

Mathematical Modeling Process as an Activity Requiring Creativity

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Abstract: The aim of this study is to address creativity as a measurable and conceptually assessable dimension by incorporating it into the processes of modeling competence. For this purpose, real-world problem situations requiring creativity were employed to evaluate the enriched structure. Usefulness, fluency, and originality were indicators of creativity in solving real-world problems. The study was conducted by addressing both qualitative and quantitative dimensions of the research process. The research participants consist of 60 students taking the modeling course in mathematics teaching at the undergraduate level. The participants, selected through criterion sampling, completed their work individually during the data collection process. The findings of the study revealed that students' mathematical modeling (MM) competencies varied depending on the difficulty level and content structure of the tasks. A significant and consistent relationship was identified among the dimensions of usefulness, fluency, and originality in students' performance across all tasks. Overall, the findings reveal that creativity indicators—such as usefulness, fluency, and originality—should be integrated into the structure of MM competencies, and they contribute to opening a new avenue for future modeling studies in this direction.

Keywords: Conceptualization, creativity, mathematical modeling, preservice mathematics teacher

Yaratıcılık Gerektiren Bir Aktivite Olarak Matematiksel Modelleme Süreci

Öz: Bu çalışmanın amacı, yaratıcılığı modelleme yeterlik süreçlerine dahil ederek ölçülebilir ve kavramsal olarak değerlendirilebilir bir boyut olarak ele almaktır. Bu amaç doğrultusunda, zenginleştirilmiş yapının değerlendirilmesi için yaratıcılık gerektiren gerçek dünya problem durumları kullanılmıştır. Gerçek dünya problemlerin çözüm sürecindeki yaratıcılık göstergeleri olarak yararlık, akıcılık ve orijinallik dikkate alınmıştır. Çalışma, araştırma sürecinin hem nitel hem de nicel boyutları dikkate alınarak yürütülmüştür. Araştırmanın katılımcıları lisans düzeyinde matematik öğretiminde modelleme dersini alan 60 lisans öğrenciden oluşmaktadır. Ölçüt örnekleme yöntemiyle belirlenen katılımcılar, veri toplama sürecinde çalışmalarını bireysel olarak gerçekleştirmiştir. Çalışmanın bulguları, öğrencilerin matematiksel modelleme (MM) yeterliklerinin, görevlerin zorluk düzeyi ve içerik yapısına bağlı olarak değişiklik gösterdiğini ortaya koymuştur. Tüm görevlerde öğrencilerin performansları incelendiğinde, yararlık, akıcılık ve orijinallik boyutları arasında anlamlı ve tutarlı ilişkiler belirlenmiştir. Genel olarak elde edilen sonuçlar, yararlılık, akıcılık ve özgünlük gibi yaratıcılık göstergelerinin MM yeterliklerinin yapısına dahil edilmesi gerektiğini ortaya koymakta ve bu doğrultuda yapılacak modelleme araştırmaları için yeni bir alan açmaktadır.

Anahtar kelimeler: Kavramsallaştırma, yaratıcılık, matematiksel modelleme, matematik öğretmeni adayı

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Introduction

Mathematical modeling (MM) is a process skill in which daily problems are solved, explained, tested, evaluated, and interpreted with the help of mathematical tools. For this reason, effective use of modeling competencies (MC) is needed in organizing real-life problems (Maaß, 2006). Integrating mathematics with daily life into mathematics teaching is essential to increase the understanding of mathematical concepts seen as complicated, complex, and abstract for students. At this point, MC requires identifying the variables, assumptions, and related connections of a real-life problem (translating it into mathematical language), organizing the solution process (showing the accuracy of the process), comparing models (analyzing), and perceiving the general scope of the model (checking the features) (Kaiser & Schwarz, 2006). This form of competence is a practical resource that individuals can benefit from in their future careers by undertaking the task of creating mathematical, concrete, and mental models that they can use in real life (Blomhøj & Jensen, 2007; Blum & Leiß, 2007; Borromeo Ferri, 2018; English, 2009). These justifications indicate that the modeling competency in the curriculum is the product of a philosophical approach that targets success in life rather than school success (Acquah & Szelei, 2020; Vorhölter, 2018). MC requires willing effort towards aim and independent modeling (Blum, 2011; Kaiser et al., 2006). Therefore, developing students' MC in mathematics learning and teaching involves several procedures. First of all, the characteristic difficulties encountered in solving real-life problems are defined by typical modeling cycles (e.g., Blum & Leiß, 2005; Galbraith & Stillman, 2006; Niss & Blum, 2020). The identified cycles contain critical competencies for solving modeling problems (Lu & Kaiser, 2022a). Although the proposal of different frameworks regarding modeling cycles may cause differences in the curricula of countries, it is considered necessary in terms of both having the potential to encourage modeling and guiding instructors (Borromeo Ferri, 2018; Niss & Blum, 2020). Having grown substantially to form a distinct research field at the international level, modeling discourses have empirically explored MM processes and students' competencies, while also defining the scope and importance of the sub-competencies associated with these processes (Kaiser, 2017; Ludwig & Xu, 2010; Sriraman, 2009). In addition, due to the nature of modeling problems, students approach the modeling problem with different solutions due to the clarity and insufficient specification of the real-world problem (Schukajlow et al., 2015). Students' preference for different solutions and being open-minded requires them to use their creativity to simplify the current problem situation and make appropriate assumptions (Lu & Kaiser, 2022a). Therefore, modeling developed for real-world problems does not require reaching solutions using standard methods; on the contrary, it teaches us to look at modeling processes differently by developing new solution methods (Blomhøj & Jensen, 2003; Blum & Leiß, 2007; Bukova-Güzel, 2021; Niss & Blum, 2020). The number of studies exploring the relationship between creativity and MM is quite limited, and there is a noticeable lack of research focusing specifically on the role of preservice mathematics teachers (PMTs) in this process (e.g., Lu & Kaiser, 2022a, 2022b; Wessels, 2014). Within the Turkish context, empirical studies that address these two concepts in an integrated manner are rarely encountered in the literature. In particular, the scarcity of studies investigating how preservice teachers engage with creativity indicators during the modeling process constitutes the main rationale of this research. This study aims to increase the visibility of the dimensions of usefulness, fluency, and originality related to creativity within the MM competency framework and to create a resource for the field by contributing to the conceptualization of MC. Especially in recent years, with the increasing need for creativity, the lack of evidence from empirical studies supporting MM has been felt more deeply (Blum, 2011; Jensen, 2007; Lu et al., 2019), emphasizing the necessity of such studies.

Mathematical Modeling Competencies

Blum and Kaiser (1997) emphasized that the MM process and MC are closely intertwined, and addressed them through a set of sub-competencies. These sub-competencies are grouped under the headings of understanding the real problem and building a model based on reality, creating a mathematical model from the real model, solving the model, interpreting mathematical results, and verifying the solution. Blomhøj and Jensen (2003) defined modeling sub-competencies within the framework of modeling cycles. They stated the sub-competencies as formulating the given problem to determine the characteristics of the real situation, selecting relevant objects and relationships, presenting relationships mathematically consistently, using mathematical methods, interpreting the results, and evaluating the accuracy of the results. Ikeda and Stephens (1998) emphasized finding answers to some questions to understand students' MC. They stated that students' competencies in determining the mathematical focus of the problem, choosing the variables correctly, idealizing the assumptions, analyzing the primary variable, reaching appropriate results, and interpreting the results should be questioned. In addition, Borromeo Ferri (2006) offered a widely cited explanation of MC, aligning it with the phases of the modeling cycle: understanding the problem, simplifying, mathematizing, working mathematically, interpreting, and verifying. The current debate about MM today is aimed at global MC, which assume that individuals must successfully carry out all MM processes and sub-competencies of modeling, which take into account individual stages in the modeling cycle (Lu & Kaiser, 2022a). Nevertheless, the widely recognized structure of the MM cycle encompasses specific sub-competencies corresponding to each phase of the process. These sub-competencies involve formulating assumptions and simplifying the real-world situation, translating the problem into mathematical terms, systematically constructing or extending the model using coherent strategies, and ultimately interpreting and validating the outcomes within the context of the original real-world scenario (Kaiser, 2007; Maaß, 2006). In summary, we can say that mathematical MC does not have standardized and common aspects. Creativity makes its presence felt at all stages of the modeling cycle, especially since there are unlimited methods to solve real-world problems and modeling processes include daily life contexts (Kaiser & Brand, 2015; Lu & Kaiser, 2022a, 2022b; Wessels, 2014).

