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Stock Market Prediction Using Nonparametric Fuzzy and Parametric GARCH Methods

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A B S T R A C T

Prediction of stock market value is one the most complicated issue during the past decades. Due to its importance, in this research, we consider the prediction of stock values based on non-parametric and parametric methods. In this first method, we use the fuzzy Markov chain procedure in order to prediction problem. In this regard, all of the rising and falling probabilities during the weekdays are calculated and then they applied to obtain the increasing and decreasing rate. Then, based on this information we model and predict the stock values. In the sequel, we implement different methods of parametric time series such as generalized autoregressive conditionally heteroskedastic (GARCH), ARIMA-GARCH, Exponential GARCH (E-GARCH) and GJR-GARCH by assuming the normal and t-student distribution for the error terms to obtain the best model in terms of minimum mean square errors. Finally, the mythologies developed here are applied for the Tehran Stock Exchange Index (TEDPIX).

1. Introduction

The prediction of the financial market is a complex task since the distribution of financial time series is changing over a period of time. There is also never-ending debate as to whether these markets are predictable or not. In other words, they are called efficient markets (EMH) if being unpredictable ones, and vice versa. In the recent years, investors have started to show interest in trading on stock markets indices as it provides an opportunity to hedge their market risk and at the same time offers a good investment opportunity for speculators and arbitrageurs.

There is a dream of the fascination of any investor to know the future asset prices and/or any financial instrument, e.g. stock exchange index. Basically, traders use several different approaches for prediction based upon the fundamental analysis, technical analysis (TA), psychological analysis, etc. The technical analysis paradigm states that all price relevant information is contained in market price itself. Hence, the instant processing of market messages plays a specific role, thus leading to permanent interactions among traders. TA concerns with identifications of both trends and trend reversals using more or less sophisticated procedures to predict future price movements from those of the recent past. So far many models have been proposed to predict stock markets, although, there is not a perfect and best model over the others for all time.

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The GARCH model is developed by [1] to extend the earlier work on ARCH models by [2]. [3], [4], and [5] fitted GARCH models to USA stock market data and found that these models have better performance for predicting stock markets in the USA. [6], it is explained that the GARCH model provides a good first approximation to the observed temporal dependencies in daily data. In [7], the GARCH Models for Australian stock markets are fitted. Regression and GARCH models to the UK stock market prediction are used in [8]. New Zealand stock market through different kinds of GARCH models is modelled by [9]. The relative out of sample predictive ability of different GARCH models, with particular emphasis on the predictive content of the asymmetric component is explored by [10]. A four regime double threshold GARCH (DTGARCH) model is introduced in [11]. In [12], a fuzzy system method to analyse clustering in GARCH models is used. In [13], it is examined the performance of a selection of GARCH models: GARCH, EGARCH, and GJR. While the results are mixed, the paper concludes that different varieties of the GJR specification outperform the others included the study. In [31], it is introduced a fractionally integrated generalized autoregressive conditional heteroscedasticity model. Estimating functions approach combining with the first order EGARCH and GJR-GARCH models to predict the volatility of two market indices from the USA and Japanese stock markets is applied by [14]. In [15], new methods for improved ex-post interdaily volatility measurements based on high-frequency intradaily data are discussed. In [16], it is suggested that the long-run dependencies in financial market volatility may be better characterized by a fractionally integrated model. The multiplicative structure of the model was suggested for volatility modelling in the univariate case by [17], who proposed using the exponential distribution. For estimation and applications of GARCH models see [18], [19], [20], and [15]. Recently, [21] applied GARCH-type forecasting models for the volatility of the stock market and they found that the GARCH using Student's t innovation model is the best model for volatility predictions of SSE380 among the six models.

Tehran Stock Exchange (TSE) is Iran's largest stock exchange, which was first opened in 1967. TSE computed and published its price index under the title of TEPIX since April 1990. As of May 2012, 339 companies with a combined market capitalization of US$104.21 billion were listed on TSE. TSE, which is a founding member of the Federation of Euro-Asian Stock Exchanges, has been one of the world's best performing stock exchanges in the years 2002 through 2013. TSE is an emerging or "frontier" market. The most important advantage that Iran's capital market has in comparison with other regional markets is that there are 37 industries directly involved in it. Industries such as the automotive, telecommunications, agriculture, petrochemical, mining, steel iron, copper, banking and insurance, financial mediation and others trade shares at the stock market, which makes it unique in the Middle East. The second advantage is that most of the state-owned firms are being privatized under the general policies of article 44 in the Iranian constitution. Under the circumstances, people are allowed to buy the shares of newly privatized firms.

