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The influence of partially τ-quasinormal subgroups on the structure of finite groups

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Abstract

A subgroup H of a group G is said to be τ-quasinormal in G if H permutes with every Sylow subgroup Q of G such that $(|H|,|Q|)=1$ and $(|H|,|Q^G|)\neq 1$; H is called partially τ-quasinormal in G if G has a normal subgroup T such that HT is S-quasinormal in G and $H\cap T\leq H_{\tau G}$, where $H_{\tau G}$ is the subgroup generated by all those subgroups of H which are τ-quasinormal in G. In this paper, we investigate the influence of some partially τ-quasinormal subgroups on the structure of finite group. Some new characterizations of p-supersoluble and p-nilpotent groups are obtained.

Keywords: S-quasinormal, partially τ-quasinormal, p-supersoluble, p-nilpotent

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1. Introduction

All groups that we consider will be finite. We use standard notions and notation, as in [2] and [4]. G always denotes a finite group, $|G|$ is the order of G and p denotes a prime. The generalized Fitting subgroup $F^*(G)$ of G is the unique maximal normal quasinilpotent subgroup of G (see [5, X, 13]). The generalized p-Fitting subgroup $F_p^*(G)$ is defined to be as the normal subgroup of G such that $F^*(G/O_{p'}(G))=F_p^*(G/O_{p'}(G))$ (see [1]).

A subgroup H of G is said to be S-quasinormal in G if H permutes with every Sylow subgroup of G (see [6]). As a generalization of S-quasinormality, a subgroup H of G is

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said to be τ-quasinormal in G if H permutes with every Sylow subgroup Q of G such that $(|H|,|Q|)=1$ and $(|H|,|Q^G|)\neq 1$ (see [9]). On the other hand, Wang extended normality as follows: a subgroup H of G is said to be c-normal in G if there exists a normal subgroup T of G such that $HT=G$ and $H\cap T\leq H_G$, where H_G is the maximal normal subgroup of G contained in H (see [12]). In order to unify the above mentioned subgroups, the authors in [8] introduced the following concept:

1.1. Definition. A subgroup H of G is partially τ-quasinormal in G if there exists a normal subgroup T of G such that HT is S-quasinormal in G and $H\cap T\leq H_{\tau G}$, where $H_{\tau G}$ is the subgroup generated by all those subgroups of H which are τ-quasinormal in G.

G is said to be p-nilpotent if it has a normal p-complement and G is said to be p-supersoluble if every p-chief factor of G is cyclic. It is easy to see that if G is p-nilpotent then G is also p-supersoluble. The aim of this paper is to take the above mentioned studies further. More precisely, we investigate the p-supersolubility and p-nilpotency of finite groups using some partially τ-quasinormal subgroups.

2. Preliminaries

2.1. Lemma ([8, Lemma 2.3])**.** Let H be a subgroup of G. Then

(1) If H is partially τ-quasinormal in G and $H\leq K\leq G$, then H is partially τ-quasinormal in K.

(2) Suppose that $N\trianglelefteq G$ and $N\leq H$. If H is a p-group and H is partially τ-quasinormal in G, then H/N is partially τ-quasinormal in G/N.

(3) Suppose that H is a p-subgroup of G and N is a normal p'-subgroup of G. If H is partially τ-quasinormal in G, then HN/N is partially τ-quasinormal in G/N.

(4) If H is partially τ-quasinormal in G and $H\leq K\trianglelefteq G$, then there exists $T\trianglelefteq G$ such that HT is S-quasinormal in G, $H\cap T\leq H_{\tau G}$ and $HT\leq K$.

2.2. Lemma ([8, Theorem 1.4])**.** Let P be a Sylow p-subgroup of G, where p is a prime divisor of $|G|$ with $(|G|,p-1)=1$. Then G is p-nilpotent if and only if every maximal subgroup of P is partially τ-quasinormal in G.

2.3. Lemma ([10, Theorem A])**.** If H is an S-quasinormal p-subgroup of G for some prime p, then $N_G(H)\geq O^p(G)$.

2.4. Lemma. If H is a τ-quasinormal p-subgroup of G for some prime p and $O_{p'}(G)=1$, then $HQ=QH$ for every Sylow q-subgroup of G $(p\neq q)$.

