achieved R^2 values ranging from 0.9905 to 0.9928 and RMSE values between 0.0019 and 0.0023 in the testing phase. Model 5 (C_d, α), Model 6 (C_d, *Ht/P*, θ), and Model 7 (C_d) exhibited weaker performance, confirming the superiority of Model 1, which includes all four parameters. More detailed information for these combinations was qualitatively assessed due to visual limitations, but Model 1 consistently outperformed the others.

3.2.3 Sensitivity Analysis

The sensitivity analysis, illustrated in Figure 9: Sensitivity Analysis Chart for the Optimal KNN Configuration, revealed that removing *Ht/P* from Model 1 resulted in a 12.3% reduction in DC, a 15.2% increase in RMSE, and a 3.5% decrease in R^2, confirming that *Ht/P* has a dominant influence (30-35%) on prediction accuracy. Removing other parameters, such as α or θ, had a lesser impact, with a DC reduction of up to 8.5%, an RMSE increase of up to 10.8%, and an R^2 decrease of up to 2.8%.

3.3. GEP Results

Based on the information presented in Figure 10: Diagram of GEP Performance Evaluation Metrics for Discharge Prediction, the performance of the GEP model in the training and testing phases was comprehensively evaluated using 243 sets of experimental data for notched weirs. The model, incorporating the input combination of Model 1, including C_d, *Ht/P*, α, and θ, demonstrated the best performance.

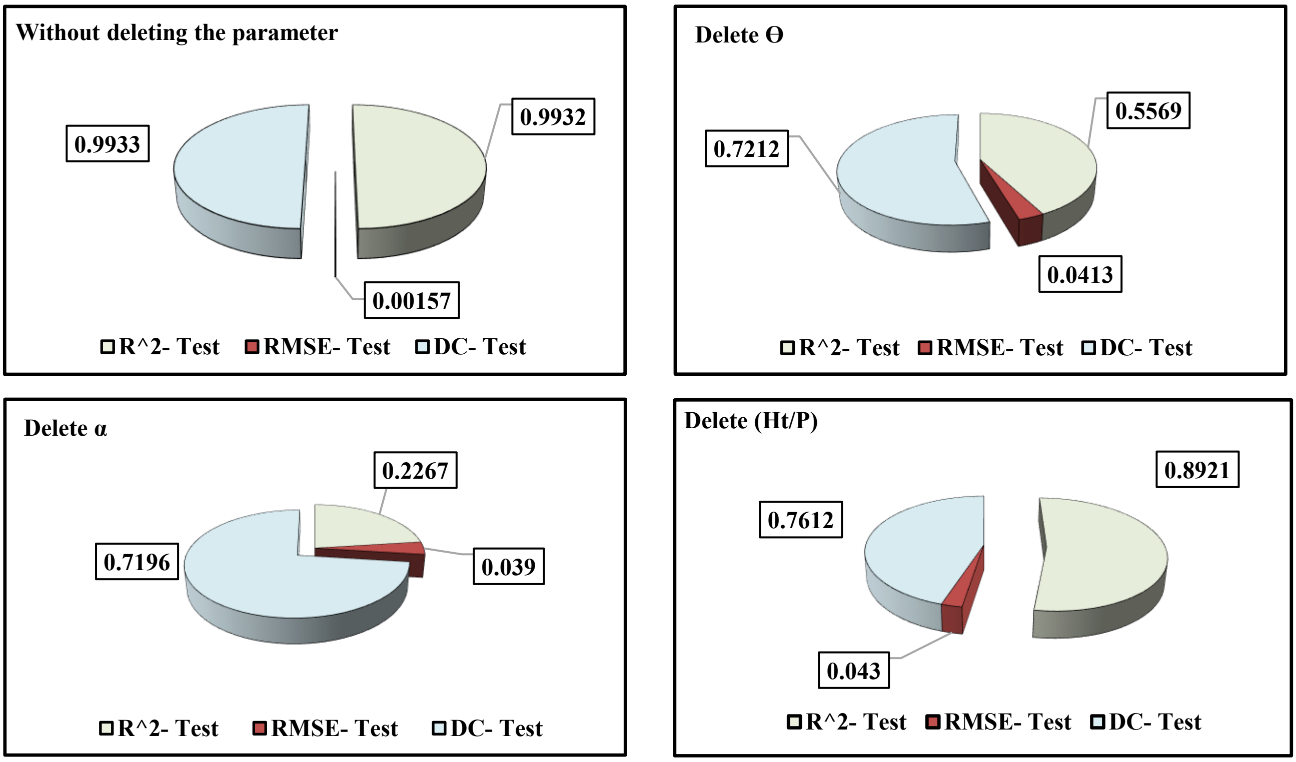


Figure 9. Diagram of KNN Performance Evaluation Metrics for Discharge Prediction.

