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DETERMINATION OF THE THEORETICAL BUCKLING LOAD OF HYBRID COMPOSITE LAMINATES HAVING DIFFERENT HOLE DIAMETERS THROUGH TAGUCHI-BASED FUZZY MODEL.

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Keywords

Buckling load, hybrid composite, notch effect, Taguchi-based fuzzy model, finite element analysis

Abstract

This study examined the determination of theoretical buckling load of hybrid composite laminates with different hole diameters using the Taguchi-based fuzzy optimization method. In the Taguchi-based fuzzy method, the buckling parameters in the theoretical approach were used as input data, and the critical buckling load value was considered the output. Multiple Performance Characteristics Index (MPCI) was utilized for determining optimization results. Moreover, theoretical results verified by experimental and numerical approaches. In the experimental approach, the hybrid composite laminates (HCGFRE) manufactured by sequential sequencing of plain woven glass fiber reinforced epoxy (GFRE) and plain woven carbon fiber reinforced laminates (CFRE). Tests were carried out at a constant loading rate of 1 mm/min at room temperature. In numerical analysis, eigenvalue buckling was performed using Ansys finite element program. Composite laminates were formed utilizing Composite Prep-Post (ACP) sub-module. Eventually, it was observed that using the Taguchi-based fuzzy method to determine the critical buckling load of hybrid composite laminates is consistent with both experimental and numerical approaches in terms of results. Furthermore, it was observed in experimental approach that the maximum critical buckling load was obtained from the CFRE composite laminate without a hole. On the other hand, the minimum critical buckling load was obtained from the GFRE composite laminate with a 9 mm hole diameter. In addition, the critical buckling load of the hybrid composite laminate (HCGFRE) was determined as an intermediate value in all experimental studies. In general, it is observed that the critical buckling load decreases in all composite laminates as the hole diameter increases due to the notch effect.

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doi : 10.46399/muhendismakina.1764046

Mühendis ve Makina / Engineer and Machinery
https://dergipark.org.tr/tr/pub/muhendismakina

FARKLI DELİK ÇAPLARINA SAHİP HİBRİT KOMPOZİT LAMİNATLARIN TEORİK BURKULMA YÜKÜNÜN TAGUCHI TABANLI BULANIK MODEL YÖNTEMİYLE BELİRLENMESİ

Anahtar Kelimeler

Burkulma yükü, hibrit kompozit, çentik etkisi, Taguchi tabanlı bulanık model, sonlu elemanlar analizi

Öz

Elektrikli otobüslerde batarya sistemlerinin yüksek kütlesi, hem yolcu kapasitesini düşürmekte hem de operasyonel menzili ciddi şekilde sınırlandırmaktadır.

Bu çalışmada, Taguchi tabanlı bulanık optimizasyon yöntemi kullanılarak farklı delik çaplarına sahip hibrit kompozit laminatların teorik burkulma yükünün belirlenmesi incelenmiştir. Taguchi tabanlı bulanık yöntemde, teorik yaklaşımdaki burkulma parametreleri girdi verisi olarak kullanılmış ve kritik burkulma yükü değeri çıktı olarak kabul edilmiştir. Optimizasyon sonuçlarının belirlenmesinde Çoklu Performans Karakteristikleri İndeksi (MPCI) kullanılmıştır. Ayrıca, teorik sonuçlar deneysel ve sayısal yaklaşımlarla doğrulanmıştır. Deneysel yaklaşımda, düz dokuma cam elyaf takviyeli epoksi (GFRE) ve düz dokuma karbon elyaf takviyeli laminatların (CFRE) ardışık olarak yerleştirilmesiyle üretilen hibrit kompozit laminatlar (HCGFRE) kullanılmıştır. Testler, oda sıcaklığında 1 mm/dak sabit yükleme hızıyla gerçekleştirilmiştir. Sayısal analizde, Ansys sonlu elemanlar programı kullanılarak özdeğer burkulması yapılmıştır. Kompozit laminatlar, Kompozit Hazırlama ve Son İşlem (ACP) alt modülü kullanılarak oluşturulmuştur. Sonuç olarak, hibrit kompozit laminatların kritik burkulma yükünü belirlemek için Taguchi tabanlı bulanık mantık yönteminin kullanılmasının, sonuçlar açısından hem deneysel hem de sayısal yaklaşımlarla tutarlı olduğu gözlemlenmiştir. Ayrıca, deneysel yaklaşımda maksimum kritik burkulma yükünün deliksiz CFRE kompozit laminattan elde edildiği gözlemlenmiştir. Öte yandan, minimum kritik burkulma yükü 9 mm delik çapına sahip GFRE kompozit laminattan elde edilmiştir. Ek olarak, hibrit kompozit laminatın (HCGFRE) kritik burkulma yükü, tüm deneysel çalışmalarda ara bir değer olarak belirlenmiştir. Genel olarak, çentik etkisi nedeniyle delik çapı arttıkça tüm kompozit laminatlarda kritik burkulma yükünün azaldığı gözlemlenmiştir.

Araştırma Makalesi
Başvuru Tarihi : 14.08.2025
Kabul Tarihi : 20.02.2026

Research Article
Submission Date : 14.08.2025
Accepted Date : 20.02.2026

INTRODUCTION

Today, composite materials are widely used in many sectors, such as aviation, transportation, and the defense industry. In addition, composite materials are used in construction, particularly in building strengthening. Different designs of composite materials are also tried with the widespread use of composite materials. The most common of these is the hybridization method. The most crucial factor in the preference for hybrid composite materials is that they are cheaper than standard composite materials and provide high quality and fatigue life (Zuo, Srinivasa, Vassilopoulos, 2021)

Studies that include hybrid composite materials tested under different loading conditions existed in the literature (Erbayrak, Gumus, Yuncuoglu, Kahraman, 2020, Erbayrak, Yuncuoglu, Kahraman, Gumus, 2021, Erbayrak, 2021). In addition, the buckling behavior of hybrid composite materials is also investigated. E. Yeter et al. (Yeter, Erklig, Bulut, 2014) examined the buckling behavior of hybrid composite laminates embedded in various cutout shapes. In their study, Carbon, S-glass, and Aramid fabrics are reinforced materials, and epoxy adhesive is the matrix in the hybridization process of composite laminates. The experimental and numerical analyses were carried out with various cutout shapes on the buckling loads. They concluded that stiffness and shear properties are affected by the buckling behavior of hybrid composite laminates. Buckling and post-buckling responses of the hybrid composite were investigated by Vummadisetti and Singh (Vummadisetti, Singh,2014). They studied the effects of fiber orientations on the buckling behavior of unidirectional carbon and glass fiber-reinforced composite laminates containing different cutout shapes. In addition, the numerical approach, including the Tsai-Hill failure criterion, was carried out using the Abaqus finite element program. Eventually, they stated that glass fiber-reinforced composite laminates provide high strength in terms of buckling load. H.B. Mutu and Z. Aslan (Mutu, Aslan,2022) experimentally studied the buckling behavior of composite materials with nanoparticle reinforcement. In their study, E-glass/epoxy composite material was examined, including five different weight ratios of nanoparticles (0.1%, 0.2%, 0.3%, 0.4%,0.5%). Consequently, they stated that the critical buckling load increased with the increase of nanoparticles. The buckling behavior of composite laminates can be examined from the relevant studies (Achour, Ovinos, Touahmia Boukendakdji, 2018, Allahbakhsh, Dadrasi ,2012, Kuehn, Pasternak, Mittelstedt, 2014)

