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Insights into Implementing Number Pattern Activities with an Inquiry-Based Learning Approach in the Education of Gifted Students

Özel Yetenekli Öğrencilerin Eğitiminde Sorgulama Temelli Öğrenme Yaklaşımıyla Sayı Örüntüleri Etkinliklerinin Uygulanmasına Yönelik İçgörüler

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ABSTRACT: This study addresses regular number patterns, which are important for algebraic, arithmetic, and spatial thinking and suitable for establishing in-depth relationships, through original activities. The aim of the study is to investigate the cognitive gains, skills, and student opinions regarding the application of number pattern activities, prepared using the inquiry-based learning approach, one of the innovative learning approaches, in mathematics lessons of gifted 7th-grade students. A qualitative case study, providing in-depth information, was determined as the research design. Data after the activities were collected using a semi-structured interview form and a number pattern task form developed by the researcher. In the analysis, data obtained from interviews, and the number patterns task form were analyzed according to content analysis. Categories were created from the data and associated with codes. In the categories of fluency, flexibility, and deep thinking, generally good results were obtained; in the categories of enrichment and originality, good to moderate results were achieved. Participants gave correct answers in algebraic and arithmetic notation, represented things with different representations, and revealed in-depth and relational information. As a result of the study, participants stated that they initially had difficulty with activities containing open-ended, complex, and inquiry-based questions because it was their first time encountering them. However, they expressed that they were able to do them when they worked on the situations in more depth. After all the activities, it was observed that they acquired skills in establishing relationships between patterns, generalizing appropriate to changing situations, showing them with different representations, creating original shapes, using different strategies, questioning, and thinking in depth.

Key words: Gifted talented student, inquiry-based learning, number patterns.

ÖZ: Bu çalışmada cebirsel, aritmetik ve uzamsal düşünme açısından önem taşıyan, derinlemesine ilişki kurmaya uygun olan kurallı sayı örüntüleri, özgün etkinlikler ile ele alınmıştır. Çalışmanın amacı yenilikçi öğrenme yaklaşımlarından sorgulama temelli öğrenme yaklaşımı ile hazırlanan sayı örüntüleri etkinliklerinin 7. sınıf özel yetenekli öğrencilerin matematik dersindeki bilişsel kazanımları, becerileri ve uygulamaya yönelik öğrenci görüşlerini araştırmaktır. Araştırma deseni olarak derinlemesine bilgi sunan nitel durum çalışması belirlenmiştir. Etkinlik sonrası veriler araştırmacı tarafından geliştirilen yarı yapılandırılmış görüşme formu ve sayı örüntüleri görev formu ile toplanmıştır. Analizde görüşmelerden ve sayı örüntüleri görev formundan elde edilen veriler içerik analizine göre analiz edilmiştir. Verilerden kategoriler oluşturulup kodlarla ilişkilendirilmiştir. Akıcılık, esneklik, derinlemesine düşünme kategorilerinde genellikle iyi; zenginleştirme, özgünlük kategorilerinde iyi ve orta düzeyde bulgular elde edilmiştir. Katılımcılar cebirsel ve aritmetik gösterimde doğru yanıtlar vermiş, farklı temsillerle göstermiş, derinlemesine ve ilişkisel bilgiler ortaya koymuştur. Çalışmanın sonucunda açık uçlu, karmaşık ve sorgulamaya dayalı sorular içeren etkinlikler konusunda katılımcılar ilk defa karşılaştığı için başlangıçta zorlandıklarını ifade etmiştir. Ancak durumlar üzerinde daha derinlemesine çalıştıklarında yapabildikleri yönünde görüş bildirmişlerdir. Tüm etkinliklerin ardından örüntüler arasında ilişki kurma, değişen durumlara uygun genelleme yapma, farklı temsillerle gösterme, özgün şekiller ortaya koyma, farklı stratejiler kullanma, sorgulama ve derinlemesine düşünme becerilerini elde ettikleri görülmektedir.

Anahtar kelimeler: Özel yetenekli öğrenci, sorgulama temelli öğrenme, sayı örüntüleri.

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In mathematics, students must be exposed to challenging and complex situations to develop original products and new strategies. Algebraic and functional studies and generalizations are examples of situations that stimulate students' thinking. Milgram & Hong (2009) state that being creative in mathematics is related to complex and relational thinking. Thus, understanding relationships and patterns leads to developing new and original strategies. Starko (2005) emphasizes that tasks in mathematics consist of doing creative things and enabling in-depth understanding of topics. Joyce et al. (2009) state that inquiry exists to explain events and reveal cognitive processes. The language used in teaching, verbal representations, and questions influence students' cognitive processes. To foster higher-order thinking and encourage creativity, teachers need to carefully select their verbal representations and questions.

Leikin et al. (2009) link creativity in mathematics primarily to understanding and deepening understanding. The importance of in-depth study and thinking in mathematics is emphasized in the present study. Similarly, the development of creative thinking and creativity in mathematics is highlighted. Two activities are planned to use an inquiry-based learning approach to both foster creativity and enable students to understand and relate through in-depth thinking in mathematics. Through these activities, the teacher is expected to guide students in developing higher-order thinking and in-depth study. Looking at literature, it is seen that many studies focus on problem-solving and problem-posing. Very few studies have been conducted on pattern generalization.

The number of studies on inquiry-based mathematics teaching has increased considerably in recent years, but the number of studies is still limited. Arıkan Güllü & Yürümezoğlu (2025) applied inquiry-based learning activities for 9 weeks in their study with gifted students. The data obtained at the end of the activities showed that the approach yielded positive results in the students' learning process and cognitive desire level. Yiğit Özüdoğru & Tosun (2023) applied inquiry-based learning activities in geography lessons to gifted students at a Science and Art Center. As a result of the study, positive results were obtained in the skill development of the students. Ülker & Bostan Sarıoğlan (2020) applied inquiry-based learning activities to gifted students on teaching the working principles of nuclear power plants. As a result of the study, it was emphasized that the students' scientific answering and inquiry skills improved.

Karaaslan (2020) examined the metacognitive skills and knowledge of gifted students in teaching complex numbers. Bal İncebacak (2019), in study, presented activities on fractions using an inquiry-based learning approach, concluded that studies in this area are insufficient and suggested that activities emphasizing the inquiry-based learning approach should be examined in other areas of mathematics as well. Işık Tertemiz et al. (2023) compared the answers given by gifted students and mathematically successful students to questions related to number sense in their study. The inquiry-based learning approach was used in the study. It is thought that number sense forms the basis for number patterns. These studies demonstrate the importance of the inquiry-based learning approach in the education of gifted students. The inquiry-based learning approach is very suitable for the Science and Art Center program. In mathematics, activities should be structured around this approach to ensure in-depth learning and foster creativity in the teaching of many topics. However, studies on the inquiry-based learning approach are quite limited. Activities should be planned to serve as a reference for teachers who are just beginning to work with gifted students. While there are activity studies in literature for different subjects, topic-based studies in mathematics are quite limited. How inquiry-based learning can be used in teaching a mathematical topic is crucial. Creating challenging and complex situations is very important for gifted students. Further research is needed on what types of questions should be used to structure the approach, and what constitutes open-ended and challenging questions. Generally, studies have used quasi-experimental methods and made comparisons. This indicates that the studies remain superficial. It is expected that this study will yield deeper and more relational data through a qualitative case study approach.

Demirci & Işık Tertemiz (2022) revealed that the most frequently discussed topics in the articles they reviewed were the development of creative thinking skills in mathematics for gifted students, providing opportunities to solve non-routine problems, and the need for enriched mathematics instruction. The study generally mentions the existence of research on creative thinking in problem-solving and mathematics in the articles presented. It emphasizes the need for studies incorporating enriched activities and highlights the perceived deficiency in this area. Syawahid et al. (2024) emphasized the lack of studies that reveal students' functional thinking skills in solving non-figurative pattern tasks.

Gifted individuals may be more advanced than their peers in terms of creativity, dedication, and intellectual ability; this can lead to them becoming bored with or unable to benefit sufficiently from standard educational programs. This situation necessitates adjustments in the education of gifted individuals (Ministry of National Education, 2021). It is considered necessary for practitioners to present more complex and challenging situations so that students can realize their full potential and conduct their own inquiries. For this reason, this study, which involves the subject-based use of an innovative approach, was needed. Mulligan (2009) emphasizes the importance of number patterns in early mathematics teaching. The topic of number patterns is particularly important for generalization. Kidd et al. (2013) emphasized that learning patterns enhances conceptual learning in mathematics and highlighted the importance of pattern activities. Bock et al. (2018) demonstrated that pattern tasks increase cognitive demand and are effective in skill development. Studies show that inquiry-based activities enhance learning in gifted students. Patterns are important for making conceptual generalizations from an early age. Assmus & Fritzlar (2022) reveal that one of the cognitive characteristics emphasized in gifted students is the generalization of mathematical structures and patterns. Pattern generalization tasks emerge as a powerful tool that supports and develops algebraic thinking. Erdoğan & Gül (2022) suggest in their study that geometric pattern tasks involving linear and nonlinear relationships can play an important role in developing functional thinking and generalization skills of gifted students in learning environments and emphasize the need for more studies that will reveal different strategies in pattern generalization tasks in the education of gifted students. Tuncer (2025) examined pattern generalization and variable assignment strategies in a pattern generalization study conducted with gifted 8th-grade students. The study highlighted that students demonstrated creative solutions when solving nonlinear generalization problems. Studies highlight the importance of pattern generalization in the education of gifted students. However, the existing research appears insufficient. It is necessary to conduct studies that explore not only pattern generalization but also different strategies. However, current literature is limited to pattern generalization, finding near and far terms, and variable assignments. This study is significant because it addresses the topic of number patterns, an important subject in the education of gifted students, in a different way. Presenting different strategies is important for establishing relationships between number patterns, which are lacking in literature, questioning generalizations in changing situations, and exploring different strategies and shape patterns through flexible thinking. When the studies are examined, it is seen that the topic of number patterns in the education of gifted students is addressed superficially and at a basic level, it is seen that inquiry-based learning in the education of gifted students is addressed in different contexts. Problem-solving, fractions, complex numbers, and finding the nearest or furthest term in number patterns, or arriving at the general rule, were covered.

