

## On the optimal search for efficient estimators of population mean in simple random sampling in the presence of an auxiliary variable

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### Abstract

This study proposes some ratio estimators of the population mean under simple random sampling schemes, in order to tackle the problem of low efficiencies of some existing estimators. An improved exponential ratio estimator of the population mean under simple random sampling scheme and its bias and mean square error have been derived. Further propositions of a generalized form of the exponential ratio estimator of the population mean under simple random sampling scheme has also been made. The Bias and Mean Square Errors of these class of estimators have also been obtained. It is observed that some existing estimators are members of this class of estimators of population mean. Analytical and numerical results indicate that, the Asymptotic Optimal Estimator (AOE) of these proposed estimators of population mean using single auxiliary variable have been found to exhibit greater gains in efficiencies than the classical regression estimators and other existing estimators in simple random sampling scheme.

**Keywords:** Ratio estimators, Efficiency, Asymptotic Optimal Estimators(AOE), generalized exponential estimators, population mean.

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## 1. Introduction

Researches in sampling theory and practice have shown that the linear regression estimator of population mean is generally more efficient than the ratio and product estimators. The equality in efficiency is always achieved if the regression line of  $Y$  on  $X$  has a zero intercept [27]. The ratio and product estimators were then limited in terms of efficiency and could not be used to give greater efficiency, since in many practical situations; the regression line of the variable of interest on the auxiliary variable does not usually pass through the origin. Consequent upon this, the linear regression estimator was considered to be the only estimator with the greatest efficiency.

In view of the limitation that engulfed the classical ratio and product estimators of population mean, many researchers have made tremendous explorations and discoveries on the improvement of efficiency of ratio estimation of population mean, either through modifications of the existing ones or proposing new ratio estimators. Works of [28],[26], [48], [46] have shown significant improvement on estimating the population Mean through the use of their proposed estimators.

Many other authors, by way of trying to make significant improvement on the efficiency of their ratio estimators, make use of the parameters of the auxiliary variable and known constants to propose new ratio estimators. Singh and Tailor [36, 37], made use of correlation coefficient of the auxiliary variable; Kadilar and Cingi [11, 12, 16, 15, 17] made use of coefficient of Kurtosis, coefficient of variation, correlation coefficient and their combinations to propose new ratio estimators of population mean. Also, [59], [50], [51], [52], [53], [54], [55], [49], [57], [45], [33] [9], [56] and many others used the Median, coefficients of kurtosis, coefficients of skewness, etc to propose classes of ratio estimators. In all these efforts none of these estimators seemed to have greater efficiency than the regression estimator, but some had greater gain in efficiency than the classical ratio estimator, while some were even less efficient than the classical ratio estimator. The important achievement here was that they created avenues for more researchers on the subject matter.

In another development, other authors came up with new ratio estimators of population mean known as exponential ratio and product estimators. Foremost among them were [1], who found that, in most cases, their exponential ratio and estimators were more efficient than the classical ratio and product estimators. Later, [24], [32], [44], [18], and many other authors were motivated in the works of [1] and they either modified the existing exponential ratio and product estimators or linear combinations of dual and ratio/product estimators. Some of their works, especially the linear combinations began to yield good results, as most of their Mean Square Errors were the same as the variance of the classical regression estimators.

Recent works have built on both the modifications of the classical ratio or regression and the exponential ratio estimators to obtain improved efficiencies in simple random sampling. These works include [24], [10], [58], [31], [35], [35], [42], [6], [7], [8]. These works showed some improvements over the Regression estimator. Although these works showed some improvement in efficiency over the regression estimates, they are not consistent in their performance for all populations.

Furthermore, their efficiencies over the regression estimator in some cases are not statistically significant. These recent discoveries have motivated many more researchers to still probe further on the efficiency of ratio and regression method of estimation in a bid to obtaining better estimation procedure with greater efficiency. It is in the light of this that this research work is carried out. The proposed estimator is intended to be more efficient than these ones or compare favourably with the best of the existing estimators.

## 2. Review of some related existing Estimators

Consider a finite population  $U = (U_1, U_2, \dots, U_N)$  of size  $(N)$ . Let  $(X)$  and  $(Y)$  denote the auxiliary and study variables taking values  $X_i$  and  $Y_i$  respectively on the  $i^{th}$  unit  $U_i (i = 1, 2, \dots, N)$  population. It is assumed that  $(x_i, y_i) \geq 0$ , (since survey variables are generally non-negative) and information on the population mean  $(\bar{X})$  of the auxiliary variable  $(X)$  is known. Let a sample of size  $(n)$  be drawn by simple random sampling without replacement (SRSWOR) from which we obtain the means  $(\bar{x})$  and  $(\bar{y})$  for the auxiliary variable  $(X)$  and the study variable  $(Y)$ .

For the above population we provide a summary of some existing estimators with its mean square error in Table 1 below.

**TABLE 1**

**Table 1.** Some related existing estimators of Population Mean in simple random sampling using a single auxiliary variable and their Mean Square Errors (MSE)

S/N	ESTIMATORS	MSE
1	$\bar{y}$ (Simple Random Sample Mean)	$\lambda \bar{Y}^2 C_y^2$
2	$\bar{y}_R = \frac{\bar{y}}{\bar{x}} \bar{X}$ (Classical Ratio Estimator)	$\lambda \bar{Y}^2 \{C_y^2 - 2P_{yx} C_y C_x + C_x^2\}$
3	$\bar{y}_{reg} = \bar{y} + b_{yx} (\bar{X} - \bar{x})$ (Classical Regression Estimator)	$\lambda \bar{Y}^2 C_y^2 (1 - P_{yx}^2)$
4	$\bar{y}_{GS}^* = [\eta_1 \bar{y} + \eta_2 (\bar{X} - \bar{x})] \Delta_j$ (Gupta and Shabbir [6])	$\bar{Y}^2 \left[ 1 - \frac{\alpha_2 \alpha_4^2 + 2\alpha_3 \alpha_4 \alpha_5 + \alpha_1 \alpha_5^2}{\alpha_1 \alpha_2 - \alpha_3^2} \right]$
5	$\bar{y}_{KCi} = [\bar{y} + b_{yx} (\bar{X} - \bar{x})] \Delta_j$ (Gupta and Shabbir [6])	$\lambda \bar{Y}^2 [a_i C_x^2 + C_y^2 (1 - \rho_{yx}^2)]$
6	$\bar{y}_{Singh}^* = t_1 \bar{y}_\beta + t_2 \bar{y}^* \left( \frac{\bar{x} - S_x}{\bar{X} - S_x} \right)$ (Singh, Rathour and Solanki [35])	$\bar{Y}^2 \left[ 1 - \frac{(C_2 - 2C_3 C_4 + C_1 C_4^2)}{C_1 C_2 - C_3^2} \right]$
7	$\bar{y}_R = \bar{y} \exp \left[ \frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right]$ (Bahl and Tuteja [1])	$\lambda \bar{Y}^2 \left[ C_y^2 + \frac{C_x^2}{4} (1 - 4K) \right]$
8	$\varphi_1^* \bar{y} \Delta_j + \varphi_1^* (\bar{X} - \bar{x}) \Delta_j^2$ (Singh and Solanki [42])	$\bar{Y}^2 \left[ 1 - \frac{(BD - 2CDE + AE^2)}{(AB - C^2)} \right]$
9	$\bar{y}_{SC} = [P_1^* \bar{y} + P_2^* (\bar{X} - \bar{x})] [\omega^* \Delta_j + (1 - \omega^*) \exp \left[ \frac{a(\bar{X} - \bar{x})}{a(\bar{X} + \bar{x}) + 2b} \right]]$ (Singh, Kumar and Chaudhary [43])	$\frac{MSE(\bar{y}_{reg})}{1 + \lambda C_y^2 (1 - \rho^2)}$
10	$\bar{y}_{Rao} = K_1^* \bar{y} + K_2^* (\bar{X} - \bar{x})$ (Rao [24])	$\bar{Y}^2 \left\{ 1 + \frac{1}{(\rho_{yx}^2 C_y^2 - 1) - 1} \right\}$
11	$\bar{y}_{GK} = [d_1^* \bar{y} + d_2^* (\bar{X} - \bar{x})] \exp \left[ \frac{\bar{X} - \bar{x}}{\bar{X} - \bar{x}} \right]$ (Grover and Kaur [7])	$\frac{\lambda \bar{Y}^2 \{ \lambda C_x^4 + 16(\rho_{yx}^2 - 1)(\rho_{yx}^2 - 1) C_y^2 \}}{64 \{ (\rho_{yx}^2 - 1) C_y^2 - 1 \}}$