Creativity and Its Relationship with Mathematical Modeling

Creativity is a highly complex concept at every moment and area of our lives. Although many definitions have been made about creativity from the past to the present, none have achieved widespread validity. However, creativity, which is increasingly accepted as an important trend of modern life, is characterized by not being afraid to think differently, discovering new relationships, creating extraordinary connections, presenting innovative perspectives, questioning conventional situations, not being afraid to experiment, seeing opposing contexts, and being willing to think (e.g., Amabile, 1996; Barker, 2002; Feldman, 1999; Robinson, 2011; Torrance, 1998). When we look at the aspects of creativity, we find that the structures of fluency (generating new ideas), flexibility (offering a different perspective), originality (designing something new), and elaboration (developing ideas) come to the fore (Guilford, 1977; Leikin, 2013; Renzulli, 2012). Silver (1997) and Torrance (1966) discussed creativity in the context of problem-solving, stating that determining multiple solutions to a problem is fluency, developing different solutions is flexibility, and producing new solutions as much as possible is originality. Leikin (2013) advanced this structure by drawing attention to the pivotal importance of multiple solutions in fostering mathematical proficiency. She framed fluency as the productive generation of numerous solutions, interpreted flexibility as the strategic grouping of those solutions, and portrayed originality as the

innovative integration and structuring of diverse problem-solving approaches. These initiatives are essential in evaluating problem-solving skills regarding flexibility, fluency, and originality (e.g., Cai & Hwang, 2002; Leikin, 2013; Lu & Kaiser, 2022b; Van Harpen & Sriraman, 2013). Therefore, it is clear that these concepts have practical implications. As Freudenthal (1991) points out, the mathematical world becomes genuinely creative when it is mathematized. Students can solve problems for a given situation and generalize their models for similar situations, enhancing their mathematical creativity (Chamberlin & Moon, 2005). This underscores the need for an understanding that transfers real-life contexts, such as mathematical modeling, to the learning environment, enabling students to find creative ways to solve problems (Hébert et al., 2002; Lesh et al., 2000; Greefrath & Vorhölter, 2016; Runco, 2010; Stillman et al., 2013).

Evaluating Creative Dimensions in Mathematical Modeling

The MM is driven by the understanding of "promoting mathematics as a human activity that answers problems of a different nature that gives rise to mathematical concepts, ideas, and procedures" (Stillman, 2019, p. 9). While the ideas, concepts, and structures developed individually are transferred to the outside in modeling activities, on the other hand, the development of the skills of creative thinking, gaining different perspectives, creating thinking, and deciding on the most effective and correct one based on various thoughts are developed through authentic problems that reflect the real-world context (Borromeo Ferri & Blum, 2014). For this reason, innovative insights that emerge in individual modeling processes can be included in the processes from a creativity perspective to be measured in the sub-competencies of modeling (Lu & Kaiser, 2022a). The three components of creativity, fluency, flexibility, and originality, are frequently used in studies on mathematical creativity (Pitta Pantazi et al., 2018). However, whether these three components are sufficient for creativity indicators in modeling studies or whether other criteria are necessary still needs to be fully clarified. In our creative modeling work, we focused on usefulness rather than flexibility. This preference stems from the belief that usefulness more directly captures the real-world applicability and relevance of modeling solutions—central goals of MM. While flexibility highlights the variety of ideas or approaches, usefulness aligns better with evaluating whether a solution is not only original but also functional in authentic contexts. In the study by Lu and Kaiser (2022a), usefulness is an essential component in MM problems. One of the theoretical frameworks in which creativity is evaluated in preservice teachers' MM is made by Wessels (2014), which considers fluency, flexibility indicators, and usability. Fluency, which reflects the productivity of the modeling process, is evaluated according to the number of unique solutions generated and is generally classified into low, medium, or high levels (Lu & Kaiser, 2022a). Flexibility emphasizes the versatility of solutions, ideas and approaches introduced during the modeling process. Wessels (2014) emphasizes that flexibility and fluency require a rich data set in his theoretical framework and mentions the importance of ideas in all possible processes. Based on the components and theoretical frameworks related to creativity and MM requires a sustainable process skill, we also included the usefulness dimension in the creativity indicator. Usefulness is essential, especially in modeling studies that seek ways to solve real-world problems and make them applicable. Since fluency and flexibility are very close, and it is challenging to determine flexibility in modeling cycles, we did not consider them. We created the creativity construct of our study by adding originality, one of the essential components of creativity. While considering all these processes, we tried to reveal the impact of the modeling study in a different geography, using the theoretical infrastructure presented by Lu and Kaiser (2022a) as a reference. We aimed to make the impact of creativity indicators in MM studies more evident and to increase the prevalence of such studies.

Research Questions

In this study, we examined the answers provided by students in MM processes, focusing on the usefulness, fluency, and originality indicators of creativity. The unique process of MM competencies and modeling cycles was explored, revealing new perspectives at each stage. We found that the indicators of creativity—usefulness, fluency, and originality—are integral features of MC in all processes of the modeling cycle. This conceptual understanding led us to evaluate the MM competencies of PMTs within the scope of the study. Creative aspects of MM processes were examined through the research questions (RQs) presented below.

RQ 1. To what extent did the modeling approaches employed by the PMTs in the three tasks reflect varying levels of MC?

RQ 2. To what extent did the PMTs demonstrate MC, as evaluated through the creativity aspects of usefulness, fluency, and originality within the context of the three modeling tasks?

RQ 3. To what extent are PMTs' competencies in modeling approaches correlated with the dimensions of creativity? If a relationship exists, how substantial is its strength?

Method

Research Model and Participants

This study draws upon three key dimensions of creativity—usefulness, fluency, and originality—which are regarded as fundamental components of MM competence and practice. The research adopted both qualitative and quantitative aspects of a situation. In this context, while the participants' responses were discussed in the qualitative dimension, the students' creativity scores, including usefulness, fluency, and originality, were evaluated in the quantitative dimension. The study's participants consist of 60 PMTs studying at a state university. Criterion sampling, among non-random sampling methods, was preferred in determining the study group. Students taking the undergraduate Modeling in Mathematics Teaching course were chosen as the criterion. As such, all participants were PMTs in their third or fourth year of study, with an average age of 21.3 years. The participants had no prior experience with MM tasks. 70% ($n=42$) of the participants were female and 30% ($n=18$) were male students. It is anticipated that PMTs possess the mathematical knowledge prescribed in the undergraduate program standards established by the Council of Higher Education (Council of Higher Education [CoHE], 2023). Modeling in mathematics teaching undergraduate course content; (i) MM and problem solving, (ii) modeling process, (iii) modeling cycle (defining problem, manipulation, prediction/verification), (iv) model development steps, (v) model development principles, (vi) implementation of modeling activities in mathematics classes and the role of the teacher, (vii) preparing MM activities and (viii) monitoring PMTs' mathematical thinking processes. After taking the modeling course in mathematics teaching, the participants gained and applied a lot of experience with MM approaches and modeling tasks.

Data Collection

Before practicing mathematical modeling tasks requiring creativity, PMTs were asked to complete a survey to determine their experiences. In the next step, PMTs were asked to work individually on three modeling tasks in 6-course hours (270 minute) without assistance. When designing MM tasks, the indicators of being able to think independently, questioning, thinking critically, to thinking flexibly, solving problems, interpreting, and experiencing were considered. Problem situations were chosen due to the tasks being open-ended, under-defined, and conducive

to multiple solution approaches (Lu & Kaiser, 2022a). The open-ended nature of these tasks was a deliberate choice to foster creativity and independent thinking among the PMTs. Modeling tasks require extensive MC, mathematical thinking, and creative solutions. Three tasks related to MM reflect daily life situations involving peeling apples, making a World Cup football, and refueling, respectively. Task 1 consists of a study called Pineapple in Mathematics. This modeling considers the situation where a pineapple seller artistically peels it and creates attractive spirals on the back. PMTs were asked to think mathematically about peeling a pineapple and produce solutions, considering why the seller peeled the pineapple in a spiral manner (Lu & Kaiser, 2022a, 2022b). Ludwig and Xu (2010) applied a similar MM situation to 1108 German and Chinese students aged 15-17. Designed as an open-ended task, this modeling scenario includes mathematical concepts like three-dimensional shapes, trigonometric rules, and polynomial functions. Figure 1 shows the problem situation for the MM task.

Figure 1

Modeling Task Named Mathematics in Pineapple (Lu & Kaiser, 2022a, pp. 295)

Task 1. Mathematics in pineapple: The situation: April is pineapple season. When we buy a pineapple, the vendor usually peels it artistically for us, leaving attractive spirals behind. Please think about this peeling process mathematically, and consider why the vendor peels the pineapple in this way. (1) Show your opinion(s); (2) Translate it/them into mathematics; (3) Provide solutions; and (4) Demonstrate your opinion(s).



The pineapple before peeling



The pineapple before removing the part inedible



The pineapple after peeling

Task 2 consists of a work called making a football ball. From a mathematical perspective, this modeling evaluates how long it takes a FIFA football ball manufacturer to produce a quality football. FIFA World Cup is an international football organization. Many countries participate in this football tournament, and a broad audience follows it. Although the PMTs were knowledgeable about the problems involving daily life situations, they needed to learn how the football ball, the most essential part of the tournament, was made. Therefore, PMTs were shown photos of the modeling task and were asked to think about the task. In this MM task, PMTs were asked to calculate the time spent to produce a football and evaluate it from a mathematical perspective (Lu & Kaiser, 2022a). Figure 2 shows the problem situation for the MM task.

