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A Mixture Partial Credit Analysis of Math Anxiety

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Abstract: The purpose of this study was to investigate a new methodology for detection of differences in middle grades students' math anxiety. A mixture partial credit model analysis was used to detect distinct latent classes based on homogeneities in response patterns. The analysis detected two latent classes. Students in Class 1 had less anxiety about *apprehension of math lessons* and *use of mathematics in daily life*, and more *self-efficacy for mathematics* than students in Class 2. Students in both classes were similar in terms of *test and evaluation anxiety*. Moreover, students in Class 1 were found to be more successful in mathematics, mostly like mathematics and mathematics teachers, and have better educated mothers than students in Class 2. Manifest variables of gender, attending private or public schools, and education levels of fathers did not differ among the latent classes. Characterizing differences between members of each latent class extends recent advances in measuring math anxiety.

1. INTRODUCTION

Identifying affective characteristics such as anger, anxiety, and depression that students experience in school settings and focusing on these characteristics in order to improve students' learning are significant challenges for educators. Math anxiety, as one such characteristic, can be defined as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551). Indicators of math anxiety include physical sensations of discomfort and distress during mathematics test-taking situations, feeling pressure to have the correct answers, fears of making mistakes and not understanding the given word problems especially in front of peers in a classroom (Luo, Wang, & Luo, 2009), and physiological reactions such as sweaty palms, being sick, and vomiting (Harper & Daane, 1998).

Math anxiety has been shown to cause low academic performance (Ashcraft, 2002), reduced cognitive information-processing (Young, Wu, & Menon, 2012), low working memories and

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spatial abilities (Novak & Tassell, 2017), and low perceptions of one's own mathematics abilities (Hembree, 1990). Low math abilities and low working memory, as well as non-supportive teachers can also be considered as important risk factors for the existence of math anxiety (Ashcraft, Krause, & Hopko, 2007). Math anxiety can lead to avoidance of taking math classes throughout the high school and college years and avoidance of selecting career paths involving mathematics (Ashcraft & Moore, 2009).

Identifying students with math anxiety in the middle grades is critical for dealing with math anxiety as early as possible because math anxiety is known to peak during the secondary grades (Hembree, 1990). Hill et al. (2016) considers the development of math anxiety as closely related to increasing educational demands when moving from lower grades to higher grades such as middle grades to secondary grades. Studies with prospective and in-service mathematics teachers have also demonstrated that math anxiety is even common in this group through their experiences with math anxiety as K-12 students, and this situation negatively influences how they teach mathematics to their students (Bekdemir, 2010; Gresham, 2017; Stoehr, 2017).

For detecting students' math anxiety, previous research has employed several methods. These include use of exploratory and confirmatory factor analysis to detect the dimensions of math anxiety (e.g., Baloğlu & Zelhart, 2007; Hopko, 2003; Kazelskis, 1998) and use of structural equation modeling to explain the relationship between math anxiety and several variables such as mathematics achievement (e.g., Harari, Vukovic, & Bailey, 2013; Krinzinger, Kaufmann, & Willmes, 2009; Meece, Wigfield, & Eccles, 1990). One common feature of these studies is that their analyses are at the total score level due to their use of the total scores on a math anxiety scale. One concern with use of total scores is that it may miss important information available in the patterns of item level responses. Another common feature of these studies is that all examinees in a given sample belong to a single population. In this study, we examine item level patterns of math anxiety with an eye to detecting differences that may exist between latent classes in the population.

Given the potential deleterious effects of math anxiety on achievement and career choice, better methods for accurately measuring math anxiety are important in order to be able to ameliorate its effects. One such method is the use of mixture item response theory (IRT) models (Mislevy & Verhelst, 1990; Rost, 1990). These models may be appropriate when distinct latent classes are suspected. In the context of the present study, a latent class indicates a statistically determined grouping of students with homogeneous response patterns. That is, distinct latent classes are reflected in the patterns of responses to items on the instrument (Bolt, Cohen, & Wollack, 2001). The classes are labeled as latent because they are not directly observable. Item parameters are allowed to differ between latent classes in a mixture IRT model reflecting the differences in response propensities between the latent classes (Izsák, Jacobson, de Araujo, & Orrill, 2012). Research on the use of mixture IRT models has shown that they can be useful in detecting latent classes of individuals that differ along one or more dimensions. Previous research, for example, has found mixture IRT models to be useful in understanding differences in teachers' mathematical reasoning (Izsák, Orrill, Cohen & Brown, 2010), differences in mathematical knowledge (Izsák et al., 2012), differences in response to test time limits (Bolt, Cohen & Wollack, 2002), differences in strategy use in solving problems (Bolt, Cohen & Wollack, 2001) and differences in personality traits such as depression (Hong & Min, 2007) and creativity (Sen, 2016). In addition, Cho, Bottge, Cohen and Kim (2011) have shown this method to provide instructionally useful information for individual latent classes. In this study, we used mixture IRT methodology to explore item level differences in patterns of math anxiety based on middle grades students' responses to a math anxiety scale.

The purpose of this study was to investigate the use of mixture IRT methodology for detecting distinct latent classes of math anxiety among middle grades students that differ in their item level patterns of math anxiety. The following research questions were addressed:

1. Are there distinct latent classes of middle grades students that differ in their math anxiety?

2. What does the existence of these latent classes imply about the different response patterns of math anxiety that exist in this population?

3. Do latent classes differ with respect to several manifest variables such as mathematics achievement, gender, liking mathematics, liking mathematics teachers, attending private or public school, education levels of mothers and fathers?

1.1. The Rasch Model

The Rasch Model is a probabilistic model which is used to express item difficulties and examinee abilities on the same scale. The probability of an examinee correctly answering an item is a function of the difference between the examinee's latent ability and the item difficulty (Rasch, 1960/1980). The probability of obtaining the correct answer with respect to each item is given as follows:

$$P(x=1|\theta_j, \beta_i) = \frac{\exp(\theta_j - \beta_i)}{1+\exp(\theta_j - \beta_i)} \qquad (1)$$

where θ_j is the latent ability parameter of an examinee j and β_i is the difficulty of item i. The probability of answering a given item correctly is expected to be relatively higher for an examinee with higher ability compared to an examinee with lower ability.

