A general procedure for estimating population variance in successive sampling using fuzzy tools

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Abstract

This paper defines a general class of estimators for estimating population variance on current occasion in two occasion successive sampling. Detail behaviors of the proposed class of estimators have been studied and its optimum replacement strategy has also been discussed. The proposed class of estimators has been compared with the sample variance estimator and the results obtained are demonstrated through empirical studies. Categorization of the dominance ranges of the proposed estimation strategies are deployed through defuzzification tools which are followed by suitable recommendations.

1. Introduction

It is well known that in sampling designs the use of auxiliary information improves the precision of estimates substantially. The theory and practice of surveying the same population at different points of time technically called repetitive sampling or sampling over successive occasions have been given considerable attention by survey statisticians. When a population is subject to change, a survey carried out on a single occasion will provide information about the characteristics of the surveyed population for the given occasion only, and cannot, of itself, give any information about (a) the rate of change of the characteristics over different occasions, and (b) the average value of the characteristics

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over all occasions or for the most recent occasion. To meet these requirements sampling is done on successive occasions. For examples; monthly data on the prices of goods are collected to determine the consumer price index and political opinion surveys are conducted at regular intervals to know the voters preference, etc. Estimates of change are wanted mainly in attempts to study the effects of forces that are known to have acted on the population. Most governments collect information regularly on the same population to find out, say the number of persons unemployed, the change in employment from time to time etc.[24]. Theory of successive sampling appears to have started with the work of [6]. It was further extended by [9], [10],[4], [2] among others. In successive sampling, it is common practice to use the information collected on previous occasion as auxiliary information to improve the precision of the estimates on current occasion. In many situations, information on an auxiliary variable may be readily available on the first as well as on the second occasion, for examples; tonnage (or seat capacity) of each vehicle or ship is known in survey sampling of transportation, number of polluting industries and vehicles are known in environmental survey. Likewise, there may be several information available, which if efficiently utilized can improve the precision of the estimates.

Variation is an inherent phenomenon of nature; it is present everywhere in our day to day life. For instance, a physician needs a full understanding of variations in the degree of human blood pressure, body temperature and pulse rate for adequate prescription. A manufacturer needs constant knowledge of the level of variations in people's reaction to his product to be able to know whether to reduce or increase the price or improve the quality of his product. Many more situations can be encountered in practice where the estimation of population variance of the study character assume importance. It is worth to be mentioned that limited number of attempts have been made to the estimation of the population variance in successive sampling. The works of [13] and [14] may be referred in this context.

Motivated by above, the present study is an attempt to introduce a general estimation procedure of population variance on the current (second) occasion in two-occasion successive sampling. Its properties have been studied and the efficacy of the proposed work has been examined through empirical studies. To categorize the dominance ranges of the proposed estimation strategies, defuzzification tools are employed. Recommendations of the proposed estimation strategy have been put forward to the survey statisticians.

2. Formulation of proposed estimation strategies

2.1. Sample structures and notations. Let $U = (U_1, U_2, U_3, ..., U_N)$ be the finite population of N units, which has been sampled over two occasions. The character under study is denoted by x(y) on the first (second) occasions, respectively. It is assumed that information on an auxiliary variable z whose population variance is known, closely related (positively correlated) to x and y on the first and second occasions, respectively, available on the first as well as on the second occasion. For convenience, it is assumed that the population under consideration is considerably large enough. A simple random sample (without replacement) of n units is drawn on the first occasion. A random sub-sample of $m = n\lambda$ units is retained (matched) for its use on the second occasion, while a fresh simple random sample (without replacement) of $u = (n - m) = n\mu$ units are drawn on the second occasion from the entire population so that the sample size on the second occasion is also n. Here λ and μ $(\lambda + \mu = 1)$ are the fractions of the matched and fresh samples, on the current(second) occasion respectively.

Hence onwards, we consider the following notations for their further use:

S_x^2, S_y^2, S_z^2:The population variances of the variables x, y and z, respectively.

s_{ym}^2: Sample variance of the variable y based on the sample of size m.

s_{xm}^2: Sample variance of the variable x based on the sample of size m.

s_{xn}^2: Sample variance of the variable x based on the sample of size n.

s_{zn}^2: Sample variance of the variable z based on the sample of size n.

s_{yu}^2: Sample variance of the variable y based on the sample of size u.

s_{zu}^2: Sample variance of the variable z based on the sample of size u.

2.2. Proposed class of estimators. To estimate the population variance S_y^2 on the current (second) occasion, two independent class of estimators are suggested. The class of estimators t_C is based on sample of size $u = n\mu$ drawn afresh on the second occasion and the class of estimators t_A is based on the matched sample of size $m = n\lambda$ common to both occasions.

2.2.1. *Estimators based on the sample drawn afresh on the second occasion.* Motivated with the conventional estimation procedures of population mean, one may propose the following ratio and product type estimators of population variance S_y^2 based on the sample of size u drawn afresh on the current (second) occasion as

$C_{1u} = s_{yu}^2 \frac{S_z^2}{s_{zu}^2}$,

$C_{2u} = s_{yu}^2 \frac{s_{zu}^2}{S_z^2}$,

$C_{3u} = s_{yu}^2 \left(\frac{s_{zu}^2}{S_z^2}\right)^\alpha$,

$C_{4u} = s_{yu}^2 [2 - \left(\frac{s_{zu}^2}{S_z^2}\right)^\alpha]$,

where α is a constant, chosen suitably, so that the mean square errors of the above defined class of estimators may be minimized.

Inspired with the above discussions and following the estimation strategies of population mean adopted [21] [22] and [16], we consider the following general class of estimators of population variance S_y^2 as

$$(2.1) \qquad t_C = C(s_{yu}^2, s_{zu}^2)$$

where $C(s_{yu}^2, s_{zu}^2)$ is a function of (s_{yu}^2, s_{zu}^2) such that

$$(2.2) \qquad C(S_y^2, S_z^2) = S_y^2 \rightarrow C_1(Q) = d_1 = \frac{\partial C(.)}{\partial s_{yu}^2} = 1$$

with $Q = (S_y^2, S_z^2)$ and $C(s_{yu}^2, s_{zu}^2)$ satisfies the following conditions :

1. Whatever be the sample chosen, (s_{yu}^2, s_{zu}^2) assume values in a bounded closed convex subset R, of the two- dimensional real space containing the points (S_y^2, S_z^2) .

2. The function $C(s_{yu}^2, s_{zu}^2)$ is continuous and bounded in R.

3. The first, second, and third partial derivatives of $C(s_{yu}^2, s_{zu}^2)$ exist and are continuous and bounded in R.