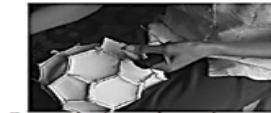
Figure 2

Modeling Task Named Making up a Football (Lu & Kaiser, 2022a, pp. 295)

Task 2. Making up a football: The 2018 FIFA World Cup was just successfully held in Russia. From group stage to quarterfinals, and to the final game, it attracted many fans' attention. Actually, there are different expectations towards the football among different groups of interest. The Adidas Company supplies the footballs for FIFA. Do you know how the FIFA footballs are made by hand? The following pictures shows how a manufacturer makes:



The manufacturer is sewing up the pieces of the football



Several pieces have been sewn up



A football is finally made



A football broken after several years and here we can see the thread

The manufacturer was paid related to the number of soccer balls of good quality. Please evaluate how long it takes to make a soccer ball from a mathematical perspective. Write down the process of thinking, and solve the problem.

The last task consists of a problem situation called refueling. In this MM, the profitability of gas station choice is discussed. It has been handled with a design frequently encountered by PMTs daily and requires mathematical understanding. The original form of the problem situation was developed by Lu and Kaiser (2022a) but was adapted to facilitate knowledge by participating PMTs. Unlike the other two tasks, it is designed to make it easier for students to generate ideas about a situation they are more familiar with. Similar situations of the modeling studies included in Task 2 and Task 3 were also applied by Blum and Leiß (2005) to investigate what a changing teaching culture would look like in concrete terms. Figure 3 presents the problem for the MM task.

Figure 3

Modeling Task Named Refuelling (Lu & Kaiser, 2022a, pp. 295)

Task 3. Refuelling: Mr. Lokman lives in Nevşehir. The nearest gas station in Nevşehir is 20 km away from his house. The nearest gas station in Kayseri is 80 km away. Mr. Lokman usually goes to Kayseri to buy gasoline for his Volkswagen CC 1.8T because the fuel price is 20,54 TL/L at Kayseri gas station and 20,97 TL/L in Nevşehir. Some information about Mr. Lokman's car, CC 1.8T, is presented below.

FAW-Volkswagen			
Length × Width × Height (mm):	4799×1855×1417	Weight of the car (kg):	1535
Fuel consumption measurement (L/100km):	7.8	Capacity of the fuel tank (L):	70.00
Warranty:	2 years 60.000 km	Capacity of the fuel trunk (L):	532

Is it worth it for Mr Lokman to go to the Kayseri station to fill up on gasoline? Please provide your opinions and demonstrate your argument.

In the determined tasks, the creativity exhibited by the PMTs at each stage of modeling competencies was tried to be prioritized. PMTs' worksheets related to the given MM tasks were collected and analyzed using a graded rubric based on a qualitative content analysis approach that includes the application of strict quality standards recommended by Mayring (2014).

Data Analysis

The PMTs' solution processes for modeling tasks were evaluated in two steps. In the first step, their solution processes for three modeling tasks were analyzed in the context of the MC levels of modeling approaches. This analysis revealed significant progress and potential for further growth, making the audience feel optimistic. In the second step, their MC were examined in the context of indicators of creativity. MC includes the individual's ability to manage the modeling process effectively, make purposeful efforts, and be willing to produce solutions. Maaß (2006) stated that MC is understanding the real problem and building a model based on reality, creating a mathematical model from the actual model, solving mathematical problems with the help of a mathematical model, and verifying the solution. Kaiser and Schwarz (2006) stated MC as solving real-life problems with mathematical expressions, activating metacognitive knowledge within the scope of modeling processes, understanding the relationship between mathematics and real life,

perceiving mathematics as a process, and being aware of the features of MM. Borromeo Ferri (2006) stated that the cognitive skills needed for a successful modeling process are understanding the problem effectively, simplifying the problem, mathematizing the problem, working mathematically, interpreting, and verifying. Based on these explanations, we can say that modeling tasks involve multiple solution scenarios and have alternative solution steps rather than a single direct line. Therefore, a three-level subcategory was created: high-medium-low, depending on the suitability of the modeling procedures. While making the rating tool, similar measurement tools in the literature were examined and reorganized (Asempapa, 2018; Lu & Kaiser, 2022a, National Council of Teachers of Mathematics [NCTM], 2020). Subcategories reflecting the competence demonstrations of MM approaches are presented in Table 1.

Table 1

Adequacy Demonstrations of Mathematical Modeling Approaches

Level	Descriptions	Examples
High	Uses the MM process effectively to formulate, represent, analyze and interpret mathematical models, successfully completes modeling tasks and reflects the modeling process in its solutions.	<p><i>Task 1#</i> A satisfactory strategy has been developed for spiral peeling of pineapple. Mathematical relationships were developed by establishing connections between the lengths between the spirals, and the results were obtained and interpreted by putting forward ideas such as simplifying the problem and representing the pineapple as a cylinder.</p> <p><i>Task 2#</i> The variables required for modeling approaches have been appropriately determined. For example, parameters such as the number of sides of the ball and the time required to sew the edges are defined correctly. By converting the data into mathematical form, the steps to obtain results and interpretation were organized and appropriate tools were used.</p> <p><i>Task 3#</i> Modeling approaches include appropriate variables. For example, the price per liter of refueling or comparative refueling costs are conveniently organised. The data were mathematicalized, simplified, analyzed and interpreted using appropriate tools.</p>
Medium	The MM process is used to formulate, represent, analyze and interpret mathematical models, but is assigned to the medium level when deficiencies are found in the modeling cycles and improvement is required.	<p><i>Task 1#</i> Approaches to spiral peeling of pineapple have been proposed, but modeling tools to develop or define mathematical relationships have not been clarified.</p> <p><i>Task 2#</i> Although modeling approaches included variables (number of edges, duration, etc.), the process was not managed effectively because they did not contain appropriate mathematical relationships.</p> <p><i>Task 3#</i> Modeling approaches include variables but fail to establish appropriate mathematical relationships between refueling at two locations or involve certain types of routine solutions.</p>

Low	It is assigned to a low level when it is not at a sufficient level in using the MM process, formulating, representing, analyzing and interpreting.	<p><i>Task 1#</i> The approaches carried out for the modeling process are not successful enough and there is no mathematical attempt on the spiral peeling of pineapple.</p> <p><i>Task 2#</i> The approaches carried out for the modeling process are not successful enough and appropriate parameter selection has not been made.</p> <p><i>Task 3#</i> Modeling approaches merely restate the problem situation and there is no attempt at the modeling process.</p>
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Note: Adapted from Asemppapa, 2018; Lu & Kaiser, 2022a; NCTM, 2020

In the next coding step, MC were analyzed independently in the context of usefulness, fluency, and originality, which were considered indicators of creativity. When evaluating solution processes involving creativity indicators, definitions and explanations in the relevant literature were used (e.g., Amabile, 1996; Guilford, 1950; Kattou et al., 2013; Klavir & Gorodetsky, 2011; Leikin & Lev, 2013; Lu & Kaiser, 2022a; Mann, 2006; Osborn, 1953; Preiser, 1976; Sriraman, 2009; Torrance, 1988).

Table 2

Classification of Creativity Aspects by High, Medium, and Low Levels

Creativity Aspects	Levels	Tasks
Usefulness <i>[It involves realizing the necessity of the approaches used for each given MM task and using them to benefit the evaluation and task solution processes]</i>	High	<p>Task 1# Recognizes the transformation of the pineapple fruit into a cylinder, a geometric object, and its expansion due to peeling, makes application(s), and understands that pineapple peeling can continue.</p> <p>Task 2# It establishes a connection between the number of sides and side lengths using geometric shapes and formulates the model by developing a mathematical understanding of the football ball's production time.</p> <p>Task 3# Applies equations and inequalities, understands functional relationships, and uses algebraic structures to express mathematical relationships in gas station choice.</p>
	Medium	<p>Task 1# It defines the structure formed by peeling the pineapple fruit, makes attempts, and depicts the relationships of the structure, but these situations are limited.</p> <p>Task 2# It specifies a procedure for planting a football that requires geometric shapes, but its use is problematic.</p> <p>Task 3# It performs arithmetical operations and certain calculations regarding the amount of gasoline consumed, but the task's results are limited.</p>
	Low	<p>Task 1# Failure to realize how the peeling marks of the pineapple fruit will benefit the solution, making incorrect inferences, producing wrong solutions, or not taking any approach at all.</p> <p>Task 2# Producing incorrect parameters or not taking any approach regarding the construction and production time of the football.</p> <p>Task 3# It exhibits limited or no practical attempts at mathematizing gasoline consumption.</p>

