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Some estimator types for population mean using linear transformation with the help of the minimum and maximum values of the auxiliary variable

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Abstract

Grover et al. [4] suggested two product type exponential estimators using linear transformation of auxiliary variable. This paper proposes ratio, product and product type exponential estimators of population mean using new three linear transformations. Transformations using the known minimum and maximum values of the auxiliary variable x have been considered. Theoretically, mean square errors (MSE) and biases equations of our proposed estimators are derived up to the first order approximation. The proposed estimators are more efficient than the classical ones under theoretical conditions. We obtain the superiority regions of the proposed product type exponential estimators. Additionally, we present the diagrams of these regions. A numerical example is perfomed to support the theoretical results.

Keywords: Transformed Auxiliary Variable, Product Type Exponential Estimators, Population Mean, Mean Squared Error, Simple Random Sampling.

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1. Introduction and terminology

The use of auxiliary information to derive the efficiency of estimates of the finite population mean has increased in the theory of sample surveys. We see that, in literature, a number of research papers on various types of estimators using auxiliary information are good illustrations in this context. The ratio and product estimators are more efficient than the regression estimators when the relation between the study variable y and the auxiliary variable x is a straight line through the origin and the variance of y about this line is proportional to x. In many application areas, this line does not passes through the origin. Some authors suggest different type of linear transformations of the auxiliary variable to eliminate this situation. For recent developments on the suggested estimators based on these transformations, several authors including Kadilar and Cingi [7],[8],[9],[10],[11], Cingi and Kadilar [2], Singh and Tailor [15], Singh et al. [16], Shabbir and Gupta [14], Grover et al. [4], Grover and Kaur [5] and Yasmeen et al. [18] have defined some estimators for population mean $\overline{Y}$.

Assume y is the positive valued study variable and x is the positive valued auxiliary variable defined on a finite population of size N by Simple Random Sampling Without Replacement. Let $\overline{X}$ and $\overline{Y}$ be the population means of auxiliary and study variables and $\overline{x}$ and $\overline{y}$ be the sample means, respectively. Suppose that the variable x is known in advance and the variable y is unknown.

The classical ratio, product and product type exponential estimators of population mean $\overline{Y}$ are described as:

$$\overline{y}_R = \frac{\overline{y}}{\overline{x}}\overline{X}, \tag{1.1}$$

$$\overline{y}_p = \frac{\overline{y}}{\overline{X}}\overline{x}, \tag{1.2}$$

and

$$\overline{y}_{pe} = \overline{y}\exp\left(\frac{\overline{x}-\overline{X}}{\overline{x}+\overline{X}}\right) \tag{1.3}$$

by Cochran [3], Robson [17] and Bahl and Tuteja [1], respectively.

Our aim is to minimize the biases and the mean square errors of estimation of unknown population mean $\overline{Y}$. For this reason, we suggest new estimators with the help of some transformations of the auxiliary variable x given by Mohanty and Sahoo [12], using X_m minimum value and X_M maximum value of its known population values. These transformations are defined by

$$a_i = \frac{x_i + X_m}{X_M + X_m}, i = 1, 2, ...N; \overline{a} = \frac{\overline{x} + X_m}{X_M + X_m}, \overline{A} = \frac{\overline{X} + X_m}{X_M + X_m}$$

and

$$b_i = \frac{x_i + X_M}{X_M + X_m}, i = 1, 2, ...N; \overline{b} = \frac{\overline{x} + X_M}{X_M + X_m}, \overline{B} = \frac{\overline{X} + X_M}{X_M + X_m}.$$

Using the transformed variables a and b, Mohanty and Sahoo [12] introduced the following ratio estimators of population mean $\overline{Y}$:

$$\overline{y}_{MSR_1} = \overline{y}\frac{\overline{A}}{\overline{a}} \text{ and } \overline{y}_{MSR_2} = \overline{y}\frac{\overline{B}}{\overline{b}}. \tag{1.4}$$

On the other hand, Singh et al. [16] suggested the following product estimators of population mean $\overline{Y}$:

$$\overline{y}_{STp_1} = \overline{y}\frac{\overline{a}}{\overline{A}} \text{ and } \overline{y}_{STp_2} = \overline{y}\frac{\overline{b}}{\overline{B}}. \tag{1.5}$$

