Research Article GU J Sci 31(3): 929-938 (2018)

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

The Distribution of Angular Flux for Slab Albedo Problem

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Article Info

Received: 20/07/2017
Accepted: 20/04/2018

Keywords

Neutron transport equation
Isotropic scattering
Angular flux
Slab
Albedo
Transmission factor

Abstract

The slab albedo problem is solved by using H_N method. The albedo, transmission coefficient and angular flux are calculated for the isotropic scattering. The behavior of these parameters according to the changing of slab thickness is obtained. The angular flux distributions are good agreement with literature. It was shown that H_N method leads to concise equations and to fast converging numerical results.

1. INTRODUCTION

Neutrons are able to escape through the boundaries while slowing down or when thermalized because of the finite dimension of a reactor core [1]. Thus, for any realistic reactor analysis, neutron transport or diffusion theory is used with spatial extent and neutron transport can be considered in a slab.

The slab albedo problem consists of a finite plane-parallel medium surrounded by vacuum on both sides. The neutrons incident on the left side and on the other side there is no neutron entrance. This problem requires the solution of one speed neutron transport equation in plane geometry. One speed neutron transport equation in a homogeneous medium for isotropic scattering is given by [2-5]

$$\mu\frac{\partial}{\partial x}\Psi(x,\mu)+\Psi(x,\mu)=\frac{c}{2}\int_{-1}^{+1}\Psi(x,\mu')d\mu'+S \tag{1}$$

Here μ is the direction cosine, x is the distance in terms of mean free path, c is number of secondary neutrons per collisions, $\Psi(x,\mu)$ is the angular distribution of neutrons and S is the source term.

If monodirectional neutrons enter the left surface of a slab and scatter isotropically without energy loss, the flux at the boundary x=0 can be written as

$$\Psi(0,\mu)=1\ ,\mu>0 \tag{2a}$$

and if there is no neutron entrance at the boundary $x=\tau$, we can write the boundary condition as

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$$\Psi(\tau,-\mu)=0 \ ,\mu>0 \tag{2b}$$

The general solution of Eq. (1) is [3]

$$\begin{aligned}\Psi(x,\mu)&=A(\nu_0)\varphi(\nu_0,\mu)\exp\left(-\frac{x}{\nu_0}\right)+A(-\nu_0)\varphi(-\nu_0,\mu)\exp\left(\frac{x}{\nu_0}\right)\\&+\int_0^1 A(\nu)\varphi(\nu,\mu)\exp\left(-\frac{x}{\nu}\right)d\nu+\int_0^1 A(-\nu)\varphi(-\nu,\mu)\exp\left(\frac{x}{\nu}\right)d\nu+\frac{S}{1-c}\end{aligned} \tag{3}$$

where $\pm\nu_0$ are the two discrete eigenvalues with eigenfunctions $\varphi(\pm\nu_0,\mu)$ and $\pm\nu$ are the continuum eigenvalues on the range [-1,+1] with eigenfunctions $\varphi(\pm\nu,\mu)$. They are the elementary solutions of Case's method [2]. $A(\pm\nu_0)$ and $A(\pm\nu)$ represent the expansion coefficients. Eigenfunctions are given by

$$\varphi(\pm\nu_0,\mu)=\pm\frac{c\nu_0}{2}\frac{1}{\pm\nu_0-\mu} \tag{4a}$$

$$\varphi(\nu,\mu)=\frac{c\nu}{2}P\frac{1}{\nu-\mu}+\lambda(\nu)\delta(\nu-\mu) \tag{4b}$$

The orthogonality relations of these eigenfunctions are

$$\int_{-1}^{1}\mu\varphi(\nu_0,\mu)\varphi(\nu_0,\mu)d\mu=N(\nu_0) \tag{5a}$$

$$\int_{-1}^{1}\mu\varphi(\nu',\mu)\varphi(\nu,\mu)d\mu=N(\nu')\delta(\nu-\nu') \tag{5b}$$

and where

$$N(\nu_0)=\frac{c\nu_0^3}{2}\left[\frac{c}{\nu_0^2-1}-\frac{1}{\nu_0^2}\right], \tag{6a}$$

$$N(\nu)=\nu\left[\left(1-c\nu\tanh^{-1}\nu\right)^2+\left(\frac{\pi c\nu}{2}\right)^2\right]. \tag{6b}$$