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12	$\bar{y}_{SHG} = \left\{ \frac{\bar{y}}{2} \left[ \exp \left[ \frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right] + \exp \left[ \frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}} \right] \right] \right. \\ \left. + l_2 \bar{y} + l_1 (\bar{X} - \bar{x}) \right\} \exp \left( \frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$ <p style="margin-left: 20px;">(Shabbir, Hag and Gupta [31])</p>	$\frac{\lambda \bar{Y}^2 \{ \lambda C_x^4 - 8(\rho_{yx}^2 - 1)(\lambda C_x^2 - 2)C_y^2 \}}{16\{(\rho_{yx}^2 - 1)\lambda C_y^2 - 1\}}$
13	$\bar{y}_{JJ}^{(1)} = \frac{1 + \lambda C_x^2}{1 + \lambda C_y^2} \bar{y}_R; \quad \bar{y}_{JJ}^{(2)} = \bar{y}_S + b_{S_{yx}} (\bar{X} - \bar{x}_S)$ <p style="margin-left: 20px;">(Jitthavech and Lorchirachoonkul [10])</p>	$\left( \frac{1 + \lambda C_x^2}{1 + \lambda C_y^2} \right) MSE(\bar{y}_R) \frac{MSE(\bar{y}_{reg})}{(1 + \lambda C_y^2)}$
14	$\bar{y}_{pr} = \Theta_1 \bar{y} + \Theta_2 (\bar{X} - \bar{x}) \exp \left[ \frac{2(\bar{X} - \bar{x})}{\bar{X} + \bar{x}} \right],$ <p style="margin-left: 20px;">(Ekpenyong and Enang [5])</p>	$\bar{Y}^2 \left[ 1 - \left( \frac{\gamma_4 + 2\gamma_2\gamma_4 + \gamma_1\gamma_4^2}{\gamma_1\gamma_4 - \gamma_2^2} \right) \right]$
15	$\bar{y}_{pr} = \theta_1 \bar{y} + \theta_2 (\bar{X} - \bar{x}) \exp \left[ \frac{(\bar{X} - \bar{x})}{\bar{X} + \bar{x}} \right],$ <p style="margin-left: 20px;">(Ekpenyong and Enang [5])</p>	$\bar{Y}^2 \left[ 1 - \left( \frac{\gamma_4 + 2\gamma_5\gamma_3 + \gamma_1\gamma_3^2}{\gamma_1\gamma_4 - \gamma_5^2} \right) \right]$

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where,

$$\lambda = \frac{1 - f}{n}, \quad f = \frac{n}{N}$$

$$\bar{X} = N^{-1} \sum_{i=1}^N x_i, \text{ population mean of the auxiliary variable;}$$

$$\bar{Y} = N^{-1} \sum_{i=1}^N y_i, \text{ population mean of the study variable;}$$

$$\bar{x} = n^{-1} \sum_{i=1}^n x_i, \text{ sample mean of the auxiliary variable;}$$

$$\bar{y} = n^{-1} \sum_{i=1}^n y_i, \text{ sample mean of the study variable;}$$

$$C_x = \frac{S_x}{\bar{X}}, \text{ the coefficient of variation of the auxiliary variable;}$$

$$C_y = \frac{S_y}{\bar{Y}}, \text{ the coefficient of variation of the study variable;}$$

$$\rho = \frac{S_{xy}}{S_x S_y}, \text{ the correlation coefficient between the auxiliary and study variables;}$$

$$K_{ij} = \frac{\rho_{ij} C_i}{C_j} \text{ and } f = \frac{n}{N}, \text{ the sampling fraction;}$$

$$S_x^2 = (N - 1)^{-1} \sum_{i=1}^N (x_i - \bar{X})^2, \text{ population variance of the auxiliary variable;}$$

$$S_y^2 = (N - 1)^{-1} \sum_{i=1}^N (y_i - \bar{Y})^2, \text{ population variance of the study variable;}$$

$$S_{xy} = \frac{\sum_{i=1}^N (x_i - \bar{X})(y_i - \bar{Y})}{(N - 1)}, \text{ population covariance between the auxiliary and study variables;}$$

$$\begin{aligned}
\beta_1(x) &= \text{Coefficient of skewness;} & \beta_2(x) &= \text{Coefficient of kurtosis;} \\
b &= \text{Sample regression coefficient;} & \alpha &= \text{Intercept on Y axis;} \\
\beta &= \text{Regression coefficient;} \\
S_x &= \text{Standard deviation of X;} & S_y &= \text{Standard deviation of Y;} \\
X &= \text{Auxiliary variable;} & Y &= \text{Study variable;} \\
\Delta_j &= \frac{a_j \bar{X} + b_j}{a_j \bar{x} + b_j}, a_j \text{ and } b_j \text{ are constants or parameters of auxiliary variables.} \\
\alpha_1 &= 1 + \lambda [C_y^2 + \epsilon C_x^2 (3\epsilon - 4K)]; \\
\alpha_2 &= \lambda C_x^2, \alpha_3 = \lambda C_x^2 (K - 2\epsilon); \\
\alpha_4 &= 1 - \lambda \epsilon C_x^2 (K - \epsilon); \\
\alpha_5 &= \lambda \epsilon C_x^2, \epsilon = \frac{a \bar{X}}{a \bar{X} + b}, K = \frac{\rho C_y}{C_x}; \\
V_1 &= \frac{\alpha_2 \alpha_4^2 + 2\alpha_3 \alpha_4 \alpha_5 + \alpha_1 \alpha_5^2}{\alpha_1 \alpha_2 - \alpha_3^2}; \\
A &= 1 + \lambda [C_y^2 + \epsilon C_x^2 (3\epsilon - 4K)], B = \lambda C_y^2, C = \lambda C_x^2 (3\epsilon - K); \\
D &= 1 + \lambda \epsilon C_x^2 (\epsilon - K), E = 2\lambda \epsilon C_x^2, V_2 = \frac{BD^2 - 2CDE + AE^2}{AB - C^2}; \\
\bar{y}_\beta &= \bar{y} + \beta (\bar{X} - \bar{x}), \\
C_1 &= 1 + \lambda C_y^2 (1 - \rho_{yx}^2), C_2 = 1 + \lambda [C_y^2 + 3\epsilon C_x^2 (\epsilon_s - 4K)]; \\
C_3 &= 1 + \lambda [C_y^2 (1 - \rho_{yx}^2) + \epsilon_s C_x^2 (\epsilon_s - K)]; \\
C_4 &= 1 + \lambda \epsilon_s C_x^2 (\epsilon - K), \epsilon_s = \frac{\bar{X}}{\bar{X} - S_x}; \\
\bar{y}_{S_y} &= \frac{\bar{y}}{1 + \lambda C_y^2}, \bar{y}_{S_x} = \frac{\bar{y}}{1 + \lambda C_x^2}, b_{S_yx} = \frac{1 + \lambda C_x^2}{1 + \lambda C_y^2} b_{xy}; \quad b_{yx} = \frac{\rho S_y}{S_x}; \\
\gamma_1 &= 1 + \lambda C_y^2, \gamma_2 = \lambda C_x^2 (K - 1), \gamma_3 = \frac{\lambda C_x^2}{2}, \\
\gamma_4 &= \lambda C_x^2, \gamma_5 = \lambda C_x^2 \left( K - \frac{1}{2} \right); \\
\Theta_1 &= \frac{\gamma_4 + \gamma_2 \gamma_4}{\gamma_1 \gamma_4 - \gamma_2^2}, \Theta_2 = \frac{R(\gamma_2 + \gamma_1 \gamma_4)}{\gamma_1 \gamma_4 - \gamma_2^2}, \theta_1 = \frac{\gamma_4 + \gamma_5 \gamma_3}{\gamma_1 \gamma_4 - \gamma_5^2}, \theta_2 = \frac{R(\gamma_5 + \gamma_1 \gamma_3)}{\gamma_1 \gamma_4 - \gamma_5^2}, \\
R &= \frac{\bar{Y}}{\bar{X}}, \quad 0 < \Theta_1, \theta_1 \leq 1 \text{ and } -\infty \leq \Theta_2, \theta_2 \leq \infty
\end{aligned}$$

$\eta_1, \eta_2, t_1, t_2, \varphi_1, \varphi_2, P_1, P_2, \omega, K_1, K_2, d_1, d_2, l_1, l_2, \theta_1, \theta_2, \Theta_1$  and  $\Theta_2$  are suitably chosen constants to minimize the mean square error of the respective estimators.