The studies were qualitative, only interviews were conducted, problem-solving tasks were applied, and the data were analyzed cognitively. In addition, it is observed that qualitative methods are generally used in studies, but these studies produce results-oriented data. The present study addresses the topics of gifted students, inquiry-based learning approach, and number patterns, which have been discussed in different studies, in depth and presents process-oriented data. It presents the topic of number patterns, which is lacking in literature, in the education of gifted students through inquiry-based learning and complex and challenging situations This study differs from others both in its implementation of the activity and in its inclusion of several patterns task form consisting of challenging and relational questions alongside the interview. The study is important in revealing the higher potential of the participants. Students can demonstrate their potential through inquiry when guided by thought-provoking and complex questions. This study offers original number pattern activities to the literature and practitioners by using thought-provoking and challenging questions. It will shed light on the application of inquiry-based learning in the education of gifted students and on future studies.

The purpose of this study is to investigate the cognitive gains and student opinions of 7th-grade gifted students in mathematics regarding the use of number pattern activities, developed using an inquiry-based learning approach, an innovative learning approach. The literature contains a limited number of subject-based studies that apply innovative approaches to the education of gifted students. This study is significant because it includes number patterns that involve inquiry skills and algebraic and functional thinking, which are crucial in the education of gifted students. Such a study was needed to fill the gap in literature and to serve as a reference for teachers working with gifted students. Studying is important as it serves as an example for teachers working with gifted students and sheds light on new research. Number pattern research with gifted students is very limited. Furthermore, it distinguishes itself from other studies by including strategies for deriving different patterns from a pattern, including the general rule changing when the first or first two terms of a pattern change, representing it with different representations, and deriving multiple related shape patterns from a pattern. The thought-provoking, complex, and challenging questions posed in the activities are expected to reveal students' high-level potential.

Research questions:

1. What are the participants' opinions on number pattern activities implemented with an inquiry-based learning approach?

2. What skills gains did the students gain because of the activities?

3. What cognitive gains did the students gain because of the activities?

4.What was the most challenging aspect for the students?

Inquiry-Based Learning

Spronken et al. (2010) emphasize inquiry-based learning as a learner-centered approach to knowledge acquisition and teaching. The inquiry-based learning approach is based on the philosophy of constructivism. John Dewey was the originator of learning through inquiry. Tan (2009) emphasizes the importance of the constructivist approach in his work. Teachers guide students by asking thought-provoking questions and enable them to construct knowledge. Piaget, Vygotsky, and John Dewey are pioneers in this field. Another researcher in the field of inquiry is G. Polya. Polya argues that while students are problem-solving, the teacher's role should be to encourage students to think deeply and to inquire. Only in this way will students generate creative solutions to problems and connect knowledge to different situations. In Polya's problem-solving process, students are guided by questions such as "Can you prove your hypothesis by checking the result?", "Can you use the method for another problem?", "Can you obtain the result with a different strategy?", and "Do you know of a related problem?" (Conway, 2004). Kogan & Laursen (2014) emphasize that in student-centered approaches, the learning environment is tailored to student interests and needs. In the study, students who are successful in mathematics experience a sense of accomplishment by using higher-level skills through inquiry-based learning. Perry & Richardson (2001) define inquiry-based learning as the process of learning by asking, analyzing, and deciphering information.

Keller (2001) emphasizes inquiry as a process in which students actively participate in learning and construct knowledge. Wu & Lin (2016) stated that students achieve lasting learning through inquiry-based learning and utilize metacognition. Lakhana (2014) states that during the inquiry process, individuals construct new knowledge by connecting it with previous knowledge. Cummins et al. (2004) emphasized in their study that inquiry-based learning allows students to question and test their own hypotheses using the scientific method. Therefore, it is important to use the inquiry-based learning approach in education to stimulate students' curiosity, encourage them to think, and connect new information to their prior knowledge, and its use in mathematics is particularly appropriate. Şensoy & Aydoğdu (2008) found in their research that the inquiry-based learning approach increased learning retention but was not effective against attitudes. Erkol & Şahintepe (2020) emphasized that students better utilize metacognition in inquiry-based learning. Atlı (2021) observed a significant difference in the experimental study using the inquiry-based approach in science education with 5th graders compared to the control group. They concluded that students' anxiety decreased, and they learned with pleasure.

Arifin et al. (2025) conducted a metasynthesis review of studies conducted using the inquiry-based learning approach. The review revealed that activities implemented with the inquiry-based approach were effective in developing higher-order thinking skills, such as critical thinking, in students. Ay & Dağhan (2023) emphasized that participation is high, and motivation increases in learning environments where the inquiry approach is applied, thus resulting in high-level performance. Öztürk & Demir (2023) examined research conducted using the inquiry-based learning approach. The review revealed disadvantages in some studies, such as lengthy sessions, teachers' inability to conduct inquiry-based instruction, and students' inability to guide students with questions. The study also addressed the limitations of inquiry-based learning. When teachers fail to guide students with appropriate questions and manage their time effectively, the approach may fail to achieve its intended purpose. Kurniati et al. (2024) stated in their study that inquiry-based learning is more effective when ideal conditions are met. The study emphasized that poorer results will be achieved if teacher competence and sufficient time are not provided. Similarly, if students are not given sufficient time to think and question, effective results may not be achieved, and students' potential may not be realized. Wan et al. (2024) emphasizes the disadvantage of cognitive overload in the inquiry-based approach. Therefore, he emphasizes the importance of teachers being good guides. Martin (2022) emphasized that more effective results are achieved in environments that present complex and challenging activities rather than in environments where structured and weak practices are implemented. As mentioned in this study, students should be encouraged to think through complex and challenging questions. Effective results are expected when students are given sufficient time and guidance to conduct their own inquiry.

Education of Gifted Students

Intelligence is defined as the sum of the mental abilities' individuals need to adapt to their environment (Garlick, 2002; Sternberg, 1997). Talent is defined as the innate ability to be developed in a specific area and the path to success by developing this ability (Gagne, 2004). There have been ongoing debates regarding giftedness and superior intelligence for years. A different perspective examines giftedness as two sub-dimensions: academic and creative superiority (Reis & Sullivan, 2009). Today, the term " gifted individual" is used. Giftedness consists of the interaction of three areas: above-average general and specific abilities, creativity, and motivation (Renzulli, 2011). The field of gifted education is grounded in the almost universally accepted fact that some students exhibit superior performance or the potential for superior performance compared to their peers in academic, creative, leadership, or artistic fields. From preschool through college, and even at the graduate and professional levels, a range of learning potential warrants the examination of differentiated opportunities and services (Renzulli, 2012). Barbara Clark, on the other hand, takes a more neurobiological approach to giftedness, explaining that it stems from the brain's ability to integrate functions in an accelerated manner (Clark, 2008). Intelligence is the sum of mental abilities, including problem-solving, abstract thinking, reasoning, memory, comprehension, applying knowledge to new situations, using information gained from past experiences (Sak, 2018). According to the most widely accepted definition within these approaches: a gifted person is someone who demonstrates superior performance or potential in many mental abilities or intelligences compared to their peers, has strong creativity, and never gives up once they start a task (Ataman, 2018). Students with good problem-solving skills, metacognitive abilities, creative mathematical thinking ability, and high aptitude or performance in mathematical problem-solving are generally considered mathematically talented. (Leikin, 2021). Special ability is not a phenomenon that exists in nature; therefore, concepts such as normal intelligence, above-average intelligence, or special ability are inventions of humans, not nature (Borland, 2021). Among the concepts considered in literature as dimensions of giftedness is the ability to learn faster than peers (Cross & Coleman, 2005). Special talent is the application of general talent in specific skill areas and the capacity to use advanced strategies and knowledge in the problem-solving process (Renzulli 1978; Sak, 2010). Renzulli (2003) divides special talent into two categories: "academic superiority" and "creative superiority." Theorists who work on special talent and intelligence include Gardner, Carroll, and Raymond John Horn. Çitil (2018) states that academic studies on gifted individuals began in the second half of the twentieth century. Winner (1996) defines gifted individuals as individuals with high potential and high performance. Milgram & Hong (2009) argue that limiting students solely to academic achievement or scores from standardized intelligence tests is insufficient. Thus, multifaceted assessments of students emphasize the discovery of their talent areas. In Türkiye, research and studies are being conducted to develop the talents of gifted individuals, primarily focusing on increasing the number of Science and Art Centers and on differentiated teaching methods and activities in specific and interdisciplinary fields (Ataman, 2004; Kaya, 2021; Özyaprak, 2016; Sak, 2011;).