The numerical approaches of the critical buckling loads of composite materials are studied in the literature. Y. Arman et al (Arman, Zor, Aksoy, 2006) studied the critical delamination diameter for laminated composite laminate was determined using the ANSYS 6.1 FEA finite element program. They stated that there was a significant reduction in the critical buckling load at a certain delamination diameter. Damghani et al (Damghani, Pir, Murphy, Fotouhet, 2022) determines the buckling failure of hybrid (CFRP-GFRP) composite laminates with circular holes. In their study, experimental and numerical studies were carried out sepa-

rately. Finally, they stated that CFRP and hybrid laminates exhibit similar failure behavior in numerical and experimental approaches. It is said that related studies (Altan, Kartal ,2009, Kolanu, Raju, Ram, 2020, Loughan, 2019) can be examined to investigate the buckling behavior of composite laminates numerically.

Taguchi-based fuzzy logic is an optimization tool used in decision-making problems with multiple data. In this method, Taguchi and fuzzy approaches are used sequentially together. Krishna et al (Krishnaa, Narasimhab, Anthony, Xavior , 2015) examined the effect of parameters such as the type of reinforcement to be applied to the aluminum alloy, the size of the reinforcement particle, and the weight percentage on the mechanical properties. They used Taguchi integrated Fuzzy Approach in the optimization approach of those parameters. They stated that the fuzzy approach is suitable for investigating the optimal combination of parameters affecting mechanical behavior. K. Palanikumar et al (Palanikumar, Karunamoorthy, Karthikeyan, Latha, 2006) optimized the parameters used in the grinding of GFRP composites with the Taguchi-based fuzzy logics. They stated that the parameters determined in the verification studies carried out with optimization increased the performance of the process. The critical impact energy of composite materials and the parameters affecting the impact force with the Fuzzy Based Taguchi Method (FBTM) was investigated (Erbayrak, Erbayrak, 2020). They stated that the results converged due to the optimum parameters determined by the optimization method. Relevant articles can be examined in the literature for different optimization techniques used in engineering approaches (Pınarbası, Külekci, Boga, Esme, 2020, Er, Külekci Esme, Boga, 2021; Boga, Koroglu ,2021).

The Taguchi-based Fuzzy Logic optimization utilization in this study, compared to existing optimization methods in the literature, achieves optimization with fewer data trials using the MPCI (Multi-Objective Optimization) index, which facilitates the simultaneous evaluation of three theoretical parameters. Notably, this research marks the first instance in the literature where Taguchi-based Fuzzy optimization has been applied to the theoretical buckling of composite materials. In addition, literature has established critical buckling load values for composite materials containing holes through finite element methods, there remains a notable gap in studies focused on the stress distributions within each layer of the composite laminate under buckling loads (Laurin, Carrere, Maire, 2007, Farooq, Myler, 2015). This study successfully elucidated the stress distributions within each layer of the composite laminate, and the findings have been rigorously verified. It was determined the constants for calculating the buckling loads of composite laminates with different hole diameters using the Taguchi-based fuzzy method. It was found that the critical buckling load obtained with these parameters closely matched the experimental and numerical results. Thus, it can be concluded that the Taguchi-based fuzzy model is effective in optimizing the buckling load of composite materials with holes of varying diameters using minimal input data.

THEORETICAL APPROACH

This section, first, the formula having the closed differential solution form of the buckling load by applying the variational approach is going to explain elaborately. Then Taguchi-based fuzzy method will be discussed.

It is known that the formulas used to determine the critical buckling load in Euler's linear buckling theorem do not have a closed-form solution in the form of an analytical procedure. Therefore, it is unsuitable for orthotropic materials in which the mechanical properties of the material change along global axes. The critical buckling load can be determined by applying the direct variational method to the linearized buckling equation in composite materials (Lopatin, Morozov, 2020);

$$\int_0^a\int_0^b\left[\left(D_{11}\frac{\partial^2 w}{\partial x^2}+D_{11}\frac{\partial^2 w}{\partial y^2}\right)\delta\left(\frac{\partial^2 w}{\partial x^2}\right)+4D_{33}\frac{\partial^2 w}{\partial x\partial y}\partial\left(\frac{\partial^2 w}{\partial x\partial y}\right)+\left(D_{12}\frac{\partial^2 w}{\partial x^2}+D_{22}\frac{\partial^2 w}{\partial y^2}\right)\delta\left(\frac{\partial^2 w}{\partial y^2}\right)-N\frac{\partial w}{\partial x}\right]dxdy \quad (1)$$

where, w is the laminate deflection, D_{11}, D_{12}, D_{22}, D_{33} are the bending stiffness matrix coefficients, δ stands for a variation. Boundary conditions for the integral solution of Equation 1 (Lopatin, Morozov, 2020);

$$w=0,\quad \frac{dw}{dx}=0 \quad (2)$$

$$D_{12}\frac{\partial^2 w}{\partial x^2}+D_{22}\frac{\partial^2 w}{\partial y^2}=0$$
$$D_{22}\frac{\partial^3 w}{\partial y^3}+(D_{12}+4D_{33})\frac{\partial^3 w}{\partial x^2\partial y}=0 \quad (3)$$

In the boundary condition, it is stated that the deflection, bending rotation, and shear force are equal to zero along the x-axis at the clamped edges. The derivative expression of the deflection in Equation 2 concerning the x-axis denotes rotation of the laminate, the second-degree derivative of the deflection in Equation 3 with respect to the x-axis is the bending moment, and the third-degree derivative represents the shear force. Meanwhile, in Equations 1-3, the deflections regarding x and y directions are expressed with the following function;

$$w(x,y)=X(x)W(y) \quad (4)$$

where $W(y)$ is the function that determines the deformed shape in the y-direction of the laminate during buckling, and $X(x)$ is the function that determines the change in the geometry of the laminate along the x-axis. The $X(x)$ function of the composite laminate buckled along the x-axis is defined as (Lopatin, Morozov, 2020);

$$X(x) = \cosh\frac{\lambda x}{a} - \cos\frac{\lambda x}{a} - \sigma(\sinh\frac{\lambda x}{a} - \sin\frac{\lambda x}{a}) \tag{5}$$

where, λ and σ are constant values for cantilever-clamped composite plates. The boundary condition of the cantilever-clamped end of composite laminate in Cartesian coordinates under the buckling load is shown in Figure 1.