Fluency	High	<p>Task 1# Solutions are produced that include expanding the cylinder shape formed by peeling the pineapple fruit, and different axioms are considered by presenting information proving the ideas.</p> <p>Task 2# Two or more models for sewing footballs are attempted, and the different processes involved in developing the models are explained in detail.</p> <p>Task 3# Considers different situations and demonstrates probabilistic thinking processes when choosing a gas station.</p>
<p><i>[It includes the speed and accuracy of generating a different number of solutions for each given MM task]</i></p>	Medium	<p>Task 1# Comparisons are made regarding the marks from peeling the pineapple fruit, but it isn't easy to produce different solutions.</p> <p>Task 2# It establishes a relationship between the number of sides of the football ball and geometric shapes, makes calculations, and presents the findings without developing different models.</p> <p>Task 3# It realizes the most suitable conditions for refueling and considers different possibilities but develops limited procedures for the cost table.</p>
	Low	<p>Task 1# It shares inconsistent information based on peeling the pineapple fruit and does not create any modeling loops or make any attempt.</p> <p>Task 2# A consistent model that evokes the sewing and production time of the football cannot be developed, or no attempt is made.</p> <p>Task 3# It does not attempt to identify the appropriate fuel station, establish the conditions for refueling, perform inconsistent mathematical manipulations of the problem, or develop an understanding.</p>
Originality	High	<p>Task 1# It effectively uses mathematical tools that prove the continuity of the peeling traces of the pineapple fruit and rarely used inequality systems, sequences, coordinate systems, analytical planes, <i>etc.</i>, associated with structures</p> <p>Task 2# It evaluates the football ball's planting time from a different perspective and calculates it using unusual methods/representations.</p> <p>Task 3# It determines additional variables taken into account by a small number of students (duration, malfunction, alternative route, <i>etc.</i>) and uses transformations between representations effectively and qualifiedly.</p>
<p><i>[It includes ideas that emerge through fluent and flexible thinking, qualified inferences, quality evaluations, and thematically rare approaches to each MM task]</i></p>	Medium	<p>Task 1# It indicates the expansion of the cylindrical structure formed by peeling the pineapple fruit using different mathematical tools, but there is a participant or participants who create similar structures.</p> <p>Task 2# It produces extraordinary solutions that consider the perfection of the football ball's sewing, but it makes incorrect/incomplete conclusions in practice.</p> <p>Task 3# It justifies the fuel station selection and applies mathematical structures (inequality, equation, rational number, <i>etc.</i>) determined for the variables, but it cannot fully determine the consistency between the parameters.</p>
	Low	<p>Task 1# Does not develop original inferences about the pineapple's genealogy and design of the process cycle or shows no initiative regarding the assigned tasks.</p>

Task 2# Solutions for planting a football are produced using a single parameter or do not show any initiative regarding the given tasks.
Task 3# Operates(s) using only the values given in the task, prefers mathematical tools commonly used among participants, or shows no initiative regarding assigned tasks.

Data Analysis

Based on the PMTs' responses, the coding of MC and creativity indicators was made independently by the first author and a field expert teaching modeling education in mathematics at the undergraduate level. In order to ensure the reliability of coding, all 60 student texts were coded. The reliability of the comparative agreement between two raters was calculated as .82 according to the kappa measure. According to Altman (1991), if the weight kappa between raters on all dimensions is greater than ≥ 0.81 , it means that there is 'very good' agreement, that is, almost perfect agreement (p. 404). This high level of agreement indicates that the coding process was consistent and reliable. For each modeling task, competency indicators and three aspects of creativity were first examined using the descriptive analysis method. In the next step, the Friedman test was used to compare the PMTs' competency and creativity performances in three modeling tasks. Friedman test is a non-parametric form of one-way analysis of variance for repeated measurements. Dunn-Bonferroni post hoc tests were used to determine the source of significant differences between indicators. Finally, to calculate the correlation coefficient between PMTs' performances on different indicators, Spearman conducted a partial correlation analysis to calculate the relationship between competency and indices representing the three aspects of creativity.

Results

PMTs' Performance Based on Their Competency in Three Modeling Tasks

PMTs' competency levels in the general modeling approach in the three given tasks include correct solutions that meet criteria such as effective use of processes, purposeful effort, providing a mathematical understanding of the current problem, and testing the model's accuracy.

Table 3

PMTs' Performance Distribution by Competency Levels in Three Distinct Modeling Tasks

Levels	Task 1	Task 2	Task 3
High	25% [15]	27% [16]	77% [46]
Medium	15% [9]	50% [30]	15% [9]
Low	60% [36]	23% [14]	8% [5]

As indicated in Table 3, approximately 77% of the PMTs presented successful approaches to the tasks named refueling (task 3), 27% to make a football (task 2), and 25% to math in pineapple (task 1). According to the Friedman test analysis findings, a significant difference was observed between the MC of the PMTs in the three types of tasks given ($\chi^2(2)=58.805, p<.001$). After Bonferroni correction, according to the Dunn-Bonferroni post hoc test analysis findings, it was determined that there were significant differences between task 1 and task 2 ($p<.001$), task 1 and task 3 ($p<.001$), and task 2 and task 3 ($p<.001$). Based on these findings, PMTs showed the highest performance in task 3 and the lowest performance in task 1.

For Task 1, many PMTs realized that the shape formed due to the peel mark of the pineapple was a cylinder, but they had problems applying the properties of the cylinder expansion, which is a solid object. They experienced difficulties, especially in later, base, and surface area applications. Approximately 60% of the PMTs could not perform effectively at the proficiency level. 15% of the PMTs provided information about the properties of the right circular ($2\pi rh$), cylinder as the lateral area of the cylinder, the base area of each circle since its bases are equal circles (πr^2), and the sum of the lateral area and two equal base areas for the surface area $2\pi rh + 2\pi r^2 = 2\pi r(h+r)$. They could not effectively manage the process by developing unclear ways to simplify the problem. For Task 2, half of the PMTs demonstrated medium competency. If half of the PMTs knew the number of sides available on a standard football (a traditional football consists of 32 panels, 20 hexagons, and 12 pentagons), they would be able to perform at a high level of competency. Many PMTs could not establish an effective relationship between the football ball's number of sides and the properties of the geometric shapes. Thus, many PMTs could not use appropriate mathematical tools to deal with the current difficulties and uncertainties. However, 27% of the students managed the modeling process effectively by successfully attempting to solve the current problem. In this context, a series of formulas [Euler's formula: $e^{ix} = \cos(x) + i\sin(x)$; C60: It has a cage like fused ring structure fullere: 60 carbon atoms are bonded to its three neighbors; trihedral pyramidal forms of honeycombs $h \{3,4,3\}$; hexagon: $d = \sqrt{3}/2 D$ etc.] were used to calculate the number of sides of the football ball. It is noteworthy that PMTs performed more effectively on Task 3 compared to other tasks. 77% of the PMTs were able to provide adequate approaches for the given task, but they were often unable to go beyond arithmetical operations when developing solutions to the problem at hand. Even though the PMTs operated the modeling processes effectively, the majority of the answers consisted of arithmetic mean and algebraic structures.

PMTs' Competency in Modeling Tasks Based on Creativity Indicators

Table 4

Analysis of PMTs at Varying Usefulness Levels in Three Modeling Tasks

Levels	Task 1	Task 2	Task 3
High	23% [14]	20% [12]	39% [23]
Medium	32% [19]	25% [15]	53% [32]
Low	45% [27]	55% [33]	8% [5]

Table 4 shows that PMTs demonstrated high levels of usefulness performance, with 23% in task 1, 20% in task 2, and 39% in task 3. On the other hand, 45% of the PMTs showed low performance in task 1, 55% in task 2, and 8% in task 3. Approximately one-third of the PMTs performed at a medium level on task 1, one-quarter on task 2, and just over half on task 3. It was observed that there were significant differences between the PMTs' usefulness performances in each task type ($\chi^2(2) = 17.745, p < .001$). The result of further analysis to determine the source of significant differences shows significant differences between task 1 and task 3 ($p < .001$) and task 2 and task 3 ($p < .001$). On the other hand, it was determined that there was no significant difference between task 1 and task 2 ($p = .074$). These findings show that PMTs showed the highest usefulness performance in task 3 and the lowest in task 2.

Table 5

Analysis of PMTs at Varying Fluency Levels in Three Modeling Tasks

Levels	Task 1	Task 2	Task 3
High	5% [3]	13% [8]	23% [14]
Medium	63% [38]	72% [43]	70% [42]
Low	32% [19]	15% [9]	7% [4]

Table 5 shows that PMTs' solution stages in the tasks are limited and do not prefer different solution approaches. In each task, the majority of PMTs demonstrated medium fluency performance. The high concentration at this level indicates that a large number of PMTs have the potential to achieve fluency in modeling approaches. As a matter of fact, in task 3, some of the PMTs tried different systems of equations, but they could not produce solutions to test their accuracy. It was observed that there were significant differences in terms of PMTs' fluency performance in the context of the tasks ($\chi^2(2)=26.957, p<.001$). The result of further analysis to determine the source of these significant differences shows that there are significant differences between task 1 and task 2 ($p<.001$), task 1 and task 3 ($p<.001$), and task 2 and task 3 ($p=0.012$).