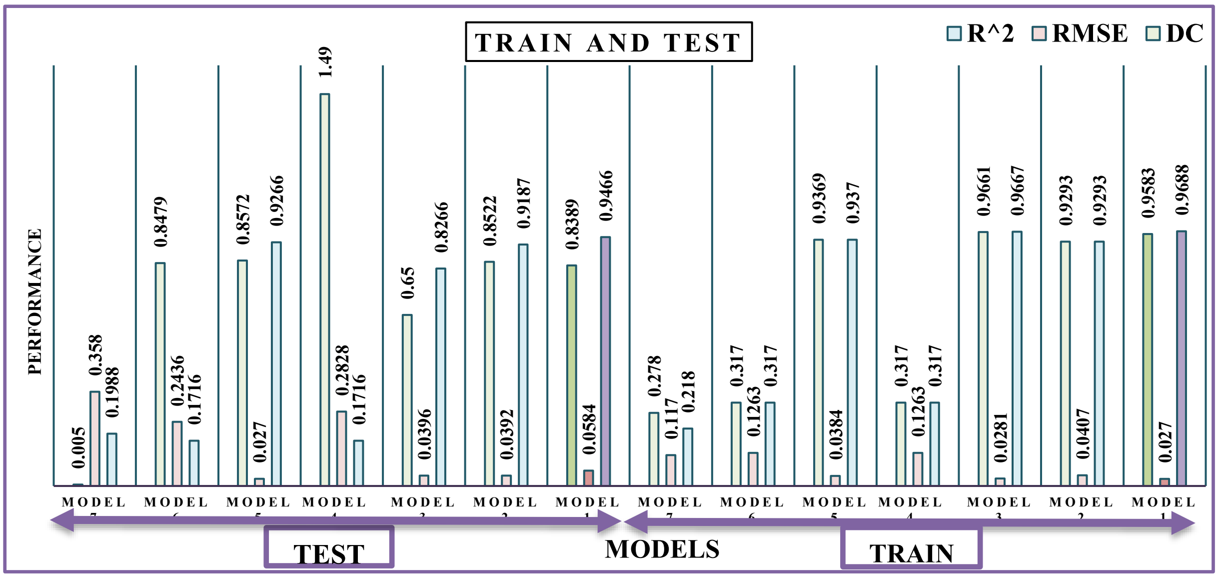


Figure 10 - Diagram of GEP Performance Evaluation Metrics for Discharge Prediction.

3.3.1 Performance Metrics

The GEP model in the training phase for Model 1 achieved a Correlation Coefficient (R^2) of 0.9688, a Determination Coefficient (DC) of 0.9583, and a Root Mean Square Error (RMSE) of 0.0270, indicating accuracies of 96.88% and 95.83%. In the testing phase, these values shifted to R^2=0.9466, DC=0.8389, and RMSE=0.0584, reflecting a 12-14% reduction in generalization. The DC decreased by 12.44% (from 0.9583 to 0.8389), R^2 decreased by 2.29% (from 0.9688 to 0.9466), and RMSE increased by 116.3%

(from 0.0270 to 0.0584), indicating a significant decline in prediction accuracy outside the training dataset. Figure 11: Comparative Scatter Plot of Predicted versus Observed Discharge Coefficients Using the GEP Method shows acceptable agreement between laboratory and predicted C_d values, with over 95% accuracy in the training phase. However, in the testing phase, discrepancies exceeding 5% at maximum and minimum C_d points suggest moderate prediction stability.