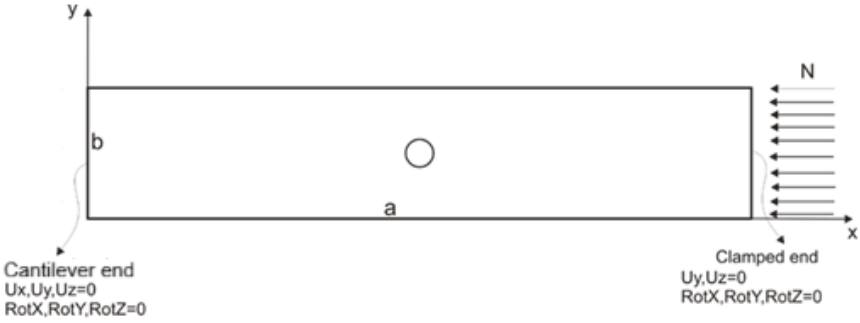

Figure 1. Boundary condition of composite laminate

Equation 1 can be expressed as follows using the boundary conditions and functions given in Equations 2-5 consecutively (Lopatin, Morozov, 2020);

$$\int_0^a\int_0^b \left\{ \begin{array}{l} \left[D_{11}W\int_0^a \left(\frac{d^2X}{dx^2}\right)^2 dx + D_{12}\frac{d^2W}{dy^2}\int_0^a X\frac{d^2X}{dx^2}dx \right]\delta W + 4D_{33}\frac{dW}{dy}\int_0^a \left(\frac{dX}{dx}\right)^2 ax\delta\left(\frac{dW}{dy}\right) + \\ \left(D_{12}W\int_0^a X\frac{d^2X}{dx^2}dx + D_{22}\frac{d^2W}{dy^2}\int_0^a X^2dx \right)\delta\left(\frac{d^2W}{dy^3}\right) - NW\int_0^a \left(\frac{dX}{dx}\right)^2 dx\delta W \end{array} \right\} dy = 0 \tag{6}$$

It is also stated that integral expressions with respect to variable x can be defined on constant values (Lopatin, Morozov, 2020);

$$\int_0^a \left(\frac{d^2X}{dx^2}\right) dx = \frac{\lambda^4}{a^3}$$
$$\int_0^a X\left(\frac{d^2X}{dx^2}\right) dx = -\frac{\gamma}{a}$$
$$\int_0^a X^2 dx = a \qquad (7)$$
$$\int_0^a X\left(\frac{dX}{dx}\right)^2 dx = \frac{\gamma}{a}$$
$$\gamma = \sigma\lambda(\sigma\lambda - 2)$$

The critical value of the distributed *N* load in Equation 6 can be determined by the equation given below (Lopatin, Morozov, 2020);

$$N_{cr} = \eta \frac{\lambda^4}{\gamma} \frac{D_{11}}{a^2} \quad where$$
$$\eta = 1 - \frac{c^2}{2}\left(\sqrt{{s_{22}}^2 + {s_{12}}^2} - s_{22}\right)$$
$$c = \frac{a}{b}, \quad s_{12} = 8\frac{\sigma\gamma}{\lambda^3}\frac{D_{12}}{D_{11}} \qquad (8)$$
$$s_{22} = \frac{D_{22}}{D_{11}}c^2 + 2\frac{\gamma^2}{\lambda^4}\frac{D_{12} + 2D_{33}}{D_{11}} + 32\frac{\sigma\gamma}{\lambda^3}\frac{D_{33}}{D_{11}}$$
$$D_{11} = E_x\frac{h^3}{12}, \quad D_{12} = E_x\nu xy\frac{h^3}{12}, \quad D_{22} = E_y\frac{h^3}{12},$$
$$D_{33} = G_{xy}\frac{h^3}{12}$$

where h is composite laminate thickness.

Taguchi-based fuzzy method

The Taguchi Method is an optimization technique that utilizes orthogonal arrays to minimize the impact of uncontrollable factors. It is commonly employed to optimize a single performance characteristic. However, the Taguchi method has limitations when optimizing multiple parameters (Tarng, Yang, Juang, 2000). In this study, the Taguchi method is combined with fuzzy logic, as the theoretical buckling load involves more than one parameter to address multiple performance characteristics. In utilizing the Taguchi-based fuzzy method, the first step involved determining the theoretical buckling load parameters (refer to Eqn 8). It should be stated here that theoretical parameters η, λ and γ were determined

as input, σ and F_{cr} as output parameters. The interval values of all parameters were determined considering the study (Lopatin, Morozov, 2020) in the literature. The orthogonal arrays are chosen based on the total degree of freedom requirement. The aggregate of each factor's individual degrees of freedom inside the group is the total degree of freedom. An array equal to or more than the whole degree of freedom is chosen as the orthogonal array when determining the array's orientation. The overall degree of freedom is nine because there are three input parameters and each parameter has four levels. Since the degree of freedom of L16 exceeds the overall degree of freedom of the input parameters, the L16 orthogonal array is chosen for this study.

The signal-to-noise (S/N) ratio value of the orthogonal array was then calculated to assess the deviation from the design. It's worth noting that the S/N ratio can also be represented as the standard deviation (noise) ratio of the average (signal). The S/N input and output data were categorized into linguistic fuzzy subsets using Mamdani fuzzy logic modeling IF-THEN rules. MPCI (The Multiple Performance Characteristic Index) value of each subset was evaluated using the fuzzy approach, and final levels were determined. The optimization process was finalized by identifying the maximum output values from these evaluations. The flow chart shows the application steps of the Taguchi-based fuzzy model given in Figure 2.

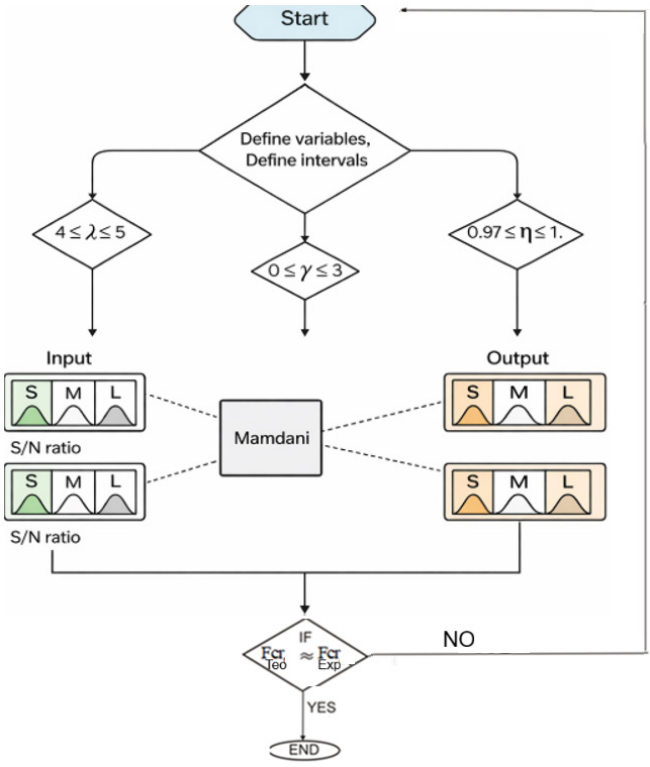

Figure 2. Flow chart of the application steps of the Taguchi-based fuzzy model

As shown in Figure 2, fuzzy rules were applied to define input and output data as given below.

