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MATHEMATICAL MODELING AND FEM-DRIVEN DESIGN TUNING OF A VOICE-COIL ACTUATOR FOR A WRIST-WORN HAPTIC DEVICE

BİLEĞE TAKILAN DOKUNSAL CİHAZ İÇİN SES BOBİNİ AKTÜATÖRÜNÜN MATEMATİKSEL MODELLEMESİ VE FEM TABANLI TASARIM AYARLANMASI

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Abstract

Voice-coil actuators are widely used in haptic interfaces due to their compact design, bidirectional controllability, and ability to generate smooth forces. However, their performance strongly depends on the electromagnetic configuration and magnetic flux within the actuation, even though the mechanical design of a prior work was heavily based on experimental observations rather than comprehensive analyses. This study presents a mathematical modeling and enhancement framework for a custom-made voice-coil actuator for a wrist-worn haptic device, named CoWrHap. This paper first presents an analytical model to describe the relationship between input current, magnetic flux density, and output force. The model parameters are identified from experimental data, revealing the effective magnetic coupling coefficient and the current saturation limits of the driver circuit. To capture spatial variations of the magnetic flux and evaluate design alternatives, a three-dimensional finite element method (FEM) simulation is performed using the measured geometric and material properties. The FEM analyses reveal a nonuniform flux distribution along the actuator axis, guiding topology modifications that result in a new prototype design, named CoWrHap-2. The combined analytical and numerical approach enables the prediction and improvement of force output in terms of linearity and efficiency. The results of this study highlight the potential for a high-performance voice-coil haptic device.

Keywords: Haptic devices, voice-coil actuation, electromagnetic field.

Öz

Ses bobini (voice-coil) aktüatörleri, kompakt yapıları, çift yönlü kontrol edilebilirlikleri ve düzgün kuvvet üretimleri sayesinde dokunsal (haptik) arayüzlerde yaygın olarak kullanılmaktadır. Ancak performansları, elektromanyetik yapılandırmaya ve etkin alan içerisindeki manyetik akı yoğunluğuna güçlü biçimde bağlıdır. Önceki çalışmalarda mekanik tasarım genellikle kapsamlı analizlerden ziyade deneysel gözlemlere dayandırılmıştır. Bu çalışma, CoWrHap adı verilen bileğe takılabilir bir haptik cihaz için özel olarak üretilmiş bir ses bobini aktüatörünün matematiksel modellenmesi ve performans iyileştirme çerçevesini sunmaktadır. Öncelikle, giriş akımı, manyetik akı yoğunluğu ve çıkış kuvveti arasındaki ilişkiyi tanımlayan analitik bir model geliştirilmiştir. Model parametreleri deneysel verilerden elde edilerek etkin manyetik bağlaşım katsayısı ve sürücü devresinin akım doyum sınırları belirlenmiştir. Manyetik akının konuma bağlı uzamsal değişimlerini yakalayabilmek ve alternatif tasarımları değerlendirebilmek amacıyla, ölçülen geometrik ve malzeme özellikleri kullanılarak üç boyutlu bir sonlu elemanlar (FEM) analizi gerçekleştirilmiştir. FEM analizleri, aktüatör ekseni boyunca akı dağılımının düzgün olmadığını göstermiş ve bu doğrultuda yapılan topoloji değişiklikleri sonucunda CoWrHap-2 adlı yeni bir prototip tasarımı elde edilmiştir. Analitik ve sayısal yaklaşımların birleşimi, kuvvet çıktısının doğrusallık ve verimlilik açısından öngörülmesini ve iyileştirilmesini sağlamış; çalışmanın sonuçları, yüksek performanslı bir ses bobini tabanlı haptik cihaz geliştirme potansiyelini ortaya koymuştur.

Anahtar Kelimeler: Haptik cihazlar, ses bobini aktüatörleri, elektromanyetik alan.