1.2. The Mixture Rasch Model

The mixture Rasch model (MRM; Rost, 1990) is a combination of a latent class model (Lazarsfeld & Henry, 1968) and a Rasch model. In contrast to the standard Rasch model, which assumes that the same Rasch model applies to all examinees in the population, the MRM assumes that distinct latent classes exist in the population and that a different Rasch model applies to each. Hence, the MRM allows different Rasch models to apply to different latent classes in the population.

In the MRM, the relative difficulty of the items is determined by a class membership parameter. The number of items which the examinee is expected to answer correctly is influenced by a continuous latent ability specific to the latent class. For each item, the MRM estimates a separate item difficulty for each latent class and for each examinee, a probability of being a member of a particular latent class.

The dichotomous form of the mixture Rasch model is employed when an item can be scored in two categories, such as agree or disagree. This form can be expressed as follows:

$$P(x_{ij}=1|g, \theta_{jg},) = \frac{\exp(\theta_{jg} - \beta_{ig})}{1+\exp(\theta_{jg} - \beta_{ig})} \qquad (2)$$

where P is the probability of a correct response in the mixture Rasch model, g is an index for the latent class ($g = 1, 2, \ldots, G$), θ_{jg} is the latent ability of an examinee j within class g, and β_{ig} is the difficulty parameter of item i for class g. When there is only one latent class, the mixture Rasch model is the same as the Rasch model in equation (1).

The polytomous form of the mixture Rasch model is used when items are scored with more than two categories. This type of items can be used when an answer is given partial credit rather than full credit or when an answer is in one of several categories such as strongly agree, agree, neutral, disagree, or strongly disagree. This form of the Rasch model is called a partial credit model (PCM; Masters, 1982). The probability of an answer for the mixture form of this model, the mixture partial credit model (MixPCM), can be written as follows:

$$P(x_{ij}=k|\theta_{jg}) = \frac{\exp[\sum_{r=1}^{k}(\theta_{jg}-\delta_{irg})]}{\sum_{t=0}^{m_i}[\exp\sum_{r=1}^{t}(\theta_{jg}-\delta_{irg})]} \quad (3)$$

where P is the probability that examinee j gives a response in category k of item i, θ_{jg} is the latent trait of an examinee j in latent class g, and δ_{irg} is a threshold parameter indicating the intersection of adjacent category response curves.

As can be seen in equation (3), the relationship between the probability of selecting a response in a given category and the latent trait is allowed to vary between latent classes. The differences in response patterns to each item of a questionnaire reflect homogeneities in characteristics of members of each latent class. In the MixPCM, the relative difficulty of a particular response category among the ordered categories is determined by a class membership parameter and the number of items answered. In this way, it is possible that the MixPCM could assign two examinees with similar test scores to different latent classes as a result of the differences in their response patterns.

2. METHOD

2.1. Participants

The sample consisted of 244 Turkish 6^{th} and 7^{th} grade students attending public and private schools in southwestern Turkey (Table 1). Parental consent was obtained through signed letters prior to the study.

Table 1. Gender and grade levels of the participants

Gender	N	%	Grade Level	N	%	School Type	N	%
Male	128	52.5	6th Grade	120	49.2	Public	152	37.7
Female	116	47.5	7th Grade	124	50.8	Private	92	62.3
Total	244							

2.2. Instruments

The *Math Anxiety Scale* (MANX; Erol, 1989) is a 45-item scale written in Turkish. Each item has four options, scored from 1 (“never”) to 4 (“always”). Scores can range from 45 to 180 points. Higher scores demonstrate a higher level of math anxiety.

Erol (1989) reported an internal consistency reliability estimate for the MANX of .91 on a sample of 380 high school students. The internal consistency reliability estimate in this study was .90. This was consistent with previous results on the MANX of .92 on a sample of 754 middle school and high school students (Erktin, Dönmez & Özel, 2006). Erktin et al. detected four factors that explained 40% of the variance. These factors were *test and evaluation anxiety*, *apprehension of math lessons*, *use of mathematics in daily life*, and *self-efficacy for mathematics*. The English translation of the MANX and its underlying factors are presented in Appendix A.

Demographic information was also obtained on a questionnaire attached to the MANX about students' mathematics grades at the end of the previous semester (i.e., grades ranging from 1 to 5), their gender (i.e., "male" or "female"), whether or not they liked mathematics (i.e., "Yes" or "No"), whether or not they liked their mathematics teacher (i.e., "Yes" or "No"), the type of school they attended (i.e., "Public" or "Private"), and their parents' education levels (i.e., "illiterate", "primary school", "secondary school", or "college"). Students were able to complete the MANX and the demographic information questionnaire in 30 minutes.

2.3. Data Analysis

Before implementing the MixPCM analysis, the dimensionality of the data was checked for ensuring the unidimensionality assumption by using exploratory factor analysis. The data were analyzed using the MixPCM as implemented in the computer program WINMIRA (von Davier, 2001). An exploratory MixPCM analysis was used to determine the number of latent classes in the data. This was done by fitting different MixPCM models with different numbers of latent classes to the data. In this way, the MPCM was estimated with one, two, three, and four latent classes. Three information indices were compared to select the best fitting model: Akaike's information criterion (AIC; Akaike, 1974), the Bayesian information criterion (BIC; Schwarz, 1978), and the consistent AIC (CAIC; Bozdogan, 1987). Each information criterion index is defined as follows:

$$\text{AIC} = -2 \log L + 2p$$

$$\text{BIC} = -2 \log L + p (\log N)$$

$$\text{CAIC} = -2 \log L + p (\log N + 1)$$

where L is the value at the maximum of the likelihood, p is the number of estimated parameters, and N is the sample size. AIC, BIC, and CAIC all include penalty functions to modify the $-2 \times$ log likelihood for either the number of parameters or the sample size or both. Because AIC has been found to be less accurate due to its sensitivity to sample size (Baghaei & Carstensen, 2013; Li, Cohen, Kim, & Cho, 2009), the model with the smallest BIC values were selected as the best fitting model. Next, the characteristics of each latent class were analyzed by examining differences in item thresholds between latent classes. In addition, differences in manifest variables between latent classes were evaluated using independent sample t-tests and chi-square tests between the latent classes.