After that, Grover et al. [4] defined the following product type exponential estimators of population mean $\overline{Y}$:

$$\overline{y}_{GKpe_1} = \overline{y}\exp\left(\frac{\overline{a}-\overline{A}}{\overline{a}+\overline{A}}\right) \text{ and } \overline{y}_{GKpe_2} = \overline{y}\exp\left(\frac{\overline{b}-\overline{B}}{\overline{b}+\overline{B}}\right). \tag{1.6}$$

2. Proposed estimators

Let X_m and X_M be the minimum and maximum values of a known positive auxiliary variable x, respectively. Using X_m and X_M, we give the following transformed auxiliary variables:

$$t_1 = \frac{x_i + X_m}{X_M - X_m},\ t_2 = \frac{x_i + X_M}{X_M - X_m},\ t_3 = \frac{x_i - X_m}{X_M - X_m},\ t_4 = \frac{x_i - X_M}{X_M - X_m}, \tag{2.1}$$

$$i = 1, 2, ...N\ \ (X_m \neq X_M).$$

If the auxiliary variable x is available in advance or in the past experience or in the pilot study, its extreme values (X_m and X_M) are easily available. Thus we can see that the new variables t_1, t_2 and t_3 are standardized in the intervals $t_1 > 0$, $t_2 > 1$ and $t_3 \in [0, 1]$. Because the variable t_4 take a negative value range, i.e. $t_4 \in [-1, 0]$, we do not use the variable t_4 in this study.

Before we propose the estimators, we give the following sample means of t_i, $i = 1, 2, 3$ and population means of $T_i, i = 1, 2, 3$ respectively:

$$\bar{t}_1 = \frac{\overline{x} + X_m}{X_M - X_m},\ \bar{t}_2 = \frac{\overline{x} + X_M}{X_M - X_m},\ \bar{t}_3 = \frac{\overline{x} - X_m}{X_M - X_m},$$

and

$$\overline{T}_1 = \frac{\overline{X} + X_m}{X_M - X_m},\ \overline{T}_2 = \frac{\overline{X} + X_M}{X_M - X_m},\ \overline{T}_3 = \frac{\overline{X} - X_m}{X_M - X_m}.$$

Now let us define ratio estimators with the help of the transformed auxiliary variables:

$$\overline{y}_{CCR_i} = \overline{y}\frac{\overline{T_i}}{\bar{t}_i},\ i = 1, 2, 3. \tag{2.2}$$

On the other hand, we propose product estimators based on the transformations:

$$\overline{y}_{CCp_i} = \overline{y}\frac{\bar{t}_i}{\overline{T_i}},\ i = 1, 2, 3. \tag{2.3}$$

Finally, product type exponential estimators are defined as follows:

$$\overline{y}_{CCpei} = \overline{y}\exp\left(\frac{\bar{t}_i - \overline{T}_i}{\bar{t}_i + \overline{T}_i}\right),\ i = 1, 2, 3. \tag{2.4}$$

To compute the biases and the mean squared errors (MSE) of proposed estimators, we define

$$e_1 = \frac{\overline{t_1} - \overline{T_1}}{\overline{T_1}}, e_2 = \frac{\overline{t_2} - \overline{T_2}}{\overline{T_2}},\ e_3 = \frac{\overline{t_3} - \overline{T_3}}{\overline{T_3}}, e_4 = \frac{\overline{y} - \overline{Y}}{\overline{Y}},\ e_5 = \frac{\overline{x} - \overline{X}}{\overline{X}}. \tag{2.5}$$

We have the following expectations:

$$\begin{aligned}
E(e_1) &= E(e_2) = E(e_3) = E(e_4) = E(e_5) = 0, \\
E(e_1e_4) &= \gamma\frac{C_{yx}}{\theta_1}, E(e_2e_4) = \gamma\frac{C_{yx}}{\theta_2}, E(e_3e_4) = \gamma\frac{C_{yx}}{\theta_3}, E(e_4e_5) = \gamma C_{yx}, \\
E(e_1^2) &= \gamma\left(\frac{S_x}{\overline{X}+X_m}\right)^2 = \gamma\frac{C_x^2}{\theta_1^2}, E(e_2^2) = \gamma\left(\frac{S_x}{\overline{X}+X_M}\right)^2 = \gamma\frac{C_x^2}{\theta_2^2}, \\
E(e_3^2) &= \gamma\left(\frac{S_x}{\overline{X}-X_m}\right)^2 = \gamma\frac{C_x^2}{\theta_3^2}, E(e_4^2) = \gamma C_y^2, E(e_5^2) = \gamma C_x^2,
\end{aligned}$$