2.2.2. *Estimators based on the matched sample which is common to both occasions.* In successive sampling, it is common practice to use the information collected on the previous occasion as auxiliary information, to improve the precision of the estimates on current occasion. In follow up of standard practice and using the concept of double sampling one may suggest ratio type estimator of S_y^2 which is based on the matched sample of size m at current (second) occasion as

$t_{1m} = s_{ym}^2 \frac{s_{xn}^2}{s_{xm}^2}$

It is well known that, if the correlation between x and z is highly positive, $\frac{s_{xn}^2}{s_{zn}^2} S_z^2$ will estimate S_x^2 more precisely than s_{xn}^2 . Following [1] technique, one may propose ratio

type estimators for population variance S_y^2 as

$$t_{2m} = \left(\frac{s_{ym}^2}{s_{xm}^2}\right)\left(\frac{s_{xn}^2}{s_{zn}^2}\right)S_z^2$$

$$t_{3m} = \left(\frac{s_{ym}^2}{s_{xm}^2}\right)\left(\frac{s_{xn}s_{zn}^2}{S_z^2}\right)$$

$$t_{4m} = \left(\frac{s_{ym}^2 s_{xm}^2}{s_{xn}^2 s_{zn}^2}\right)S_z^2$$

Motivated by the above estimation techniques, we define a class of estimators of population variance S_y^2 on current (second) occasion based on the matched sample of size m as

$$(2.3) \qquad t_A = A(s_{ym}^2, a, b, c)$$

where $a = \frac{s_{xm}^2}{s_{xn}^2}$, $b = \frac{s_{zm}^2}{s_{zn}^2}$, $c = \frac{s_{zn}^2}{S_z^2}$ and $A(s_{ym}^2, a, b, c)$ is a function of (s_{ym}^2, a, b, c) such that

$$(2.4) \qquad A(T) = S_y^2 \rightarrow A_1(T) = \frac{\partial A(.)}{\partial s_{ym}^2} = 1$$

where $T = (S_y^2, 1, 1, 1)$ and $A(s_{ym}^2, a, b, c)$ satisfy conditions similar to those given for t_C in equation (2.1).

2.2.3. *Composite class of estimators.* Combining the class of estimators t_C and t_A, we have the following composite class of estimators of population variance S_y^2 as,

$$(2.5) \qquad t = \phi t_C + (1-\phi)t_A$$

where the class of estimators t_C and t_A are respectively defined in equations (2.1) and (2.3) respectively and ϕ is a scalar quantity to be chosen suitably.

2.3. Remark. (i) The proposed class of estimators t_C of S_y^2 is very wide in sense that for any parametric function $C(s_{yu}^2, s_{zu}^2)$ satisfying regularity conditions and $C(s_{yu}^2, s_{zu}^2) = S_y^2$, for all S_y^2 can generate estimators of the class t_C. For examples; the following estimators are the member of the class of estimators t_C.

$C_{1u} = s_{yu}^2 \frac{S_z^2}{s_{zu}^2}$, $C_{2u} = s_{yu}^2 \frac{s_{zu}^2}{S_z^2}$, $C_{3u} = s_{yu}^2\left(\frac{s_{zu}^2}{S_z^2}\right)^\alpha$, $C_{4u} = s_{yu}^2\left[2 - \left(\frac{s_{zu}^2}{S_z^2}\right)^\alpha\right]$,

$C_{5u} = s_{yu}^2 \exp\left(\frac{S_z^2 - s_{zu}^2}{S_z^2 + s_{zu}^2}\right)$, $C_{6u} = s_{yu}^2 \exp\left(\frac{s_{zu}^2 - S_z^2}{s_{zu}^2 + S_z^2}\right)$ $C_{7u} = s_{yu}^2 + \alpha(S_z^2 - s_{zu}^2)$

$C_{8u} = \alpha s_{yu}^2 + (1-\alpha)s_{yu}^2 \frac{S_z^2}{s_{zu}^2}$ $C_{9u} = \alpha s_{yu}^2 + (1-\alpha)s_{yu}^2 \frac{s_{zu}^2}{S_z^2}$

and

$C_{10u} = \frac{s_{yu}^2 S_z^2}{[S_Z^2 + \alpha(S_z^2 + s_z u^2)]}$,

where α is real constant and its optimum value can be determined so that it satisfy the respective normal equations and the resulting estimators would have the same minimum mean square errors as obtained for the class of t_C.

(ii) It may be noted that the estimators

$r_1 = s_{xn}^2 \frac{S_z^2}{s_{zn}^2}$, $r_2 = s_{xn}^2 \frac{s_{zn}^2}{S_z^2}$, $r_3 = s_{xn}^2 \exp\left(\frac{S_z^2 - s_{zn}^2}{S_z^2 + s_{zn}^2}\right)$ $r_4 = s_{xn}^2 \exp\left(\frac{s_{zn}^2 - S_z^2}{s_{zn}^2 + S_z^2}\right)$

$r_5 = s_{xn}^2 + k_1(s_{zn}^2 - S_z^2)$;

where k_1 is a real constant, among others, may estiamte S_x^2 more precisely than s_{xn}^2. Further, the proposed class of estimators t_A of S_y^2 is also very wide, hence for any parametric function $A(s_{ym}^2, a, b, c)$ satisfying the regularity conditions with $A(S_y^2, 1, 1, 1) = S_y^2$ for all S_y^2, may generate asymptotically acceptable estimators of the class t_A, for examples the following estimators are the members of the class of estimators t_A.

$t_1 = s_{ym}^2 \frac{s_{xm}^2}{r_1}$, $t_2 = s_{ym}^2 \frac{s_{xm}^2}{r_2}$, $t_{im} = s_{ym}^2 + k_2(r_i - s_{xm}^2)$[i=1,2,3,4,5] and k_2 is real constant.

$t_{6m} = s_{ym}^2 \frac{r_3}{s_{xm}^2}$, $t_{7m} = s_{ym}^2 \frac{r_4}{s_{xm}^2}$ among others.

It may be easily verified that the optimum values of $k_i(i = 1, 2)$ in above estimators are determined in such a way so that they satisfy the respective normal equations and the resulting estimators should have the same minimum mean square errors of first order of approximations as derived for the class of estimators t_A .

(iii)For estimating the population variance S_y^2 of the study variable y on each occasion, the class of estimator t_C is suitable, which implies that more belief on t_C could be shown by choosing ϕ in equation (2.5), as 1 (or close to 1), while for estimating changes over occasions, the family of estimators t_A could be more useful and hence, ϕ might be chosen as 0 (or close to 0). For asserting both the problems simultaneously, the suitable (optimum) choice of ϕ is required.

3. Properties of the proposed class of estimators t

3.1. Bias and MSE of t. It may be noted from section 2 that several ratio, product and regression type estimators are members of the proposed classes of estimators t_C and t_A . Therefore, the class of estimators t_C and t_A are biased for S_y^2. This indicates that the composite class of estimators t is also biased for S_y^2. The bias B(.) and mean square error M(.) of the class of estimators t are derived up to first order of approximations under large sample assumptions in the following section.