Rule 1: Buckling load is Small and σ is Small than MPCI is very Small

Rule 2: Buckling load is Small and σ is Medium than MPCI is Small

Rule 3: Buckling load is Small and σ is Large than MPCI is Medium

Rule 4: Buckling load is Medium and σ is Small than MPCI is Small

Rule 5: Buckling load is Medium and σ is Medium than MPCI is Medium

Rule 6: Buckling load is Medium and σ is Large than MPCI is Large

Rule 7: Buckling load is Large and σ is Small than MPCI is Medium

Rule 8: Buckling load is Large and σ is Medium than MPCI is Large

Rule 9: Buckling load is Large and σ is Large than MPCI is very Large

The number of fuzzy rules is determined according to fuzzy inputs. It should be reminded here that fuzzy inputs involve S/N rules. Moreover, the number of S/N values is the same as the output values in optimization. Therefore, nine rules were determined since there are two S/N ratios and three fuzzy subsets of each input in fuzzy logic optimization (see Fig.2). Those were transferred to the MATLAB program. The MPCI values were calculated with the fuzzy logic approach.

Experimental approach

In this section, firstly, the production stages of layered woven hybrid composite materials will be explained. Then, the procedure of the buckling test will be discussed. In hybrid composite laminates, unidirectional carbon/glass fiber material was used as reinforcement material, and in thermoset Araldite LY 1564 / Aradur 3487 epoxy adhesive was used as matrix material. Composite laminates are formed by sequential stacking of prepreg fibers. Composite laminates were obtained in 8 layers using the Vacuum Assisted Resin Transfer Molding (VARTM) method. In the production stages with VARTM, the release film is first placed on the metal table having the thermocouples. Then, the prepreg fiber materials are put into the release film consecutively. Subsequently, the distribution media for the matrix material to penetrate the fiber materials is placed on the release film. Spiral hoses are connected to both ends of the distribution media. One of these hoses is connected to the vacuum pump that will provide the vacuum environment, and the other is connected to the epoxy resin to inject the matrix material into the system. During the production phase, the air inside the system is drawn by the vacuum pump, while the epoxy matrix system is penetrated through the spiral hose connected to the resin. This process continues until the liquid resin (epoxy and hardener) is impregnated. It should be stated that there is a thermocouple effect in fully integrating the matrix material with the reinforcement material by ensuring its viscosity. Composite laminates were cut to the desired dimensions using a water jet device. Water jet application was carried out using 150 MPa water pressure and garnet

silicate sand grains for obtaining a smooth cutting surface. However, the water jet device could not be used for the holes to be drilled in the centers of the composite laminates since the high pressure provided by the device causes surface damage to the composite laminate. Therefore, a hole was drilled in the composite laminates using a drilling machine tool. *Research and publication ethics were followed in this study.* All production stages are shown in Figure 3.

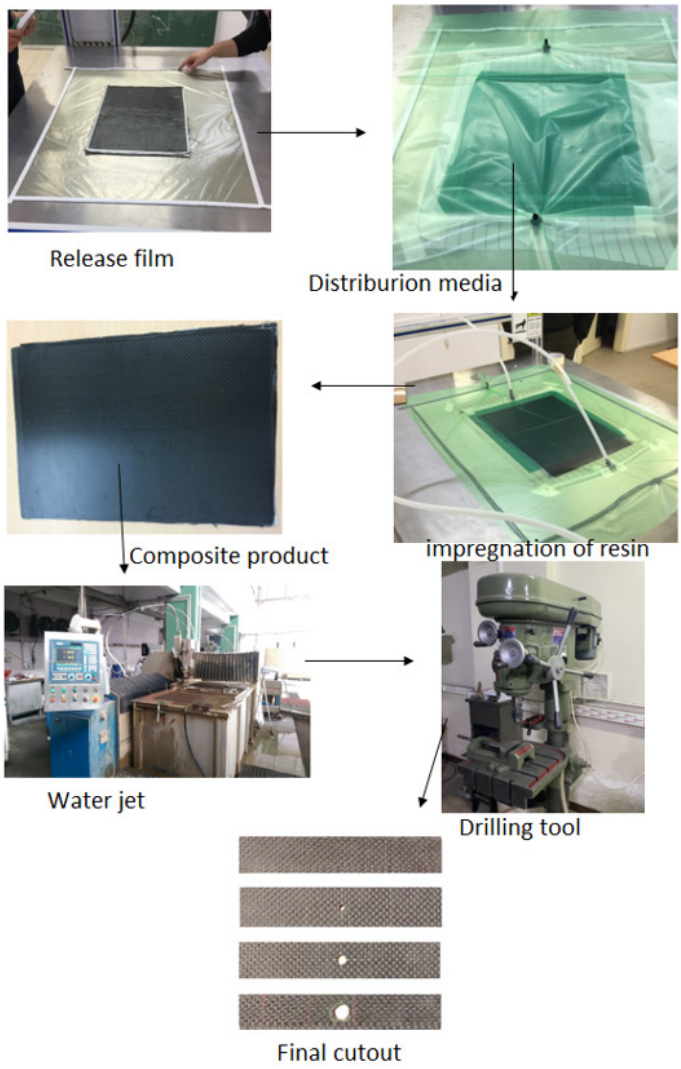

Figure 3 Production stages of composite laminate

The mechanical properties of three different composite materials (GFRE, CFRE, HCGFRE) to be used in the experimental approach are given in Table 1.

Table 1. Mechanical properties of composite laminates (Erbayrak et al. 2021)

		CFRE	GFRE	HCGFRE
Longitudinal Elasticity Modulus(E_1)	GPa	100±18	38±6	80.75±17
Transverse Elasticity Modulus (E_2)	GPa	98±16	39±5	79±16
Poisson's ratio		0.22	0.31	0.27
Tensile ultimate strength	MPa	1050±125	525±80	850±140
Elongation at break (%)		1±0.15	2.3±0.2	1.4±0.1

The buckling tests of hybrid and non-hybrid composite laminates with different hole sizes were carried out using the SHIMADZU universal tensile test device. Buckling samples were cut 120×20×2 mm dimensions. The geometric dimensions of the composite laminates were set in accordance with the ASTM D8511 standard (ASTM 2023). The loading rate of the test device ranges from 0.0001m/sec to 20m/sec. The load capacity is 10 kN (Shimadzu, 2022). In the experimental approach, buckling tests of all composite laminates were performed at a 1 mm/min loading rate. In buckling experiments, three of each composite material were tested separately for all experimental conditions. Experimental data were optimized utilizing Moving Area Method in MATLAB program to provide statistical reproducibility. Moreover, the boundary conditions are provided by the holding grips on the device. The image of the test device is shown in Figure 4.