Data in this paper are taken from the Tehran stock exchange http://www.tse.ir/en/. The index TEDPIX is the aim of this paper for prediction. We consider the related data from 1999/12/08- 2016/07/26. The rest of the paper is as follows. In Section 2, we introduce the Nonparametric Fuzzy method while the different kinds of GARCH models are given in Section 3. Finally concluding remarks are given in Section 4.

2. Nonparametric Fuzzy and GARCH Models

In [22], it is proposed that the stock exchange is a stochastic process with the following structure:

$$X\left(n+1\right)=X\left(n\right)e^{r}, \tag{1}$$

where

$$r=\frac{\sum_{n=1}^{J}\mu\left(t_{n+1}\right)-\mu\left(t_{n}\right)}{J}, J=1,2,...,N \tag{2}$$

where $\mu\left(t_n\right)=\left(x/y\right)^2$. Here x are the stock values at the day t_n and y states the maximum stock values at the week that we consider the stock. Here, we adjust the parameter r of the Fuzzy stochastic model by incorporating the Markov chain as follows.

$$r=\begin{cases} r_{11}P_{11}+r_{21}P_{21} \\ \\ r_{11}P_{11}+r_{21}P_{21} \end{cases} \tag{3}$$

We use r_{ij} to express the change rate $\{i=1,2; j=1,2\}$ from a specific day at the situation i to the next day at situation j. c denotes the probability that stock values at the day t_n decreasing and then at the day t_{n+1} decreasing, too. P_{12} denotes the probability that stock values at the day t_n decreasing and then at the day t_{n+1} increasing. P_{21} denotes the probability that stock values at the day t_n increasing and then at the day t_{n+1} decreasing. P_{22} denotes the probability that stock values at the day t_n increasing and then at the day t_{n+1} increasing. Table 1 shows the stock market values on some specific days for some weeks. Next, the rising and falling indicators are shown in Table 2.

Table 1. Stock market values at some days

Date	Days				
	Sat	**Sun**	**Mon**	**Tues**	**Wed**
	⋮	⋮	⋮	⋮	⋮
(2009/10/24-2009/10/28)	12242.8	12325.8	12342.6	12345.1	12408.4
(2009/10/31-2009/11/04)	12482.4	12545	12529.5	12483.6	12506
(2009/11/07-2009/11/11)	12576.9	12565.9	12581.5	12426.1	12381.4
(2009/11/15-2009/11/19)	12403.1	12361.7	12325.4	12241.9	12176.4
	⋮	⋮	⋮	⋮	⋮

Table 2. A portion of the stock market data

Date	Time	Stock values	μ_{index}
⋮	⋮	⋮	⋮
2009/10/24	20:07	8153.5	0.9761748
2009/10/25	20:07	8175.9	0.9815459
2009/10/26	20:07	8203.6	0.9889310
2009/10/27	20:07	8252.4	1.0000000
2009/10/28	20:07	8251.6	0.9998061
⋮	⋮	⋮	⋮

Table 3. Rising or falling indicator (1 for rising and 0 for falling)

Date	Days				
	Sat	**Sun**	**Mon**	**Tues**	**Wed**
	⋮	⋮	⋮	⋮	⋮
(2009/10/24-2009/10/28)	1	1	1	1	1
(2009/10/31-2009/11/04)	1	1	0	1	0
(2009/11/07-2009/11/11)	1	0	1	0	0
(2009/11/15-2009/11/19)	1	0	0	0	0
	⋮	⋮	⋮	⋮	⋮

Table 4 depicts the probability of decreasing or increasing for all days. For example for days, We use (times of appearance of (1, 1)/total number of entries) to obtain p_{11}. Finally, the changes in rates for every week's days are given in Table 5.