Tomlinson (2007) emphasized the transfer of knowledge to different situations and its relevance to daily life in the differentiated curriculum for gifted students. Sak (2010) focused on the differences in higher order thinking skills and learning speeds of students in the differentiated curriculum for gifted students. Another practice included in the curriculum for gifted students is enrichment. Studies conducted in Türkiye on the mathematics education of gifted students and enriched teaching activities are noteworthy (Akay, 2018; Düzgünoğlu, 2023; Mammadov & Topçu, 2014; Mertoğlu, 2010; Ünal, 2019; Yılmaz, 2019). Kaya (2021) emphasized the importance of increasing the number of methodological studies in studies conducted with gifted students. Düzgünoğlu (2023) emphasizes enrichment as a strategy that shapes the program according to the needs and levels of gifted students. Enrichment results in higher performance among gifted students. In this study, an enrichment strategy was employed to enable highly motivated gifted students to demonstrate their performance.

The Ministry of National Education's Science and Art Centers directive, published in 2022, defines gifted individuals as individuals who learn faster than their peers; are prominent in creativity, art, and leadership; possess special academic talents; can understand abstract ideas; enjoy acting independently in areas of interest; and demonstrate high-level performance (Ministry of National Education, 2022). Science and Art Centers are official institutions affiliated with the Ministry of National Education. Students are identified through nomination, preliminary assessment, and individual evaluation stages. Identified students attend Science and Art Centers. Programs for gifted individuals are developed under the guidance of relevant classroom/branch teachers, with a student-centered and interdisciplinary structure, tailored to individual learning, and differentiated and enriched according to students' interests, abilities, and potential. These programs allow students to acquire the high-level mental, social, personal, and academic skills they will need in adulthood, such as effective problem-solving, decision-making, and creativity. Programs designed to develop gifted individuals are designed and implemented with disciplinary and interdisciplinary approaches, aiming to provide in-depth and advanced knowledge, skills, and behaviors in any discipline. When preparing educational programs, attention is paid to the planning and implementation of activities that foster the development of higher-order thinking skills (Ministry of National Education, 2022). As stated in the Ministry of National Education (2022), implemented activities should aim to unlock students' potential and encourage deep thinking. The guidelines required the activities in this study to create challenging and complex situations that would foster decision-making, questioning, and relationship-building. Science and Art Centers offer educational opportunities according to students' interests, talents, and potential, enabling them to acquire high-level mental, social, personal, and academic skills they will need in adulthood, such as effective problem-solving, decision-making, and creativity. The program progresses in an in-depth manner, focusing on acquiring advanced knowledge, skills, and behaviors, and accordingly, emphasis is placed on production-oriented work (Ministry of National Education, 2025). The program aims not only to provide students with academic knowledge but also to enhance their creative thinking, analytical skills, and critical thinking abilities. It offers a multifaceted education that allows students to address problems from diverse perspectives. The student-centered approach encourages students to actively manage their own learning processes under the guidance of teachers (Ministry of National Education, 2024).

Number Patterns

Using patterns is seen as a way of approaching algebra (Mason, 1996). Patterns are one of the fundamental skills that play an important role in the development of spatial thinking, ordering, comparing, and classifying in early mathematics learning (Papic, 2007). Souviney (1994) defines patterns as a combination of shapes, symbols, and sounds. NCTM (2000) states that number patterns encourage analytical thinking by establishing relationships between the terms of the pattern; inductive and algebraic thinking by arriving at a general rule; and thinking by establishing relationships with shape patterns. From this perspective, algebraic and functional thinking are also inherent in number patterns.

Patterns are a topic that is incorporated into education at all levels, starting in preschool. NCTM (2000) focuses on teaching algebra at an early age. Tanışlı (2008) stressed that patterns should be taught from an early age because they involve algebraic and functional thinking. Studies recommend that algebra, known as a challenging subject for students, be taught with different teaching approaches and strategies. Walkoe (2015) emphasizes that the topic of patterns involves algebraic and functional thinking, problem-solving, generalization, and relationship-making skills. Erdoğan & Gül (2023) emphasized that there are very few studies in the literature on number patterns in the education of gifted students and that these studies should be increased.

Akgül (2014), in study, using a constructivist learning approach, described mathematics as establishing relationships, finding solutions to problems, and relating them to real life, rather than calculations and formulas. Karaz (2021) demonstrated that patterns facilitate understanding mathematical concepts because they contribute to algebraic thinking, stated that they contribute to the development of skills such as questioning, making connections, exploring, and problem-solving. Because number patterns require students to use analytical, geometric, and algebraic thinking skills, many students struggle. The study requires gifted students to learn number patterns, understand their general rules, create multiple patterns, and use their creativity to delve deeper into the terms within number patterns, generate new number patterns, and explore their relationships with different number patterns.

Assmus & Fritzlar (2022), Syawahidv et al. (2024), and Erdoğan & Gül (2022) worked on the generalization of number patterns with gifted students. Tuncer (2025) investigated strategies and mathematical creativity in nonlinear realistic generalization tasks in the education of gifted students. Girit Yildiz & Durmaz (2021) worked on figural reasoning in generalizing linear patterns and numerical reasoning in generalizing nonlinear patterns. Studies on number patterns with gifted students are limited. Studies have generally focused on pattern generalization, recurrent patterns, finding near and far terms, and obtaining figural patterns. The studies conducted are seen as results oriented. Number patterns should be examined in depth and relationally from a process-oriented perspective.

The reason for using inquiry-based learning in teaching number patterns is that number patterns are represented algebraically, arithmetically, and verbally. Students need to think flexibly and originally while doing this. Furthermore, challenging and in-depth questions are crucial for establishing deep connections. Enrichment, by presenting patterns in different ways and obtaining different number patterns, yields in-depth and relational results in associating number patterns. Therefore, the categories of fluency, flexibility, deep thinking, originality, and enrichment were determined in this study.

Method

Research Model

This study employed a qualitative research method. This method was chosen because it yields deeper and more effective results in mathematics education research compared to other methods. Interviews and qualitative number pattern task forms provide more detailed and in-depth findings. Qualitative research was preferred in this study not for comparison, but rather to explore the knowledge, skills, and thoughts gained by participants through an inquiry-based learning approach applied during the activity process. Alanka (2024) states that qualitative research methods provide researchers with an advantage in understanding complex issues and presenting them from different perspectives. In the research, a qualitative case study was preferred to reveal the expected potential because of the inquiry-based learning approach, which includes more in-depth learning and complex and thought-provoking questions. Within qualitative research, it is possible to obtain more in-depth information through case studies.

The study was conducted with 12 gifted students selected using a purposive sampling method. The selected 12 students were identified using criterion sampling, a purposive sampling method, and met pre-planned criteria (Yıldırım & Şimşek, 2016). Creswell (2007) describes a case study as a qualitative method that allows for in-depth investigation of a defined situation. Chmiliar (2010) states that a case study is used to gain in-depth knowledge about a topic. Brown (2008) describes a case study as obtaining rich and diverse information about events and situations. The activities involved working with inquiry-based learning approaches and complex situations. After the activities, the interview and number patterns task form were applied to the participants. This method was used to reveal the knowledge and experience gained by participants through open-ended questions resulting from their experiences in the activities, to establish relationships between the results, and to present more concrete data.

Participants

The Ministry of National Education (2022) defines gifted students as individuals who demonstrate superior performance compared to their peers in areas such as art, creativity, and academic achievement. The criteria for this study were 7th-grade male and female students taking mathematics courses at the Science and Art Center. The study included 12 gifted students attending the Science and Art Center. The students were in 7th grade. The participants were selected from 7th grade because the in-depth mathematics program was implemented during this period. This was because they were first exposed to the number patterns in the activities. The participants had taken mathematics classes for one semester (4 months) prior to the 7th grade. During this period, an enriched mathematics program was implemented, their mathematical abilities were recognized, and it was decided that they would take mathematics classes in the 7th grade. The research was conducted in the first weeks of 7th grade, that is, before the in-depth mathematics teaching program began. The participants consisted of 6 girls and 6 boys, all 12 years old.

Additionally, the students were selected for 7th grade because they were in the abstract operations stage, where mental operations such as generalization, deduction, and induction are performed (Karip, 2007), and because they had already learned number patterns and algebraic expressions in 6th grade. Therefore, the activities are suitable for gifted students to work in depth and use metacognition, based on their readiness levels. Participants were coded as U1, U2, U3, …, U12, and their identities were kept confidential. All students participated in the interview, and the number patterns task form was applied to all students.