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1. INTRODUCTION

Virtual reality (VR) technology has emerged as a powerful tool for education, training, healthcare, and entertainment, offering highly immersive and interactive environments that can simulate real-world experiences with remarkable visual and auditory fidelity (Fahl et al., 2023). VR offers several significant benefits that make it a valuable tool across different fields. One major advantage is its ability to enhance skill acquisition through safe, repeatable practice, allowing users to rehearse complex tasks in controlled environments without real-world risks and to refine their abilities at their own pace (Toyoda et al., 2022). In addition, VR enhances engagement and motivation by offering highly immersive and interactive experiences that capture users' attention, sustain their focus, and promote deeper learning and skill retention (Aliza et al., 2024; Lorenzis et al., 2022). Another important benefit is its capacity to simulate scenarios that would otherwise be too costly, dangerous, or impractical to recreate in reality, such as hazardous work conditions, emergency response situations, or large-scale industrial operations (Xu & Zheng, 2020). Finally, these systems allow educators to replicate realistic scenarios with precision, allowing controlled variations, irregularities, or randomization (Pedram et al., 2022; Xu & Zheng, 2020). Despite its advantages, VR interactions suffer from several limitations (Postuma et al., 2025).

Another critical limitation is related to the lack of haptic feedback. Haptics provides a critical channel of information to users through tactile sensation (Chapman, 1994). Humans have evolved to actively engage with their environment and manipulate objects using their hands and fingers. This pervasive behavior has led to a higher density of mechanoreceptors in the skin of our hands, particularly our fingertips, resulting in superior tactile sensitivity compared to other body parts (Dominy, 2009). While VR can convincingly simulate visual and auditory cues, it struggles to replicate the sense of touch, force, and resistance that are essential for natural interaction with objects (Lavoie et al., 2025). This shortcoming becomes especially problematic in manipulation and exploration tasks requiring fine motor skills—such as surgical training, rehabilitation, or precision assembly—where tactile cues guide accurate movement, grip, and pressure control (Tzolov et al., 2025). Without them, users often rely disproportionately on visual information, leading to less precise movements, reduced realism, and weaker skill transfer to real-world contexts.

Haptic devices aim to replicate the physical stimulation of touch for various human-robot or human-computer interaction scenarios, ranging from simple mechatronic systems to complex robotic setups. Most haptic devices are designed as wearables for the fingertips (Pacchierotti et al., 2017), leveraging their potential for high realism and the concentrated mechanoreceptor intensity in this area (Johansson & Vallbo, 1979). However, this approach also presents several significant challenges. Wearable fingertip devices necessitate compact designs, which in turn demand powerful yet miniature actuators (Zhang et al., 2022). Even with the smallest possible designs, they can severely impede the natural fluidity of hand movements and may restrict interaction with real objects, especially in augmented reality contexts.

Relocating haptic feedback from the fingertips to alternative body locations can effectively mitigate these challenges, thereby freeing the hands while still delivering valuable information during virtual interactions, particularly when rendered on the arms (Sarac, 2025). The inherent limitations of mechanoreceptor density, often a concern with such relocation, may be overcome through the deployment of more sophisticated actuators and sensors. Existing literature already provides numerous examples of wrist-worn haptic devices engineered for information transfer (Paredes et al., 2015) and VR/AR interaction scenarios (Adeyemi et al., 2024; Pezent et al., 2023). However, a significant drawback is that wrist-worn haptic devices for VR/AR applications often rely on DC/servo motors, leading to bulky device sizes (Sarac et al., 2022) or intricate mechanical designs (Pezent et al., 2023; Palmer et al., 2022). A promising alternative that addresses these issues involves the use of voice coil actuation, which has been previously implemented in various haptic contexts to generate vibration feedback (Gaudeni et al., 2019) or single-bump force feedback (i.e., tapping) (Culbertson et al., 2018; Tanacar et al., 2023).

Previously, Adeyemi et al. (2024) presented the Voice-**Co**il based **Wr**ist-worn **Ha**ptic (CoWrHap) device as a wearable, wrist-worn haptic device that leverages custom voice-coil actuation to deliver an immersive tactile experience. CoWrHap was engineered to deliver single-bump force feedback, where a predetermined current level is sent to the coil upon signal activation, causing a corresponding displacement of the magnet. Once triggered, the magnitude of this force feedback remains constant for a very short period (approximately 0.5 seconds) to prevent thermal damage to electronic components during operation. Even though our results demonstrated CoWrHap's efficacy in empowering users to instinctively explore and accurately distinguish the stiffness levels of virtual objects through adjustable force feedback, they also highlighted the limitations of realism when employing single-bump force feedback for stiffness exploration in virtual reality environments.