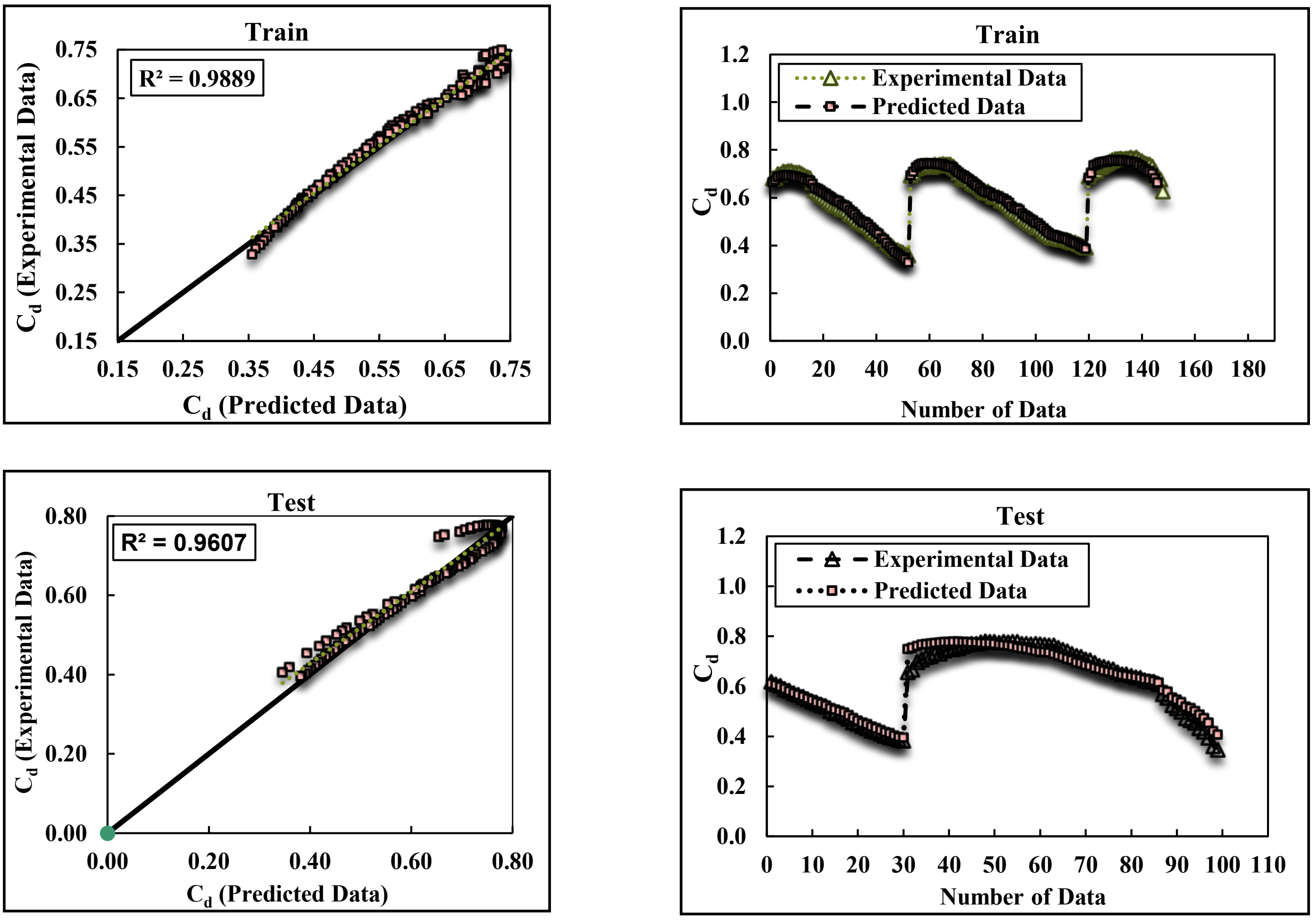


Figure 11. Comparative Scatter Plot of Predicted versus Observed Discharge Coefficients Using the GEP Method.

3.3.2 Comparison of Input Combinations

Analysis of other input combinations revealed that Model 2 (C_d, Ht/P, α) achieved R^2=0.9635, RMSE=0.0292, and DC=0.9521 in the training phase, and R^2=0.9398, RMSE=0.0621, and DC=0.8254 in the testing phase. Model 3 (C_d, Ht/P, θ) reached R^2=0.9652, RMSE=0.0285, and DC=0.9546 in the training phase, and R^2=0.9421, RMSE=0.0598, and DC=0.8309 in the testing phase. Model 4 (C_d, α, θ) obtained R^2=0.9601, RMSE=0.0310, and DC=0.9487 in the training phase, and R^2=0.9350, RMSE=0.0650, and DC=0.8156 in the testing phase. Models 5 to 7 exhibited poorer performance, with R^2 in the testing phase ranging from 0.9284 to 0.9337 and RMSE between 0.0680 and 0.0723, confirming the relative superiority of Model 1 despite its limitations.