where

$$\gamma = \frac{N-n}{Nn},\ \theta_1 = \frac{\overline{X}+X_m}{\overline{X}},\ \theta_2 = \frac{\overline{X}+X_M}{\overline{X}},\ \theta_3 = \frac{\overline{X}-X_m}{\overline{X}},$$

$$C_{yx} = \frac{S_{yx}}{\overline{Y}\overline{X}}, \rho = \frac{S_{yx}}{S_yS_x}, S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\overline{Y}\right)\left(x_i-\overline{X}\right),$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\overline{Y}\right)^2, S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\overline{X}\right)^2,$$

and C_y, C_x are the coefficients of variation of y and x respectively. Hence we obtain that $\theta_1 \in (1,2]$, $\theta_2 \in [2,+\infty)$ and $\theta_3 \in [0,1)$. Replacing (2.5) into (2.2), (2.3) and (2.4), to the first order approximation, we have

$$\overline{y}_{CCR_i} = \overline{Y}\left(1+e_4-e_i-e_4e_i+e_i^2\right),\ i=1,2,3, \tag{2.6}$$

$$\overline{y}_{CCp_i} = \overline{Y}\left(1+e_4+e_i+e_4e_i\right),\ i=1,2,3, \tag{2.7}$$

$$\overline{y}_{CCpe_i} = \overline{Y}\left(1+e_4+\frac{e_i}{2}-\frac{e_i^2}{8}+\frac{e_4e_i}{2}\right),\ i=1,2,3. \tag{2.8}$$

In order to obtain the mean square errors of $\overline{y}_{CCR_i}$, $\overline{y}_{CCp_i}$ and $\overline{y}_{CCpe_i}$, using (2.6), (2.7), (2.8) and taking square and expectation of both sides, we can write

$$E\left(\overline{y}_{CCR_i}-\overline{Y}\right)^2 \cong \overline{Y}^2E\left(e_4^2-2e_4e_i+e_i^2\right),\ i=1,2,3, \tag{2.9}$$

$$E\left(\overline{y}_{CCp_i}-\overline{Y}\right)^2 \cong \overline{Y}^2E\left(e_4^2+2e_4e_i+e_i^2\right),\ i=1,2,3, \tag{2.10}$$

$$E\left(\overline{y}_{CCpe_i}-\overline{Y}\right)^2 \cong \overline{Y}^2E\left(e_4^2+\frac{e_i^2}{4}+e_4e_i\right),\ i=1,2,3. \tag{2.11}$$

Then we derive the biases and MSE of the proposed estimators in (2.9), (2.10) and (2.11). The obtained equations, the biases and MSE of the conventional estimators are shown in Table 1.

Table 1. The biases and MSE of the proposed and traditional estimators

Estimators	Bias	MSE
$\overline{y}$	$Bias(\overline{y}) = 0$	$MSE(\overline{y}) = \gamma\overline{Y}^2 C_y^2$
$\overline{y}_R$	$Bias(\overline{y}_R) = \gamma\overline{Y}(C_x^2 - C_{yx})$	$MSE(\overline{y}_R) = \gamma\overline{Y}^2(C_y^2 + C_x^2 - 2C_{yx})$
$\overline{y}_p$	$Bias(\overline{y}_p) = \gamma\overline{Y}C_{yx}$	$MSE(\overline{y}_p) = \gamma\overline{Y}^2(C_y^2 + C_x^2 + 2C_{yx})$
$\overline{y}_{pe}$	$Bias(\overline{y}_{pe}) = \gamma\overline{Y}\left(\frac{C_{yx}}{2} - \frac{C_x^2}{8}\right)$	$MSE(\overline{y}_{pe}) = \gamma\overline{Y}^2\left(C_y^2 + \frac{C_x^2}{4} - C_{yx}\right)$
$\overline{y}_{CCR_i}$	$Bias(\overline{y}_{CCR_i}) = \gamma\overline{Y}\left(\frac{C_x^2}{\theta_i^2} - \frac{C_{yx}}{\theta_i}\right)$	$MSE(\overline{y}_{CCR_i}) = \gamma\overline{Y}^2\left(C_y^2 + \frac{C_x^2}{\theta_i^2} - 2\frac{C_{yx}}{\theta_i}\right)$
$\overline{y}_{CCpi}$	$Bias(\overline{y}_{CCpi}) = \gamma\overline{Y}\frac{C_{yx}}{\theta_i}$	$MSE(\overline{y}_{CCp_i}) = \gamma\overline{Y}^2\left(C_y^2 + \frac{C_x^2}{\theta_i^2} + 2\frac{C_{yx}}{\theta_i}\right)$
$\overline{y}_{CCpe_i}$	$Bias(\overline{y}_{CCpe_i}) = \gamma\overline{Y}\left(\frac{C_{yx}}{2\theta_i} - \frac{C_x^2}{8\theta_i^2}\right)$	$MSE(\overline{y}_{CCpe_i}) = \gamma\overline{Y}^2\left(C_y^2 + \frac{C_x^2}{4\theta_i^2} + \frac{C_{yx}}{\theta_i}\right)$

