Research Article

A New Approach to Determine the Effective Flexural Stiffness of Doubly-Reinforced Beams According to Nonlinear Behavior

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Abstract Determining the effective stiffness of structural elements made of reinforced concrete with reliability has always been an important subject of research since it provides a reliable estimate of a building’s capacity in the case of a seismic event. The load-bearing element’s design parameters, such as the longitudinal reinforcement ratio and concrete compressive strength, influence the effective stiffness of the cracked section in reinforced concrete structures, even if it is not always constant. In this study, a secure and efficient approach covering all important design parameters is proposed to determine the stiffness coefficient of cracked sections of doubly-reinforced beam models. The proposed equation for the effective stiffness coefficient has been verified by comparisons with moment–curvature relationships and data provided by various researchers and standards, based on numerical results. For beam sections, the suggested equation provides values of the effective stiffness coefficient that are reasonably accurate and consistent. Therefore, it is possible that the results of analytical solutions may differ from each other according to different researchers and standards. Analysis results have shown that an important parameter influencing the nonlinear behavior of the sections and the effective stiffness in beams is the reinforcement ratio, which includes concrete strength and compression reinforcement ratios.

Keywords: Doubly-reinforced beam, Effective stiffness, Moment-curvature, Reinforcement, Concrete.

1. INTRODUCTION

Being able to define the flexural stiffness of reinforced concrete (RC) sections as realistically as possible has become more important because the performance-based solution has emerged as an important method in determining earthquake behavior [1]. The parameters of hardness, resistance, and ductility are three basic factors in determining the response of the structure against the earthquake. The purpose of dimensioning and detailing structural load-bearing members under the influence of vertical and horizontal effects is to provide sufficient strength, rigidity, and ductility under the anticipated actions for the structure, and to satisfy the structural performance and displacement limit requirements defined by national and international seismic regulations. The results obtained from the analysis of the load-bearing members under horizontal and vertical loads may vary depending on the section stiffness. The flexural stiffness in RC structural systems is effective both in the determination of displacements and in the distribution of internal forces compared to other structural systems [2-3].

The shear and flexural stiffness of structural bearing members decrease due to cracks in the concrete. Therefore, the analyses performed without considering the cracking effect on the bearing members may not represent the actual behavior of the bearing members and structures [4]. The effective flexural stiffness values obtained after cracking for RC bearing members are the most important feature that affects the force contribution of the bearing members of RC structures [5].

The flexural stiffness varies depending on the section dimensions, material elasticity modules, and reinforcement ratio. While examining the section behavior with the moment-curvature relations, the flexural stiffness has a maximum value depending on the material elasticity modulus and the section moment of inertia in cases where the moment is small and no cracks occur in the section. With the moment reaching large values and the formation of cracks, the flexural stiffness decreases and the effective section stiffness becomes effective. The moment-curvature-based approach is used to develop expressions for estimating the flexural stiffness of RC members subjected to short-term loads [6]. Everyone accepts that during strong earthquakes, inelastic response and cracking are expected. Therefore, the column and beam moments of inertia are reduced to reflect the inelastic behavior. Various formulations and coefficients have been proposed in the regulations to take into account the effective section stiffness.

To determine the effective stiffness of the RC structural members of the cracked section, investigations and research have been carried out [7-13]. In studies on the effective stiffness of structural members, various factors and equations have been proposed for the design and analysis of structural members, considering different standards, codes, and design parameters. Different parameters, such as axial load level, reinforcement ratio, concrete strength, section geometry, and dimensions, are taken into account in studies conducted on the effective stiffnesses of RC structural members. The effective stiffness of the RC elements has been the subject of numerous investigations and analyses. The relations proposed for the design were investigated in research and studies on effective stiffnesses, taking into account the design characteristics impacting the stiffness of the members.

Studies on determining the effective flexural stiffness and nonlinear behavior of reinforced concrete beams have increased. Studies on doubly reinforced beams, in particular, have gone beyond classical elastic assumptions and combined the moment-curvature relationship, experimental observations, and numerical modelling.

Attia [14] investigated the effects of different strengthening techniques on flexural behavior using experimental and numerical methods, showing that the increases in stiffness and capacity vary depending on the strengthening method. Similarly, Bashour et al. [15] demonstrated that effective stiffness estimations can be verified by comparing the nonlinear behavior of reinforced concrete beams with moment-curvature relationships. More recent studies focus directly on effective stiffness estimation. El-Gohary [16] demonstrated that effective flexural stiffness in reinforced concrete beams can be accurately estimated using a new method based on curvature integration. Imamović [17] analyzed the moment-curvature relationship using closed-form analytical expressions and compared them with existing codes, thus developing models applicable to the design process.

Regarding doubly reinforced beams, Lofty and Hassan [18] examined the experimental performance of beams with hybrid steel lattice reinforcement and found that stiffness changes varied depending on the reinforcement type. Issa and El-Refaie [19] evaluated the effective stiffness in doubly reinforced beams using GFRP-steel hybrid reinforcement using experimental and numerical methods. These studies demonstrate that hybrid reinforcement systems improve stiffness and ductility behavior. In the field of numerical modelling, Taher and Ali [20] performed nonlinear finite element analyses on beams with different span and detailing conditions, demonstrating how stiffness and carrying capacity are affected by parameters.

In general, the literature published after 2022 indicates that the concept of effective flexural stiffness is no longer a purely theoretical parameter but is beginning to be defined more precisely through experimental, analytical, and numerical studies. Particularly for doubly reinforced beams, hybrid reinforcement systems, moment-curvature analyses, and nonlinear finite element models are prominent methods for estimating effective stiffness.

It has been determined that the stiffness coefficient of RC bearing members is different from the values predicted under seismic effects. It is known that the stiffness of RC structures exposed to earthquake effects will decrease greatly, therefore, it is recommended to use cracked section stiffness in the relevant sections of the codes to provide more realistic results. According to regulations and standards that are taken into account in the design and analysis of RC structures, the stiffness of bearing elements is expressed as a ratio of their stiffness, which is determined based on their cross-section properties. Accurately computing the structural elements’ stiffness in the cracked section is necessary to determine realistic values for the structural stiffness under seismic events. The RC beams’ flexural stiffness is a critical parameter in structural design, influencing deflections, crack widths, and overall stability. Traditional methods often rely on linear assumptions, which may not accurately capture the behavior of structures under higher loads where nonlinearities become significant.

This paper aims to bridge this gap by proposing a methodology that accurately reflects the nonlinear behavior of doubly reinforced beams. In this study, a simple equation is proposed to calculate the effective stiffness coefficients of the cracked section according to the design parameters that affect the nonlinear behavior of double-reinforced beam members. Thus, the stiffness coefficient of doubly-reinforced beams can be calculated to reflect the nonlinear behavior of the members. We determined that section geometry, reinforcement ratio, axial load ratio, and concrete strength are the main factors influencing the stiffness. The double-reinforced beam’s cracked section’s effective stiffness was compared by calculating from numerical analysis according to design parameters such as concrete compressive strength and tensile and compression reinforcement ratios.

The effective stiffness of cracked sections of double-reinforced beam models was investigated analytically by obtaining from Turkish Building Earthquake Code (TBEC) [21], Seismic Evaluation and Retrofit of Existing Buildings (ASCE/SEI41) [22], American Concrete Institute (ACI318) [23], Assessment and Retrofitting of Buildings (Eurocode8-Part3) [24], Design of Structures for Earthquake Resistance (Eurocode8) [25], Design of Concrete Structures (Eurocode2) [26], Avşar et al. [27], Biskinis [28], Biskinis and Fardis [29], moment-curvature, and the proposed simple equation. The proposed equations for doubly-reinforced beams according to different design parameters are interpreted by comparing them with the standard codes and existing equations.