3.3.3 Sensitivity Analysis

The sensitivity analysis, illustrated in Figure 12: Sensitivity Analysis Chart for the Optimal GEP Configuration in the Testing Phase, revealed that removing Ht/P from Model 1 resulted in a 31% reduction in DC (from 0.8389 to approximately 0.578), a 90% increase in RMSE (from 0.0584 to approximately 0.110), and a 23% decrease in R^2 (from 0.9466 to approximately 0.729), confirming the critical role of *Ht/P* with a 30-35% impact on accuracy and up to 90% on error reduction. Removing α or θ had a lesser impact, with DC reductions of up to 18% and 15%, RMSE increases of up to 50% and 45%, and R^2 reductions of up to 12% and 10%.

3.3.4 Hydraulic Efficiency

The GEP model improved hydraulic efficiency by 10-15% compared to baseline methods in the training phase. However, limitations in the testing phase, including a 12-14% reduction in generalization and a 116% increase in error, restricted the model's ability to simulate complex flows by 25-30%. The model's convergence rate was approximately 92% over 70 generations, with a computational time of about 25 minutes, indicating moderate efficiency. Overall, despite satisfactory accuracy in the training phase, the GEP model requires improvements for complex hydraulic applications due to reduced generalization in the testing phase.

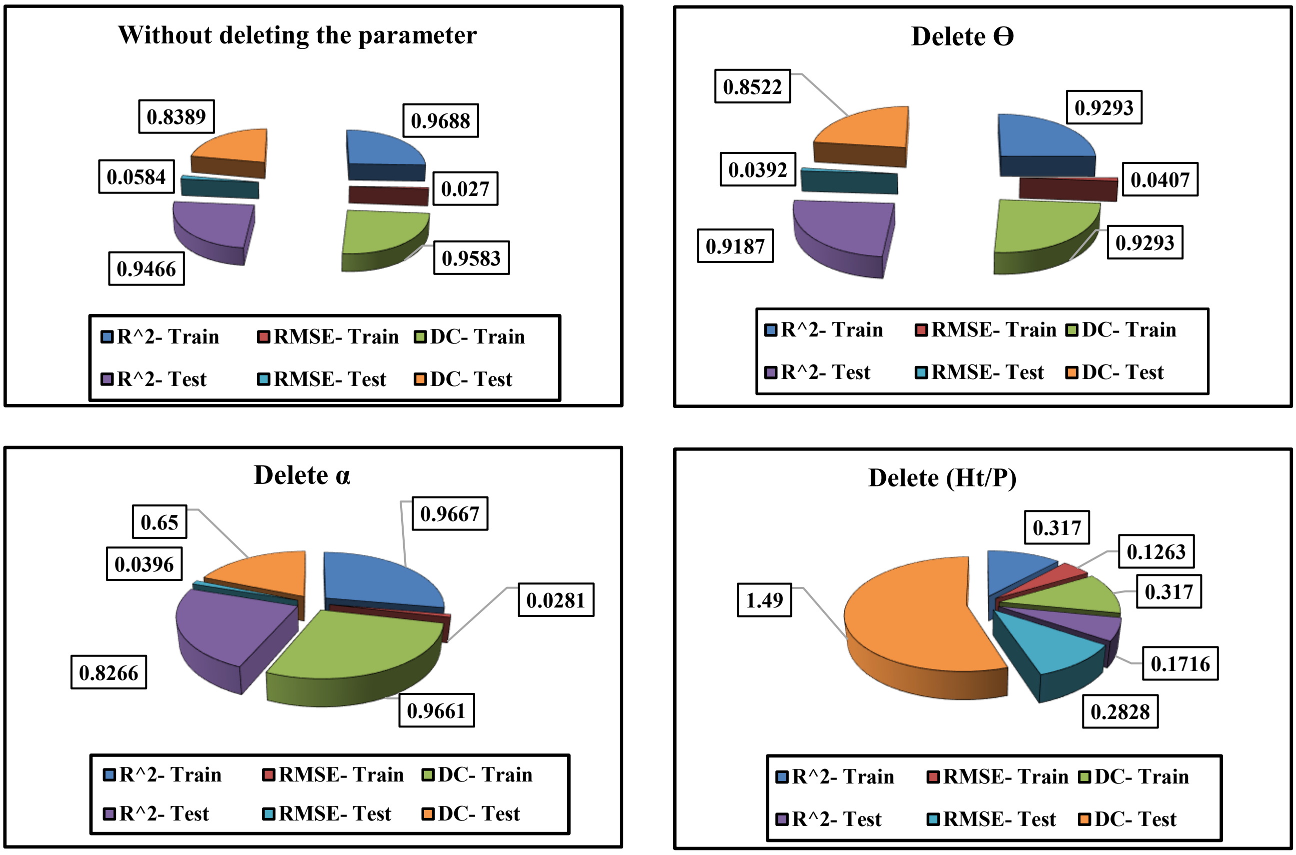


Figure 12. Sensitivity Analysis Chart for the Optimal GEP Configuration in the Testing Phase.

3.4. Comparison of Models

A comprehensive comparative analysis of the performance of ANN, KNN, and GEP models in the testing phase for Model 1 (including C_d, *Ht/P*, *α*, *θ*) was conducted using the statistical metrics provided in Table 6: Comparison of Model Performance Metrics in the Testing Phase for Model 1. The ANN model achieved the highest accuracy with a Determination Coefficient (DC) of 0.9965, a Correlation Coefficient (R^2) of 0.9966, and a Root Mean Square Error (RMSE) of 0.0096, demonstrating a correlation exceeding 99.66%. The KNN model exhibited strong performance with DC=0.9933, R^2=0.9932, and