2. SLAB ALBEDO PROBLEM WITH H_N METHOD

To apply the H_N method to the slab albedo problem, firstly, the unknown coefficients $A(\xi)$ and $A(-\xi),\xi=\nu_0$ or $\xi=\nu\in(0,1)$ in Eq. (3) are calculated [6-7]. To do this, we consider the exit distributions at the boundaries of the slab. For a slab thickness $\tau(0\le x\le\tau)$, the exit distributions for $\mu>0$ can be written from Eq.(3) as

$$\Psi(0,-\mu)=A(\nu_0)\varphi(\nu_0,-\mu)+A(-\nu_0)\varphi(\nu_0,\mu)$$
$$+\int_0^1 A(\nu)\varphi(\nu,-\mu)d\nu+\int_0^1 A(-\nu)\varphi(-\nu,\mu)d\nu+\frac{S}{1-c} \tag{7a}$$

$$\Psi(\tau,\mu)=A(\nu_0)\varphi(\nu_0,\mu)\exp\left(-\frac{\tau}{\nu_0}\right)+A(-\nu_0)\varphi(-\nu_0,\mu)\exp\left(\frac{\tau}{\nu_0}\right)$$
$$+\int_0^1 A(\nu)\varphi(\nu,\mu)\exp\left(-\frac{\tau}{\nu}\right)d\nu+\int_0^1 A(-\nu)\varphi(-\nu,\mu)\exp\left(\frac{\tau}{\nu}\right)d\nu+\frac{S}{1-c} \tag{7b}$$

We multiply both sides of Eq. (7a) and Eq. (7b) with $\mu\varphi(\nu_0,-\mu)$ and integrate over $\mu, \mu\in[-1,+1]$. Using the orthogonality relations for the discrete and continuum modes, the boundary conditions in Eqs. (2a–2b), the exit distributions for $\mu>0$ are given by [5-6]

$$\Psi(0,-\mu)=S\left[1-\exp\left(-\frac{\tau}{\mu}\right)\right]+\sum_{\alpha=0}^{N}a_\alpha P_\alpha(2\mu-1), \tag{8a}$$

$$\Psi(\tau,\mu)=S\left[1-\exp\left(-\frac{\tau}{\mu}\right)\right]+\sum_{\alpha=0}^{N}b_\alpha P_\alpha(2\mu-1), \tag{8b}$$

we obtain the unknown coefficients as

$$A(\xi)=-\frac{c\xi}{2N(\xi)}\left\{(S-1)B_0(\xi)+2S-S\int_0^1\frac{\mu}{\nu_0+\mu}\exp\left(-\frac{\tau}{\mu}\right)d\mu+\sum_{\alpha=0}^{N}a_\alpha G_\alpha(\nu_0)\right\}, \tag{9a}$$

$$A(-\xi)=-\frac{e\text{xp}\left(-\frac{\tau}{\xi}\right)}{N(\xi)}\frac{c\xi}{2}\left\{S\left(B_0(\xi)+2\right)-S\int_0^1\frac{\mu}{\nu_0+\mu}\exp\left(-\frac{\tau}{\mu}\right)d\mu+\sum_{\alpha=0}^{N}b_\alpha G_\alpha(\nu_0)\right\}. \tag{9b}$$

Here, a_α and b_α are the expansion coefficients, $P_\alpha(2\mu-1)$ is the Legendre polynomials and

$$A_\alpha(\xi)=\frac{2}{c\xi}\int_0^1\mu^{\alpha+1}\varphi(-\xi,\mu)d\mu, \tag{10a}$$

$$B_\alpha(\xi)=\frac{2}{c\xi}\int_0^1\mu^{\alpha+1}\varphi(\xi,\mu)d\mu, \tag{10b}$$

$$G_\alpha(\xi)=\frac{2}{c\xi}\int_0^1\mu\varphi(-\xi,\mu)P_\alpha(2\mu-1)d\mu, \tag{10c}$$