1.1. Effective Stiffness Factors of Reinforced Concrete Beam

The earthquake-resistant building design generally depends on the results from linear analysis techniques. Therefore, the stiffness of the load-bearing members is accepted as the uncracked section stiffness. It is known that the stiffness of RC structures exposed to earthquake effects will decrease greatly; therefore, it is recommended to use cracked section stiffness in the relevant sections of the code and standards to obtain more realistic results. For analysis, or linear or nonlinear of structures subjected to earthquake effects, current design codes and standards provide constant coefficients and different relationships for the stiffness of the structural bearing members, often providing rough indications on how to evaluate the stiffness of the members (SAP2000, Ver. 21) [30]. In current design codes, the effective shear and flexural stiffness of members can usually be calculated by reducing the moments of inertia to specified values or using some empirical equations. Most of these suggested effective stiffness formulas ignore the key design factors that influence the nonlinear behavior of the elements [31]. A critical parameter in the design and analysis of RC beams is the stiffness of the section. More accurate predictions of structural performance are ensured by taking into account elements including material characteristics, cracking, and nonlinear behavior, which results in safer and more effective designs. The effective flexural stiffness of double-reinforced beams concerning nonlinear behavior is an important issue in structural engineering. This stiffness determines how the beam behaves under load and how well it resists deformations. Nonlinear behavior refers to situations where the material goes beyond its elastic limits, and in these cases, material properties and geometric parameters affect the stiffness. With RC beams, the effective section stiffness fulfils a critical role in precisely predicting deflections, crack widths, and overall structural behavior. This component takes into account the effects of cracking and nonlinear characteristics of materials in addition to the contributions of reinforcement and concrete.

As the moment in an RC section increases, the flexural stiffness of the element gradually decreases due to concrete cracking and reinforcement yielding. While cracks occur in the regions where the moment value is greatest in a RC element, uncracked rigidity is in question in other sections. Table 1 compares the different effective stiffness relations from the literature and the design codes. ACI318 [23] gives a ratio of 0.35 for the stiffness of RC beams. It is specified as 0.3 for the stiffness of nonprestressed beams in ASCE/SEI-41 [22]. It is specified as 0.5 for the stiffness of beams in Eurocode8 [25]. In Eurocode8 [25], design parameters such as element geometry and dimension, reinforcing steel ratio, concrete strength, and axial load level affecting the section are not considered. Similarly, while effective stiffness is assumed to be constant in TBEC [21], the stiffness coefficient for RC beams is accepted as 0.35. Stiffness of designed beam models according to the stacked plastic behavior model defined in TBEC [21] computable according to the shear span of the member, yield moment, and yield rotation values of the section. Part 3 of Eurocode8 [24] considers an equation based on yield rotation and moment-shear ratio, which can be used to calculate a more accurate stiffness. Considering that effective stiffness is not constant, formulation based on parameters such as material properties and cross-section dimensions is recommended. In Eurocode2 [26], an analysis based on stiffness at the second order and nominal values ought to be used, considering the impacts of creep, the nonlinearity condition of the concrete cracking, and the material on the overall behavior.

Addressing the nonlinear behavior of RC beams is often necessary to obtain more precise results in the design and analysis process. The flexural stiffness of such beams must be calculated together with nonlinear effects. The main factors of nonlinear behavior include cracking concrete, yielding of reinforcement, and nonlinearity in the stress-strain relationships of materials.

Flexural stiffness refers to the flexural resistance of the beam under a certain load and includes the effects of cracks, reinforcement, and material behavior. In RC beams, cracks form in the tensile areas of the concrete underload. Cracking of concrete significantly reduces the flexural stiffness of the beam. Reaching the yield point of the reinforcement reduces the stiffness of the beam and is an important component of nonlinear behavior.

The overall behavior of the beam is impacted by the nonlinear stress-strain relationships obtained in both steel and concrete. These relationships, along with crack expansion and reinforcement yielding, change the stiffness of the beam. Calculating effective flexural stiffness according to the nonlinear behavior of RC beams allows more accurate analysis and design of structures. Such studies are critical, especially in structural elements that operate under high loads and where cracking is important. Nonlinear analyses are necessary to improve the safety and performance of structures [32-34].

In the Table 1; EI_e: is the effective stiffness, M_y: yield moment, $\emptyset_y$: yield curvature, EI: uncracked section stiffness, E_c and E_{cd}: concrete modulus of elasticity, E_s: is the modulus of elasticity design value of the reinforcing, I_c: is the cross-sectional moment of inertia, I_s: is the reinforcement area second moment is the area on the concrete core, θ_y: chord rotation for yield state and L_s: shear span. f_{ye}: expected yield strength of transverse reinforcement (f_{ye}=1.2f_{yk}), f_{ck}: is the concrete compressive strengths and f_{ce}: expected concrete compressive strength (f_{ce}=1.3f_{ck}). d_b and d_{bL}: is the longitudinal/tension reinforcement diameter, h: section height and $\eta = 1$ in RC beam sections. $a_V z$: is the flexural moment diagram's tension shift, z: internal lever arm length, calculated to be equal to $z = d - d'$ in beam section, d and d': are the depths of the reinforcement for compression and tension. ρ is the reinforcement ratio, A_c is the area of concrete section, A_{sl}: is the area of the reinforcement, b_w: is the width of the beam and N_{Ed}: is the axial load. K_c is a contributing element to creep and cracking and K_s is a component of the reinforcement contribution. K_1 and K_2 is a factor that depends on the axial load and the concrete's strength. φ_{ef} is the effective creep ratio, $n = N_{Ed}/(A_c f_{cd})$ and $\lambda = l_0/i$ (i: radius of rotation for the segment of uncracked concrete and l_0: effective length of the section). $a_{sl} = 1$; if longitudinal reinforcement can slip from the anchorage zone beyond the end section, or $a_{sl} = 0$; if there is no possibility of slipping.

2. MATERIAL AND METHOD

Double-reinforced beams are beams with tension and compression reinforcement. These reinforcements are located in both the upper and lower sections of the beam and increase the resistance to flexural moments. The flexural stiffness of a beam defines its resistance to flexural moment and is equal to the section stiffness of the beam multiplied by its inertia. Beyond a certain load, the beam leaves the elastic region, and plastic deformations begin. In this case, the behavior of the material ceases to be linear, and the stress-strain relationship becomes complex. Various methods and models are used to study nonlinear behavior: material modelling, section analysis, and numerical methods. The flexural stiffness is calculated taking into account nonlinear behavior. A novel method for estimating the stiffness of doubly-reinforced beams based on their nonlinear behavior is presented in this work. For the purpose of creating safe and effective structures, precise flexural stiffness measurement is crucial, especially when handling the intricacies of nonlinear material properties and structural reactions. Material properties (concrete and reinforcement), geometric properties, cracking, and nonlinear behavior are the factors affecting the section stiffness of RC elements. Cracking reduces the stiffness of the concrete section. The propagation and distribution of cracks have a significant impact on the effective stiffness. Both concrete and steel exhibit nonlinear behavior beyond certain stress levels. These nonlinearities must be taken into account to accurately determine the stiffness.