Data Collection Tools

Data was collected after the activities were implemented using two qualitative measurement tools. One of the data collection tools was a semi-structured interview form. The questions cover the activities to be implemented in the group and the implementation process. The application process was designed to obtain in-depth information from students about the research topic (Büyüköztürk et al., 2014). To increase the validity of the form, the questions were relevant to the purpose of the research, to select suitable participants, and to ensure ease of response (Büyüköztürk et al., 2014). Semi-structured interview form was developed by the researcher. The purpose of the study, the characteristics of the target audience, and the implemented activities were considered when developing the questions. Care was taken to ensure that the questions were easily understood by the target audience and appropriate to their level. Three different experts reviewed the questions for clarity, relevance to the study's purpose, and open-ended nature. Following the review, it was decided to restructure the questions and implement them. Twelve participants were interviewed for 20 minutes in a quiet environment. The instructions were read before the interview. The voluntary nature of participation was explained. The interviews were conducted in a quiet environment. The researcher took notes and audio recordings during the application.

The other data collection tool was the number patterns task form. The number patterns task form was prepared by the researcher to determine the effects of the activities on the participating students' success in number patterns. The number patterns task form includes four questions. The questions were developed based on participants' in-depth study of terms, their ability to establish relationships, their use of different representations, and their ability to enrich understanding. The questions were open-ended and designed to be consistent with the activities and research objectives.

Data Analysis

Data were analyzed using content analysis. Aziz (2015) emphasizes that content analysis involves systematically analyzing data. Yıldırım & Şimşek (2013) define content analysis as describing qualitative findings with related themes and codes. According to content analysis, data is presented in detail and directly, with the identities of individuals kept confidential, to increase validity and reliability through coding. Robert & Bouillaget (1997) emphasize content analysis as the process of systematically and objectively analyzing data using codes and themes. Alanka (2024) defines content analysis as a qualitative analysis method that provides the researcher with systematic and in-depth information. In data analysis, themes related to the research topic were identified. Themes were associated with the codes. Themes and codes were determined based on the purpose of the study. The themes included in the interview form data analysis were cognitive gain, ease, difficulty, creativity, and skill. The themes included in the number patterns task form data analysis were fluency, flexibility, originality, enrichment, and deep thinking. The answers to the questions in the number patterns task form were analyzed according to these themes. The number of correct thoughts, using multiple representations, creating different shape patterns with new strategies, creating different number patterns from the same number pattern, establishing in-depth relationships, and analysis were influential in the formation of the themes. The interview data were categorized as cognitive gain, ease, difficulty, creativity, and skill. Six codes were obtained related to cognitive gain: enabling in-depth thinking, enhancing learning, understanding complex patterns, establishing relationships between patterns, establishing relationships between number patterns and shape patterns, ensuring lasting learning. Three codes were obtained related to difficulty: creating original shapes, struggling, complexity. Four codes were obtained related to ease: creating new patterns, establishing relationships between patterns, creating shape patterns, algebraic notation. Three codes were obtained related to creativity: finding new strategies for generating number patterns, creating more different pattern shapes, originality. Six codes were obtained related to skills: decision making, problem solving, establishing relationships between numbers and shapes, algebraic thinking, creativity, establishing relationships between different number patterns. When analyzing the number patterns task form, participants' forms were scored between 0 and 5. 4 and 5 were coded as Good, 2 and 3 as Intermediate, and 0 and 1 as Low. All categories and codes are given in Tables 2, 3, 4, 5, 6, and 7 in the Results section.

Validity and Reliability

The activities were developed by the researcher. Three academics in the field of mathematics education and three mathematics teachers reviewed the suitability of the activities and their ability to enrich learning objectives. The experts submitted their reviews in written form. Following the experts' review, feedback was received regarding the ease of understanding of the language used by students, the thought-provoking nature of the open-ended questions, the two-hour duration of each activity, and the appropriateness of the activity level for the target audience. Based on this feedback, necessary revisions were made. To ensure the validity and reliability of the interview form, the opinions of three separate academics working in mathematics education and educational sciences were consulted. Opinions were collected in writing. To enhance the validity of the form, the questions were reviewed for suitability to the research objective and research questions. Necessary revisions were made. Open-ended questions were added to obtain in-depth information, and their suitability for the research was determined. To evaluate the validity and reliability of the number pattern form, three mathematics teachers with at least seven years of experience working with gifted students and three academics working in different fields of mathematics education were consulted. Opinions were obtained regarding the suitability of the questions to the study's objective and their appropriateness in terms of language and presentation. These opinions were collected in writing. The experts reviewed the questions for their alignment with the objectives and activities. Based on these opinions, the suitability of the form for use in the study was determined. To eliminate researcher bias in data analysis, the measurement instruments were re-analyzed and evaluated by two mathematics teachers with 12 and 16 years of experience and doctoral degrees in mathematics education, and participant review was used. The data were coded by two different researchers. The coding results were compared with existing codes, and a common conclusion was reached. The reliability of the data from the experts was calculated using the Miles and Huberman formula, and the confidence coefficient was found to be 86%. For participant review, the findings were shared with the participants, and their accuracy was confirmed. Both an interview form and a number patterns task form were used. The data obtained from the forms support each other.

Activity Development

In study, Ev Çimen (2008) emphasized the importance of an education system that fosters higher-order thinking skills such as interpretation, questioning, decision-making, and discovery, replacing teacher-centered education and stated that the exam-focused era in education has been replaced by a period of analysis, relationship-building, knowledge construction, and access to information. The Ministry of National Education (2022) emphasizes that programs for gifted students differ qualitatively from other programs depending on the characteristics of gifted students. Addie's instructional design model was used in the development of activities. Branch (2009) emphasizes that a good instructional design is achieved through analysis, design, development, implementation, and evaluation (ADDIE instructional design). The activities were designed using the patterns 1, 3, 7, 15, 31, 63, 127, … and 2, 6, 12, 20, 30, 42, 56, …. These patterns were chosen because they challenge students and are compatible with a challenging and inquiry-based learning approach. They challenge them to consider different strategies and are suitable for complex questions. They are also related to square numbers, cubed numbers, and exponents of 2.

The Researcher's Role

The activities were developed by a researcher experienced in activity development, revised, and implemented based on expert feedback. Semi-structured interview forms were developed by the researcher, revised, implemented, and analyzed based on expert feedback. The researcher was coded as T during the activity implementation process. The researcher played the role of developer, implementer, and evaluator of measurement tools.

Ethical Procedures

An application was submitted to the Ministry of National Education Research and Application Center, and approval letter numbered MEB.TT.2024.003563.01. An application was submitted to Social and Human Sciences Ethics Committee, and approval letter numbered 223 was obtained. The parents of the participating students were contacted, and information about the study was provided, and their consent was obtained. All of students participated in all activities.

Implementation Process

Table 1 shows the implementation process of the research. The research consisted of a total of 6 stages. The stages are detailed in Table 1. All students actively participated in the implementation process.

Table 1

Research Implementation Process

Stage	Implementation Process
1st stage	Development of activities
2nd stage	Development of interview form
3rd stage	Implementing Activity 1: The difference between the patterns 1, 3, 7, 15, 31, 63, 127, and others is highlighted, and its general rule is calculated. The shape pattern is drawn. When the first term of the pattern changes, the general rule is recalculated. Different shape patterns are constructed and related to each other. New patterns are obtained from the terms of this pattern using different methods. The general rules of these patterns are also found
4th stage	Implementation of Activity 2: The pattern 2, 6, 12, 20, 30, 42, 56, … is separated from other patterns, and its general rule is calculated. The shape pattern is drawn. When the first term of the pattern changes, the general rule is recalculated. Different shape patterns of the pattern are constructed and related to each other. Studies are conducted to develop models that will allow students to verify their own patterns using geometric and algebraic approaches. New patterns are obtained from the terms of this pattern using different methods. The general rules of the found patterns are also found. The given pattern is related to the pattern 1, 8, 27, 64, 125, 216, ….
5th stage	Application of the interview form

6th stage	Data analysis

Implementation of Activity 1

The first activity was conducted using a single number pattern (1, 3, 7, 15, 31, 63, 127, …). The researcher was coded as T, and the students as U1, U2, U3, ..., U12. Tis section, direct quotes from the application are included.

T: The pattern 1, 3, 7, 15, 31, 63, 127, … is a regular number pattern.

How would you find the general rule by examining the terms of this pattern?

U2: There is no constant increase.

U6: There is a regular increase in the differences between them, as 2, 4, 8, 16, …

U9: These numbers are exponents of 2.

T: If we want to break the numbers down, can we still find regular numbers?

U5: If we write 1+2, 3+4, they progress sequentially, but we can write 15 as 7+8 and 31 as 15+16.

T: How can we find the relationship between these numbers and exponents of 2? What strategy would you use?

U3: Like adding and subtracting numbers?

U7: If we add 1 to numbers, we get 2, 4, 8, 16, 32, ... Here, too, we have exponents of 2.