3.1.1. *Bias and MSE of t_C.* The bias B(.) and mean square error M(.) of class of estimators t_C are derived up to first order approximations under large sample assumptions and using the following transformations.
$s_{yu}^2 = S_y^2(1 + e_{1u})$, $s_{zu}^2 = S_z^2(1 + e_{2u})$ such that $E(e_{1u}) = E(e_{2u}) = 0$: and $E(e_{1u}e_{2u}) = f_u\rho_{02}C_0C_2$

Following [17], in the expressions of bias and mean square error of the class of estimators t_C, we use the following notations.
$\mu_{pqr} = \frac{1}{N}\sum_{n=1}^{\infty}(y_i\text{-}\bar{Y})^p(x_i\text{-}\bar{X})^q(z_i\text{-}\bar{Z})^r$; where p, q, r being non negetive integer.

$\lambda_{pqr} = \frac{\mu_{pqr}}{\sqrt{\mu_{200}^p\mu_{020}^q\mu_{002}^r}}$,

$C_0 = \sqrt{\lambda_{400} - 1}$, $C_1 = \sqrt{\lambda_{040} - 1}$, $C_2 = \sqrt{\lambda_{004} - 1}$,

$\rho_{01} = \frac{\lambda_{220}-1}{C_0C_1}$, $\rho_{02} = \frac{\lambda_{202}-1}{C_0C_2}$, $\rho_{12} = \frac{\lambda_{022}-1}{C_1C_2}$,

$k_{01} = \rho_{01}\frac{C_0}{C_1}$, $k_{02} = \rho_{02}\frac{C_0}{C_2}$, $k_{12} = \rho_{12}\frac{C_1}{C_2}$, $k_{21} = \rho_{21}\frac{C_2}{C_1}$

$f_1 = \frac{1}{m}$, $f_2 = \frac{1}{n}$, $f_3 = \frac{1}{m}\text{-}\frac{1}{n}$, $f_u = \frac{1}{u}$ [Finite population correlation has been ignored as N $\rightarrow \infty$

To express t_C in terms of e's we will have to consider the conditions mentioned in equation (2.2) on the function $C(s_{yu}^2, s_{zu}^2)$ and thus expanding $C(s_{yu}^2, s_{zu}^2)$ about the point $C(S_y^2, S_z^2)$ in the third order Tailor series, we have

$$(3.1) \qquad C(s_{yu}^2, s_{zu}^2) = \begin{bmatrix} C(S_y^2, S_z^2) + (s_{yu}^2 - S_y^2)d_1 + (s_{zu}^2 - S_z^2)d_2 \\ +\frac{1}{2}[((s_{yu}^2 - S_y^2)d_{11})^2 + ((s_{zu}^2 - S_z^2)d_{22})^2 \\ +2(s_{yu}^2 - S_y^2)(s_{zu}^2 - S_z^2)d_{12}] \\ +\frac{1}{6}\left[(s_{yu}^2 - S_y^2)\frac{\partial}{\partial s_{yu}^2} + (s_{zu}^2 - S_z^2)\frac{\partial}{\partial s_{zu}^2}\right]^3 C(s_{yu}^{2'}, s_{zu}^{2'}) \end{bmatrix}.$$

where, $d_1 = \frac{\partial}{\partial s_{yu}^2} C(s_{yu}^2, s_{zu}^2)$, $d_2 = \frac{\partial}{\partial s_{zu}^2} C(s_{yu}^2, s_{zu}^2)$,

$s_{yu}^{2'} = S_y^2 + \theta(s_{yu}^2 - S_y^2)$, $s_{zu}^{2'} = S_z^2 + \theta(s_{zu}^2 - S_z^2)$, $d_{22} = \frac{\partial}{\partial s_{zu}^2}(\frac{\partial C(.)}{\partial s_{zu}^2})$, and $d_{12} = \frac{\partial^2 C(.)}{\partial s_{yu}^2 \partial s_{zu}^2}$.

In light of conditions (2),

(3.2) $\quad C(S_y^2, S_z^2) = S_y^2 \rightarrow C_1(Q) = d_1 = \frac{\partial C(.)}{\partial s_{yu}^2} = 1 \quad and \quad d_{11} = \frac{\partial^2 C(.)}{\partial (s_{yu}^2)^2} = 0$

Now expressing the equation (3.1) in terms of e's and neglecting terms of e's having power greater than two we have

(3.3) $\quad C(s_{yu}^2, s_{zu}^2) = \left[\begin{array}{l} C(S_y^2, S_z^2) + (s_{yu}^2 - S_y^2) + (s_{zu}^2 - S_z^2)d_2 \\ + \frac{1}{2}[((s_{yu}^2 - S_y^2)d_{11})^2 + ((s_{zu}^2 - S_z^2)d_{22})^2 \\ + 2(s_{yu}^2 - S_y^2)(s_{zu}^2 - S_z^2)d_{12}] \end{array} \right].$

Now taking expectation on the both sides of the equation (3.3) and retaining the terms of order $o(u^{-1})$ we obtained bias and mean square error of the class of estimators t_C as

(3.4) $\quad B(t_C) = \frac{1}{2u}[S_z^4 C_2^2 d_{22} + 2S_z^2 S_y^2 \rho_{02} C_0 C_2 d_{12}]$

and

(3.5) $\quad M(t_C) = E(t_C - S_y^2)^2 = \frac{S_y^4}{u}[C_0^2 + (\frac{C_2}{R_2})^2 d_2(d_2 + 2R_2 k_{02})]$

where $k_{02} = \rho_{02}\frac{C_0}{C_2}$, $R_2 = \frac{S_y^2}{S_z^2}$

Now $M(t_C)$ is minimized for $d_2 = -R_2 k_{02}$ Here, it may be noticed that ρ_{01} is the correlation coefficient between $(y - \bar{Y})$ and $(x - \bar{X})$ and Similarly ρ_{12} is the correlation coefficient between $(x - \bar{X})$ and $(z - \bar{Z})$ and ρ_{02} is the correlation coefficient between $(y - \bar{Y})$ and $(z - \bar{Z})$ See for instance [27].
Thus, the resulting minimum mean square error is given by