Proof. Since $O_{p'}(G)=1$, it follows that $p||Q^G|$. Consequently, $HQ=QH$ from the definition of τ-quasinormal subgroup. □

2.5. Lemma ([11, Theorem C])**.** Let E be a normal subgroup of G. If every G-chief factor of $F^*(E)$ is cyclic, then every G-chief factor of E is also cyclic.

2.6. Lemma ([8, Theorem 1.6])**.** Let E be a normal subgroup of G. Suppose that for each $p\in\pi(E)$, every maximal subgroup of non-cyclic Sylow p-subgroup P of E is partially τ-quasinormal in G. Then every G-chief factor of E is cyclic.

2.7. Lemma ([7, Lemma 2.6])**.** Assume that G is p-supersoluble and P a Sylow p-subgroup of G. Then $PO_{p'}(G)$ is normal in G.

2.8. Lemma ([1, Lemma 2.10])**.** Let p be a prime divisor of $|G|$.

(1) $Soc(G)\leq F_p^*(G)$.

(2) $O_{p'}(G)\leq F_p^*(G)$.

In fact, $F^*(G/O_{p'}(G)) = F_p^*(G/O_{p'}(G)) = F_p^*(G)/O_{p'}(G)$.

(3) If $F_p^*(G)$ is p-soluble, then $F_p^*(G) = F_p(G)$.

2.9. Lemma ([8, Lemma 2.10]). Let N be a non-identity normal p-subgroup of G. If N is elementary and every maximal subgroup of N is partially τ-quasinormal in G, then some maximal subgroup of N is normal in G.

3. Results On p-supersolubility

3.1. Theorem. Let L be a normal subgroup of G such that G/L is p-supersoluble, where p is a prime divisor of $|L|$ with $(p-1, |L|) = 1$. Suppose that for a Sylow p-subgroup P of L, all maximal subgroups of P are partially τ-quasinormal in G. Then G is p-supersoluble.

Proof. Suppose that this theorem is false and consider a counterexample (G, L) for which $|G||L|$ is minimal.

(1) L is p-nilpotent.

By Lemma 2.1, it is easy to see that all maximal subgroups of P are partially τ-quasinormal in L. Applying Lemma 2.2, L is p-nilpotent.

(2) $P = L$.

According to Step (1), we know $O_{p'}(L)$ is the normal Hall p'-subgroup of L. Assume that $O_{p'}(L) \neq 1$. In view of Lemma 2.1, the hypothesis holds for $(G/O_{p'}(L), L/O_{p'}(L))$. Hence, by the minimal choice of (G, L), the theorem is true for $(G/O_{p'}(L), L/O_{p'}(L))$ and so $G/O_{p'}(L)$ is p-supersoluble. Consequently, G is p-supersoluble. This contradiction shows that $O_{p'}(L) = 1$. Hence L is a normal p-subgroup of G.

(3) The final contradiction.

Applying Lemma 2.6, all G-chief factors of L are cyclic. From the p-supersolubility of G/L, we have G is p-supersoluble, a contradiction. □

We can choose L to get some results of special interest. For example, if we choose $L = G'$ or $L = G^{U_p}$, then we obtain the following criteria for p-supersolubility of groups, where U_p is the class of all p-supersoluble groups and G^{U_p} is the U_p-residual of G, i.e., the intersection of all normal subgroups N of G with $G/N \in U_p$.

3.2. Corollary. Let p be a prime divisor of $|G^{U_p}|$ with $(p-1, |G^{U_p}|) = 1$. Suppose that for a Sylow p-subgroup P of G^{U_p}, all maximal subgroups of P are partially τ-quasinormal in G. Then G is p-supersoluble.

3.3. Corollary. Let p be a prime divisor of $|G'|$ with $(p-1, |G'|) = 1$. Suppose that for a Sylow p-subgroup P of G', all maximal subgroups of P are partially τ-quasinormal in G. Then G is p-supersoluble.