The estimators of population mean obtained by Ekpenyong and Enang (2015) did not have a mathematical method of obtaining the multiples in the exponential terms; they were arbitrarily assigned to the terms. Moreover, in obtaining the optimal values of  $\Theta_1, \theta_1, \Theta_2, \theta_2$ , the authors did not consider the ranges of values given in the proposition as constraints; they treated the minimization of the mean square errors as unconstrained minimization problem.

### 3. The Proposed Estimator

The proposed estimator of population mean in simple random sampling is given as:

$$(3.1) \quad \bar{y}_{pr} = \psi_1 \bar{y} + \psi_2 (\bar{X} - \bar{x}) \exp \left[ \frac{\delta (\bar{X} - \bar{x})}{\bar{X} + \bar{x}} \right]$$

$\psi_1$  and  $\psi_2$  are suitably chosen scalars, such that  $0 < \psi_1 \leq 1$  and  $\psi_2 \geq 0$ ,  $\delta$  is a regulating parameter. Equation (3.21) can be transformed in terms of  $e$ 's as follows:

$$\bar{y}_{pr} = \psi_1 \bar{Y} (1 + e_y) + \psi_2 [\bar{X} - \bar{X} (1 + e_x)] \exp \left\{ \frac{\delta [\bar{X} - \bar{x} (1 + e_x)]}{\bar{X} + \bar{x} (1 + e_x)} \right\}$$

where

$$(3.2) \quad \begin{aligned} e_y &= \frac{\bar{y} - \bar{Y}}{\bar{Y}}, & e_x &= \frac{\bar{x} - \bar{X}}{\bar{X}} \\ \therefore \bar{y}_{pr} &= \psi_1 \bar{Y} (1 + e_y) - \psi_2 \bar{X} e_x \exp \left[ \frac{-\delta e_x}{2} \left( 1 + \frac{e_x}{2} \right)^{-1} \right] \end{aligned}$$

To obtain the range of values for this study, we recall that for stability and convergent of  $\exp \left[ \frac{\delta (\bar{X} - \bar{x})}{\bar{X} + \bar{x}} \right]$ ,  $\left[ \frac{\delta (\bar{X} - \bar{x})}{\bar{X} + \bar{x}} \right] < 1$ . Therefore,

$$\left| \frac{\delta e_x}{2} \left( 1 + \frac{e_x}{2} \right)^{-1} \right| < 1$$

$\Rightarrow |\delta| < \left| \frac{2}{e_x} \left( 1 + \frac{e_x}{2} \right) \right|$ , but  $|e_x| < 1$

Taking  $|e_x| \rightarrow 1$ ,  $|\delta| < 3$ , but as  $|e_x| \rightarrow 0$ ,  $|\delta| < |\infty|$ .

Therefore,  $[-3 < \delta < 3] \cap [-\infty < \delta < \infty] = -3 < \delta < 3$ . Hence, for  $\delta$  being an integer  $-2 \leq \delta \leq 2$ .

Equation (3.2) could be expanded and approximated up to the first degree. This gives the expression:

$$(3.3) \quad \begin{aligned} \bar{y}_{pr} &= \psi_1 \bar{Y} (1 + e_y) - \psi_2 \bar{X} e_x \left[ 1 - \frac{\delta e_x}{2} \left( 1 + \frac{e_x}{2} \right)^{-1} \right. \\ &\quad \left. + \frac{\delta^2 e_x^2}{8} \left( 1 + \frac{e_x}{2} \right)^{-2} + \dots \right] \\ &= \psi_1 \bar{Y} (1 + e_y) - \psi_2 \bar{X} e_x \left[ 1 - \frac{\delta e_x}{2} \left( 1 - \frac{e_x}{2} + \frac{e_x^2}{4} + \dots \right) + \frac{\delta^2 e_x^2}{8} \right] \\ \therefore \bar{y}_{pr} &= \bar{Y} \left( \psi_1 + \psi_1 e_y - \psi_2 M e_x + \psi_2 M \frac{\delta e_x^2}{2} \right) \end{aligned}$$

where

$$M = \frac{\bar{X}}{\bar{Y}}$$

The bias of  $\bar{y}_{pr}$  is obtained as:

$$(3.4) \quad \begin{aligned} B(\bar{y}_{pr}) &= E \left\{ \bar{Y} \left[ (\psi_1 - 1) + \psi_1 e_y - \psi_2 M e_x + \psi_2 M \frac{\delta e_x^2}{2} \right] \right\} \\ &= \left\{ \bar{Y} \left[ (\psi_1 - 1) + \psi_2 M \frac{\delta \lambda C_x^2}{2} \right] \right\} \end{aligned}$$

The first degree approximation of its mean square error is obtained using (3) as:

$$\begin{aligned}
 MSE(\bar{y}_{pr}) &= E(\bar{y}_{pr} - \bar{Y})^2 \\
 &= E\left\{\bar{Y}^2\left[(\psi_1 - 1)^2 + 2(\psi_1 - 1)\psi_2 M \frac{\delta e_x^2}{2} + \psi_1^2 e_y^2 - 2\psi_1\psi_2 M e_y e_x + \psi_2^2 M^2 e_x^2\right]\right\} \\
 &= E\left\{\bar{Y}^2\left[1 + \psi_1^2(1 + e_y^2) - 2\psi_1 - 2\psi_1\psi_2\left(M e_y e_x - \frac{M\delta e_x^2}{2}\right) - 2\psi_2 \frac{M\delta e_x^2}{2} + \psi_2^2 M^2 e_x^2\right]\right\} \\
 &= \bar{Y}^2\left[1 + \psi_1^2(1 + \lambda C_y^2) - 2\psi_1 - 2\psi_1\psi_2 M\left(\lambda \rho C_y C_x - \frac{\delta \lambda C_x^2}{2}\right) - 2\psi_2 \frac{M\delta \lambda C_x^2}{2} + \psi_2^2 M^2 \lambda C_x^2\right] \\
 &= \bar{Y}^2\left[1 + \psi_1^2 r_1 - 2\psi_1 - 2\psi_1\psi_2 M \lambda C_x^2\left(K - \frac{\delta}{2}\right) - 2\psi_2 \frac{M\delta \lambda C_x^2}{2} + \psi_2^2 M^2 \lambda C_x^2\right] \\
 (3.5) \quad &= \bar{Y}^2(1 + \psi_1^2 r_1 - 2\psi_1 - 2\psi_1\psi_2 M r_2 - 2\psi_2 M r_3 + \psi_2^2 M^2 r_4) \\
 \text{where } r_1 &= 1 + \lambda C_y^2, r_2 = \lambda C_x^2\left(K - \frac{\delta}{2}\right), r_3 = \frac{\delta \lambda C_x^2}{2}, r_4 = \lambda C_x^2
 \end{aligned}$$

To obtain the optimum mean square error of the proposed estimator, (3.5) is differentiated partially with respect to the unknown parameters  $\psi_1, \psi_2$  and  $\delta$  subject to the following constraints:

$$\begin{aligned}
 (3.6) \quad &\psi_1 \leq 1, \psi_2 \geq 0 \text{ and } \delta \leq 2 \\
 &\Rightarrow 1 - \psi_1 \geq 0, \psi_2 = (0, \infty) \text{ and } 2 - \delta \geq 0
 \end{aligned}$$

Since (3.6) are all greater than zero, the optimization problem can be stated as follows:

$$\begin{aligned}
 (3.7) \quad &Min MSE(\bar{y}_{pr}) = \bar{Y}^2(1 + \psi_1^2 r_1 - 2\psi_1 - 2\psi_1\psi_2 M r_2 - 2\psi_2 M r_3 + \psi_2^2 M^2 r_4), \\
 &s.t. 1 - \psi_1 \geq 0, \psi_2 \geq 0, 2 - \delta \geq 0, \psi_1, \psi_2, \delta \geq 0
 \end{aligned}$$

Applying Lagrange multiplier, the problem is solved thus:

The general objective function is:

$$\begin{aligned}
 (3.8) \quad &G = \bar{Y}^2(1 + \psi_1^2 r_1 - 2\psi_1 - 2\psi_1\psi_2 M r_2 - 2\psi_2 M r_3 + \psi_2^2 M^2 r_4) \\
 &- \lambda_1(1 - \psi_1) - \lambda_2 \psi_2 - \lambda_3(\delta - 2)
 \end{aligned}$$

Therefore, the Kuhn-Tucker conditions for the minimization problem are;

$$(3.9) \quad \frac{\partial MSE(\bar{y}_{pr})}{\partial \psi_1} = 2\psi_1 r_1 - 2\psi_2 M r_2 - 2 - \lambda_1 = 0$$

$$(3.10) \quad \frac{\partial MSE(\bar{y}_{pr})}{\partial \psi_2} = -2\psi_1 r_2 - 2M r_3 + 2\psi_2 M^2 r_4 - \lambda_2 = 0$$

$$(3.11) \quad \frac{\partial MSE(\bar{y}_{pr})}{\partial \delta} = \psi_1\psi_2 M r_4 - \psi_2 M r_4 - \lambda_3 = 0$$

And

$$(3.12) \quad \lambda_1 (1 - \psi_1) = 0$$

$$(3.13) \quad \lambda_2 \psi_2 = 0$$

$$(3.14) \quad \lambda_3 (2 - \delta) = 0$$

$$(3.15) \quad 1 - \psi_1 \geq 0$$

$$(3.16) \quad \psi_2 \geq 0$$

$$(3.17) \quad 2 - \delta \geq 0$$

$$(3.18) \quad \lambda_1, \lambda_2, \lambda_3 \leq 0$$

Thus solutions corresponding to the following combinations of  $\lambda_i (i = 1, 2, 3)$  can be obtained:

- (i)  $\lambda_1 = 0, \quad \lambda_2 \neq 0, \quad \lambda_3 \neq 0$
- (ii)  $\lambda_1 \neq 0, \quad \lambda_2 = 0, \quad \lambda_3 \neq 0$
- (iii)  $\lambda_1 \neq 0, \quad \lambda_2 \neq 0, \quad \lambda_3 = 0$
- (iv)  $\lambda_1 = 0, \quad \lambda_2 = 0, \quad \lambda_3 \neq 0$
- (v)  $\lambda_1 = 0, \quad \lambda_2 \neq 0, \quad \lambda_3 = 0$
- (vi)  $\lambda_1 \neq 0, \quad \lambda_2 = 0, \quad \lambda_3 = 0$
- (vii)  $\lambda_1 \neq 0, \quad \lambda_2 \neq 0, \quad \lambda_3 \neq 0$
- (viii)  $\lambda_1 = 0, \quad \lambda_2 = 0, \quad \lambda_3 = 0$

Only solutions for combinations (iv), (v) and (vi) satisfy the Kuhn Tucker conditions and are solutions to the non-linear programming model given in equation (3.8).

- (a) **Solution for (iv)** From equation (3.14),  $\delta^* = 2$ . Also, using equation (3.9):  $2\psi_1 r_1 - 2 - 2\psi_2 M r_2 = 0$

$$(3.19) \quad \Rightarrow \psi_1 r_1 - \psi_2 M r_2 = 1$$

$$\text{From equation (3.10): } -2\psi_1 M r_2 - 2M r_4 + 2\psi_2 M^2 r_4 = 0$$

$$(3.20) \quad \Rightarrow \psi_1 r_2 - \psi_2 M r_4 = -r_4$$

Solving equations (3.19) and (3.20) simultaneously gives

$$(3.21) \quad \psi_{12}^* = \frac{r_4 + r_2 r_4}{r_1 r_4 - r_2^2}$$

$$(3.22) \quad \psi_{22}^* = \frac{R(r_2 + r_1 r_4)}{r_1 r_4 - r_2^2}$$

Therefore, the minimum mean square error of this combination of  $\lambda_i \leq 0 (i = 1, 2, 3)$  is determined by putting the values of  $\psi_1^*, \psi_2^*$  and  $\delta^*$  in equation (3.5) and simplifying to obtain:

$$(3.23) \quad MSE_{opt}(\bar{y}_{pr2}) = \bar{Y}^2 \left[ 1 - \left( \frac{r_4 + 2r_2 r_4 + r_1 r_4^2}{r_1 r_4 - r_2^2} \right) \right]$$

- (b) **Solution for (v)** Under this condition, it is seen from equation (3.13) that  $\psi_2 = 0$  and from equation (3.14),  $\delta = 2$ . Putting this in equation (3.11) gives  $\lambda_3 = 0$ , and from equation (3.9);  $\psi_1 = \frac{1}{r_1}$ . Hence, the solution, using equation (3.5) gives:

$$(3.24) \quad MSE(\bar{y}_{pr1}) = \bar{Y}^2 \left( 1 + \frac{1}{r_1} - \frac{2}{r_1} \right) = \bar{Y}^2 \left( \frac{r_1 - 1}{r_1} \right) = \bar{Y}^2 \left( \frac{\lambda C_y^2}{1 + \lambda C_y^2} \right)$$

- (c) **Solution for (vi)** With this condition,  $\psi_1 = 1$  from equation (3.12); equation (3.13) also shows that  $\lambda_3 = 0$ . Also, from equation (3.10),

$$-2\psi_1 M r_2 - 2M r_3 + 2\psi_2 M^2 r_4 = 0$$



$$\begin{aligned} &\Rightarrow -(r_2 + r_3) + \psi_2 Mr_4 = 0 \Rightarrow \frac{r_2 + r_3}{Mr_4} = \psi_2 \\ (3.25) \quad &\Rightarrow \frac{R(r_2 + r_3)}{r_4} = \psi_2 = \beta \end{aligned}$$

where  $\beta$  is the regression coefficient

In this condition, it is observed that  $\psi_2$  is a function of  $\delta$  since  $r_2$  and  $r_3$  are functions of  $\delta$ . In addition to this, it can also be seen that no matter the value of  $\delta$  under this condition, the mean square error would still be the same, but the ratio estimator will differ. Therefore, varying the values of  $\delta$  within specified constraints or conditions give various members of this class of estimators. This condition gives the following class of estimators:

$$(3.26) \quad \bar{y}_{pr3} = \bar{y} + \psi_{23}^* (\bar{X} - \bar{x}) \exp \left[ \frac{\delta (\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} \right]$$

with mean square error given as:

$$(3.27) \quad MSE_{opt3}(\bar{y}_{pr}) = \bar{Y}^2 \lambda C_y^2 (1 - \rho^2)$$

The mean square error of equation (3.27) is similar to that of the regression estimator. From the foregoing, the feasible optimal solutions to be considered for the minimization problem are solutions for conditions (iv), (v) and (vi); the only clear solution where all conditions are clearly and uniquely seen to satisfy the Kuhn Tucker conditions is the solution for condition (iv). These solutions give feasible optimal solutions at various values of the considered unknown parameters, but the solution which gives the least mean square error would be considered as the most suitable one. Moreover, it has also been observed that these feasible solutions produce existing ratio and regression estimators with their corresponding mean square errors or variances.

It is also observed that values of ( $0 \leq \delta \leq 2$ ) other than the optimal value of 2 can yield good existing estimators of population mean. For instance, if  $\delta = 1$ , the estimator would be:

$$(3.28) \quad \bar{y}_{pr4} = \psi_{14}^* \bar{y} + \psi_{24}^* (\bar{X} - \bar{x}) \exp \left[ \frac{(\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} \right]$$

where  $\psi_{14}^*$  and  $\psi_{24}^*$  can be obtained from equation (3.16)(16) by differentiating partially with respect to  $\psi_1$  and  $\psi_2$  and setting the resulting equations to zero. Then substituting  $\delta = 1$  and solving simultaneously the equations give:

$$(3.29) \quad \psi_{14}^* = \frac{r_4 + r_5 r_3}{r_1 r_4 - r_5^2}$$

$$(3.30) \quad \psi_{24}^* = \frac{R(r_5 + r_1 r_3)}{r_1 r_4 - r_5^2}$$

where  $r_5 = \lambda C_x^2 (K - \frac{1}{2})$ . This will give the mean square error as:

$$(3.31) \quad MSE_{opt}(\bar{y}_{pr4}) = \bar{Y}^2 \left[ 1 - \left( \frac{r_4 + 2r_5 r_3 + r_1 r_3^2}{r_1 r_4 - r_5^2} \right) \right]$$

Also, if  $\delta = 0$ , the same procedure is applied

$$(3.32) \quad \bar{y}_{pr5} = \psi_{15}^* \bar{y} + \psi_{25}^* (\bar{X} - \bar{x})$$

which is Rao (1991) regression type estimator. From equation (3.16), we have

$$(3.33) \quad \psi_{15}^* = \frac{r_4}{r_1 r_4 - r_6^2}$$

$$(3.34) \quad \psi_{25}^* = \frac{Rr_6}{r_1r_4 - r_6^2}$$

with its mean square error as

$$(3.35) \quad MSE_{opt}(\bar{y}_{pr5}) = \bar{Y}^2 \left[ 1 - \left( \frac{r_4}{r_1r_4 - r_6^2} \right) \right]$$

Therefore, varying the values of  $\delta, \psi_1$  and  $\psi_2$  gives alternative estimators with unique properties. Table 2 shows some forms of this proposed estimator with varying parameters.