Table 1. Comparison of Different Effective Stiffness Equations

Author and Codes	Effective Section Stiffness
Moment-curvature relations	$k_e = EI_e/EI;\quad EI_e = M_y/\emptyset_y\ , E_c = 3250\sqrt{f_{ck}} + 14000\ MPa$
TBEC (2018) according to the lumped plastic behavior	$EI_e = \frac{M_y}{\theta_y}\frac{L_s}{3},\quad \theta_y = \frac{M_y L_s}{3EI}\ (Linear\ method)\ , \varepsilon_y = f_y/E_s$ $\theta_y = \frac{\emptyset_y L_s}{3} + 0.0015\eta\left(1 + 1.5\frac{h}{L_S}\right) + \frac{\emptyset_y d_b f_{ye}}{8\sqrt{f_{ce}}}\quad (Nonlinear\ method)$
Eurocode8-Part 3 (2005)	$EI_e = \frac{M_y L_v}{3\theta_y},\quad \theta_y = \emptyset_y\frac{L_V + a_V z}{3} + 0.0014\left(1 + 1.5\frac{h}{L_V}\right) + \emptyset_y\frac{d_{bL} f_y}{8\sqrt{f_c}}$ $V_{Rd,c} = \left[C_{Rd,c}K(100\rho_1 f_{ck})^{\frac{1}{3}} + K_1\sigma_{cp}\right]b_w d$ $K = 1 + \sqrt{\frac{200}{d}} \leq 2\ ,\quad \rho_1 = \frac{A_{sl}}{b_w d} \leq 0.02\ ,\ \sigma_{cp} = \frac{N_{Ed}}{A_c} < 0.2 f_{cd}$
Eurocode2 (2004)	$EI = K_c E_{cd} I_c + K_s E_s I_s\ ,\ E_{cd,\mathrm{eff}} = \frac{E_{cd}}{(1 + \varphi_{\mathrm{ef}})}\quad , K_1 = \sqrt{\frac{f_{ck}}{20}}\ ,\ K_2 = n\frac{\lambda}{170} \leq 0.2$ $\rho \geq 0.002:\ K_s = 1\ ,\ K_c = \frac{K_1 K_2}{(1 + \varphi_{\mathrm{ef}})}\ ,\ \rho \geq 0.01:\ K_s = 0\ ,\ K_c = \frac{0.3}{(1 + 0.5\varphi_{\mathrm{ef}})}$
Avşar et al. (2014)	$\alpha_{\mathrm{eff}} = 0.271 + 0.0064 \times f_c + 0.120 \times \frac{\rho_s'}{\rho_{st}} + 37.895 \times \rho_{st}$
Biskinis (2007)	$\frac{EI_{\mathrm{eff}}}{E_c I_c} = a\left(0.8 + ln\left[max\left(\frac{L_s}{h}; 0.6\right)\right]\right)\left(1 + 0.048\ min\left(50MPa; \frac{N}{A_c}\right)\right), a = 0.10\text{: beam}$
Biskinis and Fardis (2004)	$EI_{\mathrm{eff}} = M_y L_s/3\ \theta_y\ ,\quad \theta_y = \emptyset_y\frac{L_s + a_V z}{3} + 0.0013 + a_{sl}.\frac{\emptyset_y d_{bL} f_y}{8\sqrt{f_c}}\ ,\quad L_s = M/V$ $V_{Rc} = \left\{max\left[180(100\rho_1)^{1/3}, 35\sqrt{1 + \sqrt{\frac{0.2}{d}} f_c^{1/6}}\right]\left(1 + \sqrt{\frac{0.2}{d}}\right) f_c^{1/3} + 0.15\frac{N}{A_c}\right\} b_w d$

Many design parameters influence the nonlinear behavior and effective stiffness of RC bearing members. The numerical and analytical analysis presented was performed on doubly-reinforced rectangular cross-section beams designed by considering different tensile reinforcement ratios (ρ), compression reinforcement ratios (ρ'), and concrete strength (f_{ck}). The effects of design parameters on the nonlinear behavior and effective stiffness of the doubly-reinforced beam models were investigated by changing one parameter at a time and keeping the value of the other parameters constant. In the analytical study for RC structural members, moment-curvature relations were obtained and presented graphically by taking the materials' nonlinear behavior into account. A concrete model proposed by the Mander et al. [35], mandated in TBEC [21] and universally accepted, was used to determine the nonlinear moment-curvature relations of the designed beam sections. The details and properties of the double-reinforced beam models considered in the numerical analysis are summarized in Table 2. A typical rectangular cross-section beam model with a total depth of $h = 600mm$ and a width of $b_w = 300mm$ were designed. The tensile and compression reinforcement is provided at $d = 550mm$ and $d' = 50mm$ from the top, respectively ($d = h - d'$). A concrete grade of C25 to C50 was taken into consideration in the analysis of the designed RC beam models. Concrete with a strength lower than C25 is not permitted to be used in structures that are designed and constructed by the guidelines of TBEC [21]. B420C was chosen as the transverse and longitudinal reinforcement in beam models (Table 3).

When calculating, designing, and projecting sections, it is important to make sure that the longitudinal reinforcement ratio in RC beam cross-sections is smaller than the balanced reinforcement ratio (ρ_b). The ratio of tensile reinforcement (ρ) calculated in the beam cross-sections should not be less than the minimum tensile reinforcement ratio values given in Equation (1). The longitudinal reinforcement ratio calculated to provide ductile behavior in doubly-reinforced beam members is limited to Equation (2) given in TS500 [36]. ρ in double-reinforced beams shall not be more than 2% and the maximum reinforcement ratio (ρ_{max}) given in Equation (2). In double-reinforced beams, the difference between the ρ' and ρ should not be more than $0.85\rho_b$. According to TS500 (2000), k_1 factor is provided not greater than 0.85 and not to be less than 0.70 (Equation 3). According to TBEC [21] and TS500 [36] provisions, a total of 66 double-reinforced rectangular cross-section beams were designed, taking into account different design parameters (f_{ck}, ρ and ρ'). In beam models, as the ratio of tensile reinforcement; $\rho_{max} = 0.85\rho_b$ and as the ratio of the compression reinforcement; the values of $\rho'_s = 0.0$, $0.1\rho_{max}$, $0.2\rho_{max}$, $0.3\rho_{max}$, $0.4\rho_{max}$, $0.5\rho_{max}$, $0.6\rho_{max}$, $0.7\rho_{max}$, $0.8\rho_{max}$, $0.9\rho_{max}$ and ρ_{max} are taken into account.

$$\rho = \frac{A_s}{b_w d} \geq \rho_{min} = 0.8\frac{f_{ctd}}{f_{yd}} \tag{1}$$

$$(\rho - \rho') \leq \rho_{max} = 0.85\rho_b \quad , \quad \rho_b = 0.85k_1\left(\frac{f_{cd}}{f_{yd}}\right)\left(\frac{700}{700+f_{yd}}\right) \tag{2}$$

$$k_1 = 0.85 - 0.006(f_{ck} - 25) < 0.85 \leftrightarrow f_{ck} > 25MPa \quad ; \quad k_1 = 0.85 \leftrightarrow f_{ck} \leq 30MPa \tag{3}$$

Table 2. Models of double-reinforced beams: properties and details

ρ'/ρ	Concrete:C25		Concrete: C30		Concrete:C35		Concrete:C40		Concrete:C45		Concrete:C50	
	ρ	ρ′	ρ	ρ′	ρ	ρ′	ρ	ρ′	ρ	ρ′	ρ	ρ′
0.0	0.0184	0.0	0.0213	0.0	0.0240	0.0	0.0263	0.0	0.0285	0.0	0.0303	0.0
0.1		0.0018		0.0021		0.0024		0.0026		0.0028		0.0030
0.2		0.0037		0.0043		0.0048		0.0053		0.0057		0.0061
0.3		0.0055		0.0064		0.0072		0.0079		0.0085		0.0091
0.4		0.0074		0.0085		0.0096		0.0105		0.0114		0.0121
0.5		0.0092		0.0107		0.0120		0.0132		0.0142		0.0152
0.6		0.0111		0.0128		0.0144		0.0158		0.0171		0.0182
0.7		0.0129		0.0149		0.0168		0.0184		0.0199		0.0212
0.8		0.0147		0.0171		0.0192		0.0211		0.0228		0.0243
0.9		0.0166		0.0192		0.0216		0.0237		0.0256		0.0273
1.0		0.0184		0.0213		0.0240		0.0263		0.0285		0.0303

Table 3. Parameters for materials used to reinforce concrete and steel [21]

Material	Parameters	Values
Reinforcing steel: B420C	Steel reinforcement's yield strength (f_{yk})	420MPa
	The steel reinforcement's ultimate strength (f_{su})	550MPa
	Steel reinforcement's yield strain (ε_{sy})	0.0021
	reinforcing steel's hardening strain (ε_{sp})	0.008
	Steel reinforcement's ultimate strain (ε_{su})	0.08
Concrete: C25-C50	Unconfined concrete compressive strength (f_{ck})	25-50MPa
	Strain of unconfined concrete at maximum stress (ε_{co})	0.002
	Ultimate compression strain of unconfined concrete (ε_{cu})	0.0035

For the design of RC elements, it is extremely important to know the behavior of the elements and understand the effects of the parameters that affect this behavior. The behavior of the RC element can generally be monitored from the moment-curvature relationship. Different design codes and standards provide different parameters and guidelines for modifying the stiffness of RC load-bearing members. In the analyses and calculations made according to the standards and codes, the actual behavior value is not taken into account for the sections' effective stiffness.