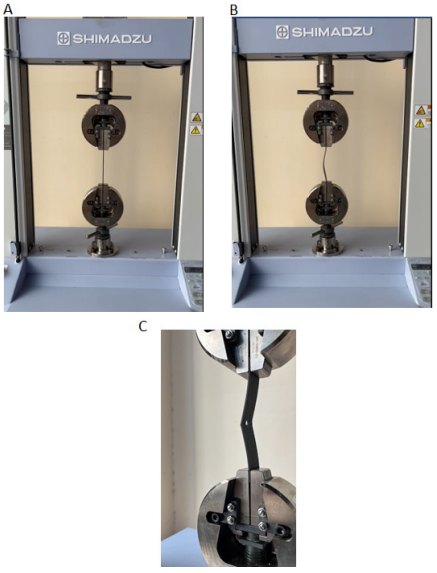

Figure 4. Experimental set up (a-Unbuckled position, b- buckled position, c-damage position)

Numerical approach

In numerical analysis, eigenvalue buckling analysis was performed using the Ansys finite element program. In general, buckling is a phenomenon in which compression force is applied to the iso-centre of the cross-sectional areas of the Euler-type rigid structures. One of the primary purposes of eigenvalue buckling analysis is to determine the critical buckling load under different boundary conditions. The applied load is expressed in incremental unit force values in eigenvalue buckling. This load value is insignificant since the critical buckling load value is determined by the load factors included in the eigenvalue buckling (Ellobody, Feng, Young. 2014). Furthermore, the deformation values corresponding to the buckling modes are not considered. In linear buckling analysis, the equivalent stress corresponding to the critical buckling load is considered (Ellobody et al. 2014).

In applying eigenvalue buckling analysis in the Ansys finite element program, the geometric properties of the composite laminates used in the experimental approach were designed using the "Space-Claim" module. Composite laminates were then formed utilizing Ansys Composite PrepPost (ACP) sub-module (Ansys 2022). In this stage of the study, the fiber and matrix constituent materials were first defined in the finite element software by assigning their corresponding mechanical properties. Subsequently, the fiber orientation angles and the stacking sequence of the composite reinforcement were specified in accordance with the laminate configuration used in the experimental program. From these definitions, a solid composite laminate model was designed by combining plies into a laminate structure. To discretize this model, the quadrilateral element (Quad Element)mesh was generated while maintaining element quality criteria (such as skewness and aspect ratio). In addition, a locally refined mesh (first-level refinement) was applied in the vicinity of the hole region to more accurately capture the high stress gradients and deformation behavior and to provide a smoother mesh transition around this area. Mesh sensitivity analysis is crucial for determining the buckling load in finite element analysis. To accurately establish the appropriate mesh size, we conducted a mesh sensitivity analysis within the scope of this study. We investigated the effects of various mesh sizes and the corresponding number of elements on the buckling load. The optimal mesh size was identified as the point at which the buckling load remained stable, and finite element analysis was conducted using that mesh size. Following the meshing procedure, boundary conditions corresponding to the experimental setup were imposed. Specifically, a cantilever–clamped boundary condition was applied to the laminate edges, ensuring that both translational and rotational degrees of freedom were constrained in the same manner as in the physical tests. After defining the material properties, laminate lay-up, mesh structure, and boundary conditions, an eigenvalue buckling analysis was performed to determine the critical buckling loads and associated buckling mode shapes of the composite laminate. In the solution part of the analysis, a static pre-buckling analysis was firstly performed under a reference load to establish the stress stiffness matrix.

An eigenvalue problem was solved afterwards, and the eigenvalues obtained were load amplification factors. The smallest positive eigenvalue of the matrices generated by the structure refers to the most critical buckling mode and would multiply the applied reference load to give the critical buckling load.

The image of the composite laminate model used in the finite element analysis is shown in Figure

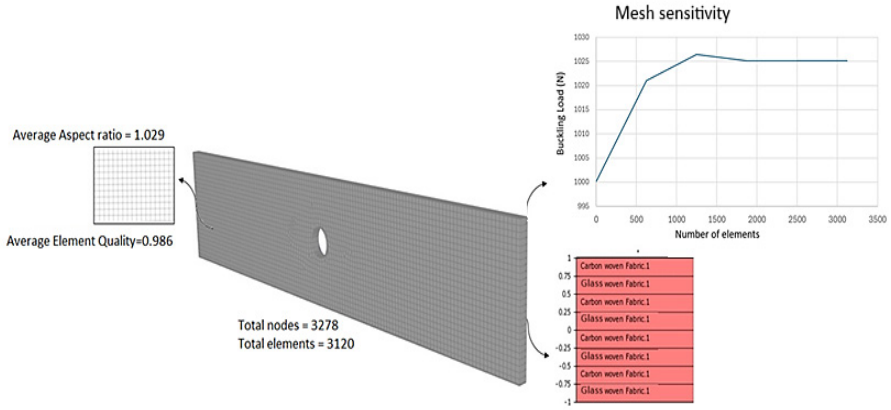

Figure 5. The finite element image of composite laminate

RESULTS AND DISCUSSION

In this section, experimental, theoretical, and numerical approaches will be evaluated. First, the fuzzy logic model results will be interpreted, then compared with the numerical and experimental results. In Taguchi-based fuzzy model, three design parameters were utilized, with each parameter being set to four levels. Therefore, the Taguchi L16 Orthogonal Array (OA) was employed. S/N values were computed for each level determined by the OA. The formulas utilized in S/N values are detailed in the accompanying study (Erbayrak, Erbayrak 2020). The L16 OA and S/N values are displayed in Table 2 as an example of the carbon composite laminate without holes. Optimization values for other composite laminates are not included here for the sake of brevity.

Table 2. The L16 OA and S/N values

Level number	parameters (η)	Parameter (λ)	Parameter (γ)	Buckling load (F_{cr})	Parameter (σ)	S/N (Buckling load)	S/N (Parameter (σ)
1 1 1	0.97	4.5	1.3	1305.47	0.559	62.32	-5.05
1 2 2	0.97	4.6	1.33	1393.286	0.549	62.88	-5.21
1 3 3	0.97	4.7	1.36	1484.954	0.539	63.43	-5.37
1 4 4	0,97	4.8	1.39	1580.559	0.53	63.98	-5.51
2 1 2	0.98	4.5	1.33	1289.179	0.561	62.21	-5.05
2 2 1	0.98	4.6	1.3	1440.134	0.547	63.17	-5.37
2 3 4	0.98	4.7	1.39	1467.883	0.541	63.33	-5.51
2 4 3	0.98	4.8	1.36	1602.078	0.528	64.25	-5.05
3 1 3	0.99	4.5	1.36	1273.606	0.563	62.10	-5.05
3 2 4	0.99	4.6	1.39	1360.63	0.553	62.67	-5.21
3 3 1	0.99	4.7	1.3	1585.52	0.535	64.00	-5.37
3 4 2	0.99	4.8	1.33	1615.92	0.526	64.54	-5.51
4 1 4	1	4.5	1.39	1258.705	0.565	62.00	-5.05
4 2 3	1	4.6	1.36	1404.692	0.551	62.95	-5.21
4 3 2	1	4.7	1.33	1565.411	0.537	63.89	-5.37
4 4 1	1	4.8	1.3	1642.249	0.524	64.82	-5.51

In Table 2, the levels are listed in three columns. The first column displays the level number for the first input parameter, the second column displays the level number for the second input parameter, and the third column displays the level number for the third input parameter. It should be noted that the level numbers and their value distribution are determined according to Taguchi orthogonal array rules (Erbayrak, Erbayrak 2020). The MPCI values were calculated using the Mamdani Fuzzy Model approach in the MATLAB program, utilizing the S/N values (The calculated MPCI output in MATLAB is given in the Appendix section). MPCI values are always defined as a single value, regardless of the number of input values used. The MPCI values corresponding to each level are shown in Table 3.