3. FINDINGS

3.1. Unidimensionality for the Scale

An exploratory factor analysis using maximum likelihood estimation as implemented in the SPSS 16.0 software (SPSS Inc., 2007) indicated eigenvalues of the first three factors as 14.1, 2.6, and 2.5. The total variance explained by the first factor was 31.4%. Even though three factors were larger than 1.0, using Reckase's (1979) criterion, the assumption of unidimensionality also could be inferred.

3.2. Model Selection

Values for the three information indices are given in Table 2. Minimum values for AIC, BIC, and CAIC of 12883.82, 13705.72, and 13978.72, respectively, all suggested a two-class solution in the data. Based on results in Table 2, the two-class MixPCM model was determined to be the best fit.

Table 2. Model fit indices of the Mixture Rasch model

Model	AIC	BIC	CAIC
One class	13757.02	14166.47	14302.47
Two classes	**12883.82**	**13705.72**	**13978.72**
Three classes	13091.45	14325.81	14735.81
Four classes	13335.07	14981.88	15528.88

Note. AIC = Akaike information criterion; BIC = Bayesian information criterion; CAIC = Consistent Akaike information criterion; the smallest information criterion index is bold.

After determining the best fitting model (see Table 2), mean assignment probabilities were calculated for each latent class (see Table 3). Students were classified into the latent class for which they had the highest mean assignment probability. Low off-diagonal values in comparison to the high diagonal values suggest that the two-class solution had good mean assignment probabilities, with Class 1 consisting of 126 students (51.5%) and Class 2 consisting of 118 students (48.5%).

Table 3. Mean assignment probabilities for the two-class solution

Latent Class	Proportion in Latent Class	Mean Assignment Probability Class 1	Mean Assignment Probability Class 2
Class 1	51.5	0.999	0.001
Class 2	48.5	0.005	0.995

Item thresholds indicate the point on the latent continuum between adjacent score categories and indicate the relative ease of endorsing each of the four categories by members of each of the two latent classes. As thresholds decrease, the likelihood of endorsement of particular response category increases. Item thresholds for each latent class are plotted in Figure 1 and Figure 2. Thresholds lower on the scale (e.g., -3, -2) indicate that examinees had a greater propensity to endorse that response category. Similarly, thresholds higher on the scale (e.g., 2, 3) indicate that examinees had a greater propensity to endorse a higher response category. Thresholds may differ by latent class. This means that, the relative propensity for endorsing a category of an item is specific to each latent class. Because the MANX has four response categories ranging from “never” to “always” for each item, there are three possible thresholds that can be used to interpret the math anxiety level as follows:

Categories:	“never”	“sometimes”	“usually”	“always”
Scores:	(1)	(2)	(3)	(4)

Thresholds: |--------T1--------|--------T2--------|--------T3--------|

For example, if an examinee’s trait level is smaller than the first threshold, then the response is expected to be “never.” If an examinee’s trait level is smaller than the second threshold but larger than the first threshold, then the response is expected to be “sometimes.”

Figure 1 and Figure 2 present plots of the item thresholds for Class 1 and Class 2 (see also Appendix A for item threshold values). It is evident that students in Class 2 were less variable in making their endorsements than students in Class 2, with thresholds ranging from -7.41 to

9.00. Students in Class 1 had lower tendency to endorse items above threshold 1, but greater tendency to endorse items above threshold 3.