Accurate and realistic stiffness factors should be determined by calculating nonlinear behavior values according to design parameters instead of defined approximate values. Since there are many beam-bearing members in a designed RC structure, the calculation of the values to be used in the analysis according to the nonlinear behavior by considering the design parameters, material properties, and cross-section information in each beam may require very detailed processes and a long time. This situation makes it almost impossible to determine the effective stiffness of the cracked section according to the nonlinear behavior of the RC-bearing members. This process is avoided because it is very laborious to create a model by calculating the stress-strain and moment-curvature relations of a real RC-bearing element in the scientific and practical environment and by calculating the effective section stiffnesses. In the analysis software, calculations and analyses are made by using the constant coefficients given in the standard and codes or by determining the effective stiffness values according to simple relations (ignoring important design parameters). In this case, more realistic results cannot be obtained without considering the cross-sectional and material properties and reinforcement ratios of the RC-bearing members.

In the design and evaluation of RC-bearing members, determining their nonlinear behavior and obtaining effective stiffness values are very important to obtain more realistic results. Information from multiple codes and sources may reveal a more accurate method of analysis for the particular RC structure that the designer is currently evaluating. In this study, a simple equation covering all design parameters has been developed to overcome very long and detailed calculations that require nonlinear behavior modelling to determine the double-reinforced beam's cracked section's effective stiffness. By computing the section in terms of effective stiffness, the effects of various design elements, including material characteristics, longitudinal reinforcement ratio, and configuration, on the nonlinear behavior of beam members were evaluated. Comparisons with alternative prediction formulas confirm the suggested predictions for the stiffness factors, which are based on the numerical and analytical results. The Research Findings section provides a detailed presentation of the stiffness factors obtained from the numerical analysis findings by the proposed equation, moment-curvature relations, and various standards, codes, and researchers.

3. RESEARCH FINDINGS

Important design parameters, including f_{ck}, ρ and ρ', were taken into consideration for determining the effective flexural stiffness due to concrete cracking. Based on the findings from the doubly-reinforced beam models' nonlinear relations, new relations based on the f_{ck}, ρ and ρ' parameters affecting the stiffness of the beams were proposed and compared with the relations proposed in the codes and by researchers. In the RC beam models designed, moment (M_y, M_u) and curvature ($\emptyset_y$, $\emptyset_u$) relations were calculated from nonlinear behaviors according to different design parameters (Figure 1).

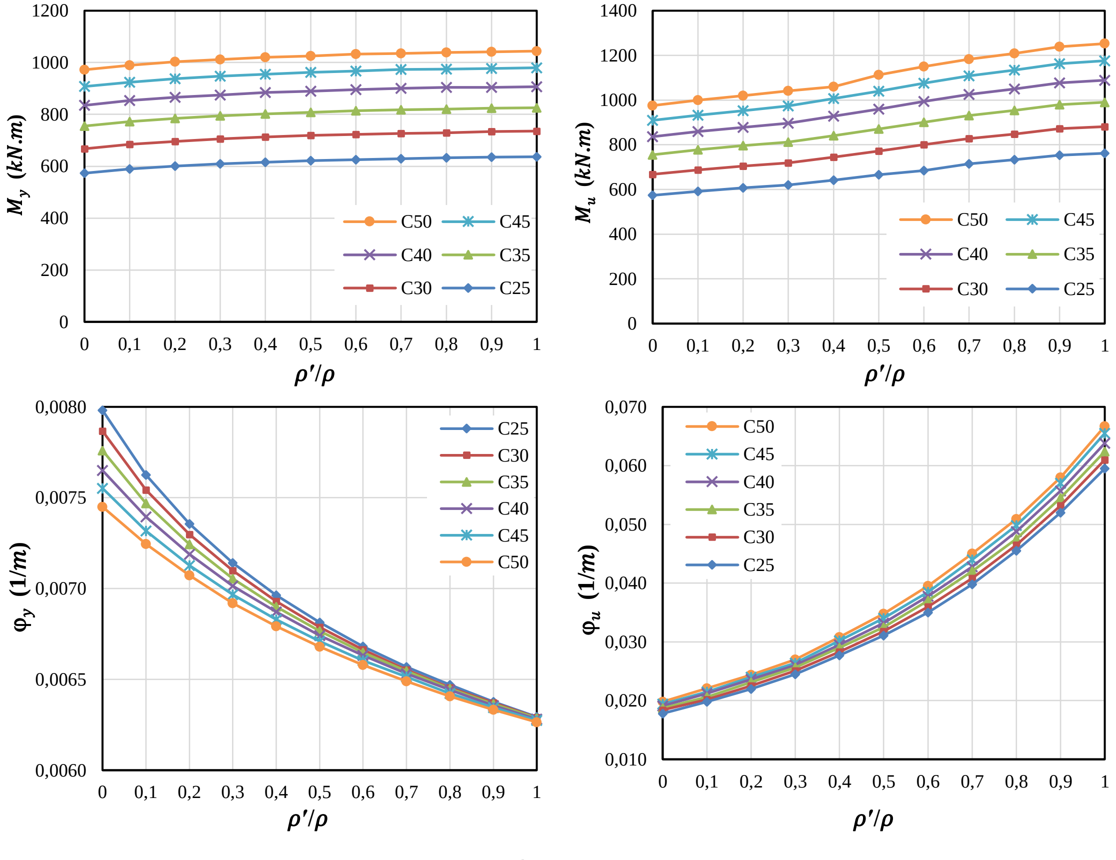


Figure 1. Influence of f_{ck} and ρ'/ρ ratios on the moment and curvature

In the beam models without ρ', it is seen that increasing the ρ values affect the moment carrying capacity (M_u), yield ($\emptyset_y$) and ultimate curvature ($\emptyset_u$) values remain almost constant. In beam sections, while the ρ and f_{ck} values are constant, the $\emptyset_y$ obtained from nonlinear relations decreases with the increase in the ρ' values, and the M_y, M_u and $\emptyset_u$ values increase. The increase in the ρ' values increase the M_u and ductility of the sections. The ρ values basically affect both the nonlinear moment-curvature relations and the section ductility. While the longitudinal reinforcement (ρ'/ρ) ratios are constant in sections, as the f_{ck} increases, so do the M_y and M_u values obtained from the moment-curvature relations.

The effective stiffness of double-reinforced beams according to nonlinear behavior is of great importance for the safe and economical design of structures. According to the nonlinear analysis results, the effective stiffnesses to be used in the design of beams should be carefully selected to increase load-carrying capacity and safety. Performing nonlinear analysis, especially for structures that will be exposed to high loads, provides safer and more economical designs.

The effective stiffness of double-reinforced beams for nonlinear behavior helps to accurately predict the actual load-carrying capacity and deformation behavior of the beam. This is critical for the safety and durability of structures. As a result, determining the effective stiffness of double-reinforced beams according to nonlinear behavior is a critical issue in structural engineering. Nonlinear analyses help increase the safety and durability of structures by more accurately predicting the actual load-carrying capacity and deformation behavior of beams. These studies will contribute to safer and more economical designs in engineering applications.