(3.6) $\quad Min M(t_C) = \frac{C_0^2 S_y^4}{u}[1 - \rho_{02}^2]$

3.1.2. *Bias and MSE of t_A.* The bias $B(.)$ and mean square error $M(.)$ of t_A are derived up to first order approximations under large sample assumptions and using the following transformations.
$s_{ym}^2 = S_y^2(1 + e_{1m})$, $s_{xm}^2 = S_x^2(1 + e_{2m})$, $s_{xn}^2 = S_x^2(1 + e_{3n})$, $s_{zn}^2 = S_z^2(1 + e_{4n})$, such that $E(e_{im}) = 0$, for i=1, 2. And $E(e_{in}) = 0$, for i=3, 4.
Following Singh *et.al* (2009), we use the following expectations in the expressions of bias and mean square error of the class of estimators t_A.
$E(e_{1m}^2) = f_1 C_0^2$, $E(e_{2m}^2) = f_1 C_1^2$, $E(e_{3n}^2) = f_2 C_1^2, E(e_{4n}^2) = f_2 C_2^2$,
$E(e_{1m}e_{3n}) = f_2 \rho_{01} C_0 C_1$, $E(e_{1m}e_{4n}) = f_2 \rho_{02} C_0 C_2$, $E(e_{3n}e_{4n}) = f_2 \rho_{12} C_1 C_2$,
$E(e_{1m}e_{2m}) = f_1 \rho_{01} C_0 C_1$, $E(e_{2m}e_{4n}) = f_2 \rho_{12} C_1 C_2$, $E(e_{2m}e_{3n}) = f_2 C_1^2$.
Proceeding as above using the conditions stated in the equations (2.3) and (2.4), we expand $A(s_{ym}^2, a, b, c)$ about $A(S_y^2, 1, 1, 1)$ in a third order Tailor's series expansion and taking expectations we have

$$(3.7) \qquad B(t_A) = \frac{1}{2} \left[\begin{array}{l} f_3 \left[C_1^2 A_{22}(T) + C_2^2 A_{33}(T) + 2S_y^2 \rho_{02} C_0 C_2 A_{13}(T) \right] \\ + f_3 \left[2S_y^2 \rho_{01} C_0 C_1 A_{12}(T) + 2S_y^2 \rho_{12} C_1 C_2 A_{23}(T) \right] \\ + f_2 \left\{ C_2^2 A_{44}(T) + 2S_y^2 \rho_{02} C_0 C_2 A_{14}(T) \right\} \end{array} \right]$$

where, $A_1(T) = \frac{\partial A(.)}{\partial s_{ym}^2}, A_2(T) = \frac{\partial A(.)}{\partial a}, A_3(T) = \frac{\partial A(.)}{\partial b}, A_4(T) = \frac{\partial A(.)}{\partial c}$. Further

$$(3.8) \qquad M(t_A) = \left[\begin{array}{l} S_y^4 f_1 C_0^2 + f_3 A_2^2(T) C_1^2 + f_3 A_3^2(T) C_2^2 \\ + f_2 A_4^2(T) C_2^2 + 2f_3 A_2 S_y^2 \rho_{01} C_0 C_1 \\ + 2f_3 A_3 S_y^2 \rho_{02} C_0 C_2 + 2f_3 A_4 S_y^2 \rho_{02} C_0 C_2 \\ + 2A_2 A_3 f_3 \rho_{12} C_1 C_2 \end{array} \right]$$

Now diffrentiating the expression $M(t_A)$ partially with respect to $A_2(T)$, $A_3(T)$, $A_4(T)$ and equating then to zero, we have the optimum values of $A_i(T)$,i=2,3,4 as

$$(3.9) \qquad A_2(T) = \frac{S_y^2 C_0(\rho_{02}\rho_{12} - \rho_{01})}{C_1(1 - \rho_{12}^2)}$$

$$(3.10) \qquad A_3(T) = \frac{S_y^2 C_0(\rho_{01}\rho_{12} - \rho_{02})}{C_2(1 - \rho_{12}^2)}$$

and

$$(3.11) \qquad A_4(T) = -\frac{S_y^2 C_0 \rho_{02}}{C_2}$$

Substitution from equation (3.9), (3.10) and (3.11) in (3.8), which yields the minimum mean square error of class of estimators t_A as,

$$(3.12) \qquad MinM(t_A) = S_y^4 C_0^2 [f_1 - f_3 \{ \frac{\rho_{01}^2 + \rho_{02}^2 - 2\rho_{01}\rho_{02}\rho_{12}}{(1 - \rho_{12}^2)} \} - f_2 \rho_{02}^2]$$

3.1.3. *Theorem.*
(i) Bias of the class of estimators t to the first order of approximations are obtained as

$$(3.13) \qquad B(t) = \phi B(t_C) + (1 - \phi)B(t_A)$$

Proof: The bias of the class of estimators t is given by
$B(t) = E(t - S_y^2)$
$= E[\phi(t_c - S_y^2) + (1 - \phi)(t_A - S_y^2)]$
$= \phi E[(t_c - S_y^2)] + (1 - \phi)E[(t_A - S_y^2)]$
$= \phi B(t_C) + (1 - \phi)B(t_A)$

Substituting the values of $B(t_C)$ and $B(t_A)$ from equations (3.4) and (3.7) in the equation (3.13), we have the expression for the bias of the class of estimators t .

(ii) Mean square error of the class of estimators t to the first order of approximations are obtained as

$$(3.14) \qquad M(t) = \phi^2 M(t_C) + (1 - \phi)^2 M(t_A)$$

Proof: The Mean square error of the class of estimators t is given by
M(t) = $E(t - S_y^2)^2$
$= E[\phi(t_c - S_y^2) + (1 - \phi)(t_A - S_y^2)]^2$
$= \phi^2 E[(t_c - S_y^2)]^2 + (1 - \phi)^2 E[(t_A - S_y^2)]^2$
$= \phi^2 M(t_C) + (1 - \phi)^2 M(t_A)$
Substituting the values of $M(t_C)$ and $M(t_A)$ from equations (3.6) and (3.12) in the equation (3.14), we have the expression for the mean square error of the class of estimators t. It should be noted that the estimators t_C and t_A may be correlated. However, the

covariance term $C(t_C, t_A)$ is of order N^{-1} and neglected for large population.

Remark :
(a) It may be seen that the bias and mean square errors of different estimators which belong to the classes of estimators t_C and t_A may be derived by substituting the suitable values of the derivatives as suggested by [15] and [16].
(b) It is to be noted from equations (3.2) and (3.3) that the optimum values of the derivatives involved in estimators depend on unknown population parameters. Thus to use such estimators one has to use the guessed or estimated values of these derivatives. Guessed values of population parameters can be obtained either from past data or experience gathered over time; see [8], [11] and [25]. If the guessed values are not known then it is advisable to use sample data to estimate these parameters as suggested by [26], [16], and [5].

3.2. Minimum mean square error of the proposed estimator t. It may be noted from Remark 2.2.3 and equation (3.14) that mean square error of class of estimators t in equation (3.14) is a function of constant ϕ, therefore, it is minimized with respect to ϕ and subsequently the optimum value of ϕ is obtained as

$$(3.15) \quad \phi_{\mathrm{opt}} = \frac{Min.M(t_A)}{Min.M(t_C) + Min.M(t_A)}$$

Substituting the value of ϕ_{opt} from equation (3.15) in equation (3.14), we get the optimum mean square error of the class of estimators t as

$$(3.16) \quad [M(t)]_{opt} = \frac{Min.M(t_C) \times Min.M(t_A)}{Min.M(t_C) + Min.M(t_A)}$$

Further, substituting the values of $Min.M(t_C)$ and $Min.M(t_A)$ from equations (3.6) and (3.12) in equation (3.16), the simplified values of $[M(t)]_{opt}$ is derived as

$$[M(t)]_{opt} = \frac{C_0^2 s_y^4 (1 - \rho_{02}^2)(f_1 - f_3 A' - f_2 \rho_{02}^2)}{u[(\frac{1}{u} + f_1) - \rho_{02}^2((\frac{1}{u} + f_2) - f_3 A']}$$

where,

$$A' = \frac{\rho_{01}^2 + \rho_{02}^2 - 2\rho_{01}\rho_{02}\rho_{12}}{1 - \rho_{12}^2}$$

put $u = n\mu, m = n(1 - \mu)$, gives,

$$(3.17) \quad [M(t)]_{opt} = \frac{C_0^2 s_y^4 (1 - \rho_{02}^2)[(1 - \rho_{02}^2) + \mu(\rho_{02}^2 - A')]}{n[(1 - \rho_{02}^2) + \mu^2(rho_{02}^2 - A')]}$$

where $\mu = \frac{u}{n}$ is the fraction of fresh sample drawn on the current (second) occasion.