3.4. Theorem. Let p be a fixed prime divisor of $|G|$ and L a p-soluble normal subgroup of G such that G/L is p-supersoluble. If all maximal subgroups of $F_p(L)$ containing $O_{p'}(L)$ are partially τ-quasinormal in G, then G is p-supersoluble.

Proof. In fact, $F_p(L) = O_{p'p}(L)$. Firstly, assume that $O_{p'}(L) \neq 1$. We consider the factor group $G/O_{p'}(L)$. Obviously, $(G/O_{p'}(L))/(L/O_{p'}(L)) \cong G/L$ is p-supersoluble. Since $O_{p'}(L/O_{p'}(L)) = 1$, we have $F_p(L/O_{p'}(L)) = O_p(L/O_{p'}(L)) = F_p(L)/O_{p'}(L)$. Let $M/O_{p'}(L)$ be a maximal subgroup of $F_p(L/O_{p'}(L))$. Then M is a maximal subgroup of $F_p(L)$ containing $O_{p'}(L)$. Since M is partially τ-quasinormal in G, we have $M/O_{p'}(L)$ is partially τ-quasinormal in $G/O_{p'}(L)$ in view of Lemma 2.1(2). Thus $G/O_{p'}(L)$ satisfies the hypotheses of the theorem. By induction, $G/O_{p'}(L)$ is p-supersoluble and so is G.

Secondly, assume that $O_{p'}(L) = 1$. Consequently, $F_p(L) = O_p(L)$. By hypothesis, all maximal subgroups of $O_p(L)$ are partially τ-quasinormal in G. By virtue of Lemma

2.6, it follows that all G-chief factors of $O_p(L)$ are cyclic. Since L is p-soluble, we have $F^*(L) = F_p^*(L) = F_p(L) = O_p(L)$ by Lemma 2.8 and so every G-chief factor of $F^*(L)$ is cyclic. Applying Lemma 2.5, every G-chief factor of L is cyclic. Since G/L is p-supersoluble, we have G is p-supersoluble □

3.5. Corollary. Let G be a p-soluble group, where p is a fixed prime divisor of $|G|$. Then G is p-supersoluble if and only if all maximal subgroups of $F_p(G^{U_p})$ containing $O_{p'}(G^{U_p})$ are partially τ-quasinormal in G.

3.6. Corollary. Let G be a p-soluble group, where p is a fixed prime divisor of $|G|$. If all maximal subgroups of $F_p(G)$ containing $O_{p'}(G)$ are partially τ-quasinormal in G, then G is p-supersoluble.

Using the arguments as in the proof of Theorem 3.4, we can prove the following Theorem.

3.7. Theorem. Let p be a fixed prime dividing the order of G and L a p-soluble normal subgroup of G such that G/L is p-supersoluble. If all maximal subgroups of Sylow p-subgroups of $F_p(L)$ are partially τ-quasinormal in G, then G is p-supersoluble.

3.8. Corollary. Let G be a p-soluble group, where p is a fixed prime divisor of $|G|$. If all maximal subgroups of Sylow p-subgroups of $F_p(G)$ are partially τ-quasinormal in G, then G is p-supersoluble.

3.9. Theorem. Let p be a fixed prime divisor of $|G|$ and L a p-soluble normal subgroup of G such that G/L is p-supersoluble. If there exists a Sylow p-subgroup P of L such that every maximal subgroup of P is partially τ-quasinormal in G, then G is p-supersoluble.

Proof. Suppose that the theorem is false and let G be a counterexample of minimal order. We will derive a contradiction in several steps.

(1) G is p-soluble.

This follows directly from the p-solubility of L and the p-supersolubility of G/L.

(2) $O_{p'}(G) = 1$.

Denote $T = O_{p'}(G)$. If $T \neq 1$, we consider the factor group G/T. Obviously, PT/T is a Sylow p-subgroup of LT/T and $(G/T)/(LT/T) \cong G/LT \cong (G/L)/(LT/L)$ is p-supersoluble by the p-supersolubility of G/L. Let M/T be a maximal subgroup of PT/T. We may assume that $M = P_1T$, where P_1 is a maximal subgroup of P. Since P_1 is partially τ-quasinormal in G, it follows that P_1T/T is partially τ-quasinormal in G/T by Lemma 2.1(3). The minimal choice of G yields that G/T is p-supersoluble, and so is G, a contradiction.