Plastic deformations and the formation of cracks significantly affect the flexural stiffness of beams. These deformations are characterized by an effective stiffness lower than the initial elastic stiffness of the beams. Accurate determination of the effective stiffness of double-reinforced beams according to nonlinear behavior plays a critical role in structural engineering for the design of safer and more durable structures.

3.1. Derivation of an Alternative Equation for Effective Stiffness Factor ($k_{e,prop}$)

The stiffness of doubly-reinforced beam models is obtained by nonlinear behavior analyses depending on section properties such as concrete compressive strength (f_{ck}) and longitudinal reinforcement ratios (ρ' and ρ). The most important design parameters determined using multilinear regression analysis were taken into consideration when developing equations for doubly-reinforced beam models to calculate the stiffness coefficient (k_e). From the analysis results, the essential parameters influencing the stiffness of beams are the f_{ck}, ρ and ρ' values. Thus, based on the nonlinear behavior with numerical analysis results, it can be observed that the use of the two important variables ratio of compression to tensile reinforcement (ρ'/ρ) and f_{ck} should be incorporated for developing an equation for stiffness factors ($k_{e,prop}$) of doubly-reinforced rectangular cross-section beams. The basic formation of the $k_{e,prop}$ for beam sections can be expressed as; $f\{(\rho'/\rho\,), f_{ck}\}$. The equation proposed for beam sections is obtained from the results obtained from numerical analysis with nonlinear relationships of models with concrete strength of 25MPa to 50MPa, yield strength of reinforcing steel 420MPa, tensile reinforcement ratio ρ=0.85ρb and compression reinforcement $\rho' = 0$ to ρ_{max}. The results of numerical analyses for ρ'/ρ=0.0 to 1.0, f_{ck}=25MPa to 50MPa and f_{yk}=420MPa indicate that the k_e increases as ρ'/ρ increases for beams. The findings of numerical analyses are used to do regression analysis in order to determine the impact of ρ'/ρ on the k_e. Figure 2 shows how ρ'/ρ affects k_e and illustrates how well the suggested equation agrees with the stiffness numerical results. The k_e is significantly affected by the ρ'/ρ. Taking into account the results of the analysis, the relationships of the $k_{e,prop}$ and the ρ'/ρ parameter were obtained from the regression analysis in Equation 4.

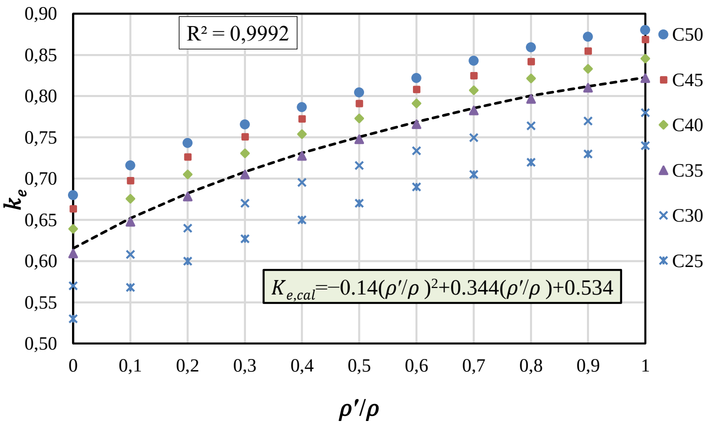


Figure 2. Influence of parameter ρ'/ρ on the effective stiffness coefficient

$$K_{e,cal\_1} = -0.14(\rho'/\rho\,)^2 + 0.344(\rho'/\rho\,) + 0.534 \tag{4}$$

The Equation (4) clearly shows that the k_e can be expressed as a function of $-0.14(\rho'/\rho\,)^2 + 0.344(\rho'/\rho\,) + 0.534$. The effect of f_{ck} on the ratio of the k_e to $-0.14(\rho'/\rho\,)^2 + 0.344(\rho'/\rho\,) + 0.534$ is shown in Figure 3. The numerical results indicate that the k_e increases with increasing of f_{ck}. By regression analysis, the relations between the ratio of k_e to $-0.14(\rho'/\rho\,)^2 + 0.344(\rho'/\rho\,) + 0.534$ and the f_{ck} of ρ'/ρ were obtained from the Equation 5.

$$k_{e,cal\_2} = \frac{k_e}{-0.14\left(\frac{\rho'}{\rho}\right)^2 + 0.344\left(\frac{\rho'}{\rho}\right) + 0.534} = -0.0002(f_{ck})^2 + 0.026(f_{ck}) + 0.455 \tag{5}$$

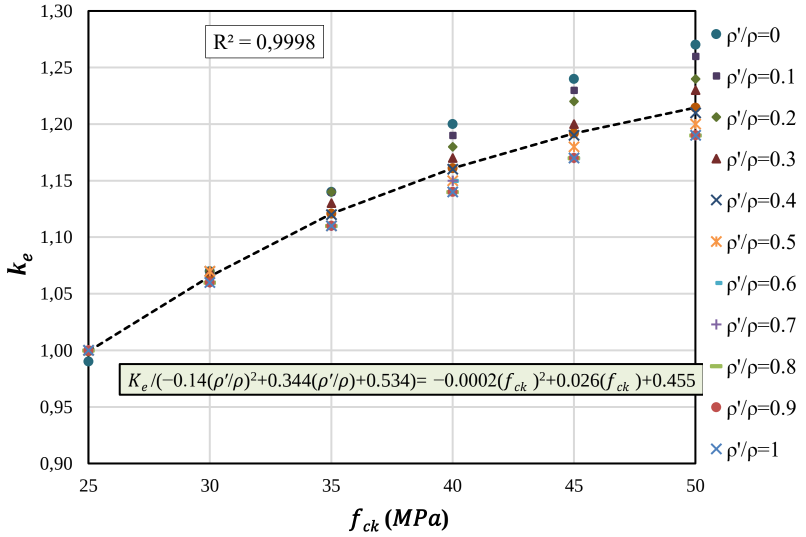


Figure 3. Influence of f_{ck} on the ratio of effective stiffness factor to Equation 4.