3.3. Optimum replacement strategy. The key design parameter affecting the estimates of change is the overlap between successive samples. Maintaining high overlap repeats of a survey is operationally convenient, since many sampled units have been located and have some experience of the survey. Hence, to determine the optimum value of μ so that population variance S_y^2 may be estimated with maximum precision, we minimize Min M(t) in equation (3.17) with respect to μ we have the optimum value of μ as,

$$(3.18) \quad \hat{\mu} = \frac{1 \pm \sqrt{1 - \frac{(\rho_{01} - \rho_{02}\rho_{12})^2}{(1 - \rho_{12}^2)(1 - \rho_{02}^2)}}}{\frac{(\rho_{01} - \rho_{02}\rho_{12})^2}{(1 - \rho_{12}^2)(1 - \rho_{02}^2)}}$$

The real values of $\hat{\mu}$ exist, if and only if the quantity under square root is positive. For any situation, which satisfies this condition, two real values of $\hat{\mu}$ are possible, hence, to choose a value of $\hat{\mu}$, it should be remembered that $0 \leq \hat{\mu} \leq 1$, all other values of $\hat{\mu}$

are inadmissible. If both the real values of $\hat{\mu}$ are admissible, lowest one will be the best choice as it reduces the cost of the survey at the same precision of estimate. Substituting the admissible value of $\hat{\mu}$ say $\hat{\mu}_0$ in equation (3.17), we have the optimum value of the mean square error of the class of estimators t which is shown as,

$$(3.19) \quad Min.M(t)_{opt} = \frac{C_0^2 s_y^4(1 - \rho_{02}^2)[1 + \sqrt{1 - \frac{(\rho_{01} - \rho_{02}\rho_{12})^2}{(1 - \rho_{12}^2)(1 - \rho_{02}^2)}}]}{2n}$$

4. Efficiency comparison

It is important to investigate situations under which our proposed estimation strategy succeeds better than the usual. Thus, to elucidate the performance of the proposed class of estimators t, we have computed the percent relative efficiency (PRE) of the proposed class of estimator t , with respect to (i)s_{yn}^2 when there is no matching and (ii)$s_{y\phi^*}^2$ when no auxiliary information is used on any occasion . When no auxiliary information is used on any occasion following estimator of population variance S_y^2 in two-occasion successive sampling may be proposed as

$$(4.1) \quad s_{y\phi^*}^2 = \phi^* s_{yu}^2 + (1 - \phi^*)s_{ylm}^2$$

where ϕ^* is unknown and

$$(4.2) \quad s_{ylm}^2 = s_{ym}^2 + b(s_{xn}^2 - s_{xm})$$

and b is suitably chosen constant to minimize the variance of the estimator s_{ylm}^2.
The mean square error of $s_{y\phi^*}^2$ up to the first degree of approximation for large population size $(N \to \infty)$ is given as

$$(4.3) \quad M(s_{y\phi^*}^2) = (\phi^*)^2 M(s_{yu}^2) + (1 - (\phi^*)^2)^2 M(s_{ylm}^2)$$

where $M(s_{yu}^2) = \frac{C_0^2 S_y^4}{u}$ and

$$(4.4) \quad M(s_{ylm}^2) = [\frac{C_0^2 S_y^4(1 - \rho_{01}^2)}{m} + \frac{C_0^2 S_y^4(\rho_{01}^2)}{n}]$$

The $M(s_{y\phi^*}^2)$ in (4.3) is minimized for

$$(4.5) \quad \phi^* = \frac{M(s_{ylm}^2)}{M(s_{yu}^2) + M(s_{ylm}^2)}$$

Thus the resulting MSE of $s_{y\phi^*}^2$ is given by

$$(4.6) \quad M(s_{y\phi^*}^2) = \frac{C_0^2 S_y^4(1 - \mu\rho_{01}^2)}{n(1 - \mu^2\rho_{01}^2)}$$

Note that if $\mu = 0$(complete matching) or $\mu = 1$(no matching) mean square error has the value $\frac{S_y^4}{n}$.Therefore optimum value of μ is found by minimizing $M(s_{y\phi^*}^2)$ in equation (4.6) w.r.t variation in μ which yields

$$(4.7) \quad \mu^* = \frac{1}{1 + \sqrt{1 - \rho_{01}^2}} \frac{m}{n} = \frac{\sqrt{1 - \rho_{01}^2}}{1 + \sqrt{1 - \rho_{01}^2}}$$

when the optimum value of μ is substituted in equation (4.6) the minimum mean square error works out as

$$(4.8) \quad M(s_{y\phi^*}^2)_{opt} = \frac{C_0^2 S_y^4}{2n}(1 + \sqrt{1 - \rho_{01}^2})$$

Thus, the percent relative efficiencies (PRE) of the proposed class of estimators t , with respect to the estimators s^2_{yn} and $s^2_{y\phi*}$ are presented below
The PRE of t, w.r.t s^2_{yn} is

$$(4.9) \qquad E_1 = \frac{V(s^2_{yn})}{MinM(t)_{(opt)}} \times 100 = \frac{2}{(1 - \rho^2_{02})(1 + \sqrt{1 + \frac{(\rho_{01} - \rho_{02}\rho_{12})^2}{(1 - \rho^2_{12})(1 - \rho^2_{02})}})} \times 100$$

where $V(s^2_{yn}) = \frac{C^2_0 S^4_y}{n}$
and the PRE of t with respect to $s^2_{y\phi*}$ is given as

$$(4.10) \qquad E_2 = \frac{V(s^2_{y\phi*})}{MinM(t)_{(opt)}} \times 100 = \frac{1 + \sqrt{1 - \rho^2_{01}}}{(1 - \rho^2_{02})(1 + \sqrt{1 + \frac{(\rho_{01} - \rho_{02}\rho_{12})^2}{(1 - \rho^2_{12})(1 - \rho^2_{02})}})} \times 100$$

In Table 1.1 and Table 1.2, we have examined the efficacy of the proposed work through the data set of natural populations. The behavior of the proposed estimation strategy for the variations in different correlations such as ρ_{01}, ρ_{02} has been demonstrated using empirical study presented in Table 2.