and

$$
\begin{aligned}
&A_0(\xi)=1-\xi\log\left(1+\frac{1}{\xi}\right),\\
&A_\alpha(\xi)=-\xi A_{\alpha-1}(\xi)+\frac{1}{\alpha+1},\qquad \alpha\geq 1,\\
&B_0(\xi)=\frac{2}{c}-1-\xi\log\left(1+\frac{1}{\xi}\right),\\
&B_\alpha(\xi)=-\xi B_{\alpha-1}(\xi)-\frac{1}{\alpha+1},\qquad \alpha\geq 1,\\
&G_0(\xi)=1-\xi\log\left(1+\frac{1}{\xi}\right),\\
&G_\alpha(\xi)=-\left(\frac{2\alpha-1}{\alpha}\right)(2\xi+1)G_{\alpha-1}(\xi)-\left(\frac{\alpha-1}{\alpha}\right)G_{\alpha-2}(\xi)+\frac{1}{2}\delta_{\alpha,2}+\delta_{\alpha,1}.
\end{aligned}
\tag{11}
$$

Now, multiplying the exit distribution $\Psi(0,-\mu)$ in Eq.(7a) by μ^{m+1}, integrating over $\mu\in(0,1)$, using boundary conditions Eqs.(2a, 2b, 8a, 8b) and the coefficients in Eqs.(9a –9b), we obtain

$$
\begin{aligned}
&\sum_{\alpha=0}^{N}\left\{\int_0^1 \mu^{m+1}P_\alpha(2\mu-1)\,d\mu+\left(\frac{c\nu_0}{2}\right)^2\frac{1}{N(\nu_0)}A_m(\nu_0)G_\alpha(\nu_0)+\left(\frac{c}{2}\right)^2\int_0^1\frac{\nu^2A_m(\nu)G_\alpha(\nu)}{N(\nu)}d\nu\right\}a_\alpha+\\
&\sum_{\alpha=0}^{N}\left\{\left(\frac{c\nu_0}{2}\right)^2\frac{\exp(-\tau/\nu_0)}{N(\nu_0)}B_m(\nu_0)G_\alpha(\nu_0)+\left(\frac{c}{2}\right)^2\int_0^1\frac{\nu^2B_m(\nu)G_\alpha(\nu)\exp(-\tau/\nu)}{N(\nu)}d\nu\right\}b_\alpha\\
&=S\left\{\frac{1}{1-c}\frac{1}{m+2}+\int_0^1\mu^{m+1}\exp(-\tau/\mu)\,d\mu+\left(\frac{c}{2}\right)^2\left[-\frac{\nu_0^2}{N(\nu_0)}\left(A_m(\nu_0)+\exp(-\tau/\nu_0)B_m(\nu_0)\right)\right.\right.\\
&\left(B_0(\nu_0)+2-\int_0^1\frac{\mu}{\nu_0+\mu}\right)\exp(-\tau/\mu)\,d\mu-\int_0^1\frac{\nu^2\left(A_m(\nu)+\exp(-\tau/\nu)B_m(\nu)\right)}{N(\nu)}\left(B_0(\nu)+2\right)d\nu\\
&\left.\left.+\int_0^1\int_0^1\frac{\nu^2\left(A_m(\nu)+\exp(-\tau/\nu)B_m(\nu)\right)}{N(\nu)}\frac{\mu}{\nu+\mu}\exp(-\tau/\mu)\,d\mu\, d\nu\right]\right\}\\
&+\left(\frac{c}{2}\right)^2\left[\frac{\nu_0^2A_m(\nu_0)B_0(\nu_0)}{N(\nu_0)}+\int_0^1\frac{\nu^2A_m(\nu)B_0(\nu)}{N(\nu)}d\nu\right].
\end{aligned}
\tag{12}
$$