Note that $i = 1, 2, 3$ in Table 1.

3. Efficiency comparisons of estimators

In this section, we compare our estimators with traditional estimators in terms of the MSE, demonstrated in Table 1 where $H = \frac{C_x}{2C_y}$ and $\delta_i = \frac{1}{\theta_i}$. When the correlation coefficients between y and $t_i, i = 1, 2, 3$ are high positive values, the ratio type estimators are more efficient than the product type estimators. Thus, we compare the ratio type estimators with each other. Firstly, comparing the MSE of our new ratio estimators with the MSE of the classical ratio estimator, we have

$$MSE(\overline{y}_{CCR_i}) < MSE(\overline{y}_R) \quad \text{if} \quad \rho < H(1+\delta_i),\ i = 1, 2, \tag{3.1}$$

$$MSE(\overline{y}_{CCR_3}) < MSE(\overline{y}_R) \quad \text{if} \quad \rho > H(1+\delta_3). \tag{3.2}$$

Secondly, a comparison of the MSE of our new product estimators with the MSE of the classical product estimator, we get

$$MSE(\overline{y}_{CCp_i}) < MSE(\overline{y}_p) \quad \text{if} \quad \rho > -H(1+\delta_i),\ i = 1, 2, \tag{3.3}$$

$$MSE(\overline{y}_{CCp_3}) < MSE(\overline{y}_p) \quad \text{if} \quad \rho < -H(1+\delta_3). \tag{3.4}$$

If the correlation coefficient between y and $t_i, i = 1, 2, 3$ have high negative values, the product type estimators are quite effective. Thus, we compare the product type estimators with each other. Finally, we compare the MSE of our new product type exponential estimators with the MSE of the classical product estimator, the product type exponential estimator given by Bahl and Tuteja [1] and our new product estimators. We obtain

$$MSE(\overline{y}_{CCpe_i}) < MSE(\overline{y}_p) \text{ if } \rho > -H(1+\frac{\delta_i}{2}) \text{ when } \frac{1}{\delta_i} > \frac{1}{2},\ i = 1, 2, \tag{3.5}$$

$$MSE(\overline{y}_{CCpe_3}) < MSE(\overline{y}_p) \text{ if } \rho < -H(1+\frac{\delta_3}{2}) \text{ when } \frac{1}{\delta_3} < \frac{1}{2}, \tag{3.6}$$

$$MSE(\overline{y}_{CCpe_i}) < MSE(\overline{y}_{pe}) \text{ if } \rho > -\frac{H}{2}(1+\delta_i),\ i = 1, 2, \tag{3.7}$$

$$MSE(\overline{y}_{CCpe_3}) < MSE(\overline{y}_{pe}) \text{ if } \rho < -\frac{H}{2}(1+\delta_3), \tag{3.8}$$