Table 4. The Probability of decreasing or increasing

Time	The Probability of decreasing (increasing)			
	p_{11}	p_{12}	p_{21}	p_{22}
Sat - Sun	0.1467	0.2934	0.2305	0.3293
Sun - Mon	0.1646	0.2125	0.3143	0.3083
Mon - Tues	0.1976	0.2814	0.2245	0.2964
Tues - Wed	0.1916	0.2305	0.2994	0.2784
Wed - Sat	0.2065	0.2844	0.2335	0.2754

Table 5. Change rates for all cases

Time	Change rate			
	r_{11}	r_{12}	r_{21}	r_{22}
Sat - Sun	0.0020	-0.0023	0.0002	-0.0035
Sun - Mon	0.0015	-0.0015	0.0012	-0.0020
Mon - Tues	0.0026	-0.0027	0.0025	-0.0006
Tues - Wed	0.0002	-0.0039	0.0008	-0.0046
Wed - Sat	0.0033	-0.0060	0.0091	-0.0139

After calculating the change rates, the main argument (r) is calculated in Table 6. Finally, the predicted values of the index based on the given formula are calculated. It should be mention here that we used 90% of the data as the training and the last 10% for examining the model. The results of prediction and mean square errors are given at the end of Section 3 in order to compare them with parametric GARCH models.

Table 6. Estimated values of *r*

Days	Increasing ***r***	Decreasing ***r***
Sat - Sun	0.0003604886	-0.001860368
Sun - Mon	0.0006678689	-0.0009611297
Mon - Tues	0.001078018	-0.0009517537
Tues - Wed	0.000312451	-0.002222598
Wed - Sat	0.002824761	-0.005589916

3. Parametric Time Series Models

In this section, we use several volatility models and use these models to predict the conditional variance about the rate of return in Iran stock prediction. These models include the GARCH model, E-GARCH model and GJR-GARCH model to analyse the rate of return and consider using two different distributions on error terms: normal distribution and student-t distribution. So, this paper is mainly capturing the forecasting performance with volatility models under different error distributions. Finally, after comparing the root mean square error (RMSE), choose the best model to predict the conditional variance. We also compare the fuzzy model with proposed volatility models. The ARCH model (Auto-regressive conditional heteroscedastic model) is proposed by [2].

$$\sigma^2{}_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_q\varepsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 \quad (4)$$

In [6], it is explained that the variation on error terms has been changed from the constant to be a random sequence. In [23], it is pointed out that ε_t has a conditional mean and variance based on the information set I_{t-1}. also $E\left(\varepsilon_t \middle| I_{t-1}\right) = 0$ and $\sigma_t^2 = E\left(\varepsilon_t^2 \middle| I_{t-1}\right)$. $\varepsilon_t = z_t\sigma_t$ and $z_t \sim N\left(0,1\right)$.

3.1. Generalized Arch (GARCH) Model

The so-called generalized ARCH (GARCH) model proposed by [1] and [24], for substituting the ARCH model as follows:

$$\sigma^2{}_t = \alpha_0 + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{i=1}^{p}\beta_i\sigma_{t-j}^2 \quad (5)$$

[25] assume $\alpha_0 > 0$, $\alpha_i \geq 0, i = 1,2,...,q$ and $\beta_j \geq 0, j = 1,2,...,q$, $j = 1,2,...,q$. Further,

$$\sum_{i=1}^{q}\alpha_i + \sum_{j=1}^{p}\beta_j < 1 \quad (6)$$

The most important point is that the GARCH model cannot capture the asymmetric performance. Later, for improving this problem, [26] proposed the EGARCH model and [27] proposed a GJR-GARCH model.

3.2. Exponential GARCH (EGARCH) Model

[26] proposed the exponential GARCH (EGARCH) model as follows:

$$\log\left(\sigma^2{}_t\right) = c + \sum_{i=1}^{p} g\left(Z_{t-i}\right) + \sum_{j=1}^{q} \beta_j \log\left(\sigma^2{}_{t-j}\right), \tag{7}$$

where $g\left(Z_{t-i}\right) = \gamma_i Z_{t-i} + \alpha_i \left\{\left|Z_{t-i}\right| - E\left|Z_{t-i}\right|\right\}$ and $Z_{t-i} = \dfrac{\varepsilon_{t-i}}{\sigma_{t-i}}$, see [22, 28] and [29] for more details on this model.