Table 6

Analysis of PMTs at Varying Originality Levels in Three Modeling Tasks

Levels	Task 1	Task 2	Task 3
High	12% [7]	22% [13]	20% [12]
Medium	18% [11]	32% [19]	38% [23]
Low	70% [42]	46% [28]	42% [25]

Table 6 indicates that PMTs' task modeling approaches are not thematically uncommon. Especially in task 1, the majority of the PMTs do not make traditional inferences and do not exhibit extraordinary approaches that will make a difference. Similarly, in task 2 and task 3, nearly half of the PMTs showed low level performance in terms of originality. 18% of the PMTs performed medium on task 1, 32% on task 2, and 38% on task 3. Additionally, there were significant differences in terms of originality between the tasks ($\chi^2(2)=14.947, p<.01$). Further analysis to determine the source of significant differences revealed that significant differences were observed between task 1 and task 2 ($p=.002$) and task 1 and task 3 ($p=.002$), while no significant difference was observed between task 2 and task 3 ($p=.868$). These findings clearly show that PMTs cannot ensure originality in using mathematical tools and that there are difficulties in reflecting different parameters into solutions in the real world. Therefore, PMTs' performance in generating new ideas and their efforts to develop innovative mathematical understanding of given problems remained low.

Correlations Between Competency in Modeling Approaches and Creativity

The conceptual structure of modeling tasks directed to PMTs contains features that encourage creativity. We also analyzed the relationships between creativity indicators and PMTs' modeling proficiency performances. This way, we tried to determine the relationship between the criteria defined for modeling approaches and usefulness, fluency, and originality with a holistic understanding. With the help of Spearman correlation analysis, we aimed to reach more in-depth findings by describing in detail the relationship between MM competence and indicators of creativity.

Table 7 shows that the correlations between MC and all identified aspects of creativity were significant. The correlation between competence and usefulness is respectively; $r_s(60)=.646, p<.01$ for task 1, $r_s(60)=.643, p<.01$ for task 2, $r_s(60)=.503, p<.01$ for task 3. While the correlation between task 1 and task 2 is very close to each other, the correlation value in task 3 is slightly weaker. The correlation between competence and fluency is respectively; $r_s(60)=.496, p<.01$ for task 1, $r_s(60)=.579, p<.01$ for task 2, $r_s(60)=.449, p<.01$ for task 3. While the correlation between task 1 and task 3 is very close to each other, the correlation value in task 2 is slightly stronger. The correlation between competence and originality is respectively; $r_s(60)=.752, p<.01$ for task 1, $r_s(60)=.724, p<.01$ for task 2, $r_s(60)=.475, p<.01$ for task 3.

Table 7

Relationships Between Different Aspects of PMTs' Performance on Modeling Tasks

Tasks	Creativity Indicator	Adequacy	Usefulness	Fluency	Originality
Task 1	Adequacy	1	.646**	.496**	.752**
	Usefulness		1	.662**	.607**
	Fluency			1	.501**
	Originality				1
Task 2	Adequacy	1	.643**	.579**	.724**
	Usefulness		1	.452**	.683**
	Fluency			1	.617**
	Originality				1
Task 3	Adequacy	1	.503**	.449**	.475**
	Usefulness		1	.608**	.279*
	Fluency			1	.301*
	Originality				1

* $p<.05$, ** $p<.01$

While the correlation between task 1 and task 2 is very close and strong, the correlation value in task 3 is weak. According to the findings, the relationship between competence and originality is higher than other indicators. Similarly, there is a strong connection between competence and usefulness. However, the relationship between proficiency and fluency is close to medium level. When we look at the relationship between creativity indicators, highest relationship was between usefulness and originality ($r_s(60)=.683, p<.01$) in task 2, while the lowest was between usefulness and originality ($r_s(60)=.279, p<.05$) in task 3. Also, strong relationships were detected between usefulness and fluency ($r_s(60)=.622, p<.01$) in task 1, fluency and originality ($r_s(60)=.617, p<.01$) in task 2, and usefulness and fluency ($r_s(60)=.608, p<.01$) in task 3.

Table 8

Partial Correlations Between PMTs' Creativity Indicators in Modeling Tasks

Tasks	Creativity Indicator	Usefulness	Fluency	Originality
Task 1	Usefulness	1	.467**	.271*
	Fluency		1	.182
	Originality			1
Task 2	Usefulness	1	.116	.489**
	Fluency		1	.364**

	Originality			1
	Usefulness	1	.533**	.152
Task 3	Fluency		1	.203
	Originality			1

* $p < .05$, ** $p < .01$

Table 8 indicates the effect of modeling capabilities, held constant, on the parameters of usefulness, fluency, and originality. In this way, the effects of more than one variable thought to be related to the variable in question were controlled. According to partial correlation coefficients, significant relationships were determined between usefulness and fluency ($r_s(60) = .467$, $p < .01$), usefulness and originality ($r_s(60) = .271$, $p < .05$) in task 1, usefulness and originality ($r_s(60) = .489$, $p < .01$), fluency and originality ($r_s(60) = .364$, $p < .01$) in task 2, and usefulness and fluency ($r_s(60) = .533$, $p < .01$) in task 3.

Discussion and Conclusion

Reasons for Considering Modeling Competencies as Creativity-Based

By examining the definitions, explanations, evaluations, inferences, and approaches in the literature from past to present regarding the concept of creativity, which is a complex structure, we showed that the current forms of creativity components are also present in the measurement tool. Based on the application of modeling tasks on Chinese students (Lu & Kaiser, 2022a), we tested the usability of creativity components for Turkish students. In this way, we emphasized that the conceptual structure of MC and the enriched creativity components of students who will engage in similar modeling tasks in different cultures and geographies can be measured. In particular, a different sub-indicator, such as the usefulness dimension, produces comparable results for creativity and is also a guide for modeling studies to be carried out in a similar direction. Although this indicates that MC that require creativity are applicable and sustainable, they are not included in many creativity explanations and definition criteria (e.g., Blomhøj & Jensen, 2003; Sriraman, 2009). This study examines the extent to which creativity impacts PMTs' MC throughout all phases of the modeling cycle, as observed in their responses to three modeling tasks. Therefore, we aimed to present qualified findings to the field by referring to current research on creativity and the few studies conducted on this subject (e.g., Lu & Kaiser, 2022a, 2022b; Wessels, 2014). Especially considering the limited number of studies on the usefulness criterion, we determined that usefulness in MC is essential and should be considered. Although usefulness remains in the background in many creativity-requiring studies, it is essential in modeling studies. It has been stated that it supports both the modeling process and sub-competencies (Lesh et al., 2000; Lu & Kaiser, 2022a). For example, in task 1, although most of the PMTs failed the task, they made attempts to understand the cylindrical structure of the pineapple fruit and, accordingly, the general properties of the cylinder, creating shapes representing the peeling marks of the pineapple and scoring them as medium level usefulness. Because PMTs tried to provide usable evidence and create strategies. Similar findings were seen in the study by Lu and Kaiser (2022a). Therefore, usefulness provides both usefulness and shareability for modeling. Similarly, in task 2, which required creativity, PMTs ingeniously emphasized the features of the geometric structure of the football ball. They attempted to establish a relationship between the number of sides, demonstrating high and medium levels of usefulness. This indicates that PMTs first apply their creative thinking to understand the nature of the problem in order to use mathematical tools effectively. Because the first step in MM studies is to understand the problem and organize possible original hypotheses (Borromeo Ferri, 2006; Maaß,

2006; Vorhölter, 2018). In addition, according to the correlation coefficients, it was determined that there was a close to medium level relationship between the usefulness of the modeling approaches and proficiency in task 3, which was less challenging for the students. Usefulness in Task 3 aims to use only mathematical structures that can be shared in similar situations, while competence only considers suitability for tasks. Although the correlation coefficients in task 1 and task 2 were above the medium level, they did not clearly demonstrate that there was a strong link between usefulness and competence. Considering the three types of tasks, depending on the difficulty level of the tasks, the usefulness indicator can be effective in revealing creativity in modeling tasks. Lu and Kaiser (2022a), drawing attention to this situation, stated that usefulness can be a unique indicator to determine creativity in MM.