$$
\begin{array}{llll}
(3.9) & MSE(\overline{y}_{CCpe_i}) & < & MSE(\overline{y}_{CCp_i}) \quad \text{if} \quad \rho > -\dfrac{3H\delta_i}{2},\ i=1,2,3,\\
(3.10) & MSE(\overline{y}_{CCpe_1}) & < & MSE(\overline{y}_{CCp_2})\\
 & \text{if either } \rho & < & -H(\dfrac{\delta_1}{2}+\delta_2) \quad \text{when} \quad \dfrac{1}{\delta_2}-\dfrac{2}{\delta_1}>0,\\
 & \text{or } \rho & > & -H(\dfrac{\delta_1}{2}+\delta_2) \quad \text{when} \quad \dfrac{1}{\delta_2}-\dfrac{2}{\delta_1}<0,\\
(3.11) & MSE(\overline{y}_{CCpe_1}) & < & MSE(\overline{y}_{CCp_3}) \quad \text{if} \quad \rho > -H(\dfrac{\delta_1}{2}+\delta_3),\\
(3.12) & MSE(\overline{y}_{CCpe_2}) & < & MSE(\overline{y}_{CCp_i}) \quad \text{if} \quad \rho > -H(\dfrac{\delta_2}{2}+\delta_i),\ i=1,3,\\
(3.13) & MSE(\overline{y}_{CCpe_3}) & < & MSE(\overline{y}_{CCp_1})\\
 & \text{if either } \rho & > & -H(\dfrac{\delta_3}{2}+\delta_1) \quad \text{when} \quad \dfrac{1}{\delta_1}-\dfrac{2}{\delta_3}<0,\\
 & \text{or } \rho & < & -H(\dfrac{\delta_3}{2}+\delta_1) \quad \text{when} \quad \dfrac{1}{\delta_1}-\dfrac{2}{\delta_3}>0,\\
(3.14) & MSE(\overline{y}_{CCpe_3}) & < & MSE(\overline{y}_{CCp_2}) \quad \text{if} \quad \rho < -H(\dfrac{\delta_3}{2}+\delta_2).
\end{array}
$$

Furthermore, we obtain the superiority regions of the proposed new product type exponential estimators. The superiority regions are derived from the results (3.5)-(3.14). In other words, these regions demonstrate reason for preference of the proposed new product type exponential estimators with regards to the MSE.

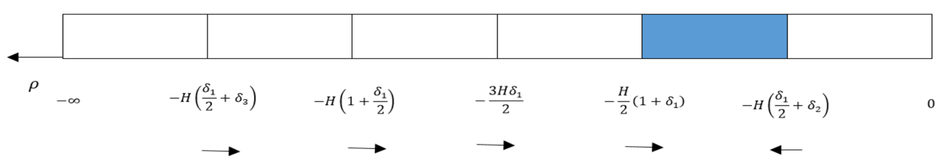

Figure 1. The superiority region of the $\overline{y}_{CCpe_1}$ when $\frac{1}{\delta_2}-\frac{2}{\delta_1}>0$.

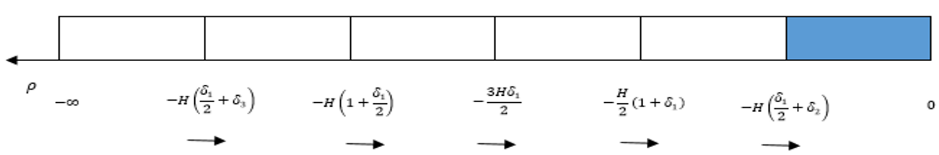

Figure 2. The superiority region of the $\overline{y}_{CCpe_1}$ when $\frac{1}{\delta_2}-\frac{2}{\delta_1}<0$.

We obtain the superiority region of the estimator $\overline{y}_{CCpe_i}$ for $i=1,2,3$ from the values of ρ, which are indicated from comparisons of mean square errors. Figure 1 and Figure 2 show the superiority regions of the $\overline{y}_{CCpe_1}$ for $\frac{1}{\delta_2}-\frac{2}{\delta_1}>0$ and $\frac{1}{\delta_2}-\frac{2}{\delta_1}<0$, respectively. If we examine Figure 1 under the condition $\frac{1}{\delta_2}-\frac{2}{\delta_1}>0$, $\overline{y}_{CCpe_1}$ is preferred over $\overline{y}_p, \overline{y}_{pe}, \overline{y}_{CCp_i}; i=1,2,3$ when $-\frac{H}{2}(1+\delta_1)<\rho<-H(\frac{\delta_1}{2}+\delta_2)$.

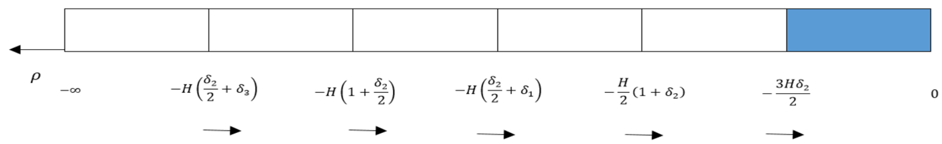

Figure 3. The superiority region of the $\overline{y}_{CCpe_2}$.