**TABLE 2**

**Table 2.** Some members of the proposed exponential estimator of population mean in simple random sampling and their MSEs

Estimators	$\psi_1$	$\psi_2$	$\delta$	MSE
$\bar{y}_{pr1} = \psi_{11}\bar{y}$ (Searls [28])	1	0	$\delta$	$\bar{Y}^2 \left( \frac{\lambda C_y^2}{1 + \lambda C_y^2} \right)$
$\bar{y}_{pr2} = \psi_{12}^*\bar{y} + \psi_{23}^*(\bar{X} - \bar{x})$ $\exp \left[ \frac{2(\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} \right]$ (Ekpenyong and Enang [5])	$\psi_{22}^*$	$\psi_{12}^*$	2	$\bar{Y}^2 \left[ 1 - \left( \frac{r_4 + 2r_2r_4 + r_1r_4^2}{r_1r_4 - r_2^2} \right) \right]$
$\bar{y}_{pr3} = \bar{y} + 1$ $\psi_{23}(\bar{X} - \bar{x}) \exp \left[ \frac{\delta(\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} \right]$			$\psi_{22}^* = \delta$ $\beta$	$\bar{Y}^2 \lambda C_y^2 (1 - \rho^2)$
$\bar{y}_{pr4} = \psi_{14}^*\bar{y} + \psi_{24}$ $\psi_{24}^*(\bar{X} - \bar{x}) \exp \left[ \frac{(\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} \right]$ (Ekpenyong and Enang [5])	$\psi_{14}$	$\psi_{24}$	1	$\bar{Y}^2 \left[ 1 - \left( \frac{r_4 + 2r_5r_3 + r_1r_3^2}{r_1r_4 - r_5^2} \right) \right]$
$\bar{y}_{pr3} = \bar{y} + 1$ $\psi_{23}(\bar{X} - \bar{x}) \exp \left[ \frac{\delta(\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} \right]$			$\psi_{22}^* = \delta$ $\beta$	$\bar{Y}^2 \lambda C_y^2 (1 - \rho^2)$
$\bar{y}_{pr5} = \psi_{15}^*\bar{y} + \psi_{25}^*(\bar{X} - \bar{x})$ , (Rao, [24])	$\psi_{15}$	$\psi_{25}$	0	$\bar{Y}^2 \left[ 1 - \left( \frac{r_4}{r_1r_4 - r_6^2} \right) \right]$

Rao [24] and Ekpenyong and Enang [5] have shown that the estimators are more efficient than the usual regression estimator of population mean in simple random sampling. The only estimator in Table 2 that is less efficient than the regression estimator but more efficient than the simple random sample mean is the estimator of Searls [28],  $\bar{y}_{pr1}$ .

#### 4. The Proposed Generalized Estimator of Population Mean in Simple Random Sampling

The general class of the proposed exponential ratio estimator of the population mean is suggested as follows;

$$(4.1) \quad \bar{y}_{prg} = \Phi_1\bar{y}U + \Phi_2(\bar{X} - \bar{x})W$$

where  $\Phi_1$  and  $\Phi_2$  are suitably chosen scalars, such that  $\Phi_1 > 0$  and  $-\infty < \Phi_2 < \infty$  and

$$U = \exp \left[ \delta_1 \left( \frac{\bar{X}^\alpha - \bar{x}^\alpha}{\bar{X}^\alpha + \bar{x}^\alpha} \right) \right], W = \exp \left[ \delta_2 \left( \frac{\bar{X}^\alpha - \bar{x}^\alpha}{\bar{X}^\alpha + \bar{x}^\alpha} \right) \right]$$

$\alpha, \delta_1$  and  $\delta_2$  are suitably chosen to align with existing forms of ratio estimators proposed by various authors such that

$$\left| \delta_i \left( \frac{\bar{X}^\alpha - \bar{x}^\alpha}{\bar{X}^\alpha + \bar{x}^\alpha} \right) \right| \leq 1, i = 1, 2$$

which is a condition for proper approximation of Taylor's series.

To obtain the bias and the mean square error of the proposed estimator, equation (4.1) is transformed and expressed in terms of  $e$ 's Taking the first term on the Right Hand Side (RHS) of equation (4.1), we have;

$$\begin{aligned} \Phi_1 \bar{y}U &= \Phi_1 \bar{Y} (1 + e_y) \exp \left[ \delta_1 \left( \frac{\bar{X}^\alpha - \bar{x}^\alpha}{\bar{X}^\alpha + \bar{x}^\alpha} \right) \right] \\ &= \Phi_1 \bar{Y} (1 + e_y) \exp \left\{ \frac{\delta_1 [1 - (1 + \alpha e_x + \alpha \frac{\alpha-1}{2} e_x^2 + \dots)]}{[1 + (1 + \alpha e_x + \alpha \frac{\alpha-1}{2} e_x^2 + \dots)]} \right\} \\ &= \Phi_1 \bar{Y} (1 + e_y) \exp \left[ \frac{-\delta_1 (\alpha e_x + \alpha \frac{\alpha-1}{2} e_x^2)}{(2 + \alpha e_x + \alpha \frac{\alpha-1}{2} e_x^2)} \right] \\ &= \Phi_1 \bar{Y} (1 + e_y) \exp \left[ \frac{-\delta_1 (\alpha e_x + \alpha \frac{\alpha-1}{2} e_x^2)}{2} \frac{1}{(1+h)} \right], \text{ where } h = \frac{\alpha}{2} e_x + \alpha \frac{\alpha-1}{4} e_x^2 \\ \therefore \Phi_1 \bar{y}U &= \Phi_1 \bar{Y} (1 + e_y) \exp \left[ \frac{-\delta_1}{2} (\alpha e_x + \alpha \frac{\alpha-1}{2} e_x^2) (1+h)^{-1} \right] \\ \Phi_1 \bar{y}U &= \Phi_1 \bar{Y} (1 + e_y) \left[ \sum_{i=0}^{\infty} \left( -\frac{\delta_1}{2} (\alpha e_x + \alpha \frac{\alpha-1}{2} e_x^2) (1+h)^{-1} \right)^i \right] \\ &= \Phi_1 \bar{Y} (1 + e_y) \left\{ 1 - \frac{\delta_1 \alpha e_x}{2} \left( 1 + \frac{\alpha-1}{2} e_x \right) \left[ 1 - \frac{\alpha e_x}{2} - \frac{\alpha(\alpha-1)}{4} e_x^2 \right] \right. \\ &\quad \left. + \left[ \frac{\alpha e_x}{2} - \frac{\alpha(\alpha-1)}{4} e_x^2 \right]^2 + \frac{\delta_1^2 \alpha^2 e_x^2}{8} \left[ \left( 1 + (\alpha-1) e_x + \frac{(\alpha-1)^2}{4} e_x^2 \right) \right. \right. \\ &\quad \left. \left. \left( 1 - \alpha e_x - \frac{\alpha(\alpha-1)}{2} e_x^2 + \dots \right) \right] \right\} \\ &= \Phi_1 \bar{Y} (1 + e_y) \left[ 1 - \frac{\delta_1 \alpha e_x}{2} + \frac{\delta_1 \alpha^2 e_x^2}{4} - \frac{\delta_1 \alpha (\alpha-1)}{4} e_x^2 + \frac{\delta_1^2 \alpha^2 e_x^2}{8} \right] \\ &= \Phi_1 \bar{Y} (1 + e_y) \left[ 1 - \frac{\delta_1 \alpha e_x}{2} + \frac{\delta_1 \alpha e_x^2}{4} + \frac{\delta_1^2 \alpha^2 e_x^2}{8} \right] \end{aligned}$$