Table 3. MPCI values for each level subset

Level number	S/N (Buckling load)	S/N (Parameter (σ)	MPCI
1 1 1	62.32	-5.05	0.528
1 2 2	62.88	-5.21	0.48
1 3 3	63.43	-5.37	0.413
1 4 4	63.98	-5.51	0.343
2 1 2	62.21	-5.05	0.5
2 2 1	63.17	-5.37	0.413
2 3 4	63.33	-5.51	0.25
2 4 3	64.25	-5.05	0.804
3 1 3	62.10	-5.05	0.5
3 2 4	62.67	-5.21	0.436
3 3 1	64.00	-5.37	0.509
3 4 2	64.54	-5.51	0.487
4 1 4	62.00	-5.05	0.5
4 2 3	62.95	-5.21	0.496
4 3 2	63.89	-5.37	0.486
4 4 1	64.82	-5.51	0.5

As can be seen in Table 4, the highest MPCI average values were calculated as level 4 in the first parameter, level 4 in the second parameter, and level 3 in the third parameter. Thus, theoretical constants and maximum buckling load of the carbon composite without hole were calculated as η=1, λ=4.8, γ=1.36 Fcr=1665.34 N and σ=5.28 respectively.

Table 4. The mean MPCI values for each level MPCI (mean)

	level 1	level 2	level 3	level 4
Parameter (η)	0.441	0.49175	0.483	0.4955
Parameter (λ)	0.507	0.45625	0.4145	0.5335
Parameter (γ)	0.4875	0.48825	0.55325	0.38225

The Taguchi based fuzzy model determined the optimal result using orthogonal array values. The buckling load obtained from the optimization represents the best outcome.

In the experimental approach, the buckling behavior of the hybrid composite (HCGFRE) and composite laminates (CFRE, GFRE) used to form the hybrid composite was investigated. In addition, the hole effect on the critical buckling loads of composite laminates is also studied. Three samples of each composite laminate with and without holes were tested during the experiment. The load-displacement graphs obtained at the end of the experiments are shown in Figure 6-9.

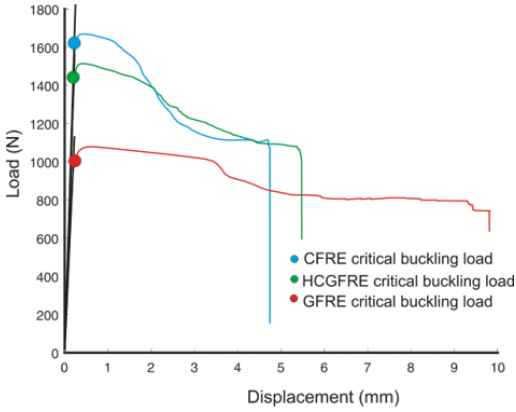

Figure 6.Load-displacement curves for composite laminates without hole

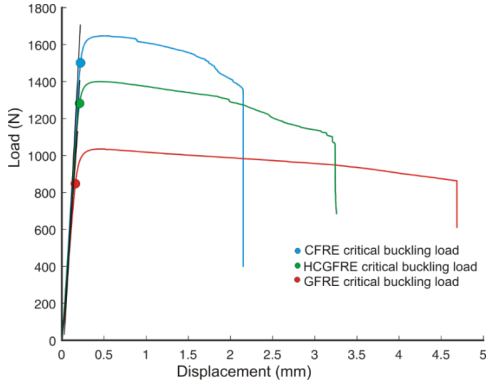

Figure 7.Load-displacement curves for composite laminates with 3 mm hole

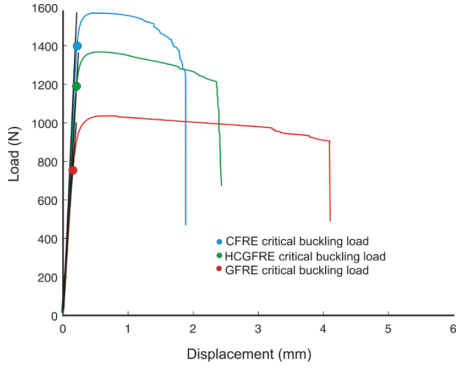

Figure 8.Load-displacement curves for composite laminates with 6 mm hole

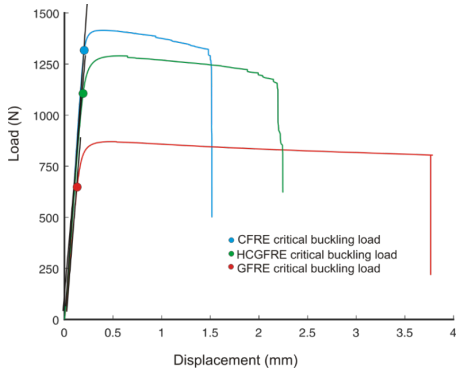

Figure 9.Load-displacement curves for composite laminates with 9 mm hole

As shown in Figure 6-9, it should be stated that the critical buckling load value was determined as the point where the slope line of the linear region cuts the curve tangentially in the load-displacement curves. This process was carried out using the basic tool in the MATLAB program. The maximum critical buckling load was obtained from the CFRE composite laminate without a hole. On the other hand, the minimum critical buckling load was obtained from the GFRE composite laminate with a 9 mm hole diameter. In addition, the critical buckling load of the hybrid composite laminate was determined as an intermediate value in all experimental studies. In other words, the critical buckling load of the hybrid composite was determined differently from both composite laminates used to create it. In general, it is observed that the critical buckling load decreases in all composite laminates as the hole diameter increases due to the notch effect.

It is also seen in Figure 6-9, the linear behavior of the curve changes after the critical buckling load is reached. After this point, the loading process is continued until damage occurs in the composite laminates. The measured critical buckling loads, however, show a certain degree of experimental scatter. The main source of this scatter is the material heterogeneity of composite laminates, the fiber volume fraction, manufacturing tolerances, minor geometrical differences of holes, layer thickness non-uniformity, and test setup imperfections like misalignment of load and boundary conditions. The experimental studies of both composite materials and perforated laminates often report such variability. The damage images of the composite laminates used in the experimental approach are shown in Figure 10. When examining the damaged images, the composite laminates were broken around the hole since the stress concentrations occur at this point due to the notch effect. Moreover, in the hole-free composite laminates, the damage occurred near the midpoint of the composite laminates due to the applied cantilever-clamped boundary condition.