3.3. GJR-GARCH Model

[27] proposed a GJR-GARCH model, another asymmetric model. Define the sequence $\{\varepsilon_t\}$ to $z_t\varepsilon_t$ and $\{\varepsilon_t\}$ have normal distribution. So the GJR-GARCH model is written as follows:

$$\sigma^2{}_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-i} + \sum_{j=1}^{p} \beta_j \sigma^2_{t-j} + \sum_{k=1}^{r} \gamma_k \varepsilon^2_{t-k} I_{t-k}\left[\varepsilon_{t-k} < 0\right], \tag{8}$$

where $I_t = \begin{cases} 1 & \varepsilon_t \le 0 \\ 0 & \varepsilon_t > 0 \end{cases}$ see [30] for more details on the GJR-GARCH models.

In this paper, we consider two popular distributions for the error namely, the normal and t results. Here we use the mean square error (MSE) of the estimators as a measure of prediction. The MSE measures the difference between the true values and estimated values, and accumulates all these difference together as a standard for the predictive ability of a model. The criterion is the smaller value of the MSE, the better the predicting ability of the model. For different GARCH models we used the log-retune of data and then implement statistical software R and some the time series packages "*rugarch*", "*fGarch*" to estimate the model parameter. The results are given in the next section. Now, we apply the proposed method to predict the stock values for 30 days. Figure 1 shows the times series plot of TEDPIX index and mentioned data.

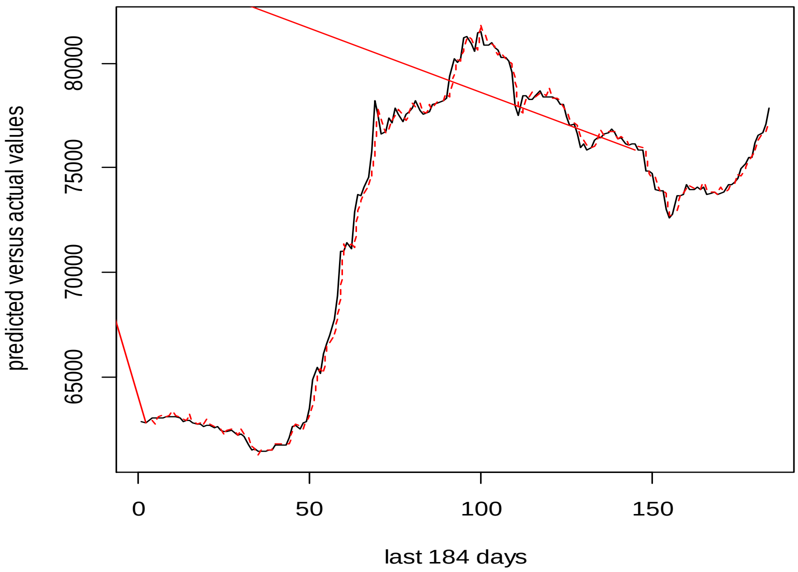

Figure 1. The times series plot of TEDPIX index of data.

It is obvious that the constant variance models such as ARIMA cannot be used for this data. Next, in the parametric GARCH models, we estimated the models' parameters in Table 7.

Table 7. Results of estimated GARCH models parameters

	GARCH(1,1)		**EGARCH(1,1)**		**GJR-GARCH(1,1)**	
	Normal	**Student-*t***	**Normal**	**Student-*t***	**Normal**	**Student-*t***
Ω	0.048948	0.012859	-0.098972	-0.037495	0.043795	0.012183
(s.e.)	0.008880	0.004861	0.021577	0.014869	0.008621	0.004743
(p-value)	0.000000	0.008156	0.000004	0.011677	0.000000	0.010207
α_1	0.210812	0.190099	0.067623	0.024326	0.257400	0.200187
(s.e.)	0.030405	0.033597	0.019818	0.019164	0.042349	0.037655
(p-value)	0.000000	0.000000	0.000644	0.204317	0.000000	0.000000
γ_1			0.407212	0.306937	-0.082249	-0.029225
(s.e.)			0.039808	0.044408	0.039381	0.039214
(p-value)			0.000000	0.000000	0.036750	0.456107
β_1	0.698507	0.808901	0.864019	0.961814	0.708689	0.813426
(s.e.)	0.036630	0.031918	0.020780	0.013364	0.036308	0.031818
(p-value)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
ν		4.698665		4.548663		4.690839
(s.e.)		0.541265		0.524758		0.540187
(p-value)		0.000000		0.000000		0.000000