Expanding, simplifying and ignoring terms of powers of  $e$  greater than 2, we proceed as follows:

$$(4.2) \quad \Phi_1 \bar{y}U = \bar{Y} \left[ \Phi_1 - \frac{\Phi_1 \delta_1 \alpha e_x}{2} + \Phi_1 \frac{(2\delta_1 \alpha + \delta_1^2 \alpha^2)}{8} e_x^2 + \Phi_1 e_y - \frac{\Phi_1 \delta_1 \alpha e_y e_x}{2} \right]$$

Similarly,

$$\Phi_2 (\bar{X} - \bar{x}) W = -\Phi_2 \bar{X} e_x \left[ 1 - \frac{\delta_2 \alpha e_x}{2} + \frac{(2\delta_2 \alpha + \delta_2^2 \alpha^2)}{8} e_x^2 \right]$$

Ignoring terms of 'e' with powers greater than 2, we have

$$(4.3) \quad \Phi_2 (\bar{X} - \bar{x}) W = -\Phi_2 \bar{X} e_x + \frac{\Phi_2 \bar{X} \delta_2 \alpha e_x^2}{2}$$

Adding equation (4.2) to equation (4.3) gives;

$$(4.4) \quad \begin{aligned} \bar{y}_{prg} = \bar{Y} & \left[ \Phi_1 - \frac{\Phi_1 \delta_1 \alpha e_x}{2} + \Phi_1 \frac{(2\delta_1 \alpha + \delta_1^2 \alpha^2)}{8} e_x^2 + \Phi_1 e_y \right. \\ & \left. - \frac{\Phi_1 \delta_1 \alpha e_y e_x}{2} - \Phi_2 M e_x + \frac{\Phi_2 M \delta_2 \alpha e_x^2}{2} \right] \end{aligned}$$

$$(4.5) \quad \begin{aligned} (\bar{y}_{prg} - \bar{Y}) = \bar{Y} & \left[ (\Phi_1 - 1) - \frac{\Phi_1 \delta_1 \alpha e_x}{2} + \Phi_1 \frac{(2\delta_1 \alpha + \delta_1^2 \alpha^2)}{8} e_x^2 \right. \\ & \left. + \Phi_1 e_y - \frac{\Phi_1 \delta_1 \alpha e_y e_x}{2} - \Phi_2 M e_x + \frac{\Phi_2 M \delta_2 \alpha e_x^2}{2} \right] \end{aligned}$$

Therefore, the Bias of  $\bar{y}_{prg}$  is given by;

$$(4.6) \quad \begin{aligned} B(\bar{y}_{prg}) = E(\bar{y}_{prg} - \bar{Y}) = & \left[ (\Phi_1 - 1) + \Phi_1 \frac{(2\delta_1 \alpha + \delta_1^2 \alpha^2) \lambda C_x^2}{8} \right. \\ & \left. - \frac{\Phi_1 \delta_1 \alpha \lambda C_y C_x}{2} + \frac{\Phi_2 M \delta_2 \alpha \lambda C_x^2}{2} \right] \end{aligned}$$

Also,

$$\begin{aligned} (\bar{y}_{prg} - \bar{Y})^2 = \bar{Y}^2 & \left[ (\Phi_1 - 1)^2 + 2(\Phi_1^2 - \Phi_1) \frac{(2\delta_1 \alpha + \delta_1^2 \alpha^2) e_x^2}{8} \right. \\ & - 2(\Phi_1^2 - \Phi_1) \frac{\delta_1 \alpha e_y e_x}{2} + 2(\Phi_1 \Phi_2 - \Phi_2) \frac{M \delta_2 \alpha e_x^2}{2} \\ & + \frac{\Phi_1^2 \delta_1^2 \alpha^2 e_x^2}{4} - \frac{2\Phi_1^2 \delta_1 \alpha e_y e_x}{2} + \frac{2\Phi_1 \Phi_2 \delta_1 \alpha M e_x^2}{2} \\ & \left. + \Phi_1^2 e_y^2 - 2\Phi_1 \Phi_2 M e_y e_x + \Phi_2^2 M^2 e_x^2 \right] \end{aligned}$$

The mean square error of the class of estimators is given as;

$$(4.7) \quad \begin{aligned} MSE(\bar{y}_{prg}) = E(\bar{y}_{prg} - \bar{Y})^2 \\ = \bar{Y}^2 & \left\{ 1 + \Phi_1^2 \left[ 1 + \lambda C_y^2 + \frac{\delta_1 \alpha \lambda C_x^2}{2} [(1 + \delta_1 \alpha) - 4K] \right] \right. \\ & - 2\Phi_1 \left[ 1 + \frac{\delta_1 \alpha \lambda C_x^2}{8} [(2 + \delta_1 \alpha) - 4K] \right] - 2\Phi_1 \Phi_2 M \lambda C_x^2 \\ & \left. \left[ K - \frac{(\delta_1 + \delta_2) \alpha}{2} \right] - \frac{2\theta_2 M \delta_2 \alpha C_x^2}{2} + \lambda \theta_2^2 M^2 C_x^2 \right\} \end{aligned}$$

$$(4.8) \quad \Rightarrow MSE(\bar{y}_{prg}) = \bar{Y}^2 (1 + \Phi_1^2 \pi_1 - 2\Phi_1 \pi_2 - 2\Phi_1 \Phi_2 \pi_3 - 2\Phi_2 \pi_4 + \Phi_2^2 \pi_5)$$

where

$$\begin{aligned} \pi_1 = 1 + \lambda C_y^2 + \frac{\delta_1 \alpha \lambda C_x^2}{2} [(1 + \delta_1 \alpha) - 4K], \quad \pi_2 = 1 + \frac{\delta_1 \alpha \lambda C_x^2}{8} [(2 + \delta_1 \alpha) - 4K] \\ \pi_3 = M \lambda C_x^2 \left[ K - \frac{(\delta_1 + \delta_2) \alpha}{2} \right], \quad \pi_4 = \frac{M \delta_2 \alpha C_x^2}{2}, \quad \pi_5 = \lambda M^2 C_x^2 \end{aligned}$$

From equation (4.8), it can be seen that the mean square error of the proposed class of exponential estimators in simple random sampling is a function of  $\delta_1, \delta_2$  and  $\alpha$ . Varying the values of  $\delta_1, \delta_2$  and  $\alpha$  gives various members of the family with their corresponding mean square errors.

When different values of  $\delta_1, \delta_2$  and  $\alpha$  substituted into equation (4.1), some members of the family with their respective mean square error obtained from equation (4.8) can be derived as shown in Table 3.

**TABLE 3**

**Table 3.** Some Members of the generalized Family of exponential Ratio Estimators

Estimator	$\delta_1$	$\delta_2$	$\alpha$	$\Phi_1$	$\Phi_2$
$\bar{y}_{prg1} = \bar{y}$ , Samplemean	0	$\delta_2$	$\alpha$	1	0
$\bar{y}_{prg2} = \bar{y} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$ , Bahland Tuteja [1]	1	$\delta_2$	1	1	0
$\bar{y}_{prg3} = \bar{y} + b(\bar{X} - \bar{x})$ , Regression estimator	0	0	$\alpha$	1	$b$
$\bar{y}_{prg4} = \bar{y} + b(\bar{X} - \bar{x}) \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	0	1	1	1	$b$
$\bar{y}_{prg5} = \Phi_1 \bar{y} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) + \Phi_2 (\bar{X} - \bar{x}) \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	1	1	1	$\Phi_1$	$\Phi_2$
$\bar{y}_{prg6} = \Phi_{11} \bar{y} + \Phi_{21} (\bar{X} - \bar{x}) \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	0	1	1	$\Phi_{11}$	$\Phi_{21}$
$\bar{y}_{prg7} = \Phi_{12} \bar{y} + \Phi_{22} (\bar{X} - \bar{x}) \exp\left[\frac{2(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right]$	0	2	1	$\Phi_{12}$	$\Phi_{22}$
$\bar{y}_{prg8} = \theta_1 \bar{y} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$ , Yadav and Kadilar [58]	1	$\delta_2$	1	$\theta_1$	0
$\bar{y}_{prg9} = \Phi_{14} \bar{y} + \Phi_{24} (\bar{X} - \bar{x})$ , Rao [24]	0	0	$\alpha$	$\Phi_{14}$	$\Phi_{24}$
$\bar{y}_{prg10} = \Phi_{16}^* \bar{y} \exp\left[\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right] + \Phi_{26}^* (\bar{X} - \bar{x}) \exp\left[\frac{2(\bar{X}^2-\bar{x}^2)}{(\bar{X}^2+\bar{x}^2)}\right]$	1	2	2	$\Phi_{16}^*$	$\Phi_{26}^*$
$\bar{y}_{prg11} = \Phi_{18}^* \bar{y} + \Phi_{28}^* (\bar{X} - \bar{x}) \exp\left(\frac{\bar{X}^2-\bar{x}^2}{\bar{X}^2+\bar{x}^2}\right)$	0	1	2	$\Phi_{18}^*$	$\Phi_{28}^*$
$\bar{y}_{prg12} = [\Phi_1^* \bar{y}]$ , Searls [28]	0	0	$\alpha$	$\Phi_1^*$	0