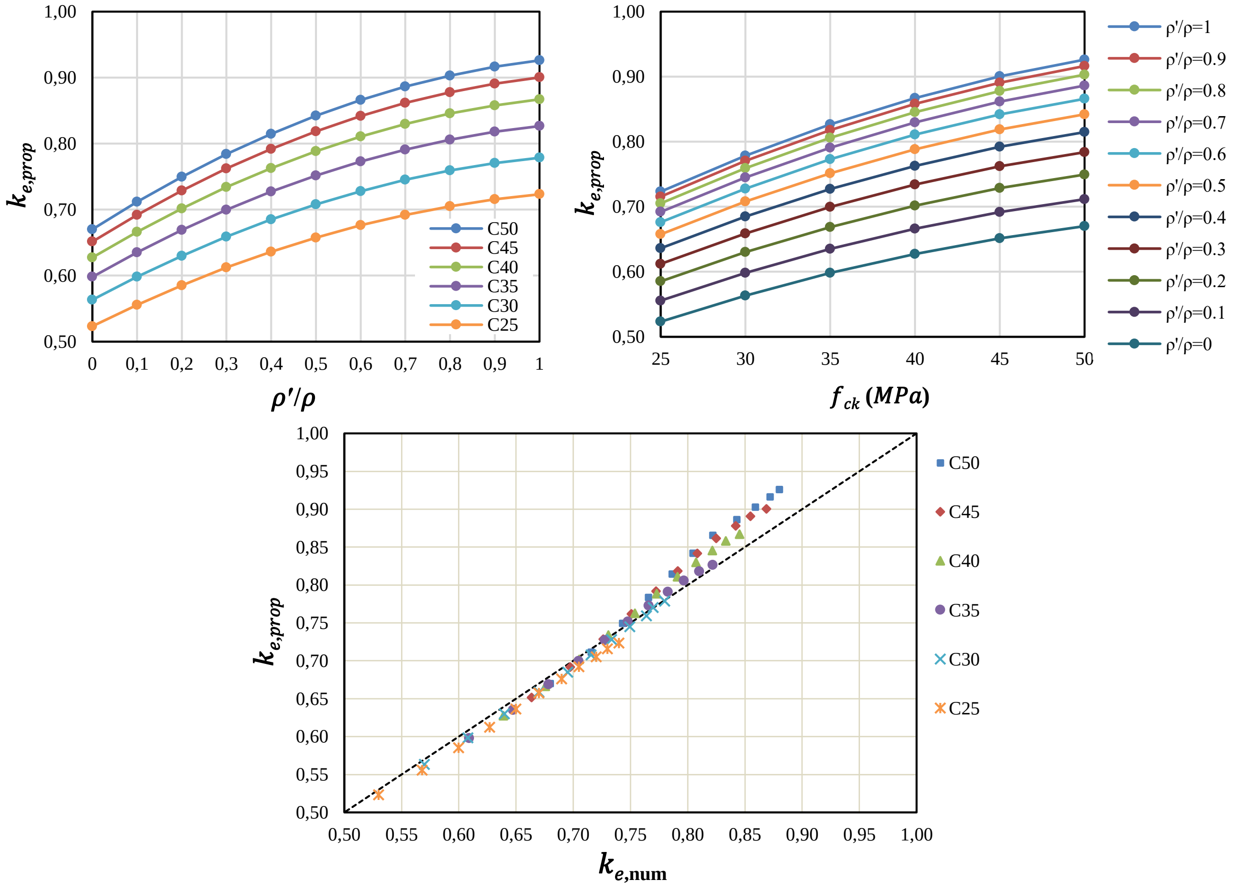


Figure 4. $k_{e,prop}-\rho'/\rho$, $k_{e,prop}-f_{ck}$ and $k_{e,prop}-k_{e,num}$ relations according to different design parameters.

Given the influence of parameters such ρ'/ρ and f_{ck} in beam models, the prediction equation of $k_{e,prop}$ can be provided using numerical analysis. Proposed $k_{e,prop}$ can be expressed as Equation (6). The analysis results calculated according to ρ'/ρ and f_{ck} parameters from the $k_{e,prop}$ equation proposed for beam sections are presented comparatively in Figure 4. The ratio of the numerical findings affected by changes in ρ'/ρ and f_{ck} and the results obtained from Equation (6). When ρ'/ρ and f_{ck} increase, there is a greater difference between the numerical results and those from Equation (6) (Figure 4).

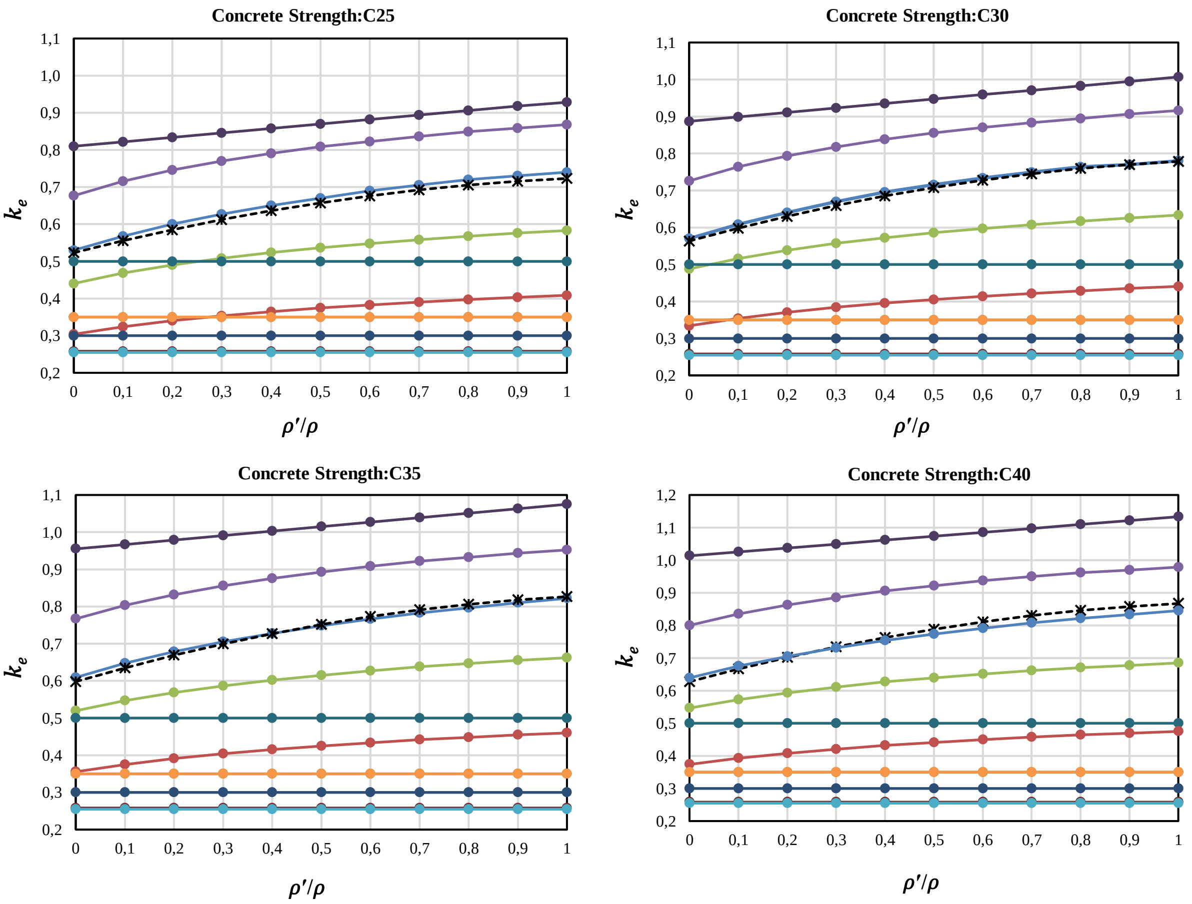
$$k_{e,prop} = \left[-0.14\left(\frac{\rho'}{\rho}\right)^2 + 0.344\left(\frac{\rho'}{\rho}\right) + 0.534\right] \times \left[-0.0002(f_{ck})^2 + 0.026(f_{ck}) + 0.455\right] \tag{6}$$

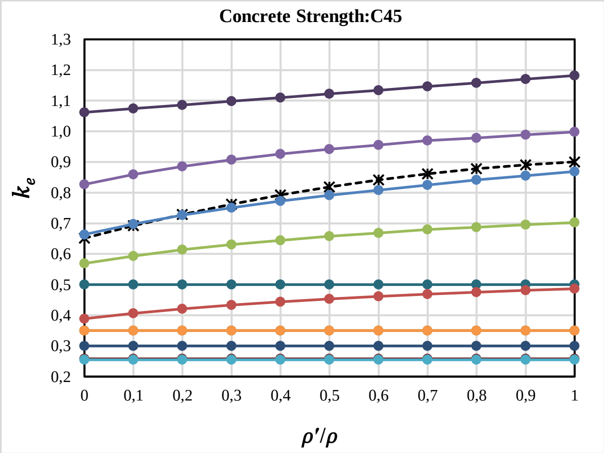
From the relations given in Figure 4; it is seen that the $k_{e,prop}$ for the stiffness factor is compatible with the results in all ranges of the important design parameters affecting the beams' nonlinear behavior. As ρ'/ρ and f_{ck} ratios and values increase, so do the differences between the numerical findings obtained from the nonlinear relations of the beams and the results obtained from the suggested equation for the stiffness factor. Excellent agreement is demonstrated by the suggested $k_{e,prop}$, as shown by coefficients correlation R^2 that are well above 0.99. The stiffness factor estimated in the proposed equation has a maximum mean value of 1.05% and a standard deviation of 4.987% for the ratio of the $k_{e,prop}$ value to the nonlinear numerical result.