Students are asked to calculate the general rule of the pattern. They are asked to check whether the rules they find apply to the pattern by finding the terms of the pattern until the general rule is found.

T: What is the general rule for exponents of 2?

U5: The answer is 2^n. Because when we replace n with 1, 2, 3, ..., we obtain the terms.

U8: So, if we subtract 1 from 2^n, we have found the general rule.

T: If we were to express the terms of this pattern with shapes, how would we express them? The numbers progress regularly, so should the shapes also progress regularly? Or would we show it with a single shape? If it progresses regularly, how would you show it?

U4: We'll get each number from a regular progression of shapes.

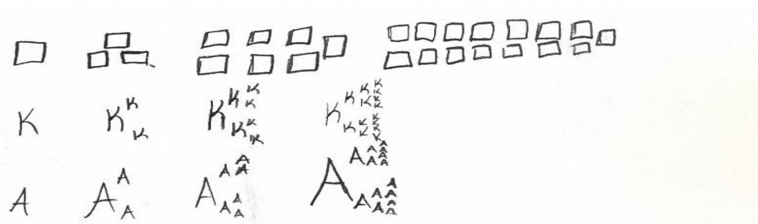
U5: I want to get it from squares, but I don't know how to place them.

T: We need to place them regularly to give each number. They should follow the same pattern.

U7: I want to do it with letters.

Figure 1

U7's Answer

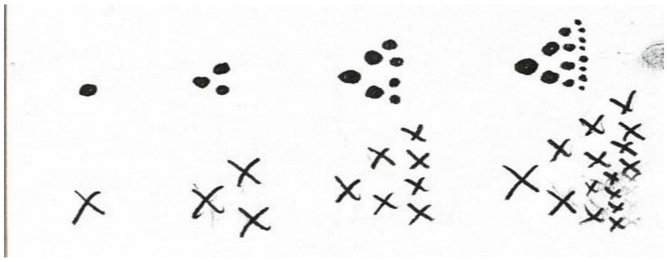


Then, they are asked to find a pattern. Different patterns are shown, and they are asked whether they fit that pattern. The relationship of the pattern found to the number pattern is discussed.

They are asked whether there are different pattern patterns that fit the number pattern. They are asked to generate different pattern patterns. The relationships between the patterns are discussed.

Figure 2

U2's Answer



T: If we remove the first term of the number pattern (3, 7, 15, 31, 63, 127, ...), the new first term becomes 3. Does this change the general rule? How would you express it?

U10: I don't think so. Because the remaining numbers are the same.

U1: The general rule for the first pattern was 2^n -1. In this pattern, when we replace n with 1, it doesn't give 3. But I don't know how to show it.

U4: The general rule seems complicated. Is it 2^n +1? I think that doesn't work either. The second term is 7, but I get 5.

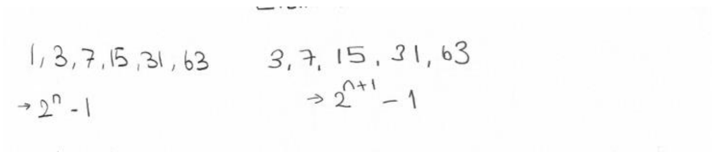
T: If it does, how would you use this information to find the new general rule if these numbers are related to exponents of 2? The exponents are sequential in 2^n -1. So, do we need to change the exponent?

U3: It works if we increase the exponent by 1.

U12: Yes, we need to increase the exponent by 1. In this case, it becomes 2^{n+1} -1.

Figure 3

U12's Answer

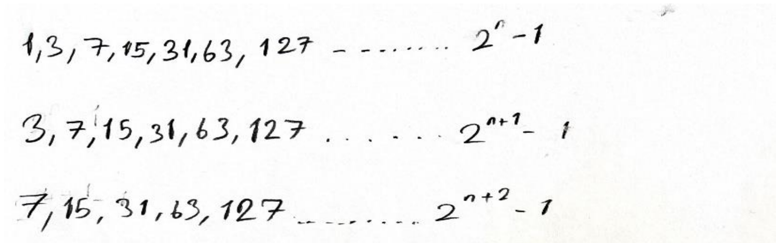


T: Can we use the same information for the general rule when the first two terms are removed? Why?

U10: Yes, this time the new general rule should be 2^{n+1}-1. Because the first term is 7.

Figure 4

U5's Answer



T: Can you obtain new regular patterns by working on the terms of the pattern 1, 3, 7, 15, 31, 63, 127, … using different methods?

U3: At the beginning of the activity, we obtained exponents of two by adding 1 to each term.

U4: If I add 2 to each term, we get 3, 5, 9, 17, ….

T: Can we do the same operation for the general rule?

U2: In this case, it becomes 2n +1. We can obtain the terms by replacing n with 1, 2, 3, 4, ….

U1: Therefore, we can find the new general rule by applying the same operation to the general rule.

U7: When we write the differences between the terms, we obtain the exponents of two.

T: Can you obtain the pattern 1, 2, 4, 8, 16, 32, 64, … from the terms of the pattern 1, 3, 7, 15, 31, 63, 127, …? Can we obtain it by breaking it down into the sum of more regularly progressing numbers?

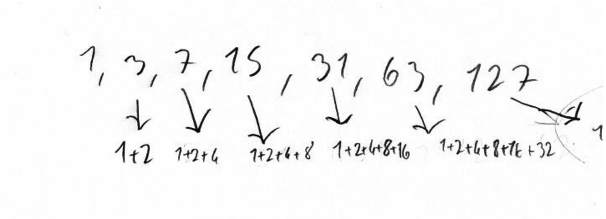
U11: 1 and 3 don't work, but we can write 7 as 2+2+3.

T: But the numbers in the sum must increase.

U9: Is it 1, 1+2, 1+2+4, 1+2+4+8, …?

Figure 5

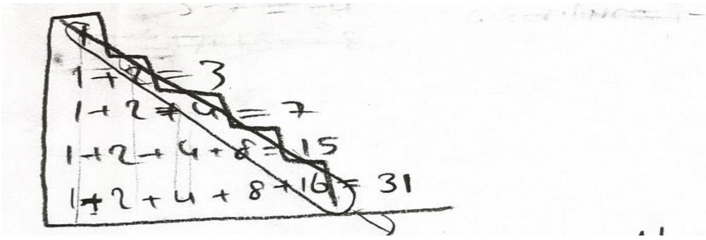
U9's Answer



U5: When we write these numbers under each other, they form a triangle. So, a triangle is formed with exponents of two.

Figure 6

U5's Answer



Implementation of Activity 2

The second activity was conducted using a single number pattern (2, 6, 12, 20, 30, 42, 56, …). The researcher was coded as T, and the students as U1, U2, U3, ..., U12. Tis section, direct quotes from the application are included.

T: The pattern 2, 6, 12, 20, 30, 42, 56, … is a regular number pattern.

How would you derive the general rule for this pattern? Would looking at the differences between the terms be helpful?

U2: When looking at the differences, it progresses as 4, 6, 8, 10, …. It increases regularly. All are even numbers. The terms of the pattern are also even numbers.

U5: We can't find it from here.

T: Can we find it by factoring the terms or by breaking them down into sums?

U7: We can write it as the sum of two numbers. I wrote it as 0+2, 2+4, 4+8, 8+12, but I can't find the rule.

U11: We can write it as two factors by factoring.

T: Can you write it as consecutive factors?

U9: Is it like this: 1x2, 2x3, 3x4, 4x5, …?

U6: It's like this: consecutive factors.

T: How can we represent this consecutive multiplication with variables?

U3: We can write n.(n+1). When we replace n with 1, 2, 3, 4, …, we find all the terms.

T: The numbers progress regularly, so should the shapes representing these numbers also progress regularly? Or would we represent them with a single shape? If they progress regularly, how would you represent them?

U8: Should we create shapes like the squares we know for each one?

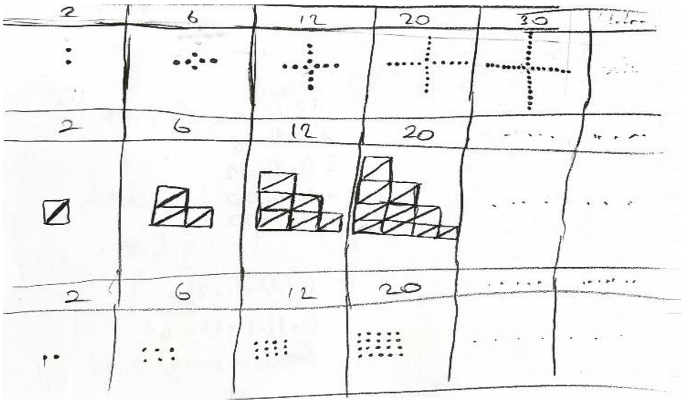
T: You can also create shapes other than the known ones.

U7: I will create shapes and draw squares inside them.

U12: I will create triangles from squares and draw squares inside them.