Table 3 indicates some members of the class of generalized exponential ratio estimator of the population mean in simple random sampling. It is observed from the Table that estimators of Yadav and Kadilar [58], Rao [24], Bahl and Tuteja [1], regression estimator and simple random sample mean are members of this class of estimator. To obtain the optimality conditions for the mean square error (MSE) for the proposed family of estimators, equation (3.19) is partially differentiated with respect to  $\Phi_1$  and  $\Phi_2$  and set to zero.

Therefore,

$$\begin{aligned} \frac{\partial MSE(\bar{y}_{prg})}{\partial \Phi_1} &= 2\Phi_1\pi_1 - 2\pi_2 - 2\Phi_2\pi_3 = 0 \\ (4.9) \quad &\Rightarrow \Phi_1\pi_1 - \Phi_2\pi_3 = \pi_2 \end{aligned}$$

Also,

$$\begin{aligned} \frac{\partial MSE(\bar{y}_{prg})}{\partial \Phi_2} &= -2\Phi_1\pi_3 - 2\pi_4 + 2\Phi_2\pi_5 = 0 \\ (4.10) \quad &\Rightarrow \Phi_1\pi_3 - \Phi_2\pi_5 = -\pi_4 \end{aligned}$$

Solving equations (4.9) and (4.10) simultaneously gives the following expressions for  $\Phi_1$  and  $\Phi_2$ .

$$(4.11) \quad \Phi_1^* = \frac{\pi_2\pi_5 + \pi_3\pi_4}{\pi_1\pi_5 - \pi_3^2}$$

$$(4.12) \quad \Phi_2^* = \frac{\pi_1\pi_4 + \pi_2\pi_3}{\pi_1\pi_5 - \pi_3^2}$$

Substituting equations (4.11) and (4.12) in equation (4.8) gives the optimum mean square error as;

$$\begin{aligned} MSE_{opt}(\bar{y}_{prg}) = \bar{Y}^2 \left[ 1 + \left( \frac{\pi_2\pi_5 + \pi_3\pi_4}{\pi_1\pi_5 - \pi_3^2} \right)^2 \pi_1 - 2\pi_2 \left( \frac{\pi_2\pi_5 + \pi_3\pi_4}{\pi_1\pi_5 - \pi_3^2} \right) \right. \\ \left. - 2\pi_3 \left( \frac{\pi_2\pi_5 + \pi_3\pi_4}{\pi_1\pi_5 - \pi_3^2} \right) \left( \frac{\pi_1\pi_4 + \pi_2\pi_3}{\pi_1\pi_5 - \pi_3^2} \right) - 2\pi_4 \right. \\ \left. \left( \frac{\pi_1\pi_4 + \pi_2\pi_3}{\pi_1\pi_5 - \pi_3^2} \right) + \pi_5 \left( \frac{\pi_1\pi_4 + \pi_2\pi_3}{\pi_1\pi_5 - \pi_3^2} \right)^2 \right] \end{aligned}$$

After simplification, the mean square error becomes:

$$(4.13) \quad MSE_{opt}(\bar{y}_{prg}) = \bar{Y}^2 \left[ 1 - \frac{(\pi_2^2\pi_5 + 2\pi_2\pi_3\pi_4 + \pi_1\pi_4^2)}{(\pi_1\pi_5 - \pi_3^2)} \right]$$

### Remark:

- (1) equation (4.13) gives the mean square error (MSE) for optimum  $\Phi = (\Phi_1^*, \Phi_2^*)$ .
- (2) When different values of  $\delta_1, \delta_2$  and  $\alpha$  are substituted in the proposed family of exponential ratio estimator, different ratio estimators would be obtained with their corresponding optimum mean square error.

### 4.1. Efficiency Comparison.

- (a) A member  $\bar{y}_{prgi}$  of the proposed estimator would be more efficient than another member  $\bar{y}_{prgj}$  if;

$$\begin{aligned} & MSE(\bar{y}_{prgj}) - MSE(\bar{y}_{prgi}) > 0 \\ \Rightarrow \bar{Y}^2 \left[ 1 - \frac{(\pi_{2j}^2\pi_{5j} + 2\pi_{2j}\pi_{3j}\pi_{4j} + \pi_{1j}\pi_{4j}^2)}{(\pi_{1j}\pi_{5j} - \pi_{3j}^2)} \right] - \bar{Y}^2 \left[ 1 - \frac{(\pi_{2i}^2\pi_{5i} + 2\pi_{2i}\pi_{3i}\pi_{4i} + \pi_{1i}\pi_{4i}^2)}{(\pi_{1i}\pi_{5i} - \pi_{3i}^2)} \right] > 0 \\ \Rightarrow x \left( \frac{\pi_{2i}^2\pi_{5i} + 2\pi_{2i}\pi_{3i}\pi_{4i} + \pi_{1i}\pi_{4i}^2}{\pi_{1i}\pi_{5i} - \pi_{3i}^2} \right) - \left( \frac{\pi_{2j}^2\pi_{5j} + 2\pi_{2j}\pi_{3j}\pi_{4j} + \pi_{1j}\pi_{4j}^2}{\pi_{1j}\pi_{5j} - \pi_{3j}^2} \right) > 0 \end{aligned}$$

(4.14)  $\Rightarrow q_i - q_j \geq 0$

where

$$q_i = \frac{\pi_{2i}^2\pi_{5i} + 2\pi_{2i}\pi_{3i}\pi_{4i} + \pi_{1i}\pi_{4i}^2}{\pi_{1i}\pi_{5i} - \pi_{3i}^2}, q_j = \frac{\pi_{2j}^2\pi_{5j} + 2\pi_{2j}\pi_{3j}\pi_{4j} + \pi_{1j}\pi_{4j}^2}{\pi_{1j}\pi_{5j} - \pi_{3j}^2}.$$

When equation (4.14) holds, then  $\bar{y}_{prgi}$  will be more efficient than  $\bar{y}_{prgj}$ .

- (b) Any member  $\bar{y}_{prgi}$  of the proposed estimator is said to be more efficient than the classical ratio estimator if;

$$\begin{aligned}
 &MSE(\bar{y}_R) - MSE(\bar{y}_{prgi}) > 0 \\
 \Rightarrow &\lambda \bar{Y}^2 (C_y^2 - 2\rho C_y C_x + C_x^2) - \bar{Y}^2 \left[ 1 - \frac{(\pi_{2i}^2 \pi_{5i} + 2\pi_{2i} \pi_{3i} \pi_{4i} + \pi_{1i} \pi_{4i}^2)}{(\pi_{1i} \pi_{5i} - \pi_{3i}^2)} \right] > 0 \\
 \Rightarrow &\bar{Y}^2 [1 + \lambda C_y^2 + \lambda C_x^2 (1 - 2K) - 1] - \bar{Y}^2 (1 - q_i) > 0 \\
 \Rightarrow &\bar{Y}^2 \{ [\lambda C_y^2 + \lambda C_x^2 (1 - 2K) - 1] - \bar{Y}^2 q_i \} > 0 \\
 \Rightarrow &\{ [\lambda C_y^2 + \lambda C_x^2 (1 - 2K) - 1] - q_i \} > 0 \\
 (4.15) \quad &\Rightarrow (r_1 + r_2 - 1) - (1 - q_i) > 0
 \end{aligned}$$

When equation (4.15) holds, then  $\bar{y}_{prgi}$  will be more efficient than the classical ratio estimator.