To further investigate the implications of this constraint, Ercan et al. (2024) conducted a subsequent study to compare user perceptual capabilities created by CoWrHap over a wrist-worn device equipped with a linear actuator, capable of providing continuous force feedback (Sarac et al., 2022). The findings from this study revealed that continuous force feedback enhanced users' discrimination sensitivity, boosted their confidence during exploration, and improved overall user experience. Despite these advances, the literature still lacks device designs that can offer compelling and believable haptic feedback on users' wrists without compromising on affordability and portability.

However, both of the earlier CoWrHap studies focused primarily on user perception and behavioral evaluation, and relied on an experimentally derived actuator without a rigorous examination of its electromagnetic characteristics. The underlying voice-coil design had never been analytically modeled, its force-generation efficiency had not been quantified, and its geometric topology had not been optimized. As a result, key performance limitations—such as nonlinear force response, flux nonuniformity, and current saturation—remained unexplained.

This work addresses these gaps by providing the first detailed analytical and numerical characterization of the CoWrHap actuator and by proposing a new optimized design. The main motivation of this paper is to improve the actuation performance of the original CoWrHap device through a combined analytical and numerical investigation of its electromagnetic design. Unlike previous studies that focused primarily on user experience and behavioral validation, the present work introduces a rigorous modeling and simulation framework to guide topology changes and enable the development of a new prototype of the device. The main hypothesis is that the wearability and efficacy of the wearable haptic device can be noticeably improved through numerical/analytical analyses or design/topology changes. Specifically, this study contributes the following:

- An analytical formulation of the CoWrHap actuation principle based on the Lorentz force law.
- A comparison between the experimental and the theoretical (simulation) results developed for the original design of CoWrHap and a validation of the chosen parameters based on the previous experimental force measurements.
- A detailed three-dimensional FEM analysis of the magnetic field and force output, revealing the spatial dependence of flux density along the actuation axis and explaining the nonlinearity observed in the original prototype.
- An iteration-based redesign process to create CoWrHap-2 by changing the design topology to a dual-coil configuration that can potentially result in a more linear force transmission and reduced overall mass.

By combining analytical modeling, empirical parameter estimation, and high-fidelity FEM simulations, this work provides a comprehensive and principled foundation for optimizing compact voice-coil actuators for haptic wearables. This study bridges a critical gap left by earlier CoWrHap studies and establishes a structured pathway for further performance improvements. The rest of the paper proceeds as follows: Section 2 details the design concept of CoWrHap, expands on its mathematical modeling and simulation-based analyses, and proposes a new topology based on a dual-coil design that could potentially improve force transmission in terms of linearity and wearability. Section 3 discusses the findings of these results and their implications for future work. Finally, Section 4 concludes the paper.

2. METHODOLOGY

2.1. CoWrHap Design

Figure 1 shows the design mechanism of the Voice-**Co**il based **Wr**ist-worn **Ha**ptic device (CoWrHap). It operates on the principle of electromagnetic actuation through the interaction between a current-driven coil and permanent magnets. The system consists of a cylindrical core around which insulated copper wire was wound to form a coil, while three axially aligned permanent magnets, that were rigidly attached, were placed inside the cylinder. These magnets were securely mounted onto the user's wrist via a protective fixture. By supplying the coil with time-varying currents of different amplitudes, a controllable magnetic field was generated. This magnetic field interacted with the wrist-mounted permanent magnets, inducing a mechanical motion consistent with the Lorentz force principle. Consequently, a force was applied to the user's wrist, with its magnitude and temporal profile directly determined by the characteristics of the input current signal.