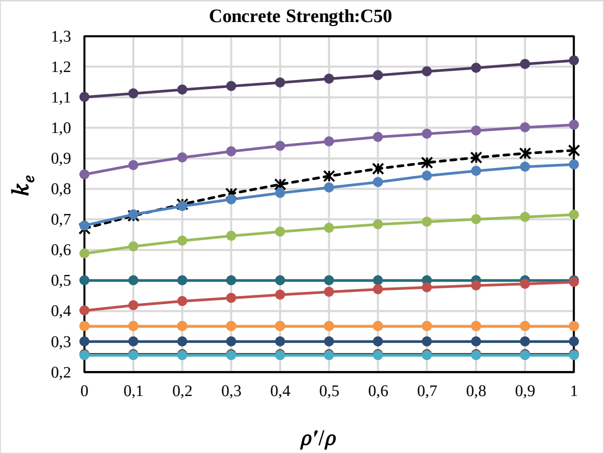
3.2. Verification of the Proposed Equation for Effective Stiffness Factor

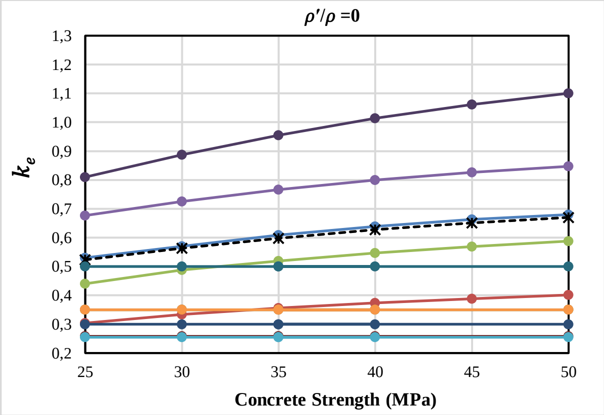
The ρ', ρ and f_{ck} are the key parameters affecting the stiffness of doubly-reinforced beams. The effect of each factor on the stiffness of beam sections is presented in the following. In this part of the study, it was calculated analytically the k_e of models designed with different ρ', ρ and f_{ck} according to the relations by proposed effective stiffness factor equation. Comparative $k_e - \rho'/\rho$ and $k_e - f_{ck}$ relations according to proposed equation for effective stiffness factor, different researchers and seismic codes are given in Figure 5.

The comparison analysis results show that the k_e values calculated from the nonlinear relation, from various codes, and the relationships proposed by various researchers differ from each other. It has been obtained from the comparison of the numerical results that the stiffness coefficient values calculated from the equation proposed for the effective stiffness factor, different codes and from the relationships suggested by the researchers, increase with the increase of ρ' for constant ρ and f_{ck} values. In beam models with constant ρ' and ρ, k_e values increase with increasing f_{ck}. It has been proven that the ρ' beam models have an effect on the moment capacity, stiffness and ductility.




Concrete Strength:C45
k_e
1,3
1,2
1,1
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1
ρ'/ρ


Concrete Strength:C50
k_e
1,3
1,2
1,1
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1
ρ'/ρ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k_e
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1,2
1,1
1,0
0,9
0,8
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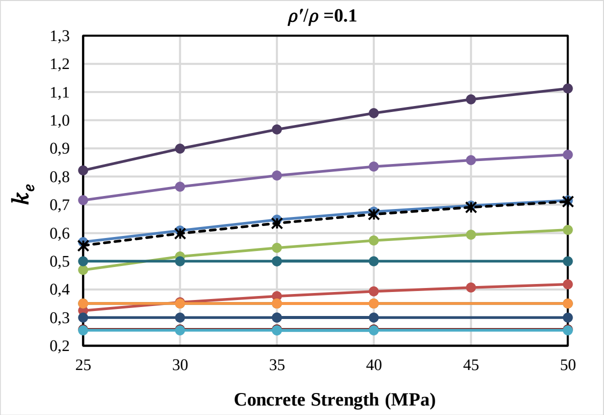
0,6
0,5
0,4
0,3
0,2
25
30
35
40
45
50
Concrete Strength (MPa)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k_e
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1,2
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1,0
0,9
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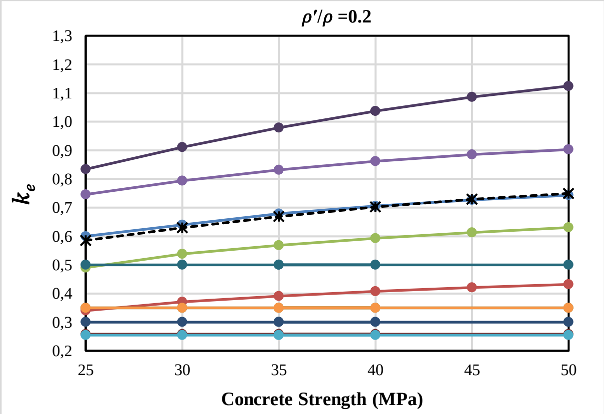
0,7
0,6
0,5
0,4
0,3
0,2
25
30
35
40
45
50
Concrete Strength (MPa)


ρ'/ρ =0.2
k_e
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1,2
1,1
1,0
0,9
0,8
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0,5
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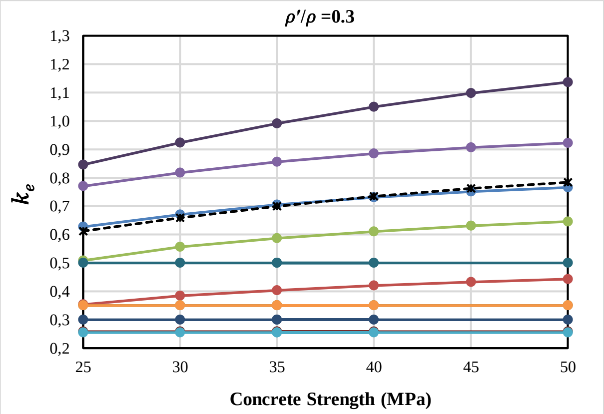
Concrete Strength (MPa)


ρ'/ρ =0.3
k_e
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1,2
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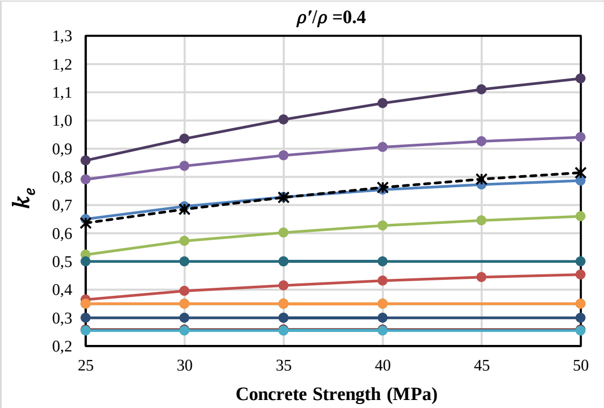
Concrete Strength (MPa)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k_e
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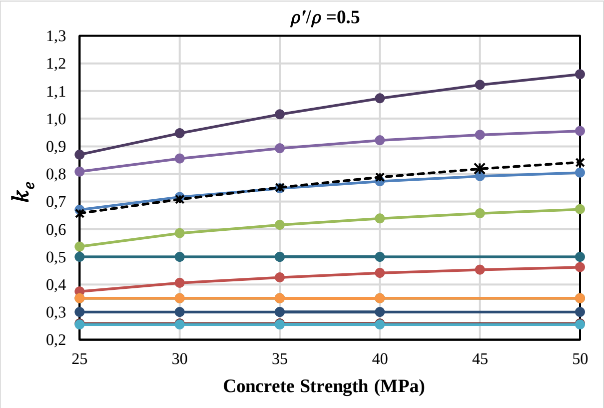
Concrete Strength (MPa)


ρ'/ρ =0.5
k_e
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1,2
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0,9
0,8
0,7
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0,5
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0,3
0,2
25
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Concrete Strength (MPa)