Similarly using the exit distribution $\Psi(\tau,\mu)$ in Eq. (7b) and multiplying by μ^{m+1}, integrating over $\mu\in(0,1)$, we get

$$\sum_{\alpha=0}^{N}\left\{\left(\frac{c\nu_0}{2}\right)^2\frac{\exp\left(-\tau/\nu_0\right)}{N(\nu_0)}B_m(\nu_0)G_\alpha(\nu_0)+\left(\frac{c}{2}\right)^2\int_0^1\frac{\nu^2B_m(\nu)G_\alpha(\nu)\exp\left(-\tau/\nu\right)}{N(\nu)}d\nu\right\}a_\alpha+$$
$$\sum_{\alpha=0}^{N}\left\{\int_0^1\mu^{m+1}P_\alpha\left(2\mu-1\right)d\mu+\left(\frac{c\nu_0}{2}\right)^2\frac{1}{N(\nu_0)}A_m(\nu_0)G_\alpha(\nu_0)+\left(\frac{c}{2}\right)^2\int_0^1\frac{\nu^2A_m(\nu)G_\alpha(\nu)}{N(\nu)}d\nu\right\}b_\alpha$$
$$=S\left\{\frac{1}{1-c}\frac{1}{m+2}+\int_0^1\mu^{m+1}\exp\left(-\tau/\mu\right)d\mu+\left(\frac{c}{2}\right)^2\left[-\frac{\nu_0^2}{N(\nu_0)}\left(A_m(\nu_0)+\exp\left(-\tau/\nu_0\right)B_m(\nu_0)\right)\right.\right.$$
$$\left(B_0(\nu_0)+2-\int_0^1\frac{\mu}{\nu_0+\mu}\right)\exp\left(-\tau/\mu\right)d\mu-\int_0^1\frac{\nu^2\left(A_m(\nu)+\exp\left(-\tau/\nu\right)B_m(\nu)\right)}{N(\nu)}\left(B_0(\nu)+2\right)d\nu \tag{13}$$
$$\left.\left.+\int_0^1\int_0^1\frac{\nu^2\left(A_m(\nu)+\exp\left(-\tau/\nu\right)B_m(\nu)\right)}{N(\nu)}\frac{\mu}{\nu+\mu}\exp\left(-\tau/\mu\right)d\mu\, d\nu\right]\right\}$$
$$+\left(\frac{c}{2}\right)^2\left[\frac{\nu_0^2B_m(\nu_0)B_0(\nu_0)}{N(\nu_0)}\exp\left(-\tau/\nu_0\right)+\int_0^1\frac{\nu^2B_m(\nu)B_0(\nu)}{N(\nu)}\exp\left(-\tau/\nu\right)d\nu\right].$$

Finally, the two equations to be solved are

$$\sum_{\alpha=0}^{N}T_{m\alpha}a_\alpha+\sum_{\alpha=0}^{N}R_{m\alpha}\,b_\alpha=S_m\,, \tag{14a}$$

$$\sum_{\alpha=0}^{N}R_{m\alpha}a_\alpha+\sum_{\alpha=0}^{N}T_{m\alpha}\,b_\alpha=M_m\,, \tag{14b}$$

where,

$$T_{m\alpha}=\int_0^1\mu^{m+1}P_\alpha\left(2\mu-1\right)d\mu+\left(\frac{c\nu_0}{2}\right)^2\frac{1}{N(\nu_0)}A_m(\nu_0)G_\alpha(\nu_0)+\left(\frac{c}{2}\right)^2\int_0^1\frac{\nu^2A_m(\nu)G_\alpha(\nu)}{N(\nu)}d\nu\,, \tag{15}$$

$$R_{m\alpha}=\left(\frac{c\nu_0}{2}\right)^2\frac{\exp\left(-\tau/\nu_0\right)}{N(\nu_0)}B_m(\nu_0)G_\alpha(\nu_0)+\left(\frac{c}{2}\right)^2\int_0^1\frac{\nu^2B_m(\nu)G_\alpha(\nu)\exp\left(-\tau/\nu\right)}{N(\nu)}d\nu\,, \tag{16}$$