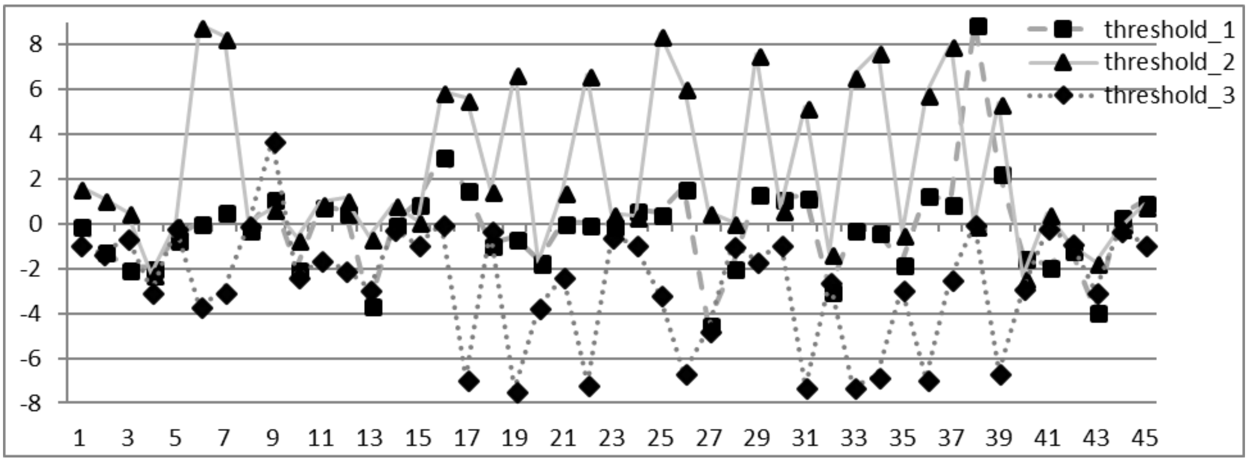

Figure 1. Item thresholds for Class 1

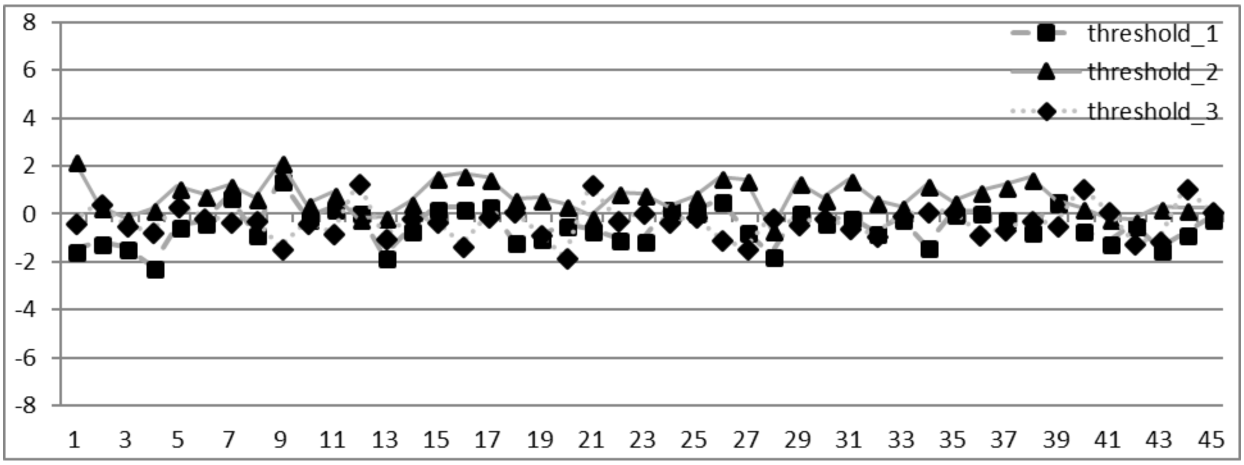

Figure 2. Item thresholds for Class 2

3.3. Analyses of Mean Item Thresholds and Item Response Distributions

The mean item threshold is the mean of all item thresholds for an item (Masters, 1982). Higher mean item thresholds indicate lower propensities of endorsement. In addition to the mean item thresholds, item response distributions were compared between the two latent classes to examine similarities and differences in item responses for each latent class. Analyses of the mean item thresholds and the item response distributions led to three main results: (1) Students in Class 1 were less anxious than students in Class 2 in terms of having anxiety about *apprehension of math lessons* and *use of mathematics in daily life*. Students in Class 1 also expressed feelings of more *self-efficacy for mathematics*, as evidenced by greater propensity to endorse items such as feeling comfortable asking the teacher questions in class. Students in both latent classes, however, had similar levels of *test and evaluation anxiety*.

Differences in the mean item thresholds of 1 logit or more were considered as reflecting differences between the two latent classes. The mean item thresholds are provided for the two latent classes in Appendix B. Items 4, 6, 7, 9, 10, 13, 16, 20, 25, 27, 29, 32, 35, 37, 38, 40, and 43 appeared to be have different response propensities for Class 1 and Class 2. Moreover, these response propensity differences in the mean item thresholds are plotted in Figure 3 along with descriptions of the content of each item in ascending order of the mean item thresholds.

Logit	Class 1	Class 2
3	Bothered by math book (16); Uneasiness with calculations in daily life (38)	
	Rejecting help (29)	
	Uneasiness with the next class (37)	
2	Butterflies in one's stomach (7)	
	Feeling uneasy (25)	
	Panic (6); Confusing the change (9)	
1		Confusing the change (9); Butterflies in one's stomach (7)
		Rejecting help (29); Feeling uneasy (25)
		Bothered by math book (16); Liking math problems (40); Liking math sections (35)
		Uneasiness with calculations in daily life (38);
		Uneasiness because the next class is math (37);
0		Panic (6)
		Keeping the account (10)
		Easy to ask teacher questions (27); Enjoying geometry (32)
		First math class with hope (20)
		Being relaxed while waiting for exam results (43)
		Liking math homework (4);
-1		Liking numbers or graph (13)
	Liking math sections (35); Keeping the account (10)	
	Liking math problems (40); Enjoying geometry (32);	
-2	First math class with hope (20)	
	Liking math homework (4); Liking numbers or graph (13)	
	Being relaxed while waiting for exam results (43)	
	Easy to ask teacher questions (27)	
-3		

Figure 3. Ordering items by mean item thresholds for each class

Note. Items with difference of 1 logit or more are ordered. Parentheses refer to item numbers. Vertical axis is the logit scale.

Items 6, 7, 16 and 37 reflect anxiety in the form of *apprehension of math lessons*. Students in Class 1 were less likely than students in Class 2 to select a higher numbered category. This is represented in Figure 3 by showing the item text for these items listed under Class 1 and Class 2. For example, Item 16, "Bothered by Math book," is shown under the columns of Class 1 and Class 2 in Figure 3. The mean item threshold for Class 1 was 3.00 but for Class 2, it was .25 (see Appendix B). The proportions selecting the options of "never" and "always" in Class 1 were 98.7% and 0% respectively, in contrast to 53.3% and 15.1% in Class 2, respectively. Similarly, for Item 37, "Uneasiness because the next class is math," the mean item threshold for Class 1 was 2.19, and for Class 2, it was .15. These values indicate that students in Class 1 were less likely to endorse a higher numbered category for Item 37 than students in Class 2.

Item 6 asked students to indicate how much they "Panic" when a lot of mathematics problems are given as homework and Item 7 asked whether they get "Butterflies in one's stomach" when studying a hard mathematics topic. For Item 6 and Item 7, Class 1 mean item thresholds were 1.78 and 2, respectively, and for Class 2, the mean item thresholds were .14 and .58, respectively. This suggests that students in Class 1 mostly agreed with the option "never" (80.2% for Item 6 and 87.1% for Item 7) in comparison to students in Class 2 (38.6% for Item 6 and 64.2% for Item 7).

On the other hand, students in Class 1 more strongly endorsed items 4, 10, 13, 20, 32, 35, and 40. These contained positive statements related to mathematics lessons such as liking mathematics sections of the social classes, liking mathematics problems and homework, and liking numbers and geometry. For example, for Item 40, "Liking math problems," the mean item threshold for Class 1 was -2.21 and for Class 2, it was .25. The proportion selecting the option "always" was 70.3% in Class 1 as compared to 7.8% in Class 2.

Items 9, 29 and 38 focused on anxiety about the *use of mathematics in daily life*. Students in Class 1 were less likely to select a higher category for these items than for students in Class 2 (see Figure 3). As an example, a marked difference occurred in Item 38, "Uneasiness with calculations in daily life" with Class 1 mean item threshold was 3.00 and Class 2 mean item threshold was .19. For this item, all students in Class 1 selected the option "never," while 34% of the students in Class 2 made this choice.