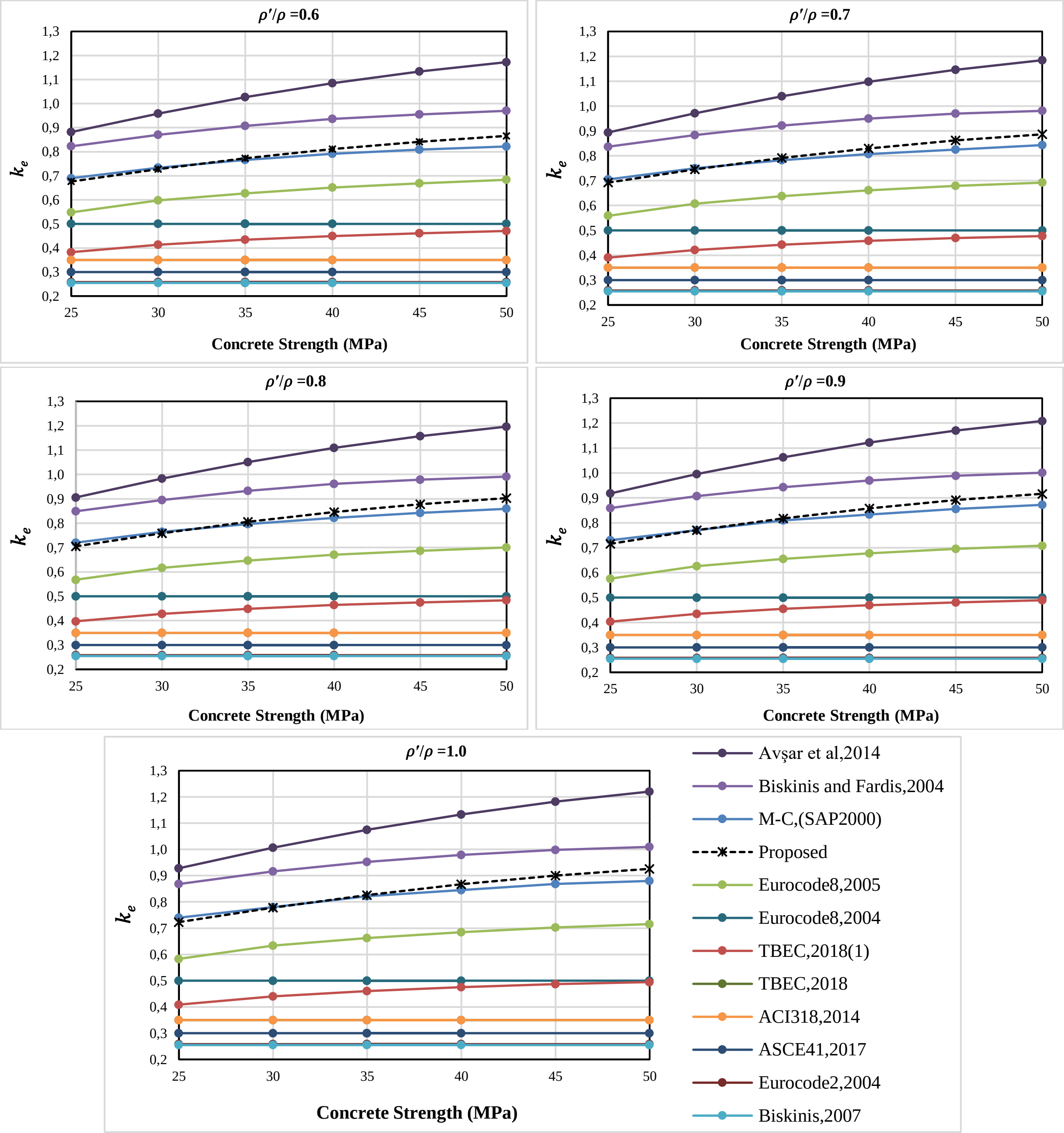

Figure 5. Comparison of $k_e - \rho'/\rho$ and $k_e - f_{ck}$ relations based on $k_{e,prop}$, various researchers, and codes

The comparison graphics show that the ρ'/ρ ratio has increased and f_{ck} value in double-reinforced beams increases the stiffness of the sections. It is seen that as the ρ'/ρ ratio and f_{ck} values increase, correspondingly increases the difference between the findings of the nonlinear analysis of sections and the stiffness values calculated from the suggested equation for the stiffness factor. The comparison of all the numerical results shows for the effective section stiffness of the double-reinforced beam models; Almost the same result is obtained from the Equation (6) proposed with the moment-curvature relations. This new approach to determining the stiffness of doubly-reinforced beams offer significant improvements in accuracy over traditional methods by incorporating nonlinear material behavior and the effects of cracking and plastic deformations. Adopting this methodology in structural design can enhance the reliability and safety of RC structures while optimizing material usage and overall cost-efficiency. Further research and development of this approach could refine its application and expand its use to other types of elements and materials.

4. CONCLUSIONS

Effective stiffness of double-reinforced beams according to different design parameters; a comprehensive parametric study was carried out according to the relations given in the literature, nonlinear relations, and the equation obtained from the numerical analysis results. The values recommended by the regulations for the effective stiffness factor of the cracked section are quite different from the values obtained from the analysis results. However, considering the criteria proposed in Part 3 of Eurocode8 (2005), values close to the analysis results compared to other regulations are obtained. The results of analytical solutions may differ from each other according to the different regulations, the relations proposed by researchers, and the new equation for the effective stiffness factor. The analysis results are different because different limit values are taken into account for parameters such as material properties, design parameters, and longitudinal and transverse reinforcement in studies and regulations in literature. In the analyses made by adhering to the regulations, the effective stiffness of the section is not a realistic behavior value. Overestimating effective stiffness will result in an overestimation of the structural stiffness. Errors will thus occur during the calculation and assessment of structures. To determine the stiffness factor of RC members, instead of using the factor values defined in the standards and codes, it is of great importance to calculate the stiffness values by considering the nonlinear behavior of the structural element sections.

Depending on the increase in the concrete grade and the ratio of longitudinal reinforcement, the effective section stiffness values of the beam sections increased. From the analysis results, since the concrete grade and the ratio of compression reinforcement to tensile reinforcement are effective on the nonlinear behavior and effective stiffness of beam members, a simple equation for the stiffness factor has been proposed considering these parameters. Each analytical calculation performed includes an approximation within the framework of the assumptions made for the cross-section and material.

In the design and assessment of RC load-bearing members, determining their nonlinear behavior and obtaining effective stiffness values are very important to obtain more realistic results. The stiffness factor values calculated from the numerical analysis of double-reinforced beam models were compared according to different design parameters, and it is seen that the nonlinear moment-curvature relations with the proposed equation have similar results. Correlation coefficients R^2 are significantly above 0.99, which indicates that the proposed equation exhibits high agreement. Additionally, the ratio of the suggested effective stiffness factor, which was determined by Equation (6), to the numerical result has a maximum mean value of %1.05 and a standard deviation of %4.98, respectively. The effective stiffness factor equation, which is proposed based on numerical analysis results according to design parameters such as different longitudinal reinforcement ratios and concrete grades, is accurate to within %4.98 error for practical applications. The stiffness factor values calculated from the proposed equation were verified by comparing the numerical results from the nonlinear relations and the other estimates defined in the literature, codes, and standards. The equation proposed for the stiffness factor takes into account the most important design parameters that affect the nonlinear behavior of double-reinforced beam sections and as can be seen from the examination of the numerical results, offers fairly accurate and consistent effective stiffness factor estimation. By using equation (6), the stiffness factors of the cracked section of double-reinforced beam sections according to different design parameters can be conveniently and simply calculated and compared.

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Authors' Contributions

In this study, authors contributed equally to the study.

Competing Interests

The authors declare that they have no conflict of interest.

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