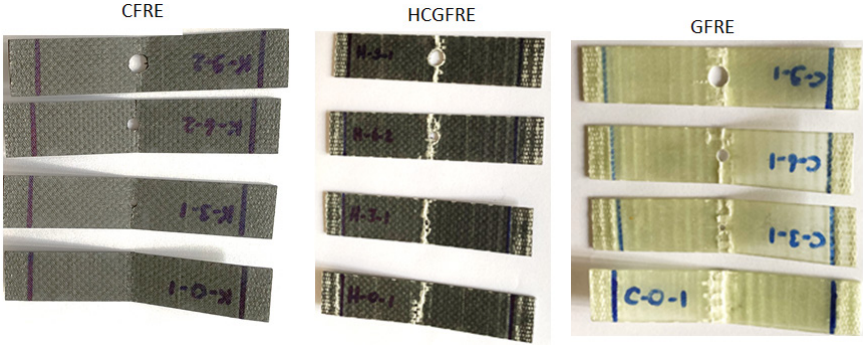

Figure 10.The damage view of composite laminates after the buckling load

The implementation steps of the numerical analysis were explained elaborately in the numerical approach. The critical buckling loads of composite laminates with different hole diameters were determined by applying these steps. The critical buckling load and deformation of a CFRE composite laminate featuring a 9 mm hole structure depicted in Figure 11a. It's important to note that since the numerical analysis was conducted under a single boundary condition, similar buckling mode deformations will be seen in other composite laminates. Due to space constraints, we have not provided the buckling deformations of the other composite laminates here. Furthermore, in numerical analysis, the composite laminates were modeled with Ansys Composite PrepPost, a sub-application of the Ansys finite element program, and merely composite modeling can be done. One of the advantages of the model is to enable layered modeling of composite laminates using appropriate fiber and matrix materials. Thus, the analysis program can calculate the equivalent stress values created on the layers caused by the critical buckling load. It is known that stress values acting on the layers are essential, particularly when determining the damage criteria of layered composites (Soutis, 2012- Whitworth, Aluko O, Tomlinson, 2008). In the verified finite element model, equivalent stresses occurring in the layers of the composite laminate under the buckling load are shown in Figure 11b.

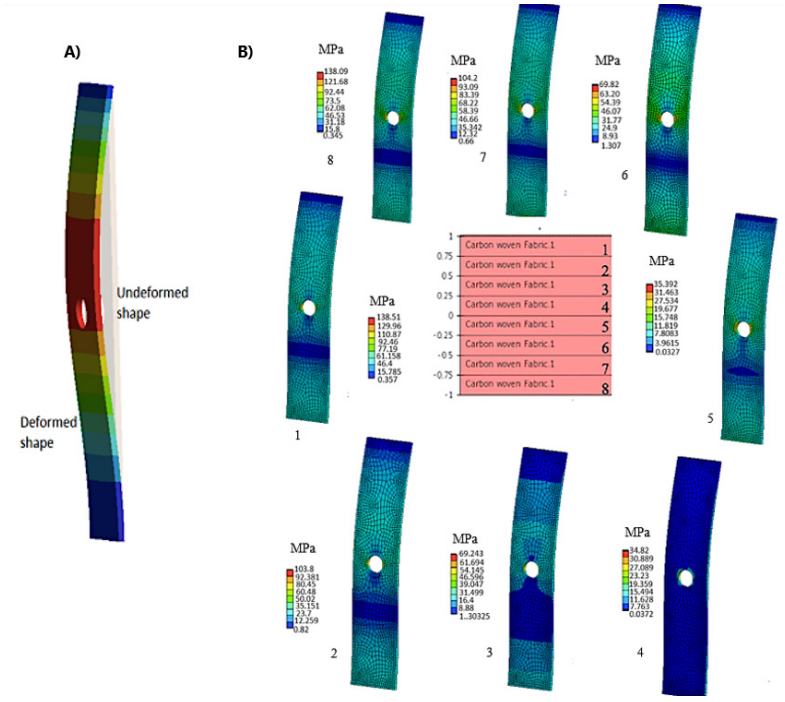

Figure 11.A) Buckling shape of composite laminate B) The equivalent stress occurring at each composite layer under the critical buckling load

As shown in Figure 11b, the top/bottom surfaces of composite laminates have high strength, whereas the interface layers are weak. Therefore, laminated composites are very sensitive to interlayer damage under different loading conditions (Diniz, Cunha, Gomes, 2019) The maximum equivalent stress occurs in the upper and lower layers and decreases towards the middle layers as expected. The maximum and minimum equivalent stress occurring on the composite layer for all composite laminates with different hole diameters are given in Table 5. Also, the buckling shape of the composite laminate was given in the figure to demonstrate its similarity to the deformation observed in the experimental results presented in Figure 4.

Table 5. Equivalent stresses obtained from numerical analyses

		Without hole	3 mm hole	6 mm hole	9 mm hole
CFRE	Minimum equivalent stress (MPa)	14	27	29	34.8
	Maximum equivalent stress (MPa)	57	106	118	128
GFRE	Minimum equivalent stress (MPa)	10.5	18	20	23
	Maximum equivalent stress (MPa)	42	71	80	88
HCGFRE	Minimum equivalent stress (MPa)	11.5	17	19.6	22
	Maximum equivalent stress (MPa)	53	76	84	92

It is seen in Table 5, the minimum and maximum stress values occurring in the plies of the composite laminates increase as the hole diameter increases due to the point stress failure criterion (Soutis, 2012- Whitworth et al. 2008). In addition, the highest stress value was obtained from the plies of CFRE composite laminate for all different hole diameters. On the other hand, the lowest stress values were obtained from the plies of the GFRE composite laminate. It should be stated here that the same is true for the critical buckling load values of the two composite laminates. This situation can be explained by the buckling load resistance of CFRE composite laminates being higher than other composite laminates. The minimum and maximum stress formed in the layers of the HCGFRE composite laminate were determined as the intermediate value of the CFRE and GFRE composite laminates.

Eventually, the critical buckling load determined by the Taguchi-based Fuzzy model was compared with the experimental and numerical results and those are presented in Figure 12.

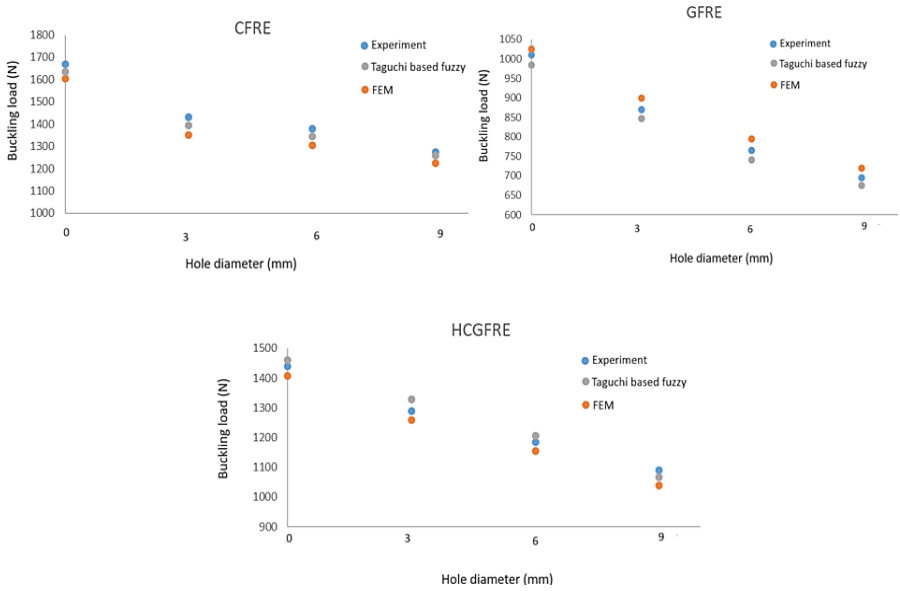

Figure 12. The comparison results between all approaches

As shown in Figure 12, the critical buckling load, determined through all approaches converged with each other. Hence, it was demonstrated that the Taguchi-based fuzzy model can optimize the buckling load values of composite materials with holes of various diameters. This method involves testing a small number of input parameters to determine a buckling load value that aligns with other approaches.