The superiority region of the $\overline{y}_{CCpe_2}$ is shown in Figure 3 for all cases. If we examine Figure 3, $\overline{y}_{CCpe_2}$ is preferred over $\overline{y}_p, \overline{y}_{pe}, \overline{y}_{CCp_i}; i = 1, 2, 3$ when $-\frac{3H\delta_2}{2} < \rho < 0$.

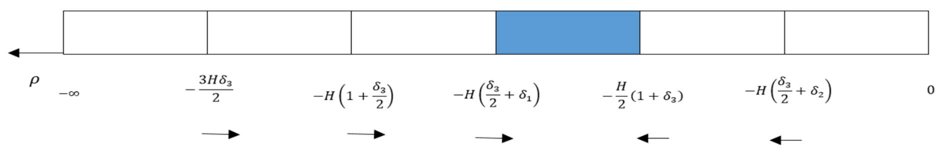

Figure 4. The superiority region of the $\overline{y}_{CCpe_3}$ when $\frac{1}{\delta_1} - \frac{2}{\delta_3} < 0$.

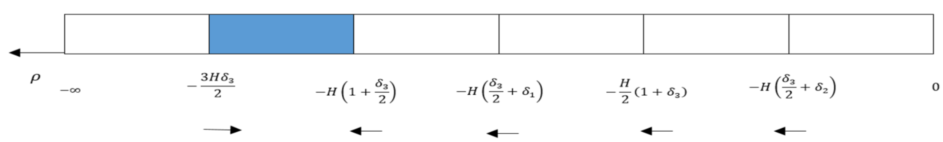

Figure 5. The superiority region of the $\overline{y}_{CCpe_3}$ when $\frac{1}{\delta_1} - \frac{2}{\delta_3} > 0$.

The superiority regions of the $\overline{y}_{CCpe_3}$ are shown in Figure 4 and Figure 5 for $\frac{1}{\delta_1} - \frac{2}{\delta_3} < 0$ and $\frac{1}{\delta_1} - \frac{2}{\delta_3} > 0$, respectively. If we examine Figure 5 under the condition $\frac{1}{\delta_1} - \frac{2}{\delta_3} > 0, \overline{y}_{CCpe_3}$ is preferred over $\overline{y}_p, \overline{y}_{pe}, \overline{y}_{CCp_i}; i = 1, 2, 3$ when $-\frac{3H\delta_3}{2} < \rho < -H(1 + \frac{\delta_3}{2})$.

4. Numerical example

We compare numerically the superiority between the suggested and classical estimators with regard to relative efficiencies. The data set I consists of the egg production as the study variable and the egg prices as the auxiliary variable for 50 states in the USA for 1990 and 1991 [6]. On the other hand, the data set II consists of the foreign exchange rates in United Kingdom as the study variable and in Switzerland as the auxiliary variable for years 1977-1998 [6]. We use simple random sampling and assume sample sizes of $n = 20$ and $n = 9$ for the population I and the population II, respectively. We give the descriptions of populations in Table 2.

Table 2. Data Statistics

Population I			Population II		
$N = 50$	$C_y = 1.21$	$C_x = 0.29$	$N = 22$	$C_y = 0.14$	$C_x = 0.22$
$\rho = -0.31$	$H = 0.12$		$\rho = -0.17$	$H = 0.31$	
$\theta_1 = 1.60$	$\theta_2 = 2.96$	$\theta_3 = 0.40$	$\theta_1 = 1.70$	$\theta_2 = 2.45$	$\theta_3 = 0.30$

The relative efficiency value is described by the following equation:

$$RE(t,\overline{y}) = \frac{MSE(\overline{y})}{MSE(t)}$$

where t is the relative efficiency of an estimator and $\overline{y}$ is the considered usual unbiased estimator. In Table 3, we calculate the relative efficiency values of the concerned estimators for these data sets. If these values are greater than one, the concerned estimators are more efficient than the compared estimators. Moreover, the superiority regions of the suggested product type exponential estimators are given in Table 4.