$$S_m = S\left\{\frac{1}{1-c}\frac{1}{m+2}+\int_0^1 \mu^{m+1}\exp\left(-\tau/\mu\right)d\mu+\left(\frac{c}{2}\right)^2\left[-\frac{\nu_0^2}{N(\nu_0)}\left(A_m(\nu_0)+\exp\left(-\tau/\nu_0\right)B_m(\nu_0)\right)\right.\right.$$
$$\left(B_0(\nu_0)+2-\int_0^1\frac{\mu}{\nu_0+\mu}\right)\exp\left(-\tau/\mu\right)d\mu-\int_0^1\frac{\nu^2\left(A_m(\nu)+\exp\left(-\tau/\nu\right)B_m(\nu)\right)}{N(\nu)}\left(B_0(\nu)+2\right)d\nu$$
$$\left.\left.+\int_0^1\int_0^1\frac{\nu^2\left(A_m(\nu)+\exp\left(-\tau/\nu\right)B_m(\nu)\right)}{N(\nu)}\frac{\mu}{\nu+\mu}\exp\left(-\tau/\mu\right)d\mu d\nu\right]\right\} \tag{17}$$
$$+\left(\frac{c}{2}\right)^2\left[\frac{\nu_0^2A_m(\nu_0)B_0(\nu_0)}{N(\nu_0)}+\int_0^1\frac{\nu^2A_m(\nu)B_0(\nu)}{N(\nu)}d\nu\right],$$

$$M_m = S\left\{\frac{1}{1-c}\frac{1}{m+2}+\int_0^1 \mu^{m+1}\exp\left(-\tau/\mu\right)d\mu+\left(\frac{c}{2}\right)^2\left[-\frac{\nu_0^2}{N(\nu_0)}\left(A_m(\nu_0)+\exp\left(-\tau/\nu_0\right)B_m(\nu_0)\right)\right.\right.$$
$$\left(B_0(\nu_0)+2-\int_0^1\frac{\mu}{\nu_0+\mu}\right)\exp\left(-\tau/\mu\right)d\mu-\int_0^1\frac{\nu^2\left(A_m(\nu)+\exp\left(-\tau/\nu\right)B_m(\nu)\right)}{N(\nu)}\left(B_0(\nu)+2\right)d\nu$$
$$\left.\left.+\int_0^1\int_0^1\frac{\nu^2\left(A_m(\nu)+\exp\left(-\tau/\nu\right)B_m(\nu)\right)}{N(\nu)}\frac{\mu}{\nu+\mu}\exp\left(-\tau/\mu\right)d\mu d\nu\right]\right\} \tag{18}$$
$$+\left(\frac{c}{2}\right)^2\left[\frac{\nu_0^2B_m(\nu_0)B_0(\nu_0)}{N(\nu_0)}\exp\left(-\tau/\nu_0\right)+\int_0^1\frac{\nu^2B_m(\nu)B_0(\nu)}{N(\nu)}\exp\left(-\tau/\nu\right)d\nu\right].$$

3. NUMERICAL RESULTS

The expansion coefficients a_α and b_α can be obtained from Eqs. (14a-14b) and then we can compute the albedo [3,8]

$$A^* = 2\left\{S\left[\frac{1}{2}-\int_0^1\mu\exp\left(-\frac{\tau}{\mu}\right)d\mu\right]+\sum_{\alpha=0}^{N}\left[\int_0^1\mu P_\alpha\left(2\mu-1\right)d\mu\right]a_\alpha\right\} \tag{19}$$

which is the ratio of the current of neutrons emerging from the medium to the incident current at the boundary $x=0$. The transmission factor is similarly at the boundary $x=\tau$

$$B^* = 2\left\{S\left[\frac{1}{2}-\int_0^1\mu\exp\left(-\frac{\tau}{\mu}\right)d\mu\right]+\sum_{\alpha=0}^{N}\left[\int_0^1\mu P_\alpha\left(2\mu-1\right)d\mu\right]b_\alpha\right\}. \tag{20}$$

In addition, the angular flux values can be obtained from Eqs. (8a-8b).