Our research has uncovered a unique perspective on the relationship between fluency and MC. We found a medium level relationship, which differs from the strong connection between fluency and creativity suggested by previous studies (e.g., Hébert et al., 2002; Runco, 2010). In contrast to the study by Lu & Kaiser (2022a) that suggests a lower relationship, our research indicates a concentrated relationship at a medium level. Although it is generally accepted to follow multiple solution processes in modeling studies (e.g., Galbraith & Stillman, 2006; Stillman et al., 2013), it is also very important that the general view of the model suggests multiple solution paths (Kaiser & Brand, 2015). One of the study's striking findings is the strength of the relationship between MC and originality in tasks 1 and 2. The fact that tasks 1 and 2 are more difficult has highlighted the originality of modeling tasks.

Students' Mathematical Modeling Performances as an Activity Requiring Creativity

While Turkish PMTs were able to follow the stages of the modeling process more effectively than peers without modeling experience, their MM competency, with the exception of one task, fell significantly short of the expected standard. Especially considering the presence of mathematical operations in task 3, the PMTs mostly remained at the operational level. Although there are many reasons for this situation, adopting more operational approaches in the curriculum is effective. The findings revealed in the context of MC confirmed Leikin's (2013) statement that the fit criterion effectively produces valid results. Similarly, in the study conducted by Lu and Kaiser (2022a), the suitability criterion was evaluated as an essential structure in MC. Only 25% of the PMTs demonstrated adequate approaches in task 1 and 27% in task 2. These tasks contained unusual content that brought creativity to the fore. Therefore, PMTs could not organize and apply their mathematical knowledge effectively in this modeling task, which has yet to be encountered. However, this differed for PMTs who encountered similar situations (task 3). The most important reason for this situation is that task 3 contains more information than other tasks. While the correlation values between usefulness, fluency, and originality, indicators of MC and creativity in the study, were at a medium level or above in task 1 and task 2, the correlation values in task 3 remained at a medium level. This emphasizes that task 1 and task 2 need to follow richer processes for usefulness, fluency, and originality. PMTs' performance well below expectations in these challenging tasks indicates the impact of their creativity. The relationship between proficiency and creativity indicators in task 3 confirms this rationale. Indeed, in task 3, only one-fifth of the students performed with indicators of originality. For this reason, the difficulty of the tasks restricts PMTs from doing different experiments and developing different perspectives. These findings are also reflected in the study conducted by Lu and Kaiser (2022a).

As a result, PMTs could have performed better regarding creativity indicators. This situation was observed mainly in the fluency and originality dimensions. This situation shows that PMTs do not make enough effort and have difficulty reaching multiple solutions. These findings are consistent with the results of modeling studies but reveal that PMTs have difficulty producing alternative ways in modeling studies (Bukova-Güzel, 2021; Lu & Kaiser, 2022a; Schukajlow et al., 2015). However, considering the innovative approaches in the teaching programs of nations (e.g., NCTM, 2020), this undesirable situation may vary over time. In particular, the increasing importance of fluency and originality in many nations and expressing their necessity with vital discourses will undoubtedly lead to a growing awareness of creativity in modeling studies (Blum, 2011; Lu & Kaiser, 2022a; Wessels, 2014).

Limitations and Agenda and Implications for Future Research

It is generally recommended that MM studies be carried out and organized in groups (Ludwig & Xu, 2010). In our study, the tasks directed to the PMTs to evaluate their performances were carried out individually. This can be considered a limitation of our study, but revealing individual creativity in group modeling studies poses specific difficulties. Among these difficulties, modeling studies require process skills, and the inability to determine who organized the usefulness, fluency, and originality can be cited. Another limitation is the difficulty in the multidimensional evaluation of modeling processes. Because techniques such as thinking aloud, clinical interviews, and interviews are needed to detail students' mental processes and better elaborate their thoughts (Clement, 2000). However, our study evaluated the modeling processes based on the PMTs' answers. To minimize this limitation, we tried to address creativity indicators holistically. In addition, although the tasks directed at students are implemented in different countries (e.g., China and Germany), the validity of originality in other languages needs to be tested. Tasks 1 and 2, which require creativity and were developed by Lu and Kaiser (2022a), are similar to the modeling problem in task 3, developed by Blum and Leiß (2005). A further limitation is the restricted number and variety of modeling tasks, which may limit the applicability of the creativity indicators across different contexts and content domains. Since our study was aimed at Turkish students, changes were made in the proper names to make the task contents understandable. Therefore, as the number of similar studies increases, richer knowledge of MC that require creativity and the reliability of the assessment tool will also increase. Furthermore, the study was conducted with a limited sample from a single state university in Turkish, which restricts the generalizability of the findings to broader populations. Conducting this study, especially at the undergraduate and high school levels, will be an essential opportunity for future researchers, underscoring the significance of our work. In this way, comparisons between different reference groups and measures that can be taken for future modeling studies can be more easily achieved. In addition, processes can be further detailed by conducting longitudinal studies. At the same time, effect levels can be determined by changing creativity indicators.

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References

- Acquah, E. O., & Szelei, N. (2020). The potential of modelling culturally responsive teaching: Pre-service teachers' learning experiences. *Teaching in Higher Education*, 25(2), 157-173. <https://doi.org/10.1080/13562517.2018.1547275>
- Altman, D. G. (1991). *Practical statistics for medical research*. Chapman and Hall.
- Amabile, T. M. (1996). *Creativity in context: Update to the social psychology of creativity*. West-View Press.
- Asempapa, R. S. (2018). Assessing teachers' knowledge of mathematical modeling: Results from an initial scale development. *Journal of Mathematics Education*, 11(1), 1-16. <https://doi.org/10.26711/007577152790017>
- Barker, A. (2002). *The alchemy of innovation: Perspectives from the leading edge*. Spiro Press.
- Blomhøj, M., & Jensen, T. H. (2007). What's all the fuss about competencies? Experiences with using a competence perspective on mathematics education to develop the teaching of mathematical modelling. In *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 45-56). Springer.
- Blomhøj, M., & Jensen, T. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching Mathematics and Its Applications*, 22(3), 123-139. <https://doi.org/10.1093/teamat/22.3.123>
- Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds), *Trends in teaching and learning mathematical modelling* (pp. 15-30). Springer. https://doi.org/10.1007/978-94-007-0910-2_3
- Blum, W., & Leiß, D. (2007). How do students and teachers deal with mathematical modelling problems? The example sugaloaf und the DISUM project. In C. Haines, P. L. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA12)-Education, Engineering and Economics*. Horwood Publishing.
- Blum, W., & Leiß, D. (2005). Modellieren im Unterricht mit der "Tanken"-Aufgabe. *Mathematik Lehren*, 128, 18-21.
- Blum, W., & Kaiser, G. (1997). *Vergleichende empirische Untersuchungen zu mathematischen Anwendungsfähigkeiten von englischen und deutschen Lernenden*. Unpublished application to Deutsche Forschungsgesellschaft.
- Borromeo Ferri, R. (2018). *Learning how to teach mathematical modelling in school and teacher education*. Springer.
- Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *Zentralblatt für Didaktik der Mathematik* 38(2), 86-95. <https://doi.org/10.1007/BF02655883>

- Borromeo Ferri, R., & Blum, W. (2014). Barriers and motivations of primary teachers for implementing modelling in mathematics lessons. In B. Ubuz, C. Haser, & M. A. Mariotti (Eds.), *Proceedings of CERME 8* (pp. 1000-1009). Middle East Technical University.
- Bukova-Güzel, E. (Ed.) (2021). *Mathematical modeling in mathematics education. For researchers, educators and students* (4th ed.). Pegem Akademi Publishing.
- Cai, J., & Hwang, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. *The Journal of Mathematical Behavior, 21*(4), 401-421. [https://doi.org/10.1016/S0732-3123\(02\)00142-6](https://doi.org/10.1016/S0732-3123(02)00142-6)
- Chamberlin, S. A., & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *Journal of Secondary Gifted Education, 17*(1), 37-47. <https://doi.org/10.4219/jsge-2005-393>
- Clement, J. (2000) Analysis of clinical interviews: Foundations and model viability. In R. Lesh, & A. Kelly, (Eds.), *Handbook of research methodologies for science and mathematics education* (pp. 341-385). Lawrence Erlbaum Publishing.
- Council of Higher Education (CoHE) (2023). *Elementary mathematics teaching undergraduate program*. Retrieved from <https://www.yok.gov.tr/kurumsal/idari-birimler/egitim-ogretim-dairesi>
- English, L. D. (2009). Promoting interdisciplinarity through mathematical modelling. *ZDM Mathematics Education, 41*(1), 161-181. <https://doi.org/10.1007/s11858-008-0106-z>
- Feldman, D. H. (1999). The development of creativity. In R. J. Sternberg (Ed.), *Handbook of creativity* (pp. 169-186). Cambridge University Press.
- Freudenthal, H. (1991). *Revisiting mathematics education*. Kluwer Academic.
- Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *Zentralblatt für Didaktik der Mathematik 38*(2), 143-162. <https://doi.org/10.1007/BF02655886>
- Greefrath, G., & Vorhölter, K. (2016). Teaching and learning mathematical modelling: Approaches and developments from German speaking countries. In G. Greefrath, & K. Vorhölter (Eds.), *Teaching and learning mathematical modelling* (pp. 1-42). Springer. https://doi.org/10.1007/978-3-319-45004-9_1
- Guilford, J. P. (1977). *Way beyond the IQ*. Creative Synergistic Associates.
- Guilford, J. P. (1950). Creativity. *American Psychologist, 5*(9), 444-454. <http://dx.doi.org/10.1037/h0063487>
- Hébert, T. P., Cramond, B., Neumeister, K. L. S., Millar, G., & Silvian, A. F. (2002). *E. Paul Torrance: His life, accomplishments, and legacy. Research monograph series*. National Research Center on the Gifted and Talented, Storrs. Retrieved from <https://eric.ed.gov/?id=ED480289>
- Ikeda, T., & Stephens, M. (1998). The influence of problem format on students' approaches to mathematical modelling. In P. Galbraith, W. Blum, G. Booker, & I. Huntley (Eds.), *Mathematical modelling, teaching and assessment in a technology-rich world* (pp.223-232). Horwood Publishing.