4.1. Investigation through natural populations.
Two natural population (based on positive correlation) data sets have been chosen to illustrate the efficiency of our proposed estimation procedure. The source of the populations, the nature of the variables y, x, z and the values of the various parameters are as follows:
Population I-Source:Murthy (1967), (Page Number- 399)
y: Area under wheat in 1964.
x: Area under wheat in 1963.
z: Cultivated area in 1961.
$N = 80, C_0 = 1.1255, C_1 = 1.6065, C_2 = 1.3662,$
$\rho_{01} = 0.7319, \rho_{02} = 0.794, \rho_{12} = 0.9716, S^2_z = 715055.82136$

Population II-Source: Sukhatme and Sukhatme (1970), (Page Number -185)
y: Area (acres) under wheat in 1937.
x: Area (acres) under wheat in 1936.
z: Total cultivated area (acres) in 1931.
$N = 34, C_0 = 1.5959, C_1 = 1.5105, C_2 = 1.32,$
$\rho_{01} = 0.6251, \rho_{02} = 0.8007, \rho_{12} = 0.5342, S^2_z = 222931.3868$
Utilizing the above data set, we have examined the performances of the proposed class of estimators t in Table 1.1.

Table 1.1 PRE of t with respect to $s^2_{yn}, s^2_{y\phi*}$

population	E_1	E_2
I	275.9056	231.9555
II	290.1144	258.2809

4.2. Investigation on artificial population.
We have generated three sets of independent random numbers of size N (N = 100) namely x'_k, y'_k, z'_k where k=1,2,3,......,N from a standard normal distribution with the help of R - software. Further, motivated by the artificial population generation techniques adopted by Singh and Deo (2003) and Singh *et al.* (2001), we have generated the following transformed variables of the population U with the values of $\rho_{xy}=0.8, \rho_{xz}=-0.7, \sigma^2_y=100, \mu_y=40, \sigma^2_x=225, \mu_x=50, \sigma^2_z=25, \mu_z=30$ as

$$y_k = \mu_y + \sigma_y[\rho_{xy}x'_k + \sqrt{(1-\rho^2_{xy})}y'_k], \ x_k = \mu_x + \sigma_y x'_k, z_k = \mu_z + \sigma_z[\rho_{xz}x'_k + \sqrt{(1-\rho^2_{xz})}z'_k]$$
where

σ_x, σ_y, σ_z : Standard deviations of the variables y, x and z respectively based on the population of size N.

μ_x, μ_y, μ_z: Mean of the variables y, x and z respectively based on the population of size N.

Utilizing the above data set, we have examined the performances of the proposed class of estimators t are presented in Table 1.2.

Table 1.2 PRE of t with respect to artificial generate population

population	E_1	E_2
Artificial generate population	134.7082	116.6528

4.3. Empirical Study. To have a tangible idea about the performance of the class of estimators t we have computed the PRE of t with respect to s^2_{yn} and $s^2_{y\phi*}$ for various choices of ρ_{01} and ρ_{02} are presented in Table 2. Here for the sake of conveniences we have considered the assumption $\rho_{12} = \rho_{02}$

Table 2. Optimum values of PRE of t for different choices of ρ_{01} and ρ_{02}

ρ_{01}	ρ_{02}	E_1	E_2	ρ_{01}	ρ_{02}	E_1	E_2	ρ_{01}	ρ_{02}	E_1	E_2
	-0.7	238.4832	237.8855		-0.7	203.3995	198.7152		-0.7	205.1791	175.8532
	-0.6	163.2909	162.8817		-0.6	156.5948	152.9884		-0.6	169.1736	144.9938
	-0.5	134.6940	134.3564		-0.5	133.4818	130.4027		-0.5	148.1481	126.9735
	-0.4	119.1999	118.9011		-0.4	119.8860	117.1250		-0.4	134.8228	115.5528
	-0.3	109.8934	109.6180		-0.3	111.3934	108.8280		-0.3	126.1609	108.1289
	-0.2	104.2686	104.0073		-0.2	106.1503	103.7056		-0.2	120.6901	103.4000
0.1	-0.1	101.2197	100.9660	0.4	-0.1	103.2752	100.8968	0.7	-0.1	117.6519	100.8361
	0.1	101.2197	100.9660		0.1	103.2752	100.8968		0.1	117.6519	100.8361
	0.2	104.2686	104.0073		0.2	106.1503	103.7056		0.2	120.6901	103.4400
	0.3	109.8934	109.6180		0.3	111.3934	108.8280		0.3	126.1669	108.1289
	0.4	119.1999	118.9011		0.4	119.8860	117.1250		0.4	134.8228	115.5528
	0.5	134.6940	134.3564		0.5	133.4818	130.4077		0.5	148.1481	126.9735
	0.6	163.2909	162.8817		0.6	156.5948	152.9884		0.6	169.1736	144.9938
	0.7	238.4832	237.8855		0.7	203.3995	198.7152		0.7	205.1791	175.8532
	-0.7	215.1640	212.9904		-0.7	196.0973	182.9613		-0.7	218.5866	174.8693
	-0.6	158.7708	162.8817		-0.6	158.1653	147.5702		-0.6	181.0351	144.8281
	-0.5	133.4818	132.1334		-0.5	137.2583	128.0637		-0.5	158.7425	126.9940
	-0.4	119.1152	117.9119		-0.4	124.3692	116.0381		-0.4	144.5025	115.6020
	-0.3	110.2945	109.1803		-0.3	116.1168	108.3384		-0.3	135.2073	108.1658
	-0.2	104.9003	103.8406		-0.2	110.9499	103.5177		-0.2	129.3230	103.4584
0.2	-0.1	101.9578	100.9278	0.5	-0.1	108.0945	100.8536	0.8	-0.1	126.0512	100.8409
	0.1	101.9578	100.9278		0.1	103.0945	100.8536		0.1	126.0512	100.8409
	0.2	104.9003	103.8406		0.2	110.9499	103.5177		0.2	129.3230	103.4584
	0.3	110.2945	109.1803		0.3	116.1168	108.3384		0.3	135.2073	108.1658
	0.4	119.1152	117.9119		0.4	124.3692	116.0381		0.4	144.5025	115.6020
	0.5	133.4818	132.1334		0.5	137.2583	128.0637		0.5	158.7425	126.9940
	0.6	158.7708	157.1669		0.6	158.1653	147.5702		0.6	181.0351	144.8281
	0.7	215.1640	212.9904		0.7	196.0973	182.9613		0.7	218.5866	174.8693
	-0.7	203.3995	198.7152		-0.7	198.4135	178.5721		-0.7	245.9072	176.5478
	-0.6	156.5948	152.9884		-0.6	162.1671	145.9504		-0.6	203.3530	145.9963
	-0.5	133.4818	130.4027		-0.5	141.5103	127.3592		-0.5	177.9097	127.7294
	-0.4	119.8860	117.1250		-0.4	128.5726	115.7153		-0.4	161.6172	116.0322
	-0.3	111.3934	108.8280		-0.3	120.2170	108.1953		-0.3	150.9743	108.3912
	-0.2	106.1503	103.7056		-0.2	114.9595	103.4635		-0.2	144.2362	103.5536
0.3	-0.1	103.2752	100.8968	0.6	-0.1	112.0458	100.8412	0.9	-0.1	140.4899	100.8640
	0.1	103.2752	100.8968		0.1	112.0458	100.8412		0.1	140.4899	100.8640
	0.2	106.1503	103.7056		0.2	114.9595	103.4635		0.2	144.2362	103.5536
	0.3	111.3934	108.8280		0.3	120.2170	108.1953		0.3	150.9743	103.3912
	0.4	119.8860	117.1250		0.4	128.5726	115.7153		0.4	161.6172	116.0322
	0.5	133.4818	130.4077		0.5	141.5103	127.3592		0.5	177.9097	127.7294
	0.6	156.5948	152.9884		0.6	162.1671	145.9504		0.6	203.3530	145.9963
	0.7	203.3995	198.7152		0.7	198.4135	178.5721		0.7	245.9072	176.5478