Figure 7

U12's Answer

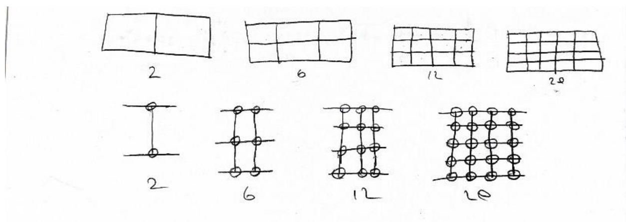


T: We can create different shapes with dots or lines.

Then, the students find a pattern. Different patterns are shown, and they are asked whether they fit into this pattern. The relationship between the found pattern and the number pattern is discussed.

Figure 8

U4's Answer



T: If we remove the first term of the number pattern (6, 12, 20, 30, 42, 56, ...), the new first term will be 6. Does this change the general rule?

U3: The general rule in the first pattern was n.(n+1). That is, it was the product of consecutive numbers. Here, the terms are the product of consecutive numbers.

U10: I thought it wouldn't change, but when I start from 1 and write values instead of n, the first term, 6, doesn't become 6.

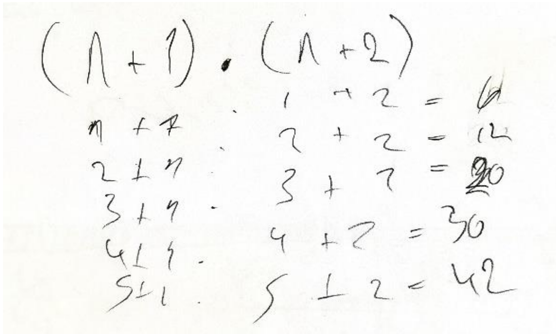
U8: It does. However, I'm having difficulty calculating the new general rule.

U5: The next term, 6, can be written as 2x3. In this case, we'll rewrite the rule as consecutive multiplication. We can start from n.(n+1).

U6: We can increase the variables by 1. If we write (n+1). (n+2), it works.

Figure 9

U6's Answer



T: Can we use the same information when the first two terms are removed? Why?

U3: This time, it will be (n+2). (n+3). When I replace n with 1, 2, 3, 4, …, we find the terms.

T: Can you obtain other regular patterns from the terms of the pattern 2, 6, 12, 20, 30, 42, 56, …?

U3: I can add numbers to these terms.

U11: We can look at the differences between the terms. In this case, we obtain the even numbers 4, 6, 8, 10, ….

T: Can you obtain the square numbers (1, 4, 9, 16, 25, 36, …) from the terms of the pattern 2, 6, 12, 20, 30, 42, 56, …?

U10: No. This is too difficult.

U5: Can we do it by adding numbers again? I'm trying, but it doesn't work.

T: If we add the same number every time, it might not work. We can add numbers in different orders to each term.

U4: Can consecutive numbers work?

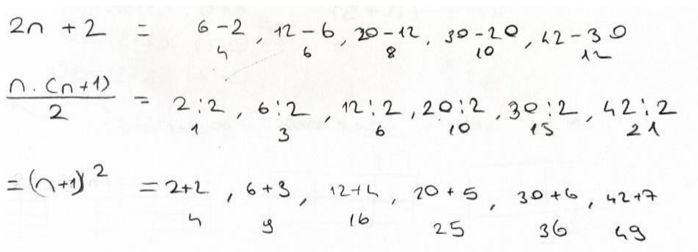
U10: But if we add starting from 1, it doesn't.

U1: If we add starting from 2, it works. 2+2, 3+6, 4+12, 5+20, 6+30, …

U4: When we take their differences, we find even numbers.

Figure 10

U4's Answer



T: Is there a relationship between the cubed numbers 1, 8, 27, 64, 125, 216, … and the pattern 2, 6, 12, 20, 30, 42, …? If so, can you explain how?

U1: It seems very difficult, but maybe we can do it using consecutive numbers.

U5: The first pattern is cubed numbers; the second pattern is multiples of 2. Finding a relationship between the two is very difficult.

U8: Cubed numbers are multiplied by themselves three times in a row.

U7: We obtained square numbers by adding consecutive numbers. If we multiply the result again by the consecutive numbers we added, we obtain cubed numbers. In other words, (2+2).2, (6+3).3, (12+4).4, (20+5).5, …

U12: Cubed numbers were obtained. This is correct.

Results

This section presents the analysis of the number patterns task form data and interview data. The interview data were categorized as “cognitive gain, ease, difficulty, creativity, and skill”. These categories were associated with codes. Direct participant opinions were also included. Content analysis was conducted for each student's form based on the themes of "fluency, flexibility, originality, enrichment, and deep thinking" which were determined for the analysis of the Number Patterns Task Form findings. During the activity implementation process, teachers T and participants were coded as U1, U2, U3, …, U12. Participants' number patterns were scored on a scale of 0 to 5 based on their responses to the task forms. Scores were categorized as “4 and 5” indicating “Good”, “2 and 3” indicating “Intermediate”, and “0 and 1” indicating “Low”. The initial scoring, with adjustments, is shown in Table 6, and the scores are categorized under themes and codes in Table 7.

Table 2

Student Opinions from the Intervention Group Regarding the Contribution of the Activities To Learning Number Patterns

Theme	Code	Frequency(f)
Cognitive gain	Enabling in-depth thinking	12
	Enhancing learning	11
	Understanding complex patterns	9
	Establishing relationships between patterns	10
	Establishing relationships between number patterns and shape patterns	11
	Ensuring lasting learning	11

Even though I encountered it for the first time, I learned about number patterns. The patterns 1, 3, 7, 15, 31, ...weres the most challenging. However, here, I broke down the numbers into sums and established relationships with exponents of 2. Each was 1 less than the exponent of 2. That's how I arrived at the general rule. I was able to create more than one shape pattern. This was possible for every number pattern. I created new number patterns. It seemed complicated at first. But with practice, I was able to do it better. (U3's comments)

The number patterns given were initially difficult. However, as I worked and thought about them, I was able to do them. Patterns that changed the first term changed the general rule even though the remaining terms were the same. When I found the general rule, I saw this very clearly. I saw that the pattern 1, 4, 9, 16, … was related to the pattern 2, 6, 12, 20, …. When I added the consecutive terms in the pattern 2, 6, 12, 20, … and divided them by 2, I obtained perfect square numbers. Multiple shape patterns could be obtained. It just took a little more thought. (U4's comments)

I obtained new numbers and shape patterns from number patterns. Although it seemed very difficult at first, multiple shape patterns can be created for each pattern. It just takes effort and imagination. I also learned to obtain new patterns by applying the same operation to all the terms of the patterns. In patterns 1, 3, 7, 15, ..., the differences between the terms have exponents of 2. Furthermore, when I added 1 to each term, I obtained exponents of 2. (U5's comments)

Table 2 examines the contribution of activities to cognitive learning, revealing related codes derived from data obtained from interviews regarding cognitive gain. Most of students responded positively to understanding complex patterns, facilitating learning, establishing relationships, reflective thinking, and fostering lasting learning. According to U3, U4, and U5, they initially struggled but became more successful as they worked on them more thoroughly.

Table 3

Student Opinions of the Intervention Group Regarding the Ease and Difficulty of the Activities

Theme	Code	Frequency(f)
Difficulty	Creating original shapes	8
	Struggling	11
	Complexity	9
Ease	Creating new patterns	5
	Establishing relationships between patterns	5
	Creating shape patterns	7
	Algebraic notation	8

The given number patterns were difficult at first. But as I worked and thought about them, I was able to do them... Multiple shape patterns could be obtained. It just took a little more thought... I was able to derive new number patterns from the given number patterns using different strategies. For example, when I added or subtracted the same number, new patterns emerged. I had difficulty applying the general rules in the first number patterns task, but I gained experience with the activities and was able to do them better… (U2's comments)

Representing number patterns with shape patterns was something I did for the first time. I was able to show it with a single shape. Perhaps it could have been shown with more shapes if I had worked harder... I had a bit of a hard time deriving a new pattern. I was able to derive a single pattern. Given the first pattern, I was able to find the general rule, how the numbers increase, and the subsequent terms. However, I thought a lot about whether the general rules would change when the first term changed; it seemed complicated to me... (U1's comments)

I calculated the general rules of the patterns... Then I checked whether the terms were given. I was able to obtain perfect square and cube numbers from the number pattern 2, 6, 12, 20, ... I did this with the help of addition, subtraction, multiplication, and division operations. I created both triangular and rectangular shape patterns for the pattern 2, 6, 12, 20, ... It was difficult for me to create more shape patterns. But I was able to do it because I worked on it more. (U5's comments)

Table 3 shows codes related to creating original shapes, complexity, effort, deriving new patterns, establishing relationships, algebraic representation, and obtaining shape patterns, based on the themes of difficulty and ease. Most students stated that they struggled to create original shapes due to their complexity and struggled. Only five students said that deriving new patterns and establishing relationships between patterns was easy. Eight students found algebraic representation easy, while seven students said creating shape patterns was easy. When we look at the opinions of U2, U1, and U5, we see that they initially struggled but improved with practice.