Table 7. Results of estimated GARCH models parameters (continued)

	GJR-GARCH(2,1) Threshold=2		**GJR-GARCH(1,2) Threshold=2**		**GJR-GARCH(2,2) Threshold=2**	
	Normal	**Student-*t***	**Normal**	**Student-*t***	**Normal**	**Student-*t***
Ω	0.053147	0.010095	0.052434	0.014242	0.053145	0.010095
(s.e.)	0.012221	0.004131	0.009808	0.005584	0.012922	0.003028
(p-value)	0.000014	0.014532	0.000000	0.010760	0.000039	0.000855
α_1	0.191688	0.185790	0.317492	0.250616	0.191679	0.185803
(s.e.)	0.047710	0.061807	0.053236	0.055444	0.049519	0.059160
(p-value)	0.000059	0.002647	0.000000	0.000006	0.000108	0.001686
α_2	0.142114	0.000000			0.142115	0.000000
(s.e.)	0.066064	0.066017			0.067260	0.033450
(p-value)	0.031463	0.999996			0.034608	0.999999
γ_1	-0.025450	0.061675	-0.117541	-0.045160	-0.025446	0.061654
(s.e.)	0.060257	0.087465	0.051302	0.051215	0.062667	0.035186
(p-value)	0.672760	0.480725	0.021953	0.377906	0.684706	0.079735
γ_2	-0.083497	-0.109265			-0.083498	-0.109257
(s.e.)	0.068112	0.089044			0.068416	0.043622
(p-value)	0.220248	0.219788			0.222297	0.012257
β_1	0.637235	0.837005	0.223879	0.455737	0.637245	0.836999
(s.e.)	0.059265	0.031723	0.114954	0.255164	0.209443	0.377259
(p-value)	0.000000	0.000000	0.051469	0.074090	0.002346	0.026512
β_2			0.425180	0.315226	0.000000	0.000000
(s.e.)			0.110967	0.229871	0.170582	0.302625
(p-value)			0.000127	0.170277	1.000000	1.000000
ν		4.672250		4.692637		4.672339
(s.e.)		0.536530		0.539747		0.532948
(p-value)		0.000000		0.000000		0.000000

Then these models are used to predict the stock values in Table 8. In this Table, the mean square errors are calculated. The results show that for TEDPIX prediction the best model with minimum MSE is GJR-GARCH(2,2)-Normal. Next, we compare this model with a fuzzy model in terms of minimum mean square error. Finally, based on Fuzzy model MSE for 30 days forecasting is 7428405 while based on GJR-GARCH (2,2)-Normal the value of the mean square error is 17890446 which to too large, so we recommend the fuzzy Markov chain model for Iran stock prediction.

Table 8. RMSE of different models.

Models	RMSE
GARCH(1,1) - Normal	0.9566516
GARCH(1,1) - Student-*t*	0.9928256
EGARCH(1,1) - Normal	0.9604817
EGARCH(1,1) - Student-*t*	0.9918592
GJR-GARCH(1,1) - Normal	0.9528201
GJR-GARCH(1,1) - Student-*t*	0.9926979
GJR-GARCH(2,1) - Normal	0.9513464
GJR-GARCH(2,1) - Student-*t*	0.9926126
GJR-GARCH(1,2) - Normal	0.9598617
GJR-GARCH(1,2) - Student-*t*	0.9927451
GJR-GARCH(2,2) - Normal	**0.9513463**
GJR-GARCH(2,2) - Student-*t*	0.9926125

4. Conclusion

Prediction of stock is an important and complicated task. Due to its importance, in this paper, we discussed different methods of stock prediction. It is observed that for Iran stock market the Fuzzy method work better than other Parametric GARCH models in terms of minimum mean square error (MSE) in prediction.

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