Items 2, 3, 8, 11, 14, 18, 19, 21, 22, 24, 25, 28, 30, 33, 41, 42, and 44 asked students to rate their ideas about *test and evaluation anxiety*. Mean item thresholds as well as the distributions of responses were similar for both latent classes. The one exception to this trend occurred on Item 25, "Feeling uneasy." The item asked students to rate if they felt uneasy the week before a math exam. The mean item threshold for Class 1 was 1.95 and for Class 2, it was .27. For this item, 85.8% of the students in Class 1 selected the option "never," and 47.7% of the students in Class 2 selected this choice.

Finally, Items 27 and 43 involved *self-efficacy for mathematics*. Mean item thresholds and the distribution of responses were different across response categories for the two latent classes (see Figure 3). Item 27 asked whether students found it "Easy to ask teacher questions." The mean item threshold for Class 1 was -2.87 and for Class 2, it was item -.23. The "always" option was selected by 76.3% of the Class 1 students, but only 26.2% of the Class 2 students answered "always." For Item 43, "Being relaxed while waiting for exam results," the mean item threshold for Class 1 was -2.84 and for Class 2 it was -.73. For this item, the proportion selecting the option "always" in Class 1 was 72.8% compared to 39.5% in Class 2. Thus, Item 43 was easier Class 1 students to respond "always" and harder to respond "never."

3.4. The Relationships between Manifest Variables and Latent Class Membership

To obtain detailed information about the characteristics of each latent class, the relationships between the manifest variables and latent class membership were examined using independent sample t-tests and chi-square tests. Based on students' responses, the mean ability logit score in Class 1 (M = -1.34) was significantly lower than the mean score in Class 2 (M = -.27) (t (df = 148) = -12.94, $p < .01$). This indicated that students in Class 1 were less anxious than those in Class 2. Regarding mathematics achievement, there was a significant difference between the two latent classes (t (df = 111) = 3.71, $p < .01$), suggesting that the students in Class 1 were more successful in mathematics than students in Class 2. Furthermore, mother's education level was significantly higher for students in Class 1 than Class 2 (t (df = 136) = 2.36, $p < .05$), but there was no significant difference between the latent classes with respect to fathers' education level (t (df= 136) = 1.07, $p = .29$).

A chi-square test between the two latent classes for gender was not significant ($\chi^2(1) = .98$, $p = .32$). On the other hand, the associations between students' liking mathematics and liking their mathematics teachers, and latent class membership were found as significant ($\chi^2(1) = 11.83$, $p < .01$ and $\chi^2(1) = 6.30$, $p < .01$, respectively). This indicated that students' being in Class 1 or Class 2 is related to their liking mathematics and liking their mathematics teachers. Finally, there was no association between students' attending private or public schools and latent class membership ($\chi^2(1) = .57$, $p = .45$).

4. DISCUSSION AND CONCLUSION

In the present study, a mixture partial credit model (MixPCM) was used to detect differences in math anxiety of middle grades students. With respect to the first research question regarding existence of latent classes, two latent classes were detected, indicating the presence of distinct latent classes exist of math anxiety. The classes were similar in size but had different patterns of math anxiety.

With respect to the second research question regarding characteristics of latent classes, Class 1 consisted of students who were less anxious about *apprehension of math lessons* and *use of mathematics in daily life*, and who had more *self-efficacy for mathematics* than students in Class 2. However, there did not exist any differences between Classes 1 and 2 in terms of *test and evaluation anxiety*.

With respect to the third research question regarding the effects of manifest variables on class membership, students in Class 1 also appeared to be less anxious than students in Class 2, as evidenced by their mean scores on the MANX. In addition, students in Class 1 were reported being more successful in mathematics, liked mathematics and mathematics teachers, and had better educated mothers in comparison to students in Class 2. No significant association was found between the two latent classes for either gender, attending private or public schools, or fathers' education level.

The results reported here on the relationships between math anxiety and the manifest variables were consistent with findings in the literature. As previous research indicates that math anxiety was negatively related to mathematics achievement (e.g., Hembree, 1990), students in Class 1, in the present study, reported being less anxious and more successful in mathematics. Moreover, previous research on the effects of positive attitudes and education levels of mothers on math anxiety has found that positive attitudes towards mathematics and education levels of mothers were negatively associated with math anxiety (e.g., Engelhard, 1990; Meece, Wigfield, & Eccles, 1990). Similarly, students in Class 1 reported being less anxious and being more likely to have positive attitudes such as enjoying mathematics and liking their mathematics teachers and to have mothers with higher education levels than students in Class 2. On the other hand, there did not appear to be consensus from previous research regarding the effects of gender, type of school attended, or education levels of fathers on math anxiety (Alkan, 2018). Some studies reported significant effects of gender on math anxiety (e.g., Luo et al., 2009; Wigfield & Meece, 1988), but others found no associations (e.g., Birgin, Baloglu, Catlioglu, & Gurbuz, 2010). Incorporating the analysis of manifest variables into the MixPCM analysis and obtaining consistent results with previous research strengthen the validity of the interpretation about the characteristics of each latent class reported in this study.

The results of this study suggest that within a population of students there exist latent classes that differ in their math anxiety. Use of the MixPCM provided information at the item level that revealed potentially useful distinctions that may not be easily detectable at the total score level. As such, this methodology provides a useful tool for identifying latent classes of students with different patterns of math anxiety. In school settings, teachers can use results from the MixPCM to detect those students and provide appropriate interventions specific to the needs of students in each latent class. For example, they can provide a classroom environment supporting students in each latent class instead of highlighting their mistakes; focus on reducing some particular students' anxiety levels towards mathematics lessons by not calling on these students to solve a problem at the board; engage some students with more mathematics related activities in daily life by presenting simulated real-life situations and asking word problems in a real-life context; and help some students build self-confidence for mathematics through asking mathematical problems from simple to more complex.

The present study takes a step towards detecting math anxiety by examining the utility of a new methodology for detection of distinct latent classes based on different patterns of math anxiety. The results provide evidence that the MixPCM, when applied to a math anxiety scale, can provide fine-grained information about latent classes of middle grades students and their differences in math anxiety.

Future studies focusing on detecting and characterizing latent classes of students with respect to math anxiety in different populations would be helpful. Such studies will help provide a more complete understanding about middle grades students' math anxiety.