Table 4

Student Opinions in the Intervention Group Regarding the Contribution of the Activities to Developing Creativity

Theme	Code	Frequency(f)
Creativity	Finding new strategies for generating number patterns	11
	Creating more different pattern shapes	10
	Originality	9

> I created new pattern patterns by working on them more. This required imagination. I tried many ways to derive different number patterns from the existing number patterns. Like adding consecutive terms, summing the sums and differences of consecutive terms, etc. (U2's comments)
>
> …More than one pattern could be obtained. It just took a little more thought… I was able to derive new number patterns from the given number patterns using different strategies. For example, when I added or subtracted the same number from all of them, new patterns emerged. (U4's comments)
>
> I was able to obtain cubed numbers from the number pattern 2, 6, 12, 20, ... I struggled at first. More number patterns could have been obtained. I did this using addition, subtraction, multiplication, and division. For the pattern 2, 6, 12, 20, ..., I created both a triangular and a rectangular shape pattern. (U5's comments)

In Table 4, the theme of creativity is associated with codes such as finding new strategies for creating number patterns, originality, and creating more different shapes. Most students emphasized that their creativity had improved. The views of U2, U4, and U5 indicate that they created more than one shape pattern and employed different strategies when creating number patterns.

Table 5

Student Opinions In The Practice Group Regarding The Skill Development Of The Activities

Theme	Code	Frequency(f)
Skills	Decision making	9
	Problem solving	10
	Establishing relationships between numbers and shapes	12
	Algebraic thinking	11
	Creativity	10
	Establishing relationships between different number patterns	12

> When the first term was removed in number patterns, the second term became the first. Here, I sought an answer to the question of whether the general rule remained the same or changed. However, when I substituted 1 for the variable in the first general rule, it didn't give the first term in the second pattern because the first term had changed. I decided that the rules had changed here. I created shapes representing the numbers. I imagined different shapes to create them. (U1's comments)

> It was difficult to derive multiple pattern shapes, but I was able to do this for all of them... Some patterns seemed confusing to me. I couldn't reach the general rule because of the differences between the terms. When I worked on them, I factored them, broke them down into sums, and wrote them as exponents. This made it easier to reach the general rules. Some of the patterns given were related to each other. For example, 2, 6, 12, 20, … and so on were related to the perfect square numbers... (U3's comments)

> I associated number patterns with shapes, trying to use different shapes and checking if they were the right ones... Although it seemed very difficult at first, each pattern can be created with multiple shapes. I calculated general rules using letter expressions. I associated number patterns with other number patterns, creating new number patterns... It seemed difficult at first, but after working on it, I got it right. (U4's comments)

Table 5 presents data on the skill development of the activities. The skill theme was associated with codes such as decision making, creativity, problem solving, establishing relationships, and algebraic thinking. Most of students believed that the higher-order thinking skills covered in the codes had improved. All students reported establishing relationships between different number patterns and between numbers and shapes. The views of U1, U3, and U4 indicate that they used their creativity to create shapes related to numbers, create new number patterns, solve problems, and demonstrate skills such as algebraic expression. We see that the activities enabled students to achieve high-level performance.

Questions and student responses in the number patterns task form

The number patterns task form consists of 4 questions. The questions and the participants' direct answers are provided this section.

1. Find the general rule for the triangular number pattern that goes as follows: 1, 3, 6, 10, 15, 21, …

Figure 11

U1's Answer To The First Question On The Number Patterns Task Form

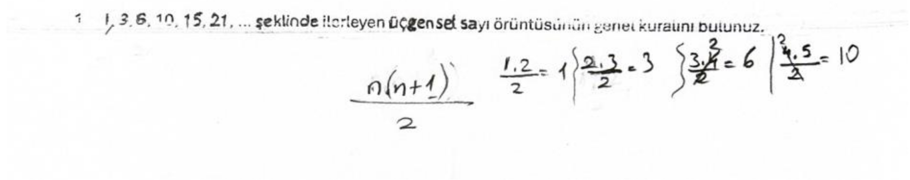
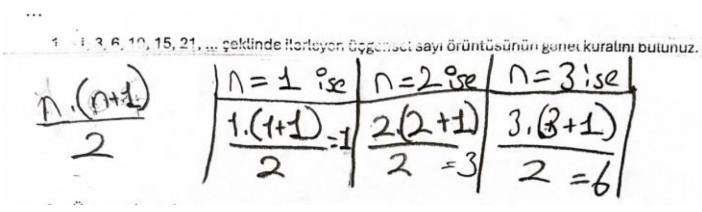


Figure 12

U7's Answer To The First Question On The Final Number Patterns Task Form



2. Does the general rule change when the first term is written as 3 in the form of triangular numbers 3, 6, 10, 15, 21, ...? If so, find the new general rule, stating how it changes.

Figure 13

U9's Answer To The Second Question On The Number Patterns Task Form

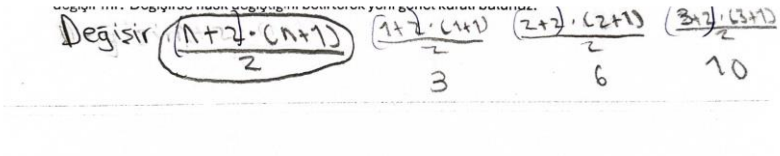
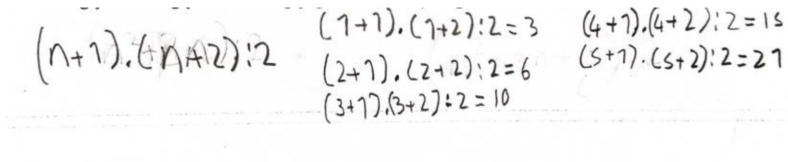


Figure 14

U8's Answer To The Second Question On The Number Patterns Task Form



3. Create multiple shape patterns for the triangular numbers 1, 3, 6, 10, 15, 21, ... Would the shape pattern change if the first term were 3? If so, show how.

Figure 15

U6's Answer To The Third Question On The Number Patterns Task Form

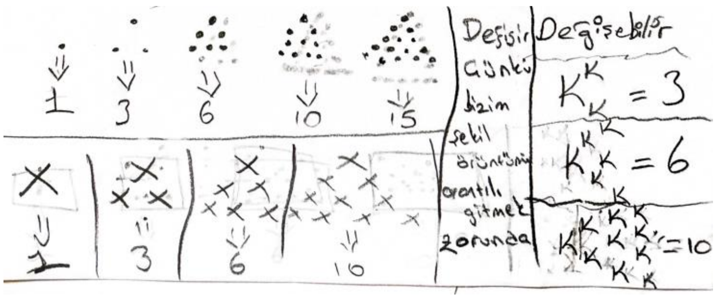


Figure 16

U11's Answer To The Third Question On The Number Patterns Task Form

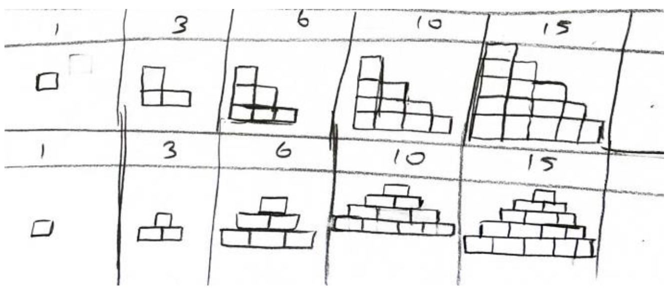
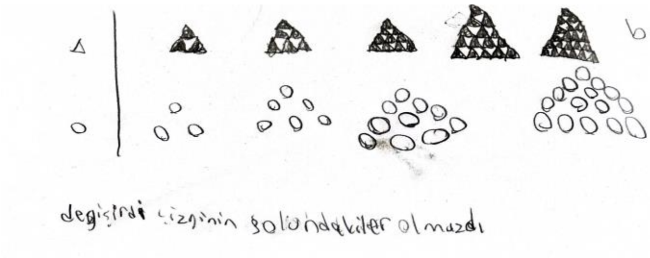


Figure 17

U10's Answer To The Third Question On The Number Patterns Task Form



4. Can you obtain different number patterns from the terms of the triangular numbers 1, 3, 6, 10, 15, 21, ...? If so, write down the number patterns you obtained and their general rules.