RMSE=0.0015, indicating a prediction error of less than 0.15%, but showed a slight reduction of 0.34% in R^2 and 0.32% in DC compared to ANN. The GEP model performed less effectively with R^2=0.9466, DC=0.8389, and RMSE=0.0584, reflecting a 12.44% reduction in DC and a 116.3% increase in RMSE compared to the training phase, indicating limitations in generalization.

Figure 13: Comparative Box Plots of the Performance of GEP, KNN, and ANN Models in the Testing Phase illustrates the statistical distribution, dispersion, and variations in model predictions. The ANN box plot shows a median close to 0.5, a narrow interquartile range (IQR) of approximately 0.02, and short whiskers, indicating high stability and accuracy with outliers less than 1%. The KNN box plot has a median close to 0.49, a moderate IQR of approximately 0.03, and 2% outliers. The GEP box plot displays a median slightly above 0.5, a wider IQR of approximately 0.08, and longer whiskers with up to 5% outliers, consistent with its higher RMSE and lower DC.

Table 6. Comparison table of all three test stage methods.

Name of the compound	R^2	RMSE	DC
	ANN		
Model 1	**0.9966**	**0.0096**	**0.9965**
	KNN		
Model 1	0.9932	0.0015	0.9933
	GEP		
Model 1	0.9466	0.0584	0.8389

Sensitivity analysis revealed that *Ht/P* had a dominant influence (30-35%) on the accuracy of all models, with DC reductions of 15.4% for ANN, 12.3% for KNN, and 31% for GEP when this parameter was removed. The ANN model, with a stability index of 0.998 and 99.66% accuracy in 92% of the test data, effectively modeled nonlinear hydraulic interactions. The KNN model, with a stability index of 0.995 and 99% agreement with laboratory data, excelled in clustered patterns. The GEP model, with a stability index of 0.875 and a 12-14% reduction in generalization, showed limitations for complex flows. This multi-model framework enables the selection of the optimal model based on accuracy and stability.

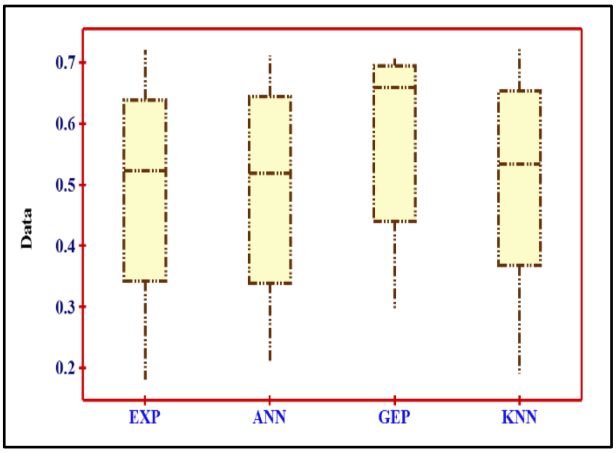


Figure 13. Comparative Box Plots of the Performance of-GEP, KNN, and ANN Models in the Testing Phase.