CONCLUSION

In this study, the determination of theoretical buckling values of hybrid composite laminates with different hole diameters using the Taguchi-based fuzzy optimization method was investigated. Experimental and numerical studies were also carried out to verify the results which are obtained from the optimization method. The results can be explained as follows;

• It was determined the constants for calculating the buckling loads of composite laminates with different hole diameters using the Taguchi-based fuzzy method. It was found that the critical buckling load obtained with these parameters closely matched the experimental and numerical results. Thus, it can be concluded that the Taguchi-based fuzzy model is effective in optimizing the buckling load of composite materials with holes of varying diameters using minimal input data.

• It was observed in experimental approach that the maximum critical buckling load was obtained from the CFRE composite laminate without a hole. On the other hand, the minimum critical buckling load was obtained from the GFRE com-

posite laminate with a 9 mm hole diameter. In addition, the critical buckling load of the hybrid composite laminate (HCGFRE) was determined as an intermediate value in all experimental studies. In general, it is observed that the critical buckling load decreases in all composite laminates as the hole diameter increases due to the notch effect.

• In numerical analysis result, the critical buckling loads decrease as the hole diameter increases for all composite laminates. However, the minimum and maximum stress values occurring in the plies of the composite laminates increase as the hole diameter increases due to the point stress failure criterion. In addition, the highest stress value was obtained from the plies of CFRE composite laminate for all different hole diameters. On the other hand, the lowest stress values were obtained from the plies of the GFRE composite laminate. Furthermore, the minimum and maximum stresses formed in the layers of the HCGFRE composite laminate were determined as the intermediate value of the CFRE and GFRE composite laminates. The reason of the composite laminates exhibit different stress values is that it can be attributed to their differing mechanical properties, different fiber and matrix damage behaviors, and different anisotropies.

CONFLICT OF INTEREST

Authors approve that to the best of their knowledge, there is not any conflict of interest or common interest with an institution/organization or a person that may affect the review process of the paper.

CRediT AUTHOR STATEMENT

Engin ERBAYRAK Conceptualization, Methodology, Writing- original draft. **Seda ERBAYRAK** Data curation, Investigation, Writing- review and editing**.**

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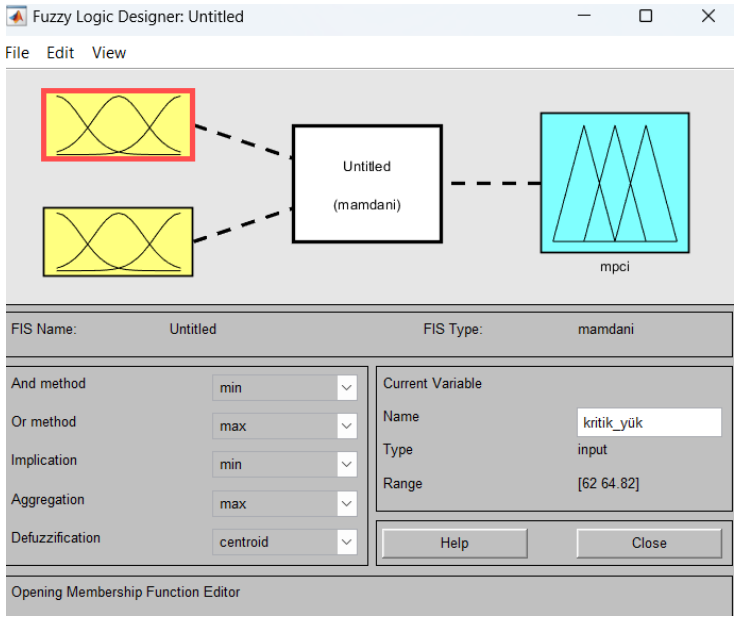
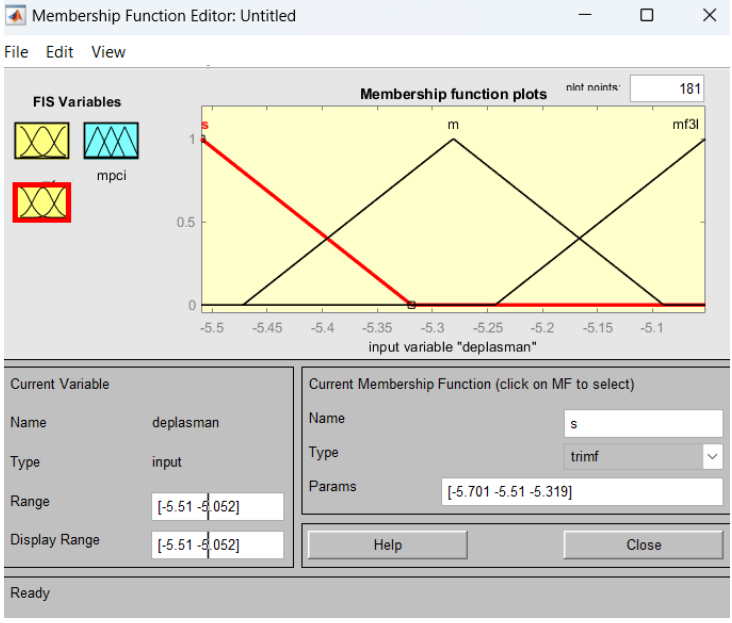
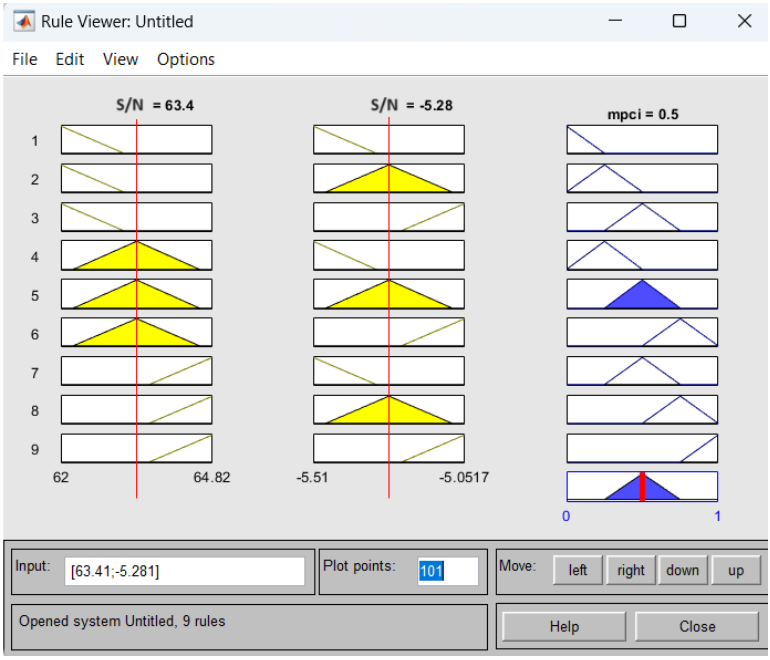
Appendix

1-The steps for calculating MPCI output in Matlab



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