From above empirical study, it is to be noted that the trend in PRE may be obtained for different choices of correlations ρ_{01} and ρ_{02} .However, the specific ranges ρ_{01} and ρ_{02} where proposed estimator performs extremely well or dominate mildly over the sample variance estimator s_{yn}^2 may not be clearly obtained from the above empirical analysis. This situation helps us in choosing the suitable population where our proposed work may be applied effectively which is very essential for the recommendations of our proposed work. Motivated with this arguments we proceed to build up a decision making machinery through fuzzy tools which will enable us to measure the degree of efficiency of the estimator for different choices of correlations ρ_{01} and ρ_{02}.

5. Analysis of empirical study through fuzzy tools

Construction of the Fuzzy Logic Controller (FLC) is based on the empirical study furnished in Table 2 where the FLC checks the degree of efficiency for a given range of ρ_{01} and ρ_{02}. ρ_{01} and ρ_{02} are conceived as to be the two input fuzzy variables having 9 and 14 linguistics respectively (listed in Tables 4a and 4b). Entire range of $\rho_{01}[0.1 \leq \rho_{01} \leq 0.9]$ is divided into 9 equal parts and that for $\rho_{02}[-0.7 \leq \rho_{02} \leq 0.7]$ is divided into 14 equal parts and for each part a linguistic is assigned suitably for both cases. Also the E_1, E_2, and PRE are taken as the two output fuzzy variables having the same set of 20 linguistics for both (listed in Table 3) in the descending degree of efficiency.The range$[100 \leq E_1, E_2 \quad or \quad PRE \leq 230]$, as it obtained from the Table 2, is divided into 20 equal parts as shown in Table 3.

Table 3

Ling	Range	A	C
E20	220-229	224.5	4.5
E19	210-219	215	4.5
E18	200-209	205	4.5
E17	190-199	195	4.5
E16	180-189	185	4.5
E15	174-179	176.5	2.5
E14	168-173	170.5	2.5
E13	162-167	164.5	2.5
E12	156-161	158.5	2.5
E11	150-155	152.5	2.5
E10	144-149	146.5	2.5
E9	138-143	140.5	2.5
E8	132-137	134.5	2.5
E7	126-131	128.5	2.5
E6	120-125	122.5	2.5
E5	116-119	117.5	1.5
E4	112-115	113.5	1.5
E3	108-111	109.5	1.5
E2	104-107	105.5	1.5
E1	100-103	101.5	1.5

The Mamdani Inference Model is followed here as it is the most commonly used fuzzy methodology and was one among the first few control systems built using fuzzy set theory. It was proposed by Ebrahim Mamdani (1975)[7] in order to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. Mamdani's effort has its root in LotfiZadeh's paper on

fuzzy algorithms for complex systems and decision processes (1973)[28]. The following standard operator set is used in this model:

Operators	Type	Default Function
AND	BINARY	MIN(a,b)
OR	BINARY	MAX(a,b)
IMPLICATION	BINARY	MIN(a,b)
ALSO	BINARY	MAX(a,b)
NOT	UNARY	1-a
STRONGLY	UNARY	a^2
MODERATELY	UNARY	$a^{\frac{1}{2}}$
SLIGHTLY	UNARY	4.a.(1-a)
DEFUZZIFICATION	DEFUZZIFICATION	Centre Of Area

A three-parameter (a, b, c) bell shaped continuous membership grade function has been chosen for each linguistic of both input and output variables (This is a direct generalization of Cauchy Distribution) so that membership functions can be fine grained according to the necessity. The parameters a, b, c being respectively the middle point of bell shaped curve (where the grade is max), the degree of peakedness (resembling the Kurtosis in Normal distribution) and half width of the membership function.c is kept constant=0.044444 for ρ_{01} and =0.05 for ρ_{02} and it takes three different values (4.5, 2.5, 1.5) for E_1 and E_2 and b is kept constant=1 throughout.

The function is given by $f(x; a, b, c) = \frac{1}{1+|\frac{x-a}{c}|^{2b}}$

The set of values for the parameters are computed from the set of data generated in Table 2.

<div align="center">Table: 4a</div>

Sl.No.	Left end of ρ	Right end of ρ	Mid Point(a)	Linguistics	Interpretation	Half-Width
1	0.1	0.2	0.15	Mcp	mildly correlated positive	0.04
2	0.2	0.3	0.25	mpc3	mildly positively correlated 3	0.04
3	0.3	0.4	0.35	mpc2	mildly positively correlated 2	0.04
4	0.4	0.5	0.45	mpc1	mildly positively correlated 1	0.04
5	0.5	0.6	0.55	pc3	positively correlated 3	0.04
6	0.6	0.7	0.65	pc2	positively correlated 2	0.04
7	0.7	0.8	0.75	pc1	positively correlated 1	0.04
8	0.8	0.9	0.85	Spc2	strongly positively correlated 2	0.04
9	0.9	1	0.95	Spc1	strongly positively correlated 1	0.04

Table: 4b

Sl.No.	Left end of ρ	Right end of ρ	Mid Point(a)	Linguistics	Interpretation	Half-Width
1	-0.7	-0.6	-0.65	Nc1	negatively correlated 1	0.05
2	-0.6	-0.5	-0.55	nc2	negatively correlated 2	0.05
3	-0.5	-0.4	-0.45	nc3	Negatively correlated 3	0.05
4	-0.4	-0.3	-0.35	Mnc1	Mildly Negatively correlated 1	0.05
5	-0.3	-0.2	-0.25	Mnc2	Mildly negatively correlated 2	0.05
6	-0.2	-0.1	-0.15	Mnc3	Mildly negatively correlated 3	0.05
7	-0.1	0	-0.05	Mcn	Mildly correlated negative	0.05
8	0.0	0.1	0.05	Mcp	Mildly correlated positive	0.05
9	0.1	0.2	0.15	Mpc3	Mildly positively correlated 3	0.05
10	0.2	0.3	0.25	mpc2	Mildly positively correlated 2	0.05
11	0.3	0.4	0.35	mpc1	mildly positively correlated 1	0.05
12	0.4	0.5	0.45	Pc3	Positively correlated	0.05
13	0.5	0.6	0.55	pc2	Positively correlated	0.05
14	0.6	0.7	0.65	Pc1	Positively correlated	0.05