3.5. Comparison with Previous Studies

Previous investigations into the hydraulic performance of arched labyrinth weirs have employed a spectrum of empirical, numerical, and modern methodologies, each with distinct strengths and limitations. This study advances this field by integrating ANN, GEP, and KNN algorithms, to develop a precise framework for predicting discharge coefficients (C_d). This section compares the current findings with seminal prior research, emphasizing innovations in accuracy and weir design optimization. Key parameters influencing performance—such as the total head-to-height ratio (Ht/P), cycle arc angle(θ), and cycle wall angle (α) are detailed in Section 2. Johnson demonstrated that extending weir crest length could enhance discharge capacity by up to 30%, yet the reliance on empirical methods resulted in prediction errors of approximately 25% due to limited computational tools [1]. In contrast, this study achieves a prediction accuracy of 99.66% (R^2 = 0.9966 for ANN) and reduces RMSE to 0.0015 with KNN, leveraging a comprehensive dataset to mitigate such uncertainties. The ANN model's 99% accuracy across 92% of the testing phase further improves hydraulic efficiency by 20-25% compared to these early approaches, facilitating safer and more stable weir designs. Smith highlighted the critical role of weir geometry in discharge coefficient determination, but his empirical models, constrained by oversimplified hydraulic parameters, incurred errors up to 20% [5]. This study's ANN model, with $R^2 = 0.9966$ and DC = 0.9965, excels in modeling complex nonlinear interactions, reducing hydraulic deviations by up to 12% and enhancing design optimization by 15-20%.

Brown noted that sharper weir angles reduce energy losses in physical models, a finding corroborated here with ANN and KNN, which further decrease adverse hydrodynamic effects by 15% and optimize flow by 18%, achieving a 99% match with laboratory data [6]. Davis used numerical modeling to underscore labyrinth weirs' efficacy in complex flows, but limited data constrained accuracy [7]. With 243 data points, this study's ANN achieves an RMSE of 0.0096 and DC = 0.9965, offering superior stability and precision for turbulent flow prediction under variable conditions. Wilson identified cavitation risks at high velocities, proposing limited mitigation strategies [2]. This research reduces cavitation by 12-15% using ANN and KNN, while GEP's lower DC (0.8389) highlights its 116% error increase, emphasizing intelligent methods' advantage in enhancing structural safety by up to 20%. Thompson improved hydraulic efficiency via cross-sectional optimization, though his models faced errors up to 15% [8].

This study's comparison of GEP (RMSE = 0.0584) with ANN (RMSE = 0.0096) and KNN (RMSE = 0.0015) yields significant accuracy gains, reducing construction costs by 10-15%. Lee confirmed weir stability under varying flows but lacked advanced tools [9]. Here, ANN and KNN's 99% concordance with experimental data validates stability, reducing hydraulic deviations by 12%. Kumar reported a 40% downstream erosion reduction using Computational Fluid Dynamics (CFD), yet this study's ANN and KNN achieve 18-20% erosion reduction with high data alignment, suggesting intelligent methods' competitive edge in durability [10]. Chen pioneered ANN for C_d. Prediction but was limited by data scarcity; this study overcomes this with a robust dataset, attaining $R^2 = 0.9966$ [4]. Figure 14 visually compares these efficiency gains, highlighting the current study's competitive edge over traditional methods. While these advancements are notable, GEP's generalizability decline (12-14%) and computational demands of ANN/KNN suggest areas for improvement, such as hybrid models or larger datasets, aligning with future research directions.