- Jensen, T. H. (2007). Assessing mathematical modelling competency. In C. P. Haines, P. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 141–148). Horwood Publishing.
- Kaiser, G. (2017). The teaching and learning of mathematical modelling. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 267–291). National Council of Teachers of Mathematics.
- Kaiser, G. (2007). Modelling and modelling competencies in school. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA12) education, engineering and economics* (pp. 110-119). Horwood Publishing.
- Kaiser, G., & Brand, S. (2015). Modelling competencies: Past development and further perspectives. In G. A. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice: Cultural, social and cognitive influences* (pp. 129-149). Springer.
- Kaiser, G., Blomhøj, M., & Sriraman, B. (2006). Towards a didactical theory for mathematical modelling. *Zentralblatt für Didaktik der Mathematik*, 38(2), 82-85. <https://doi.org/10.1007/BF02655882>
- Kaiser, G., & Schwarz, B. (2006). Mathematical modelling as bridge between school and university. *ZDM Mathematics Education*, 38(2), 196-208. <https://doi.org/10.1007/BF02655889>
- Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2013). Connecting mathematical creativity to mathematical ability. *ZDM Mathematics Education*, 45(2), 167-181. <https://doi.org/10.1007/s11858-012-0467-1>
- Klavir, R., & Gorodetsky, M. (2011). Features of creativity as expressed in the construction of new analogical problems by intellectually gifted students. *Creative Education*, 2(3), 164-173. <https://doi.org/10.4236/ce.2011.23023>
- Leikin, R. (2013). Evaluating mathematical creativity: The interplay between multiplicity and insight. *Psychological Test and Assessment Modeling*, 55(4), 385-400.
- Leikin, R., & Lev, M. (2013). Mathematical creativity in generally gifted and mathematically excelling adolescents: What makes the difference? *ZDM Mathematics Education*, 45(2), 183-197. <https://doi.org/10.1007/s11858-012-0460-8>
- Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought-revealing activities for students and teachers. In A. Kelly & R. Lesh (Eds.), *Handbook of research in mathematics and science education* (pp. 113-149). Erlbaum Publishing.
- Lu, X., Cheng, J., Xu, B., & Wang, Y. (2019). Xuesheng shuxue jianmo suyang de pingjia gongju yanjiu [A research of the assessment tool of students' mathematical modelling competency]. *Kecheng Jiaocai Jiaofa [Curriculum, Teaching Materials, and Method]*, 39(2), 100-106.
- Lu, X., & Kaiser, G. (2022a). Creativity in students' modelling competencies: Conceptualisation and measurement. *Educational Studies in Mathematics*, 109(2), 287-311. <https://doi.org/10.1007/s10649-021-10055-y>

- Lu, X., & Kaiser, G. (2022b). Can mathematical modelling work as a creativity-demanding activity? An empirical study in China. *ZDM Mathematics Education*, 54(1), 67-81. <https://doi.org/10.1007/s11858-021-01316-4>
- Ludwig, M., & Xu, B. (2010). A comparative study of modelling competencies among Chinese and German students. *Journal für Mathematik-Didaktik*, 31(1), 77-97. <https://doi.org/10.1007/s13138-010-0005-z>
- Maaß, K. (2006). What are modelling competencies? *Zentralblatt für Didaktik der Mathematik*, 38(2), 113-142. <https://doi.org/10.1007/BF02655885>
- Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted*, 30(2), 236-260. <https://doi.org/10.4219/jeg-2006-264>
- Mayring, P. (2014). *Qualitative content analysis: Theoretical foundation, basic procedures and software solution*. Klagenfurt. Retrieved from <https://nbnresolving.org/urn:nbn:de:0168-ssoar-395173>
- National Council of Teachers of Mathematics (NCTM) (2020). *NCTM 2020 standards for mathematics teacher preparation*. Retrieved from <https://www.nctm.org/caep/>
- Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling*. Routledge. <https://doi.org/10.4324/9781315189314>
- Osborn, A. (1953). *Applied imagination: Principles and procedures of creative problem solving*. Charles Scribner's Sons.
- Pitta Pantazi, D., Kattou, M., & Christou, C. (2018). Mathematical creativity: Product, person, process and press. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness: Enhancing creative capacities in mathematically promising students* (pp. 27-53). Springer.
- Preiser, S. (1976). *Kreativitätsforschung [Research on creativity]*. Wissenschaftliche Buchgesellschaft.
- Renzulli, J. S. (2012). Reexamining the role of gifted education and talent development for the 21st century: A four-part theoretical approach. *Gifted Child Quarterly* 56(3), 150-159. <https://doi.org/10.1177/0016986212444901>
- Robinson, K. (2011). *Out of our minds: Learning to be creative*. Capstone Publishing.
- Runco, M. A. (2010). Divergent thinking, creativity, and ideation. In J. C. Kaufman & R. J. Sternberg (Eds.), *The Cambridge handbook of creativity* (pp. 413-446). Cambridge University Press.
- Schukajlow, S., Krug, A., & Rakoczy, K. (2015). Effects of prompting multiple solutions for modelling problems on students' performance. *Educational Studies in Mathematics*, 89(3), 393-417. <https://doi.org/10.1007/s10649-015-9608-0>
- Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *Zentralblatt für Didaktik der Mathematik*, 29(3), 75-80. <https://doi.org/10.1007/s11858-997-0003-x>
- Sriraman, B. (2009). The characteristics of mathematical creativity. *ZDM Mathematics Education*, 41(1-2), 13-27. <https://doi.org/10.1007/s11858-008-0114-z>

- Stillman, G. A. (2019). State of the art on modelling in mathematics education-lines of inquiry. In G. A. Stillman, & J. Brown (Eds.) *Lines of inquiry in mathematical modelling research in education. ICME-13 monographs*. Springer. https://doi.org/10.1007/978-3-030-14931-4_1
- Stillman, G., Kaiser, G., Blum, W., & Brown, J. (Eds.). (2013). *Teaching mathematical modeling: Connecting research to practice*. Springer.
- Torrance, E. P. (1998). *The Torrance test of creative thinking norms-technical manual figural, forms A and B*. Scholastic Testing Service, Inc.
- Torrance, E. P. (1988). The nature of creativity as manifest in its testing. In R. J. Sternberg (Ed.), *The nature of creativity: Contemporary psychological perspectives* (pp. 43-75). Cambridge University Press.
- Torrance, E. P. (1966). *Torrance tests of creative thinking: Directions manual and scoring guide*. Personnel Press.
- Van Harpen, X. Y., & Sriraman, B. (2013). Creativity and mathematical problem posing: An analysis of high school students' mathematical problem posing in China and the USA. *Educational Studies in Mathematics*, 82(2), 201-221. <https://doi.org/10.1007/s10649-012-9419-5>
- Vorhölter, K. (2018). Conceptualization and measuring of metacognitive modelling competencies: Empirical verification of theoretical assumptions. *ZDM Mathematics Education*, 50(1-2), 343-354. <https://doi.org/10.1007/s11858-017-0909-x>
- Wessels, H. (2014). Levels of mathematical creativity in model-eliciting activities. *Journal of Mathematical Modelling and Application*, 1(9), 22-40.