A 9x14x20 Fuzzy Association Matrix (FAM) is constructed which is the basis of FLC engine and the 'Centre of Area' method (which resembles the expected value computation in probability distribution) is adopted for defuzzification which is most widely used method and defined by

$z_{COA} = \int z\mu(z)dz / \int \mu(z)dz$

All computations are done with the help of standard fuzzy software named XFuzzyVs3.0from IMSE-CNM which is available on internet (vide: $xfuzzy - team@imse.cnm.es$).

6. Categorization of Efficacy of Proposed Work

The above analysis of empirical study using fuzzy tools gives the advantage to find out the specific ranges of ρ_{01} and ρ_{02} where our suggested estimator dominates
i. extremely
ii. mildly and
iii. equally.
the sample variance estimator s_{yn}^2 .
To elucidate these particular regions of ρ_{01} and ρ_{02},the same graph of PRE of t in different views are presented below:
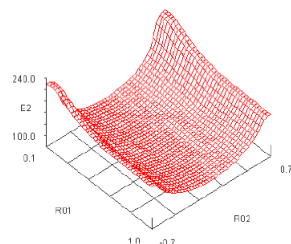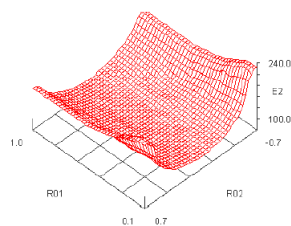
709

Fig. 1 (Top View 1)
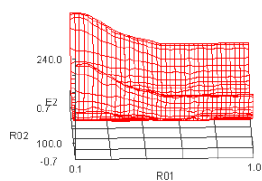

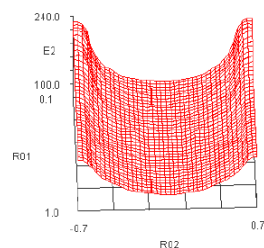**Fig. 2** (Top View 2)

Fig. 3 (Front View)

Fig. 4 (Side View)

Surface Plot of Specification of PRE against from different angles

6.1. Remark. It may be noted that the graphical demonstrations of PRE of the proposed class of estimator t may not be presented correctly through graphical simulations techniques. Because, it subject to the existence of the optimal (admissible) values of μ i.e. $0 \leq \mu \leq 1$ in the entire range of ρ_{01} and ρ_{02}. Motivated with this argument, we analyze the performance of the class of estimators t through fuzzy tools and obtained the approximate graphical demonstrations of it's PRE.

7. Conclusions

The following interpretations can be read out from the present study:

a) Table-1.1 and Table 1.2 exhibit that our suggested estimator t is superior to the sample variance estimator s_{yn}^2 . This result justifies the use of auxiliary information at estimation stage.

b) Table-2 interprets that

i. For fixed values of ρ_{01}, the values of μ_0 are increasing and PRE of t is decreasing with the increasing negative values of ρ_{02} and the values of μ_0 are decreasing whereas PRE of t is increasing with the increasing positive values of ρ_{02}. The same behavior of μ_0 and PRE of t is reflected when the values of ρ_{01} are increasing by keeping ρ_{02} as fixed.

ii. The decreasing value of μ_0 with the increasing values of ρ_{01} and ρ_{02} indicates more the negative (and positive) values of the correlation coefficients, less the fraction of fresh sample is required at the current occasion which enhances the precision of the estimates. This pattern is highly desirable as it pays in terms of enhanced precision of estimates as well as reduces the cost of survey.

iii. Minimum value of μ_0 is obtained 0.24, which indicates that only about 24 percent of the total sample size is to be replaced at the second (current) occasion for the corresponding choices of correlations.

c) The above graphical representation of PRE against ρ_{01} and ρ_{02} gives a clear idea about the efficiency of our proposed estimator t over the sample variance estimator s_{yn}^2. It itself describes the specific ranges of ρ_{01} and ρ_{02} at where our estimator t dominates (extremely, mildly or equally)s_{yn}^2. From the different views of the graph (taken from top and different sides), it is clear that the portion of the graph which is almost horizontal denotes that the proposed class of estimators t is equally efficient with s_{yn}^2. Whereas the uprising portions denote the mildly efficient range and the peaks of the graph along with its neighborhoods denote the extremely efficient range of the class of estimators t.

The following conclusions may be drawn about the performance of t over s_{yn}^2 :

i. Fig.1 and Fig.2 indicates that t is superior to in the regions and s_{yn}^2.

$0.1 \le \rho_{01} \le 1.0$, $-0.7 \le \rho_{02} \le -0.42$, and $0.1 \le \rho_{01} \le 1.0$, $0.42 \le \rho_{02} \le 0.7$.

ii. It is cleared from Fig.1, Fig.2 and Fig.3 that t extremely dominates s_{yn}^2 within the corner regions of plane of the graph, i.e., within the regions

$0.1 \le \rho_{01} \le 0.28$, $-0.7 \le \rho_{02} \le -0.42$; $0.1 \le \rho_{01} \le 0.28$, $0.42 \le \rho_{02} \le 0.7$; $0.82 \le \rho_{01} \le 1.0$, $-0.7 \le \rho_{02} \le -0.42$; $0.82 \le \rho_{01} \le 1.0$, $0.42 \le \rho_{02} \le 0.7$.

iii. It can be observed from Fig.4 that t is mildly efficient than s_{yn}^2 in the regions

$0.1 \le \rho_{01} \le 1.0$, $-0.42 \le \rho_{02} \le -0.14$ and $0.1 \le \rho_{01} \le 1.0$, $0.14 \le \rho_{02} \le 0.42$ and t is equally efficient with s_{yn}^2 in the region $0.1 \le \rho_{01} \le 1.0$, $-0.14 \le \rho_{02} \le 0.14$.

Thus it is erected that the use of an auxiliary character is highly rewarding in terms of the proposed class of estimators. It is seen that if a highly correlated auxiliary variable is used, relatively, only a smaller fraction of the sample on the current (second) occasion is required to be replaced by a fresh sample, which is reducing the cost of the survey. Moreover, the proposition of the class of estimators in the present study is justified as it unifies several desirable results including efficiently finding the dominance range of the proposed strategy. Looking on the nice behavior of the proposed strategy, they are recommended to the survey statisticians for their applications in real life problems.

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