Table 1. *Albedo values for S=0*

τ	$c=0.1$	$c=0.3$	$c=0.5$	$c=0.7$	$c=0.9$
0.0	$2.0394x10^{-7}$	$7.0518x10^{-12}$	$-2.6124x10^{-9}$	$4.7578x10^{-9}$	$-4.0589x10^{-11}$
0.5	0.017771	0.058389	0.107676	0.168948	0.247494
1.0	0.020748*	0.070073*	0.134165*	0.222070*	0.352712*
1.5	0.021444	0.073159	0.142361	0.242390	0.407139
2.0	0.021626	0.074052	0.145085	0.250616	0.437147
2.5	0.021676	0.074324	0.146025	0.254036	0.454206
3.0	0.021691	0.074410	0.146357	0.255479	0.464064
3.5	0.021696	0.074438	0.146476	0.256093	0.469814
4.0	0.021697	0.074447	0.146519	0.256357	0.473186
4.5	0.021697	0.07445	0.146535	0.256471	0.475169
5.0	0.021698	0.074451	0.146541	0.256519	0.476339

* Siewert, 1978 [3]

Table 2. *Transmission factor values for S=0*

τ	$c=0.1$	$c=0.3$	$c=0.5$	$c=0.7$	$c=0.9$
0.0	1.000000	1.000000	1.000000	1.000000	1.000000
0.5	0.458010	0.492387	0.534993	0.589094	0.659920
1.0	0.231854*	0.263095*	0.306709*	0.371195*	0.474746*
1.5	0.122115	0.145009	0.180049	0.238777	0.352350
2.0	0.065856	0.081381	0.107055	0.155071	0.265581
2.5	0.036096	0.046231	0.064175	0.101235	0.201867
3.0	0.020021	0.026496	0.038690	0.066299	0.154168
3.5	0.011208	0.015289	0.023424	0.043511	0.118066
4.0	0.006321	0.008870	0.014228	0.028597	0.090564
4.5	0.003587	0.005168	0.008665	0.018815	0.069536
5.0	0.002045	0.003022	0.005288	0.012389	0.053421

* Siewert, 1978 [3]

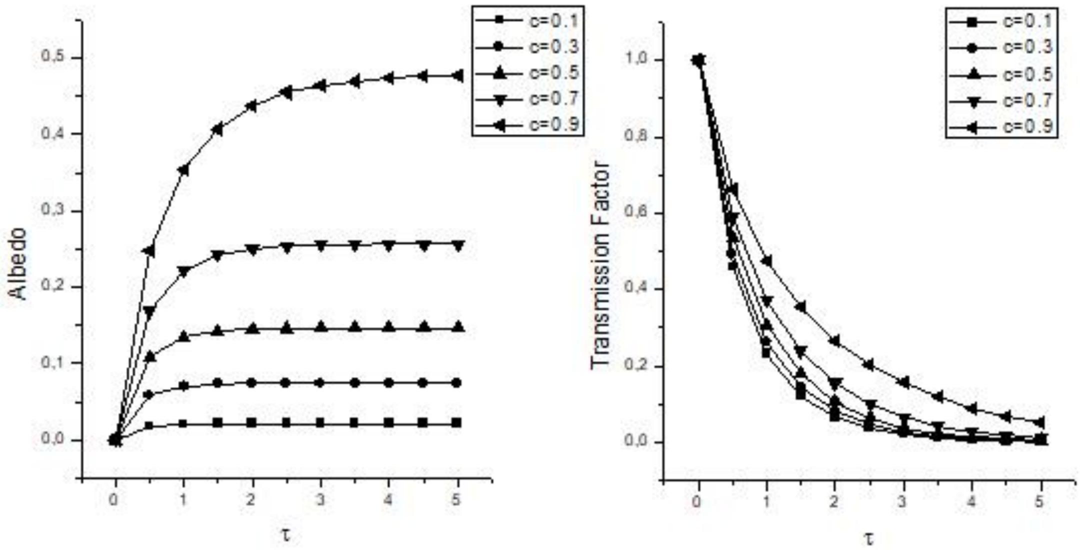

Figure 1. *Albedo and transmission factors versus slab thickness for different values of c and S=0*