(3) If N is a minimal normal subgroup of G, then N is an elementary abelian p-group. This follows from Steps (1) and (2).

(4) G has a unique minimal normal subgroup N contained in L such that G/N is p-supersoluble.

Let N be a minimal normal subgroup of G contained in L. Obviously, $N \leq P$ and P/N is a Sylow p-subgroup of L/N. Let P_1/N be a maximal subgroup of P/N. Then P_1 is a maximal subgroup of P. By hypothesis, P_1 is partially τ-quasinormal in G and so P_1/N is partially τ-quasinormal in G/N by Lemma 2.1(2). Since $(G/N)/(L/N) \cong G/L$ is p-supersoluble, G/N satisfies all the hypotheses of our theorem. It follows that G/N is p-supersoluble by the minimality of G. Noticing that the class of all p-supersoluble groups is a saturated formation, we have N is the unique minimal normal subgroup of G contained in L.

(5) N is not cyclic.

If N is cyclic, then G is p-supersoluble from the p-supersolubility of G/N, a contradiction.

(6) The final contradiction.

If N is contained in all maximal subgroups of G, then $N \leq \Phi(G)$. Since G/N is p-supersoluble, we have $G/\Phi(G)$ is p-supersoluble. Noticing that the class of all p-supersoluble groups is a saturated formation, it follows that G is p-supersoluble. This contradiction shows that there exists a maximal subgroup M of G such that $G = NM$ and $N \cap M = 1$. Let G_p be a Sylow p-subgroup of G containing P. Then $G_p = N(G_p \cap M)$ and $G_p \cap M < G_p$. Take a maximal subgroup G_1 of G_p containing $G_p \cap M$ and set $P_1 = G_1 \cap P$. Then $|P : P_1| = |P : G_1 \cap P| = |PG_1 : G_1| = |G_p : G_1| = p$ and so P_1 is a maximal subgroup of P. By hypothesis, P_1 is partially τ-quasinormal in G. In view of Lemma 2.1(4), G has a normal subgroup T such that P_1T is S-quasinormal in G, $P_1 \cap T \leq (P_1)_{\tau G}$ and $P_1T \leq L$.

Firstly, we have $N \nsubseteq P_1$. If not, $P = P \cap G_p = P \cap NG_1 = N(G_1 \cap P) = NP_1 = P_1$, a contradiction.

Secondly, we have $N \cap P_1 \neq 1$. If not, $|N : P_1 \cap N| = |NP_1 : P_1| = |P : P_1| = p$ and so $P_1 \cap N$ is a maximal subgroup of N. Therefore $|N| = p$, which contradicts Step (5).

Thirdly, we have $T > 1$. If not, $T = 1$ and P_1 is S-quasinormal in G. By Lemma 2.3, $O^p(G) \leq N_G(P_1)$. Since $P = G_p \cap L$, we have $P \trianglelefteq G_p$ and so $P_1 = G_1 \cap P \trianglelefteq G_p$. Hence P_1 is normal in $G_pO^p(G) = G$. Consequently, $P_1 \cap N \trianglelefteq G$. Step (4) shows that $P_1 \cap N = 1$ or $N \leq P_1$, a contradiction.

Finally, we have $N \leq T$ by Step (4). Then $P_1 \cap N = (P_1)_{\tau G} \cap N$ from $P_1 \cap T \leq (P_1)_{\tau G}$. For any Sylow q-subgroup G_q of G $(p \neq q)$, $(P_1)_{\tau G}G_q = G_q(P_1)_{\tau G}$ by Lemma 2.4. Then $(P_1)_{\tau G} \cap N = (P_1)_{\tau G}G_q \cap N \trianglelefteq G_q(P_1)_{\tau G}$. Obviously, $P_1 \cap N \trianglelefteq G_p$. Therefore $P_1 \cap N$ is normal in G, a same contradiction as above. □

3.10. Corollary. A p-soluble group G is p-supersoluble if and only if all maximal subgroups of any Sylow p-subgroup of G^{U_p} are partially τ-quasinormal in G.