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5. REFERENCES

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, *19*, 716–723.

Alkan, V. (2018). A systematic review research: 'Mathematics anxiety' in Turkey. *International Journal of Assessment Tools in Education*, 5(3), 567–592.

Ashcraft, M. H. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current Directions in Psychological Science*, *11*(2), 181–185.

Ashcraft, M. H., Krause, J. A., & Hopko, D. R. (2007). Is math anxiety a mathematical learning disability? In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 329–348). Baltimore: Brookes.

Ashcraft, M. H., & Moore, A. M. (2009). Mathematics anxiety and the affective drop in performance. *Journal of Psychoeducational Assessment*, *27*(3), 197–205.

Baghaei, P., & Carstensen, C. H. (2013). Fitting the mixed Rasch model to a reading comprehension test: Identifying reader types. *Practical Assessment, Research & Evaluation, 18*(5), 1–12.

Baloğlu, M. & Zelhart, P. F. (2007). Psychometric properties of the revised mathematics anxiety rating scale. *The Psychological Record, 57*, 593–611.

Bekdemir, M. (2010). The pre-service teachers' mathematics anxiety related to depth of negative experiences in mathematics classroom while they were students. *Educational Studies in Mathematics, 75*, 311–328.

Birgin, O., Baloglu, M., Catlioglu, H., & Gurbuz, R. (2010). An investigation of mathematics anxiety among sixth through eighth grade students in Turkey. *Learning and Individual Differences, 20*, 654–658.

Bolt, D. M., Cohen, A. S., & Wollack, J. A. (2001). A mixture item response for multiple-choice data. *Journal of Educational and Behavioral Statistics, 26*, 381–409.

Bolt, D. M., Cohen, A. S., & Wollack, J. A. (2002). Item parameter estimation under conditions of test speededness: Application of a mixture Rasch model with ordinal constraints. *Journal of Educational Measurement, 39*(4), 331–348.

Bozdogan, H. (1987). Model selection and Akaike's information criterion (AIC): The general theory and its analytic extensions. *Psychometrika, 52*, 345–370.

Cho, S.-J., Bottge, B. A., Cohen, A. S., & Kim, S.-H. (2011). Detecting cognitive change in the math skills of low-achieving adolescents. *The Journal of Special Education, 45*(2), 67–76.

Cohen, A. S., & Bolt, D. M. (2005). A mixture model analysis of differential item functioning. *Journal of Educational Measurement, 42*(2), 133–148.

Engelhard, G. (1990). Math anxiety, mother's education, and the mathematics performance of adolescent boys and girls: Evidence from the U.S. and Thailand. *Journal of Psychology, 124*(3), 289–298.

Erktin, E., Dönmez, G., & Özel, S. (2006). Psychometric characteristics of the mathematics anxiety scale. *Education and Science, 31*(140), 26–33.

Erol, E. (1989). *Prevalence and correlates of math anxiety in Turkish high school students*. Unpublished master thesis, Bogazici University.

Gresham, G. (2017). Preservice to inservice: Does mathematics anxiety change with teaching experience. *Journal of Teacher Education, 69*(1), 90–107.

Harari, R. R., Vukovic, R. K., & Bailey, S. P. (2013). Mathematics anxiety in young children: An exploratory study. *The Journal of Experimental Education, 81*(4), 538–555.

Harper, N. W., & Daane, C. J. (1998). Causes and reduction of math anxiety in preservice elementary teachers. *Action in Teacher Education, 19*(4), 29–38.

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education, 21*, 33–46.

Hill, F., Mammarella, I. C., Devine, A., Caviola, S., Passolunghi, M. C., & Szücs, D. (2016). Math anxiety in primary and secondary school students: Gender differences, developmental changes and anxiety specificity. *Learning and Individual Differences, 48*, 45–53.

Hong, S., & Min, S. (2007). Mixed Rasch modeling of the Self-Rating Depression Scale: Incorporating Latent Class and Rasch Rating Scale models. *Educational and Psychological Measurement*, *67*(2), 280–299.

Hopko, D. R. (2003). Confirmatory factor analysis of the Math Anxiety Rating Scale-Revised. *Educational and Psychological Measurement*, *63*(2), 336–351.

Izsák, A., Jacobson, E., de Araujo, Z., & Orrill, C. H. (2012). Measuring mathematical knowledge for teaching fractions with drawn quantities. *Journal for Research in Mathematics Education, 43*(4), 391–427.

Izsák, A., Orrill, C. H., Cohen, A. S., & Brown, R. E. (2010). Measuring middle grades teachers' understanding of rational numbers with the mixture Rasch model. *The Elementary School Journal, 110*, 279–300.

Kazelskis, R. (1998). Some dimensions of mathematics anxiety: A factor analysis across instruments. *Educational and Psychological Measurement, 58*, 623–633.

Krinzinger, H., Kaufmann, L., & Willmes, K. (2009). Math anxiety and math ability in early primary school years. *Journal of Psychoeducational Assessment, 27*(3), 206–225.

Lazarsfeld, P. F., & Henry, N. W. (1968). *Latent structure analysis*. Boston, MA: Houghton Mifflin.

Li, F., Cohen, A. S., Kim, S. H., & Cho, S. J. (2009). Model selection methods for dichotomous mixture IRT models. *Applied Psychological Measurement, 33*(5), 353–373.

Luo, X., Wang, F., & Luo, Z. (2009). Investigation and analysis of mathematics anxiety in middle school students. *Journal of Mathematics Education, 2*(2), 12–19.

Masters, G. N. (1982). A Rasch model for partial credit scoring. *Psychometrika, 47*, 149-174.

Meece, J. L., Wigfield, A., & Eccles, J. S. (1990). Predictors of math anxiety and its influence on young adolescents' course enrollment intentions and performance in mathematics. *Journal of Educational Psychology, 82*(1), 60–70.

Mislevy, R. J., & Verhelst, N. (1990). Modeling item responses when different subjects employ different solution strategies. *Psychometrika, 55*, 195–215.

Novak, E., & Tassell, J. L. (2017). Studying preservice teacher math anxiety and mathematics performance in geometry, word, and non-word problem solving. *Learning and Individual Differences, 54*, 20–29.