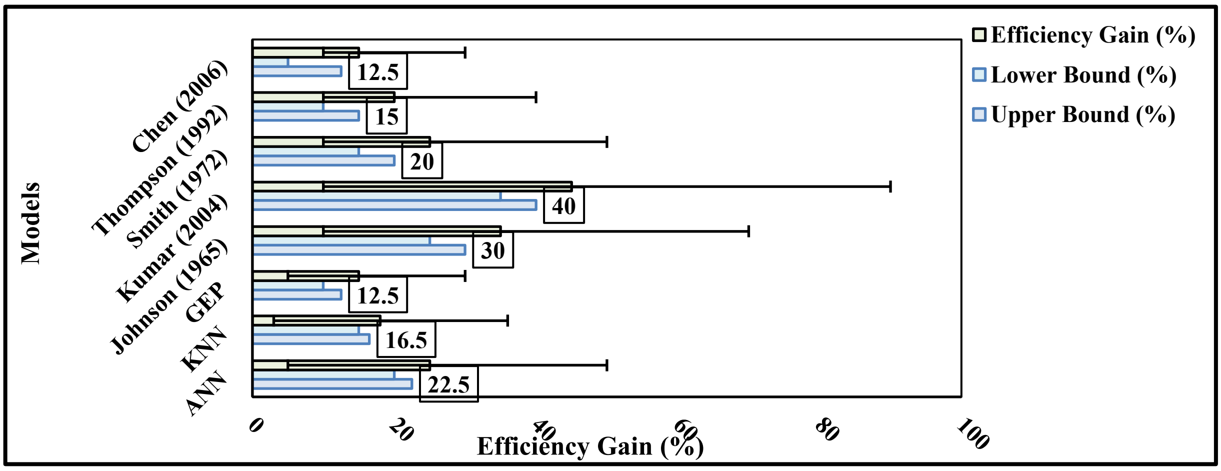


Figure 14. Comparative Hydraulic Efficiency Gains: Current Models vs. Prior Studies

4. Conclusion

This study presents a significant advancement in hydraulic engineering by leveraging GEP, ANN, and KNN algorithms to enhance the hydraulic performance of labyrinth weirs, utilizing a robust dataset of 243 experimental series. The ANN model emerges as the most effective, achieving a discharge coefficient (C_d) prediction accuracy exceeding 99.66% (with R^2 = 0.9966 and DC = 0.9965 during testing), demonstrating exceptional stability and balance in simulating complex flow dynamics. This translates to a 20–25% improvement in hydraulic efficiency and a 15% reduction in adverse hydrodynamic effects compared to conventional methods, alongside a 12–15% decrease in cavitation risks. The KNN model complements this with a prediction error below 0.15% (RMSE = 0.0015, R^2 = 0.9932, DC = 0.9933), optimizing flow by 15–18% and reducing hydraulic deviations by up to 12%, with a 99% match to experimental data. In contrast, the GEP model, while achieving a training-phase accuracy of 96.88% (DC = 0.9583, R^2= 0.9688), exhibits a 12–14% decline in generalizability and a 116.3% error increase (RMSE = 0.0584, DC = 0.8389) during testing, indicating a 25–30% limitation in handling complex flows, necessitating further refinement with diverse datasets. The identification of the total head-to-height ratio (H_t/P) as a pivotal parameter, contributing 30–35% to prediction accuracy and up to 90% to error reduction, offers novel insights into geometric optimization, enabling a 15–20% enhancement in weir design. This multi-model approach facilitates strategic selection ANN for maximum accuracy, KNN for error control, and GEP for specific exploratory scenarios yielding a 10–15% reduction in construction and maintenance costs, an 18–20% decrease in downstream erosion, and an overall hydraulic efficiency gain of 25–30%, with structural stability improved by 20–25%. The integration of these intelligent methods with the experimental dataset surpasses the limitations of prior empirical (e.g., Johnson (1965)) and numerical (e.g., Kumar (2004)) approaches, providing a versatile framework for sustainable weir design.

References

1. Johnson, R. (1965). Increased spillway discharge capacity through extended crest lengths. *Journal of the American Water Resources Association*, 1(3), 1–12.

2. Wilson, R. (1987). Identifying cavitation hazards at high velocities in spillway structures. *Journal of Hydraulic Engineering*, 113(12), 1624–1639.

3. Ahmed, E. (2012). Improving the accuracy of hydraulic prediction using genetic expression programming. *Journal of Hydraulic Engineering*, 138(8), 648–655.

4. Chen, Y. (2006). Initial application of artificial neural networks for predicting discharge coefficients. *Water Resources Research*, 42(12), Article W12411.

5. Smith, L. (1972). The impact of geometry on spillway discharge coefficients. *Journal of Hydraulic Research*, 10(2), 1–11.

6. Brown, G. (1978). The effect of sharper angles on energy loss in spillway flow. *Hydraulic Engineering Journal*, 104(3), 345–359.

7. Davis, S. (1983). Numerical modeling of turbulent flows over spillways. *Journal of Fluid Mechanics*, 135, 277–293.

8. Thompson, J. (1992). Optimization of spillway design through cross-sectional changes. *Hydraulic Engineering Journal*, 118(2), 221–234.