3.11. Corollary. Let P be a Sylow p-subgroup of a p-soluble group G, where p is a fixed prime divisor of $|G|$. If all maximal subgroups of P are partially τ-quasinormal in G, then G is p-supersoluble.

4. Results On p-nilpotency

4.1. Theorem. Let p be a prime dividing the order of G and P a Sylow p-subgroup of G. If $N_G(P)$ is p-nilpotent and all maximal subgroups of P are partially τ-quasinormal in G, then G is p-nilpotent.

Proof. It is easy to see that the theorem holds when $p = 2$ by [8, Theorem 1.4], so it suffices to prove the theorem for the case of odd prime. Suppose that the theorem is false and let G be a counterexample of minimal order.

(1) G is not p-supersoluble.

If G is p-supersoluble, then $PO_{p'}(G)$ is normal in G in view of Lemma 2.7. It follows that

$$G/O_{p'}(G) = N_{G/O_{p'}(G)}(PO_{p'}(G)/O_{p'}(G)) = N_G(P)O_{p'}(G)/O_{p'}(G).$$

Since $N_G(P)$ is p-nilpotent, we have $G/O_{p'}(G)$ is also p-nilpotent. Consequently, G is p-nilpotent, a contradiction.

(2) G is not p-soluble.

By hypothesis, all maximal subgroups of P are partially τ-quasinormal in G. If G is p-soluble, then G is p-supersoluble by Corollary 3.11, contrary to Step (1).

(3) If K is a proper subgroup of G with $P \leq K < G$, then K is p-nilpotent.

It is easy to see that $N_K(P) \leq N_G(P)$ and hence $N_K(P)$ is p-nilpotent. By Lemma 2.1(1), all maximal subgroups of P are partially τ-quasinormal in K. Hence K satisfies the hypothesis of our theorem. The minimal choice of G implies that K is p-nilpotent.

(4) $O_p(G) \neq 1$.

Let $J(P)$ be the Thompson subgroup of P. Then $N_G(P) \leq N_G(Z(J(P))) \leq G$. If $N_G(Z(J(P))) < G$, then, in view of Step (3), $N_G(Z(J(P)))$ is p-nilpotent and so G is p-nilpotent by [3, Theorem 8.3.1], a contradiction. Hence $N_G(Z(J(P))) = G$, which shows that $Z(J(P))$ is a normal p-subgroup of G and so $O_p(G) \neq 1$.

(5) The final contradiction.

It is easy to see that the factor group $G/O_p(G)$ satisfies the hypothesis of our theorem. Now, by the minimality of G, we see that $G/O_p(G)$ is p-nilpotent. In particular, $G/O_p(G)$ is p-soluble and so is G, which contradicts Step (2). □

4.2. Corollary. Let p be a prime dividing the order of G and L a normal subgroup of G such that G/L is p-nilpotent. Suppose that there exists a Sylow p-subgroup P of L such that all maximal subgroups of P are partially τ-quasinormal in G and $N_G(P)$ is p-nilpotent. Then G is p-nilpotent.

Proof. It is obvious that $N_L(P)$ is p-nilpotent and all maximal subgroups of P are partially τ-quasinormal in L. Applying Theorem 4.1, L is p-nilpotent. Let $L_{p'}$ be the normal Hall p'-subgroup of L. Obviously, $L_{p'}$ is a normal subgroup of G. If $L_{p'} \neq 1$, we consider the factor group $G/L_{p'}$. Firstly, $(G/L_{p'})/(L/L_{p'}) \cong G/L$ is p-nilpotent and all maximal subgroups of $PL_{p'}/L_{p'}$ are partially τ-quasinormal in $G/L_{p'}$ by Lemma 2.1(3). Secondly, $N_{G/L_{p'}}(PL_{p'}/L_{p'}) = N_G(P)L_{p'}/L_{p'}$ is p-nilpotent. Hence $G/L_{p'}$ satisfies the hypothesis of our corollary. By induction, $G/L_{p'}$ is p-nilpotent and so is G, as desired. Hence we may assume $L_{p'} = 1$, i.e., $L = P$. By hypothesis, $N_G(P) = G$ is p-nilpotent. □

4.3. Theorem. Let p be a prime dividing the order of G and P a Sylow p-subgroup of G. If every maximal subgroup P_1 of P is partially τ-quasinormal in G and $N_G(P_1)$ is p-nilpotent, then G is p-nilpotent.