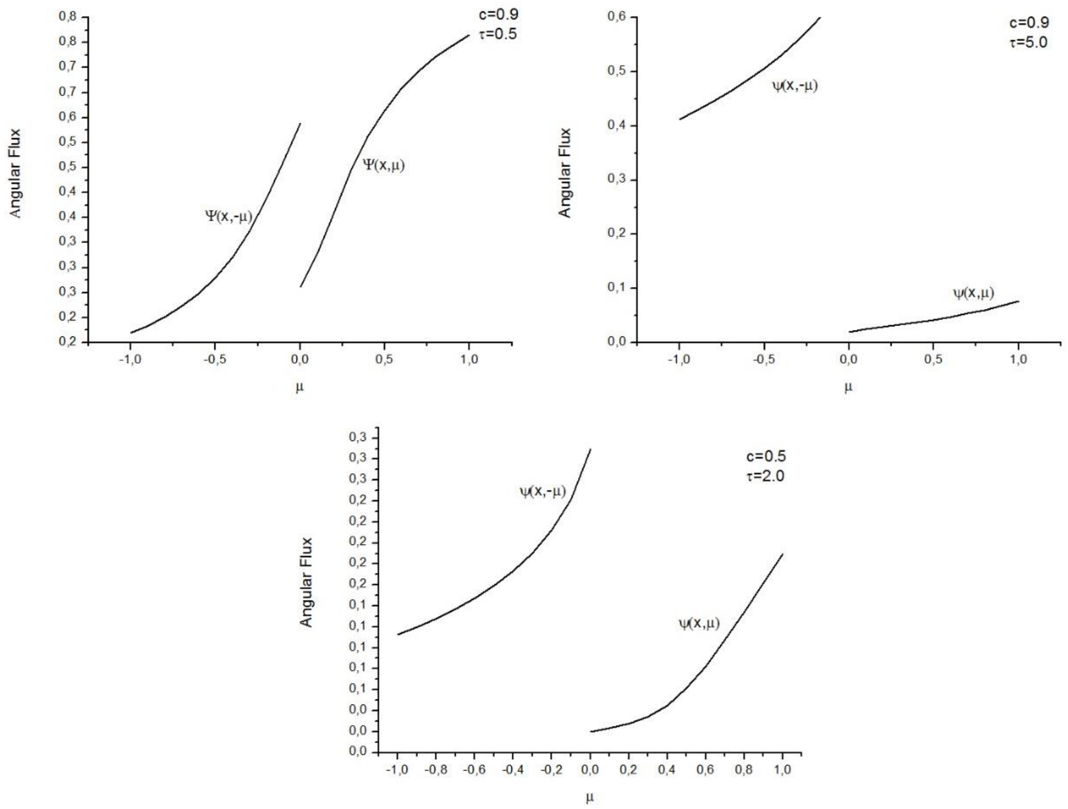

Figure 2. *Angular flux for isotropic scattering with S=0 [9]*

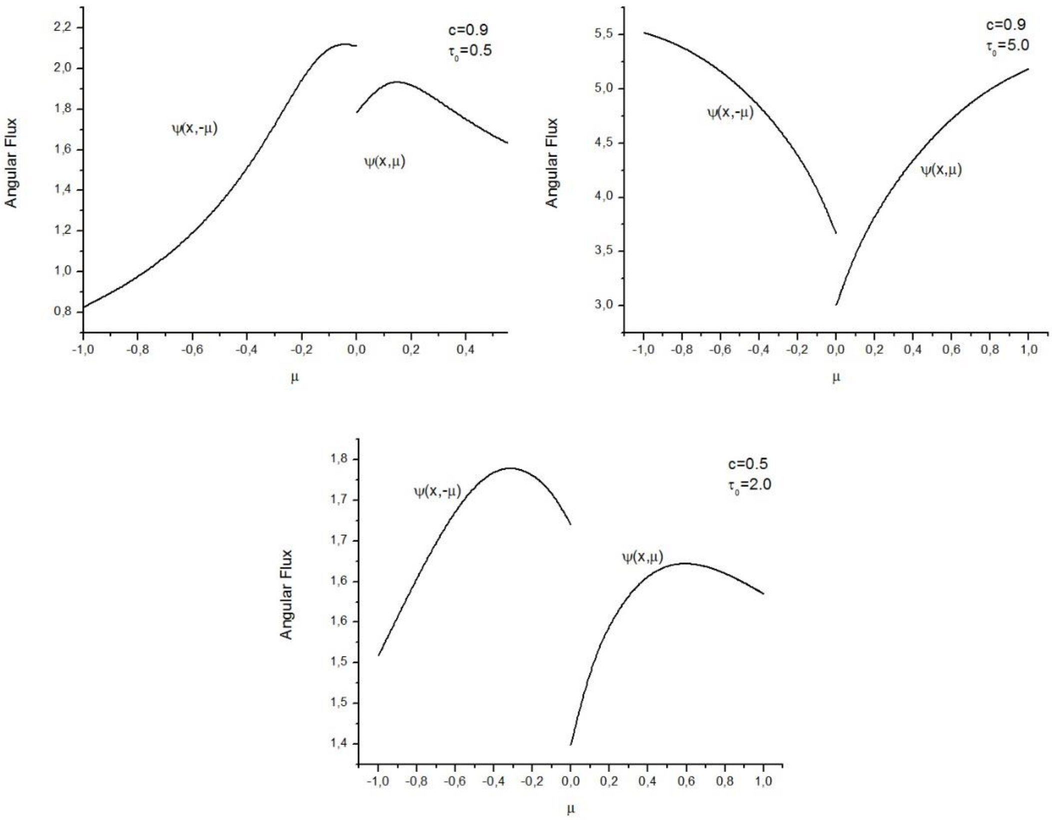

Figure 3. *Angular flux for isotropic scattering with S=1*

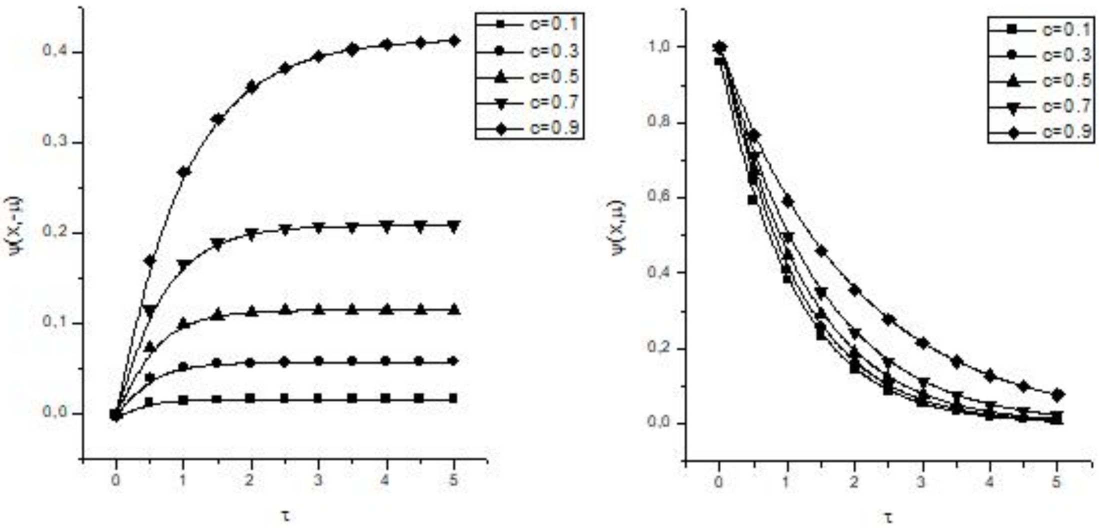

Figure 4. *Angular flux versus slab thickness for different values of c and S=0*

4. CONCLUSIONS

The slab albedo problem is solved by using H_N method depending on the use of the singular eigenfunctions of the method of elementary solutions. The albedo and transmission coefficients are calculated, shown as Table 1 and Table 2. Some values are compared with literature and they are in good agreement with it. The results can be obtained even in the lowest order of approximation of N. This is the success of the method. Albedo and transmission factor versus slab thickness with different values of c is given in Figure 1. The behavior of angular flux according to the changing of the direction cosine for some values of slab thickness are shown in Figure 2 and Figure 3, respectively for S=0 and S=1. Figure 2 is compatible with Ref.[9]. This paper is written on a problem in the radiative transfer theory as similar to the neutron transport theory. In Figure 4, the angular flux versus slab thickness is also given for different values of c with S=0.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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