- (c) Any member of the proposed family of estimators  $\bar{y}_{prgi}$  is said to be more efficient than the Gupta and Shabbir (2012) estimator if;

$$(4.16) \quad q_i - v_1 \geq 0$$

## 5. Numerical Illustration

To validate our theoretical claims and assess the efficiencies of our proposed estimators over the existing ones considered in this work under certain optimal conditions, data from the following ten populations are used.

**TABLE 4**

**Table 4.** Populations and Parameters considered for the proposed exponential ratio estimators in simple random sampling

Source of Population	Parameter						
	N	n	$\rho$	$C_y$	$C_x$	$\bar{Y}$	$\bar{X}$
I (Murthy, [23])	80	20	0.9413	0.3542	0.7507	51.8264	11.2646
II (Murthy, [23])	80	20	0.9150	0.3542	0.9484	51.8264	2.8512
III (Cochran, [3])	10	4	0.6515	0.1449	0.1281	101.1	58.8
IV (Kadilar and Cingi, [14])	200	50	0.9	15	2	500	25
V (Koyuncu and Cingi, [19])	923	180	0.9543	1.7183	1.8645	436.435	11440.5
VI (Kadilar and Cingi, [16])	106	20	0.86	5.22	2.1	2212.59	27421.7
VII (Kadilar and Cingi, [17])	104	20	0.865	1.866	1.653	625.37	13.93
VIII (Kadilar and Cingi, [13])	204	50	0.71	2.4739	1.7171	966	26441
IX (Kadilar and Cingi, [16])	256	100	0.887	1.42	1.4	56.47	44.45
X (Das, [4])	278	25	0.7313	1.4451	1.6198	39.068	25.111

TABLE 5

Table 5. Percent Relative Efficiencies (PRE) of the proposed and related estimators of population mean in simple random sampling

Estimators	Population									
	I	II	III	IV	V	VI	VII	VIII	IX	X
$\bar{y}_{prg1}$	11.40	16.28	57.55	19.00	8.93	26.04	25.18	49.59	21.32	47.97
$\bar{y}_{prg2}$	89.05	47.55	92.92	21.51	34.50	37.50	58.56	79.01	57.86	94.88
$\bar{y}_{prg3}$ (Regression Estimator)	100	100	100	100	100	100	100	100	100	100
$\bar{y}_{prg4}$	100	100	100	100	100	100	100	100	100	100
$\bar{y}_{prg5}$	101.91	103.37	100.25	166.64	100.83	135.06	107.12	105.83	100.65	106.61
$\bar{y}_{prg6}$	105.34	105.59	100.41	220.54	105.70	196.26	125.34	110.50	102.48	113.03
$\bar{y}_{prg7}$	122.55	124.89	<b>100.73</b>	349.49	<b>122.65</b>	598.67	<b>203.08</b>	<b>120.01</b>	<b>107.23</b>	<b>132.84</b>
$\bar{y}_{prg8}$ (Yadav and Kadilar [58])	102.71	108.15	100.18	134.94	100.00	109.84	101.54	103.45	100.11	104.74
$\bar{y}_{prg9}$ (Rao [24])	100.07	100.01	100.18	164.13	100.12	128.78	103.59	104.58	100.26	103.65
$\bar{y}_{prg10}$	<b>135.59</b>	<b>371.30</b>	100.08	<b>364.08</b>	100.04	<b>697.05</b>	105.48	105.83	100.08	100.04
$\bar{y}_{prg11}$	122.55	124.89	<b>100.73</b>	349.49	<b>122.65</b>	598.67	<b>203.08</b>	<b>120.01</b>	<b>107.23</b>	<b>132.84</b>
$\bar{y}_{prg12}$	11.45	16.36	57.73	83.13	9.05	54.82	28.72	54.17	21.59	51.62
(Shabbir, et al) [31]	106.75	112.40	100.33	169.26	102.22	142.52	112.15	107.26	101.21	110.66
(Singh, Rathour and Solanki) [35]	100.07	100.10	100.18	169.91	100.15	152.11	117.87	113.32	100.26	104.10
(Singh, Kumar and Chaudhary) [43]	100.07	100.01	100.18	164.13	100.12	128.78	103.59	104.58	100.26	103.65
(Jittavech and Lorchira-choonkul) [10]	100.94	100.94	100.63	<b>1914.06</b>	102.66	443.25	130.10	119.33	102.47	115.78
(Singh and Solanki) [42]	<b>135.59</b>	<b>371.30</b>	100.08	364.08	100.04	<b>697.05</b>	105.48	105.83	100.08	100.04
(Gupta and Shabbir) [6]	100.49	100.10	100.18	168.22	100.12	135.06	103.98	104.80	100.26	104.03
(Grover and Kaur) [7]	101.91	103.37	100.25	166.64	100.83	135.06	107.12	105.83	100.65	106.61

\*Figures in bold indicate the largest PRE in each population.

### 6. Discussion of Results

The proposed exponential ratio-type estimator of population mean under simple random sampling scheme, in the presence of one auxiliary variable, given in equation (3.1) contains some unknown parameters  $\psi_1, \psi_2$  and  $\delta$ , whose range of values have been defined. Significantly, the range of values of the regulating parameter  $\delta$  obtained through appropriate mathematical proof and solving a formulated non-linear programming model are used to obtain the Asymptotic Optimal Estimators for the proposed family, which are shown with their Mean Square Errors in Table 2. This approach shows advancement over the works of [43] and [31], whose choice of parameters were given intuitively without any concrete mathematical backup. From Table 2, it has also been observed that Asymptotic Optimal Estimators (AOE) include some existing estimators of [28] and [24].

A generalization of this proposed exponential estimator of the population mean is proposed in equation (36) with appropriate choices of unknown parameters  $\varphi_1, \varphi_2, \alpha, \delta_1$  and  $\delta_2$  to produce members of this general family of estimators as shown in Table 3. Table 3 shows that even the Asymptotic Optimal Estimators of the first proposed exponential estimator of population mean in Table 2 are all members of this generalized exponential estimator; estimator of [1], classical regression estimator, [58] and other generated estimators are also members of this proposed family of estimators of population mean under simple random sampling scheme. The optimal Mean Square Error of this proposed



general family of exponential estimators is given in (4.13) from where it can be observed that the optimality condition is dependent upon the other three parameters  $\alpha, \delta_1$  and  $\delta_2$ , whose choices leads to various Asymptotic Optimal Estimators (AOE's) with their different Mean Square Errors (MSE's).

Ten (10) populations presented in Table 5 have been used in empirical analysis. The results presented in Table 5 indicate the Percent Relative Efficiencies (PRE) of some existing estimators and members of the proposed family of exponential estimators obtained with respect to the classical regression estimator. Table 5 shows that estimator denoted by  $\bar{y}_{prg10}$ , which is the same as  $\bar{y}_{pr4}$  in Table 2 has the greatest PRE of 135.9%, 371.3%, 697.05% in populations I, II, and VI respectively among all estimators considered (both existing and proposed), except for the estimator of [42], which has the same PRE. Also,  $\bar{y}_{prg11}$ , which is the same as  $\bar{y}_{pr5}$  in Table 2, has the greatest PRE of 100.73%, 122.65%, 203.08%, 120.01%, 107.23% and 132.84% in populations III, V, VII, VIII, IX, X respectively among all estimators considered. The only deviation here is in population IV, where [10] estimator has the greatest efficiency. All other members except  $\bar{y}_{prg1}$ ,  $\bar{y}_{prg2}$ , and  $\bar{y}_{prg12}$  have their efficiencies greater than the classical Regression estimator. On the whole, Table 5 has indicated that  $\bar{y}_{prg10}$  ( $\bar{y}_{pr4}$ ) and  $\bar{y}_{prg11}$  ( $\bar{y}_{pr5}$ ) have significant gains in efficiencies in the ten (10) populations except population IV. However, [42] estimator and  $\bar{y}_{prg15}$  ( $\bar{y}_{pr4}$ ) have the same performance in all populations. Hence, there are greater gains in efficiency among the proposed estimators of population mean in simple random sampling.

The proposed estimators  $\bar{y}_{prg15}$  ( $\bar{y}_{pr4}$ ) and  $\bar{y}_{prg17}$  ( $\bar{y}_{pr5}$ ) have demonstrated tremendous gains in efficiencies under simple random sampling strategies. They have therefore been found useful for estimating the population mean in simple random sampling strategies under certain optimal conditions.

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