Rasch, G. (1960). *Probabilistic models for some intelligence and attainment tests*. Copenhagen, Denmark: The Danish Institute of Education Research. (Expanded edition (1980) with foreword and afterword by B.D. Wright. Chicago, IL: The University of Chicago Press)

Reckase, M. D. (1979). Unifactor latent trait models applied to multifactor tests: Results and implications. *Journal of Educational Statistics, 4*, 207–230.

Richardson F. C., & Suinn, R. M. (1972). The mathematics anxiety rating scale: Psychometric data. *Journal of Counseling Psychology, 19*(6), 551–554.

Rost, J. (1990). Rasch models in latent classes: An integration of two approaches to item analysis. *Applied Psychological Measurement, 14*, 271–282.

Schwarz, G. (1978). Estimating the dimension of a model. *Annals of Statistics, 6*, 461–464.

Sen, S. (2016). Applying the mixed Rasch model to the Runco ideational behavior scale. *Creativity Research Journal, 28*(4), 426–434.

SPSS Inc. (2007). SPSS for Windows, Version 16.0. Chicago, SPSS Inc.

Stoehr, K. J. (2017). Mathematics anxiety: One size does not fit all. *Journal of Teacher Education, 68*(1), 69–84.

Von Davier, M. (2001). WINMIRA [Computer Software]. St. Paul, MN: Assessment Systems Corporation.

Wigfield, A. & Meece, J. L. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology, 80*, 210–216.

Young, C. B., Wu, S. S., Menon, V. (2012). The neurodevelopmental basis of math anxiety. *Psychological Science, 23*(5), 492–501.

Appendix A. Item information of the MANX across three latent classes

Items	Content	Underlying Factor	Class 1 threshold 1	Class 1 threshold 2	Class 1 threshold 3	Class 2 threshold 1	Class 2 threshold 2	Class 2 threshold 3
Item 1	When my friend is asked to answer a question in Math class, I feel glad that I am not in his/her shoes.	Apprehension of Math Lessons	-0.02	1.67	-0.91	-1.46	2.25	-0.33
Item 2	I panic when I start the Math part of a common test.	Test and Evaluation Anxiety	-1.14	1.14	-1.31	-1.15	0.32	0.49
Item 3	I am intimidated when asked to answer a question that I do not completely know the answer to.	Test and Evaluation Anxiety	-1.96	0.55	-0.59	-1.40	-0.15	-0.44
Item 4	I like doing Math homework.***	Apprehension of Math Lessons	-1.92	-2.21	-3.00	-2.19	0.21	-0.73
Item 5	I dislike the formulas in the science classes.	Apprehension of Math Lessons	-0.66	-0.04	-0.17	-0.46	1.15	0.37
Item 6	I panic when I am assigned homework which includes a lot of math problems.	Apprehension of Math Lessons	0.11	8.89	-3.67	-0.30	0.82	-0.10
Item 7	I feel butterflies in my stomach when I prepare to study a hard math topic.	Apprehension of Math Lessons	0.64	8.36	-3.00	0.75	1.25	-0.26
Item 8	I become unable to think about anything one hour prior to math exam.	Test and Evaluation Anxiety	-0.19	0.06	-0.06	-0.81	0.71	-0.22

Item 9	I feel confused when I try to count the change I received from my purchase from the school cafeteria; most of the time I just get what is given to me without counting the change.	Use of Mathematics In Daily Life	1.19	0.73	3.72	1.45	2.21	-1.37
Item 10	I would like to keep the accounts for a school club or activity that I am participating in.***	Apprehension of Math Lessons	-1.96	-0.63	-2.34	-0.16	0.43	-0.32
Item 11	I feel intimidated to check my math score when I am given my class report.	Test and Evaluation Anxiety	0.82	0.97	-1.60	0.31	0.88	-0.78
Item 12	I feel reluctant to explain the problems even the ones I can solve.	Self-efficacy for Mathematics	0.54	1.16	-2.03	0.10	-0.16	1.34
Item 13	I like any topic explained to me in numbers and graphs rather than verbal explanations.***	Apprehension of Math Lessons	-3.54	-0.58	-2.895	-1.73	-0.11	-0.97
Item 14	I feel terrible a day before the math exam.	Test and Evaluation Anxiety	0.07	0.89	-0.23	-0.60	0.51	-0.13
Item 15	Even if I think that a shop clerk gave me the wrong change, I say nothing because I cannot make calculations while somebody is watching me.	Use of Mathematics In Daily Life	0.95	0.14	-0.92	0.28	1.55	-0.27
Item 16	The math book bothers me.	Apprehension of Math Lessons	3.08	5.92	0.00	0.31	1.70	-1.27
Item 17	I cannot even make an addition operation while somebody is watching.	Use of Mathematics In Daily Life	1.60	5.61	-6.89	0.39	1.52	-0.05
Item 18	I become so nervous before the important math exams that I forget all I know.	Test and Evaluation Anxiety	-0.89	1.51	-0.28	-1.08	0.64	0.13

Item 19	I feel afraid when the teacher gives a pop quiz on math.	Test and Evaluation Anxiety	-0.60	6.78	-7.41	-0.97	0.68	-0.81
Item 20	I always enter the first math class of the year with hope.***	Apprehension of Math Lessons	-1.62	-1.68	-3.68	-0.43	0.41	-1.79
Item 21	While studying for a math exam, I may not prepare enough at times because of worrying about the score I will get.	Test and Evaluation Anxiety	0.11	1.49	-2.36	-0.60	-0.07	1.26
Item 22	I feel an inability to succeed while going through the pages of the math book.	Test and Evaluation Anxiety	0.04	6.69	-7.14	-1.01	0.93	-0.22
Item 23	I cannot dare to ask the points I do not get in the math class.	Self-efficacy for Mathematics	-0.01	0.49	-0.55	-1.04	0.87	0.08
Item 24	I feel uneasy even when I calculate the GPA for my class report.	Test and Evaluation Anxiety	0.67	0.38	-0.92	0.27	0.29	-0.29
Item 25	I feel uneasy a week before the math exam.	Test and Evaluation Anxiety	0.52	8.48	-3.15	0.14	0.76	-0.08
Item 26	Even making calculation related to time gives me discomfort.	Use of Mathematics In Daily Life	1.63	6.01	-6.64	0.61	1.56	-1.02
Item 27	I can easily ask something that I did not understand to the math teacher after the class.***	Self-efficacy for Mathematics	-4.41	0.54	-4.73	-0.70	1.44	-1.43
Item 28	I feel nervous and pessimistic while waiting for the announcement of the result for a math exam that I think I failed at.	Test and Evaluation Anxiety	-1.90	0.10	-0.95	-1.67	-0.62	-0.11