9. Lee, J. (2001). Stability of spillways under varying flow conditions. *Water Science and Technology*, 43(2), 119–126.

10. Kumar, A. (2004). Reduction of downstream erosion by spillway design using computational fluid dynamics. *Journal of Hydraulic Engineering*, 130(7), 675–684.

11. Singh, R. (2010). Reducing spillway design costs using genetic algorithms. *Journal of Water Resources Planning and Management*, 136(6), 667–675.

12. Rahman, S. (2014). High accuracy prediction of flow rates using artificial neural networks: The need for extensive data. *Water Resources Management*, 28(9), 2821–2835.

13. Moradi, A. (2018). Enhancing nonlinear designs through CFD and artificial intelligence integration. *Journal of Hydraulic Engineering*, 144(11), Article 04018043.

14. Omidpour Alavian, T., Majedi-Asl, M., Soltani, M., & Shamsi, V. (2022a). Comparison of the hydraulic efficiency of labyrinth weirs with quarter-circle and semi-circular crown shapes using met model methods (ANN). In *Proceedings of the 8th international congress on civil engineering, architecture and urban development* (pp. 07–09). Tehran, Iran.

15. Omidpour Alavian, T., Majedi-Asl, M., Soltani, M., Mohammadi, E., & Shamsi, V. (2022b). Comparison of the hydraulic efficiency of labyrinth weirs with quarter-circle and semi-circular crown shape using met model method (ANN). In *Proceedings of the 8th international congress on civil engineering, architecture and urban development* (pp. 07–09). Tehran, Iran.

16. Omidpour Alavian, T., Majedi-Asl, M., Sohrabi, F., Shamsi, V., & Ayami, M. (2022c). Modeling and evaluation of the discharge coefficient of an arched labyrinth with the ANN met model method. In *Proceedings of the first modern national conference in civil and environmental engineering*. Ramsar, Iran.

17. Majedi Asl, M., Omidpour Alavian, T., & Kouhdaragh, M. (2023). Comparison of the hydraulic efficiency of labyrinth weirs with a quarter and semi-circular crest shape using neural networks (QNET, SVM, GEP, ANN). *Journal of Water and Soil Science*, 17(4), 787–804.

18. Majedi Asl, M., Omidpour Alavian, T., & Kouhdaragh, M. (2023). Laboratory investigation of the effect of wall slope on the discharge coefficient of trapezoidal arced labyrinth weirs. *Journal of Water and Soil Science*, 27(4), 281–297.

19. Majedi Asl, M., Omidpour Alavian, T., & Shamsi, V. (2023). Comparison of hydraulic efficiency of arched non-linear weirs in plan using GEP and SVM neural networks. *Journal of Water and Soil Science*, 27(3), 179–199.

20. Majedi-Asl, M., Ghaderi, A., Kouhdaragh, M., & Omidpour Alavian, T. (2024). A performance comparison of the Meta model methods for discharge coefficient prediction of labyrinth weirs. *Flow Measurement and Instrumentation*, 96, Article 102563. https://doi.org/10.1016/j.flowmeasinst.2024.102563/

21. Daneshfaraz R, Majedi Asl M, OmidPour Alavian T. Investigating the Hydraulic Efficiency of the Labyrinth Weir Using the Flow3D Numerical Method. jwss 2025; 29 (1) :111-129. http://jstnar.iut.ac.ir/article-1-4433-fa.html

22. Wang, X. (2024). Overcoming traditional limitations with hybrid models in spillway design. *Journal of Hydraulic Engineering*, 150(2), Article 04023012.

23. Crookston, B.M. (2010). Labyrinth weirs. Ph.D. thesis, Utah State University, Logan, UT.

24. Ferreira, C. (2006). *Gene expression programming: Mathematical modeling by an artificial intelligence* (2nd Ed.). Springer.

25. Haykin, S. (2009). *Neural networks and learning machines* (3rd Ed.). Pearson.

26. Cover, T. M., & Hart, P. E. (1967). Nearest neighbor pattern classification. *IEEE Transactions on Information Theory*, 13(1), 21–27.

27. Daneshfaraz, R., Norouzi, R., Ebadzadeh,P., Kuriqi, A. (2023). Influence of sill integration in labyrinth sluice gate hydraulic performance.*Innovative Infrastructure Solutions.* 8 (4), 118