Geniş Özet

Giriş

Matematiksel modelleme, günlük yaşamda karşılaşılan problemlerin matematiksel araçlar yardımıyla çözüldüğü, açıklandığı, sınındığı, değerlendirildiği ve yorumlandığı süreç becerisidir. Bu nedenle, gerçek yaşam problemlerini düzenlerken etkili bir şekilde modelleme yeterliklerinin kullanılmasına ihtiyaç duyulmaktadır (Kaiser, 2007; Maaß, 2006). Öğrenciler tarafından karmaşık, soyut ve anlaşılması güç olarak görülen matematiksel kavramların daha iyi kavranabilmesi için matematiğin günlük yaşamla ilişkilendirilerek öğretime entegre edilmesi büyük önem taşır. Bu bağlamda modelleme yeterliği; gerçek yaşam problemine ilişkin değişkenlerin, varsayımların ve ilişkili bağlantıların tanımlanmasını, bu problem durumunun matematiksel dile çevrilmesini, çözüm sürecinin düzenlenmesini, modellerin karşılaştırılmasını ve modelin genel yapısının kontrol edilmesini gerektirir (Kaiser & Schwarz, 2006; Lu & Kaiser, 2022a). Bu yeterlik, bireylerin gerçek yaşamda kullanabilecekleri somut ve zihinsel modeller oluşturma görevini üstlenmeleri açısından gelecekteki kariyerlerinde faydalanabilecekleri pratik bir kaynak olarak değerlendirilmektedir (Blomhøj & Jensen, 2007; Blum & Leiß, 2007; Borromeo Ferri, 2018; English, 2009). Tüm bu gerekçeler, modelleme yeterliğinin müfredattaki varlığının, yalnızca okul başarısını değil yaşam başarısını da hedefleyen felsefi bir yaklaşımın ürünü olduğunu göstermektedir (Acquah & Szelei, 2020; Vorhölter, 2018). Modelleme yeterliği, amaca yönelik istekli çaba ve bağımsız modelleme süreci gerektirir (Blum, 2011; Kaiser vd., 2006; Wessels, 2014). Modelleme problemlerinin doğası gereği, öğrenciler belirsizlik içeren durumlarla karşılaştıklarında farklı çözümler üretebilir ve bu

durum onların yaratıcı düşünme becerilerini devreye sokmalarını zorunlu kılar (Lu & Kaiser, 2022b). Ancak matematiksel modelleme ile yaratıcılık arasındaki ilişkiye odaklanan çalışmaların sınırlı olması, bu alanda yeni araştırmalara duyulan ihtiyacı da artırmaktadır (Blum, 2011; Jensen, 2007; Lu vd., 2019). Bu çalışma, yaratıcı düşünmenin orijinallik, akıcılık ve yararlık boyutlarını matematiksel modelleme yeterliği çerçevesinde görünür kılmayı ve bu kavramsal anlayışa katkı sunmayı amaçlamaktadır.

Yöntem

Bu araştırma, matematiksel modelleme yeterliği bağlamında yaratıcılığın temel bileşenleri olan yararlık, akıcılık ve orijinallik boyutlarını temel almaktadır. Araştırmada durumun hem nitel hem de nicel boyutları ele alınmıştır. Bu bağlamda, katılımcıların verdikleri yanıtlar nitel boyutta tartışılmış, yararlık, akıcılık ve orijinallik gibi yaratıcılık göstergelerini içeren puanlamalar ise nicel boyutta değerlendirilmiştir. Araştırmanın çalışma grubunu, bir devlet üniversitesinde öğrenim gören 60 matematik öğretmeni adayını oluşturmaktadır. Çalışma grubunun belirlenmesinde seçkisiz olmayan örnekleme yöntemlerinden ölçüt örnekleme tercih edilmiştir. Ölçüt olarak matematik öğretiminde modelleme lisans dersini alan öğrenciler esas alınmıştır. Katılımcıların %70'i ($n=42$) kadın, %30'u ($n=18$) erkek öğrencilerden oluşmaktadır. Öğretmen adaylarının, Yükseköğretim Kurulu (YÖK, 2023) tarafından belirlenen lisans program standartlarında öngörülen matematiksel bilgiye sahip oldukları varsayılmaktadır. Bu ders kapsamında modelleme ve problem çözme, modelleme süreci ve döngüsü, model geliştirme ilkeleri, sınıf içi uygulamalar ve öğretmen rolü, öğretmen adaylarının matematiksel düşünme süreçlerinin izlenmesi gibi içerikler yer almaktadır. Ders sonunda katılımcılar, modelleme yaklaşımlarını deneyimlemiş ve uygulamaya aktarmışlardır. Ayrıca, açık uçlu, farklı çözüm yollarına olanak tanıyan ve bağımsız düşünmeyi teşvik eden üç modelleme görevine bireysel olarak cevap vermişlerdir.

Bulgular

Çalışmada, matematik öğretmeni adaylarının üç farklı matematiksel modelleme görevinde gösterdikleri performanslar, modelleme yeterliği çerçevesi ve yaratıcılık göstergeleri bağlamında değerlendirilmiştir. Öğretmen adaylarının görevlerdeki yeterlik düzeyleri; süreci etkili kullanma, amaca yönelik çaba gösterme, problemi matematiksel açıdan anlama ve modeli doğrulama kriterlerine dayalı olarak analiz edilmiştir. Elde edilen bulgulara göre, öğretmen adayları en yüksek performansı yakıt ikmal (Görev 3) görevinde gösterirken, en düşük performansı ise ananasın soyulması (Görev 1) görevinde göstermiştir. Görev 1'de öğretmen adaylarının büyük bir bölümü silindirin geometrik özelliklerini tanımlamakta başarılı olsa da bu temel bilgileri matematiksel modellemeye dönüştürmede yetersiz kalmıştır. Görev 2'de, öğrencilerin yarısı orta düzey yeterlik göstermiştir; bu görevde futbol topunun geometrik yapısını anlamlandırmakta ve matematiksel ilişkilendirmeler kurmakta zorluklar yaşanmıştır. Temel süreçler yaratıcılık göstergeleri açısından değerlendirildiğinde, öğretmen adaylarının en yüksek yararlılık ve akıcılık performansını Görev 3'te sergilediği, en düşük yararlılık performansını ise Görev 2'de gösterdiği tespit edilmiştir. Orijinallik düzeyinde ise tüm görevlerde düşük performans dikkat çekmiştir. Öğretmen adaylarının tematik olarak sıradışı ya da yenilikçi fikirler üretme konusunda zorlandıkları görülmektedir. Friedman testi ve Dunn-Bonferroni sonrası yapılan analizlerde ise hem yeterlik hem yaratıcılık göstergelerinde görevler arasında anlamlı farklar olduğunu göstermiştir. Spearman korelasyon analizleri sonucunda, yeterlik ile özellikle orijinallik ve yararlılık arasında güçlü, akıcılıkla ise orta düzeye yakın ilişkiler bulunmuştur. Ancak yeterlik etkisi kontrol altına alındığında, yaratıcılık

göstergeleri arasındaki ilişkilerin gücünde azalma olduğu, yani bu boyutlar arasında doğrudan ilişkinin zayıfladığı belirlenmiştir.

Sonuç ve Tartışma

Modelleme süreçlerinde öğrencilerin performansları, ele alınan problem durumlarının yapısına göre anlamlı şekilde değişiklik göstermektedir. Özellikle yaratıcı düşünmeyi gerektiren ve önceden deneyimlenmemiş problem durumlarında öğrencilerin yeterlik düzeylerinin düştüğü, buna karşın işlem temelli yapıların yer aldığı durumlarda daha yüksek performans sergiledikleri gözlemlenmiştir (Lu & Kaiser, 2022a). Yararlılık göstergesi, öğrencilerin matematiksel çözüm yollarını değerlendirme, uygulama ve problem durumuna uygun bir şekilde kullanma kapasitelerini ölçmek açısından önemli bir belirteç olarak öne çıkmıştır. Ancak bu gösterge, Sriraman (2009) gibi bazı çalışmalarda yaratıcılığın tanım kriterleri arasında yer almasa da matematiksel modelleme bağlamında uygulanabilir ve paylaşılabilir yapısıyla (Lesh vd., 2000) önem kazanmaktadır. Lu ve Kaiser (2022b) da yararlılığı, modelleme süreçlerinde yaratıcı düşünmeyi belirlemek için özgün bir gösterge olarak değerlendirmiştir. Akıcılık ile modelleme yeterliği arasında orta düzeyde ilişki saptanmış, bu durum önceki araştırmalarla (Hébert vd., 2002; Runco, 2010) kısmen örtüşmektedir. Ayrıca, özellikle zorlayıcı ve özgün yapılandırma gerektiren durumlarda orijinallik ile yeterlik arasında daha güçlü ilişkiler kurulmuştur (Barbosa, 2006; English, 2009; Kaiser & Schwarz, 2006). Bu bağlamda, öğretim programlarında yaratıcı düşünmeyi destekleyen günlük yaşam durumlarını yansıtan modelleme etkinliklerine daha fazla yer verilmesi ve öğrencilerin çoklu çözüm yollarını deneyimleyebileceği öğrenme ortamlarının yaygınlaştırılması önerilmektedir. Nitekim bulgular, yararlılık, akıcılık ve orijinallik gibi temel yaratıcılık göstergelerinin modelleme yeterliklerinin yapısına dahil edilmesinin önemli olduğunu ortaya koymakta; bu doğrultuda yürütülecek benzer çalışmalara yeni bir bakış açısı kazandırmaktadır.