Figure 18

U4's Answer To The Fourth Question On The Number Patterns Task Form

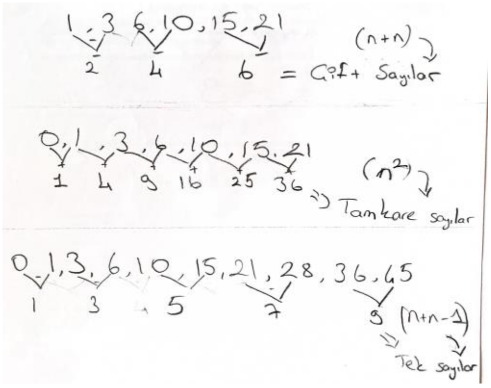
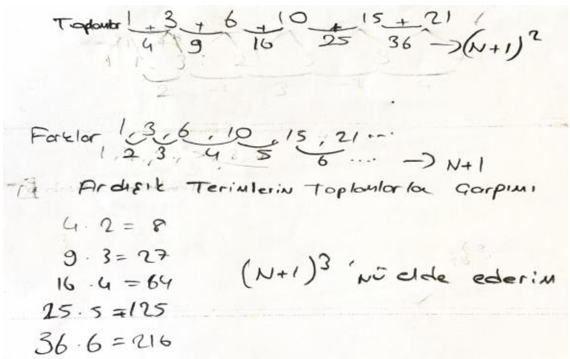


Figure 19

U6's Answer To The Fourth Question On The Number Patterns Task Form



The first question asked for the general rule for the number pattern 1, 3, 6, 10, 15, ... Looking at the answers of U1 and U7, they expressed it using an algebraic method. The second question asked for the general rule for the pattern 3, 6, 10, 15, ... when the first term is removed. Students initially struggled with the activity. However, when we look at the answers of U9 and U8, we see that they made both arithmetic and algebraic generalizations. The third question asked for more than one shape pattern of the numbers 1, 3, 6, 10, 15, ... The question asked about the change in the shape pattern when the first term was removed. U6 showed two different shapes. He drew the new shape pattern when the first term was removed. U11 showed two different shapes. However, he did not draw the shape pattern that would have formed when the first term was removed. The fourth question asked students to create new patterns from the number patterns 1, 3, 6, 10, 15, … and so on. U4's answer shows that he obtained odd, even, and perfect square numbers by using addition and subtraction operations and performing operations between consecutive terms. He expressed the number patterns he found algebraically. U6's answer shows that he obtained perfect square numbers by adding consecutive terms and cube numbers by multiplying consecutive numbers with their sums.

Table 6

Number Patterns Task Form Scoring Analysis

	Fluency	Flexibility	Originality	Enrichment	Deep Thinking
U1	5	5	4	4	4
U2	5	4	4	3	4
U3	4	4	4	3	3
U4	5	5	3	3	4
U5	3	4	3	4	3
U6	5	5	4	4	4
U7	4	4	4	5	4
U8	5	4	4	4	4
U9	4	4	3	3	3
U10	5	5	3	3	4
U11	4	5	4	4	4
U12	5	4	3	3	4

Table 7

Number Patterns Task Form Themes and Codes

	Fluency	Flexibility	Originality	Enrichment	Deep Thinking
U1	Good	Good	Good	Good	Good
U2	Good	Good	Good	Intermediate	Good
U3	Good	Good	Good	Intermediate	Intermediate
U4	Good	Good	Intermediate	Intermediate	Good
U5	Intermediate	Good	Intermediate	Good	Intermediate
U6	Good	Good	Good	Good	Good
U7	Good	Good	Good	Good	Good
U8	Good	Good	Good	Good	Good
U9	Good	Good	Intermediate	Intermediate	Good

U10	Good	Good	Intermediate	Intermediate	Good
U11	Good	Good	Good	Good	Good
U12	Good	Good	Intermediate	Intermediate	Good

Looking at Table 6 and Table 7, we can say that students were generally successful in the number pattern task. They correctly represented verbal, algebraic, and geometric representations. They obtained geometric shapes and number patterns using new strategies. They created multiple shape patterns belonging to the same number pattern and established relationships with multiple number patterns.

Discussion and Conclusion

This study investigated the effects of number pattern activities, designed using an inquiry-based learning approach, on the cognitive gains and higher-order thinking skills of gifted 7th-grade students. Qualitative data analysis revealed that students achieved significant cognitive gains through the activities, including deep thinking, understanding complex patterns, establishing relationships between numbers and shapes, and lasting learning. The improvements in decision-making, problem-solving, and algebraic thinking skills are particularly consistent with the studies by Karademir (2017), Yılmaz (2024), and Butcher et al. (2023), which emphasize that the inquiry-based learning approach enhances students' research, critical thinking, and inquiry abilities. Furthermore, the participants' 7th-grade level provided a suitable foundation for abstract and functional thinking skills, supporting their cognitive potential and enabling them to more easily perceive higher-order relationships.

One of the most striking findings of the study is the cognitive resilience and increased effort shown by students in the face of initially complex and challenging original activities. Participants reported experiencing initial confusion and difficulty (e.g., U1, U2, and U3) in generating original shapes and analyzing nonlinear number patterns. However, with sufficient time and guidance, they overcame these difficulties by developing new strategies. This directly supports the studies of Leikin et al. (2017), who argue that gifted students unleash their full potential by expending more mental effort as complexity increases, and Silver & Mesa (2011) and Tirosh et al. (2019), who emphasize the need to expose these students to challenging situations. Stacey (1989) and Aslan (2011) noted that students struggle with finding near/far terms and algebraic generalizations; while Warren (2005) and Dayan (2017) stated that repeating patterns are easier to model. However, even though our study consciously selected complex, non-repetitive, changing, and non-linear patterns, students were able to overcome these obstacles and achieve a high level of performance thanks to their flexible thinking skills.

The fact that participants generated multiple diagrams and derived entirely new patterns starting from a single number pattern is another critical finding for the development of creativity and flexible thinking. Students used verbal, algebraic, and geometric representations together; and obtained new number and shape patterns using different strategies such as adding, subtracting, squaring, or cubing consecutive terms (e.g., U4, U5, U6). The students' overall success in terms of fluency, flexibility, and deep thinking parallels the studies of Hong & Aqui (2004) and Taucei et al. (2015), which show that mathematical creativity can be developed with enriched strategies and stimulating learning environments. Although Karaz (2021) stated that multiple representations support flexible thinking, he was limited to the use of a single diagram. Our study, however, overcame the limitation mentioned by Karaz (2021) by forcing students to derive more than one diagram from the same pattern, and took creativity to a much higher level. However, the fact that performance in the enrichment and originality categories remained at a more moderate level compared to other skills may be attributed to the students' first encounter with such complex pattern derivation tasks and their lack of prior experience.

Another important finding reflecting the ability to think deeply in the study is the students' discovery of how the general rule changes algebraically when the first term of the pattern changes. Initially, students mistakenly believed that the general rule would not change when the first term of the triangular numbers "1, 3, 6, 10, 15..." was removed (becoming 3, 6, 10, 15...). However, as the inquiry process progressed, they noticed this change and correctly expressed the new rule arithmetically and algebraically. In contrast to Topbaş Tat (2020), who stated that teachers found it difficult for students to derive the general rule, and Düzgünoğlu (2023), who indicated that enrichment practices remained superficial and results-oriented; this study qualitatively and in-depthly examined the process, revealing the students' relational analysis capacity. This result aligns with Tuncer (2025), who stated that numerical variable assignment and algebraic generalization strategies support flexibility in nonlinear patterns, and Erdoğan & Gül (2023), who argued for the necessity of advanced cognitive tasks.

In conclusion, integrating inquiry-based approaches to number patterns in mathematics education for gifted students is a strategic tool in maximizing the student's intellectual potential. Our findings that students enjoy complex situations, challenging processes, and conducting their own research are consistent with the observations of Arıkan (2021) and Türkman (2020) regarding the nature of gifted individuals. This research provides a strong response to the need to increase inquiry-based studies that facilitate the transition from arithmetic to algebra and blend different forms of representation, as also pointed out by Arifin et al. (2025). These findings, filling a gap in literature, academically demonstrate that the applied innovative approach plays a critical role in the multidimensional development of gifted students.

Limitations and Recommendations

The study is limited to notes taken during the activity, activity forms, and audio recordings. The researcher in this study played the role of developer, implementer, and analyzer of the measurement tools and activities. Analysis was conducted by different individuals to eliminate research bias. Member checking was conducted. Audio recordings were made during the activity implementation process and during the interviews. In this study, no video recording was made because the participants did not request it. In other studies, video recording can be used. Video recording can be effective in preventing researcher bias, increasing data diversity, and presenting more concrete data. A different researcher may also be involved during the implementation process. Furthermore, because the target audience is special education, the number of participants is limited to 12. This study could be conducted with more students. This study could be conducted using an experimental design with pretest and posttest groups.

Inquiry-based studies with gifted students are limited in number in doctoral studies. The number of such studies could be increased. Different innovative approaches can be implemented with original activities based on the subject.

NCTM (2000) emphasizes that students should be given the opportunity to analyze and question the information they need. The mathematics curriculum for gifted students includes a multitude of topics, and teachers struggle to provide enrichment and depth in every subject. However, in every subject, students must be actively engaged in thinking and foster in-depth relationships to ensure high-level performance. Other studies have focused on problem posing and problem solving.

This study examined number patterns in depth with gifted students. Further studies could include different mathematical topics. New activities that utilize an inquiry-based learning approach can be designed for teaching gifted students. This will create reference activities for teachers working with gifted students. Activities that challenge students and reveal and develop their metacognitive demands for higher performance can be designed.

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Statement of Responsibility

HB: Conceptualization, Methodology, Software, Writing – Original Draft, Investigation, Data Curation, Formal Analysis, Validation, Writing – Review & Editing, Visualization. **DS:** Resources, Supervision, Project administration, Funding acquisition. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

There is no conflict of interest among the authors of the study.

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