Proof. Assume that the assertion is false and let G be a counterexample of minimal order. Then:

(1) If $P \leq K < G$, then K is p-nilpotent.

By Lemma 2.1(1), every maximal subgroup P_1 of P is partially τ-quasinormal in K. Obviously, $N_K(P_1) \leq N_G(P_1)$. By hypothesis, we have $N_K(P_1)$ is p-nilpotent. Therefore, K satisfies the hypothesis of the theorem, and so K is p-nilpotent by the choice of G.

(2) $O_{p'}(G) = 1$.

If $O_{p'}(G) \neq 1$, we consider the factor group $G/O_{p'}(G)$. Obviously, $PO_{p'}(G)/O_{p'}(G)$ is a Sylow p-subgroup of $G/O_{p'}(G)$. Let $M/O_{p'}(G)$ be a maximal subgroup of $PO_{p'}(G)/O_{p'}(G)$. We may assume that $M = P_1O_{p'}(G)$, where P_1 is a maximal subgroup of P. Since P_1 is partially τ-quasinormal in G, it follows that $P_1O_{p'}(G)/O_{p'}(G)$ is partially τ-quasinormal in $G/O_{p'}(G)$ by Lemma 2.1(3). It is easy to see that

$$N_{G/O_{p'}(G)}(P_1O_{p'}(G)/O_{p'}(G)) = N_G(P_1)O_{p'}(G)/O_{p'}(G)$$

is p-nilpotent. Hence $G/O_{p'}(G)$ satisfies the hypothesis of our theorem. The minimal choice of G yields that $G/O_{p'}(G)$ is p-nilpotent and so is G, a contradiction.

(3) $F(G) = P$.

If $N_G(P) < G$, then $N_G(P)$ is p-nilpotent by Step (1). Applying Theorem 4.1, G is p-nilpotent. This contradiction implies P is normal in G. By Step (2), $F(G) = P = O_p(G)$.

(4) G is p-soluble.

This follows from Step (3) directly.

(5) Let N be a minimal normal subgroup of G. Then $N < P$.

In view of Steps (2) and (4), $N \leq P$. Assume that $N = P$. By hypothesis, every maximal subgroup of N is partially τ-quasinormal in G. By Lemma 2.9, $|N| = p$. This shows that the maximal subgroup of P is 1. By hypothesis, $G = N_G(1)$ is p-nilpotent, a contradiction.

(6) Final contradiction.

If N is a maximal subgroup of P, then, by hypothesis, $G = N_G(N)$ is p-nilpotent, a contradiction. Hence we may assume that $|P : N| \geq p^2$. By Lemma 2.1(2), it is easy to see that G/N satisfies the hypothesis of the theorem. Hence G/N is p-nilpotent by the minimal choice of G. Since the class of all p-nilpotent groups is a saturated formation, it follows that N is a unique minimal subgroup of G and $\Phi(G) = 1$. Consequently, $F(G) = N$. By Step (3), $N = P$, which contradicts step (5). □

4.4. Corollary. Let p be a prime dividing the order of G and L a normal subgroup of G such that G/L is p-nilpotent. Suppose that there exists a Sylow p-subgroup P of L such that every maximal subgroup P_1 of P is partially τ-quasinormal in G and $N_G(P_1)$ is p-nilpotent. Then G is p-nilpotent.

Proof. Using the arguments as in the proof of Corollary 4.2, we may assume that $L = P$. Let V/P be the normal p-complement of G/P. By the Schur-Zassenhaus Theorem, there exists a Hall p'-subgroup $V_{p'}$ of V such that $V = P \rtimes V_{p'}$. Since V is p-nilpotent by Lemma 2.1(1) and Theorem 4.3, $V = P \times V_{p'}$. This induces that $V_{p'}$ is the normal p-complement of G. Therefore, G is p-nilpotent. □

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