Table 3. The relative efficiencies of the concerned estimators

Population I ($\rho = -0.31$)			Population II ($\rho = -0.17$)		
No	Estimators	RE	No	Estimators	RE
1	$\overline{y}$	1.0000	1	$\overline{y}$	1.0000
2	$\overline{y}_R$	0.8265	2	$\overline{y}_R$	0.2548
3	$\overline{y}_p$	1.1018	3	$\overline{y}_p$	0.3468
4	$\overline{y}_{pe}$	0.9172	4	$\overline{y}_{pe}$	0.5372
5	$\overline{y}_{CCR_1}$	0.8952	5	$\overline{y}_{CCR_1}$	0.4671
6	$\overline{y}_{CCR_2}$	0.9455	6	$\overline{y}_{CCR_2}$	0.6200
7	$\overline{y}_{CCR_3}$	0.5698	7	$\overline{y}_{CCR_3}$	0.0344
8	$\overline{y}_{CCp_1}$	1.0769	8	$\overline{y}_{CCp_1}$	0.6548
9	$\overline{y}_{CCp_2}$	1.0464	9	$\overline{y}_{CCp_2}$	0.8420
10	$\overline{y}_{CCp_3}$	1.0075	10	$\overline{y}_{CCp_3}$	0.0390
11	$\overline{y}_{CCpe_1}$	1.0432	11	$\overline{y}_{CCpe_1}$	0.9478
12	$\overline{y}_{CCpe_2}$	1.0244	12	$\overline{y}_{CCpe_2}$	1.0063
13	$\overline{y}_{CCpe_3}$	1.1077	13	$\overline{y}_{CCpe_3}$	0.1487

It is shown in Table 3, the relative efficiency values are greater than one for the last six proposed estimators (i.e. between the eighth and thirteenth estimators) in the population I. Also for the population II, we have only one proposed estimator (i.e. the twelfth estimator) which the relative efficiency value is greater than one. The number of the proposed estimators, having relative efficiency values are less than one, is more than the number of other estimators in Table 3, because the correlations in the population I and II are low negative values, which are the same as Grover et al.'s [4] results. It is important to determine the estimator of the minimum MSE by means of the value of ρ.

Table 4. The superiority regions of the $\overline{y}_{CCpe_i}, i = 1,2,3$ estimators

Population I ($\rho = -0.31$)			Population II ($\rho = -0.17$)		
Estimators	Superiority regions	RE	Estimators	Superiority regions	RE
$\overline{y}_{CCpe_1}$	$(-0.113, -0.098)$	1.0432	$\overline{y}_{CCpe_1}$	$(-0.2740, -0.2464)$	0.9478
$\overline{y}_{CCpe_2}$	$(-0.061, 0)$	1.0244	$\overline{y}_{CCpe_2}$	$(-0.1898, 0)$	**1.0063**
$\overline{y}_{CCpe_3}$	$(-0.459, -0.274)$	**1.1077**	$\overline{y}_{CCpe_3}$	$(-1.5397, -0.8233)$	0.1487

As it is seen from Table 4, the bold values are the best values among the relative efficiency of the proposed estimators. The superiority regions in Table 4 provides the best estimator through the value of ρ. In population I, the best estimator is $\overline{y}_{CCpe_3}$, because the superiority regions of $\overline{y}_{CCpe_3}$ include the value of ρ. Otherwise, the best relative efficiency value is the estimator of $\overline{y}_{CCpe_2}$, because the superiority regions of $\overline{y}_{CCpe_2}$ include the value of ρ in population II.

5. Results

In this study, we have proposed new ratio, product and product type exponential estimators for simple random sampling by mean of linear transformation of auxiliary variables. From the theoretical section of this study, we conclude that the suggested estimators are more efficient than the considered estimators under the theoretical conditions. From the results in Table 4, we see that the value of ρ is -0.31 for the data set I. This value is in the superiority region of the $\overline{y}_{CCpe_3}$ and it is the biggest among the relative efficiency values of all compared estimators. On the other hand, the value of ρ for the data set II is in the superiority region of the $\overline{y}_{CCpe_2}$. Similarly, the $\overline{y}_{CCpe_2}$ estimator has bigger relative efficiency values than all compared estimators. If the value of ρ is known in advance, we can determine the estimator of the minimum MSE among the compared estimators using the superiority regions. As a result, we can find the most efficient estimator among all the studied estimators easily when the value of ρ is available.

6. Conclusions

In the forthcoming studies, new estimators should be proposed taking power of the defined transformations in this study. Furthermore, the estimators should be suggested using transformations with the help of the known median value of the auxiliary variable. Moreover, they should be proposed in different sampling methods such as systematic sampling, stratified sampling, two phase sampling and successive sampling etc.

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