Item 29	When I am asked to help a primary school student with his/her homework, I may refuse to help because I feel afraid that there may be some problems I couldn't solve.	Use of Mathematics In Daily Life	1.41	7.59	-1.63	0.12	1.34	-0.40
Item 30	When I think of the math subjects I have to learn before graduating from high school, I doubt if I am ever going to finish school.	Test and Evaluation Anxiety	1.17	0.71	-0.88	-0.33	0.68	-0.11
Item 31	Dealing with numbers makes me annoyed.	Apprehension of Math Lessons	1.22	5.25	-7.26	-0.11	1.44	-0.54
Item 32	Geometry questions remind me of fun puzzles.***	Apprehension of Math Lessons	-2.93	-1.29	-2.56	-0.72	0.56	-0.86
Item 33	I feel tense when my friend solves a problem and I cannot understand his/her solution.	Test and Evaluation Anxiety	-0.20	6.66	-7.27	-0.12	0.32	-0.03
Item 34	I feel confused in math class.	Apprehension of Math Lessons	-0.30	7.75	-6.77	-1.31	1.25	0.13
Item 35	The most likable part of the social classes are the parts that consist of math, even if they are miniscule.***	Apprehension of Math Lessons	-1.73	-0.40	-2.89	0.07	0.53	0.13
Item 36	I struggle with listening to the teacher in the math class.	Apprehension of Math Lessons	1.36	5.84	-6.90	0.14	0.98	-0.79
Item 37	I feel uneasy when I know that the following lesson is math.	Apprehension of Math Lessons	0.99	8.01	-2.43	-0.15	1.20	-0.60
Item 38	I feel bothered by the necessity of making calculations by solving mathematical problems in my daily life even if they are simple.	Use of Mathematics In Daily Life	9.00	0.00	0.00	-0.70	1.52	-0.25

Item 39	I feel depressed by the math book.	Apprehension of Math Lessons	2.35	5.40	-6.62	0.61	0.56	-0.43
Item 40	Opening any book on math and looking at one of its pages full of mathematical problems makes me happy.***	Apprehension of Math Lessons	-1.43	-2.35	-2.86	-0.64	0.27	1.13
Item 41	When I am given a problem to solve, I panic if I cannot remember the formula necessary for the solution.	Test and Evaluation Anxiety	-1.85	0.49	-0.16	-1.16	-0.16	0.14
Item 42	Five minutes before the math exam, my heart starts starts beating fast.	Test and Evaluation Anxiety	-1.09	-0.94	-0.82	-0.41	-0.23	-1.17
Item 43	When I think that I succeeded at a math exam, I feel relaxed and peaceful while waiting for the announcement of the results.***	Self-efficacy for Mathematics	-3.86	-1.70	-2.99	-1.44	0.29	-1.05
Item 44	If the teacher asks me to solve a math problem that I have been working on for a while at the blackboard, I forget what I have done out of excitement.	Test and Evaluation Anxiety	0.39	-0.13	-0.26	-0.80	0.24	1.12
Item 45	If a friend asks me to solve a math problem that has been published in a magazine, I am afraid of being embarrassed by not being able to solve even the easiest problems.	Use of Mathematics In Daily Life	0.10	0.85	-0.88	-0.12	0.24	0.14

Note. *** = Reverse-coded; MANX = Math Anxiety Scale (Erol 1989).

Appendix B. Mean item thresholds of each item for Class 1 and Class 2

Items	Class 1 Mean Item Thresholds	Class 2 Mean Item Thresholds	Difference
Item 1	0.246	0.155	0.091
Item 2	-0.436	-0.114	-0.322
Item 3	-0.669	-0.659	-0.010
Item 4	-2.375	-0.900	-1.474*
Item 5	-0.285	0.352	-0.637
Item 6	1.776	0.141	1.635*
Item 7	1.999	0.579	1.420*
Item 8	-0.060	-0.106	0.046
Item 9	1.882	0.763	1.119*
Item 10	-1.645	-0.017	-1.628*
Item 11	0.063	0.134	-0.071
Item 12	-0.106	0.427	-0.534
Item 13	-2.339	-0.934	-1.405*
Item 14	0.243	-0.072	0.315
Item 15	0.057	0.520	-0.462
Item 16	3.000	0.247	2.753*
Item 17	0.106	0.617	-0.510
Item 18	0.117	-0.104	0.221
Item 19	-0.410	-0.363	-0.047
Item 20	-2.327	-0.605	-1.721*
Item 21	-0.252	0.196	-0.446
Item 22	-0.137	-0.103	-0.034
Item 23	-0.022	-0.030	0.008
Item 24	0.043	0.089	-0.046
Item 25	1.950	0.271	1.679*
Item 26	0.364	0.382	-0.018
Item 27	-2.866	-0.229	-2.637*
Item 28	-0.915	-0.802	-0.113
Item 29	2.457	0.352	2.105*
Item 30	0.332	0.080	0.252

Item 31	-0.263	0.261	-0.524
Item 32	-2.258	-0.341	-1.917*
Item 33	-0.273	0.053	-0.327
Item 34	0.225	0.022	0.203
Item 35	-1.675	0.241	-1.916*
Item 36	0.099	0.112	-0.013
Item 37	2.188	0.149	2.039*
Item 38	3.000	0.193	2.807*
Item 39	0.379	0.246	0.133
Item 40	-2.211	0.252	-2.463*
Item 41	-0.508	-0.394	-0.114
Item 42	-0.948	-0.601	-0.347
Item 43	-2.847	-0.733	-2.114*
Item 44	0.001	0.186	-0.185
Item 45	0.324	0.086	0.238

Note. * : significance at 1 or greater.