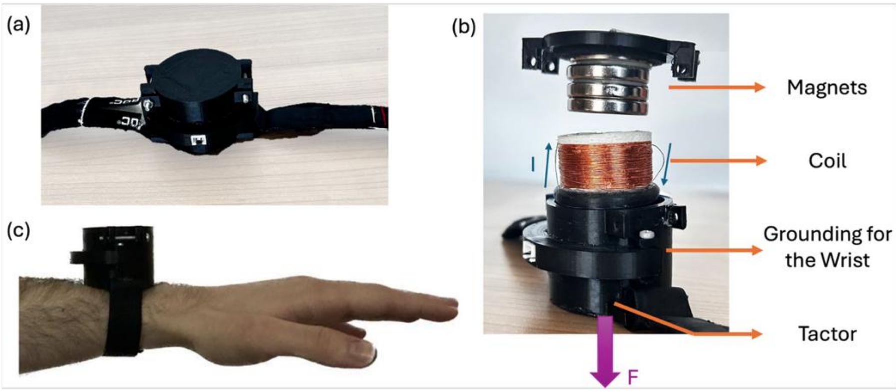

Figure 1. Voice-**Co**il based **Wr**ist-worn **Ha**ptic device (CoWrHap) structure presented in previous work (Adeyemi et al., 2024). (a) Its overall design resembles a closed box to prevent any breakage or interruptions from the outside world during operation. (b) It consists of multiple magnets and a custom-made coil with a wire wrapped around a cylinder. As electrical current passes through the two ends of the coil, the occurring magnetic field creates displacements and haptic forces. (c) It can be worn on users' wrists through Velcro straps.

2.1.1. Mathematical Model

According to Lorentz's force law, when a wire carries current inside a magnetic field, the moving charges in the wire experience a force, as:

$$F = I\,L\,x\,B \tag{1}$$

where

- I is the current passing through the wire,
- L is the length of the wire that lies inside the magnetic field, and
- B is the magnetic field strength, directly related to the magnets chosen for the voice coil design.

According to Equation (1), the achieved level of magnetic force is directly related to the current passing through the wire or the magnetic field. The voice coil design requires a single straight line to be wrapped around a cylinder, ultimately resulting in a higher total length. In this scenario, Equation (1) turns into:

$$F = I\,L\,B\,N \tag{2}$$

where N is the number of wire turns around the cylinder. It would be beneficial to consider Equation (2) as if N wires experience the magnetic field-induced forces during operation. During operation, it is quite common to modulate the achieved current for the voltage supply using the duty cycles set by pulse-width modulation (PWM) rather than the actual current supply, so Equation (2) can be adjusted as

$$F = D_{PWM} \, I_{max} \, L \, B \, N \tag{3}$$

where D_{PWM} is the duty cycle of the PWM signal (defined as a percentage).

2.1.2. Experimental Force Characterization

The original CoWrHap prototype was developed through empirical exploration rather than formal electromagnetic analysis, as reported previously (Adeyemi et al., 2024). Therefore, its performance inherently reflects the cumulative effect of manufacturing simplifications and practical loss mechanisms. Regardless of the imperfections caused by the physical implementation of the setup, it is important to have a reasonable relationship between the theoretical simulations and the experimental setup. This relationship necessitates characterizing the effective magnetic coupling coefficient and the constant loss term that must be incorporated into the analytical model.

Experimental validation plays a critical role in establishing the connection between the analytical formulation and the actual behavior of a physical voice-coil actuator. Even though analytical expressions provide valuable insights, real electromagnetic systems rarely behave as idealized models predict. In practice, deviations arise from a wide range of nonidealities, such as the physical friction in the sliding mechanism, preload forces introduced by the wearable form factor, component tolerances in the coil and magnet assembly (especially if the coil has been wrapped around manually), off-axis motion, electrical factors, or overall heating around the coil. These imperfections tend to exhibit a systematic offset or reduced magnitude relative to theoretical predictions, which are inevitable while working with physical setups.

Adjusted Analytical Model Accounting for Physical Imperfections

Ultimately, the experimentally measured forces deviate from the idealized analytical force model defined in Equation (3). To account for nonidealities, the effective force can be expressed as:

$$F_{ef} = D_{PWM} \, I_{max} \, L \, B \, N - F_{loss} \tag{4}$$

where F_{loss} is the constant loss of force, which might be due to friction, preload, mechanical bias, or practical imperfections. Originally, CoWrHap was designed intuitively by exploring different combinations of magnets (with different materials and diameters), coil diameter, coil height, and the number of turns of the wire around the coil (Adeyemi et al., 2024). Our main goal was to achieve easily detectable forces with a compact mechanical design that promotes high wearability. Overall, the decisions regarding the mechanical parameters are listed below:

- 0.25 mm copper wire,
- 60 turns (N),
- 20 mm (0.02 m) coil height (L),
- 35 mm cylindrical base diameter, and
- 4 neodymium (NdFeB) magnets axially attached with 1-2 mm distancing between them (B ≅ 1.2 T, as supported by Katter (2025)).

The initial geometric and electromagnetic parameters of CoWrHap were selected through an engineering-driven exploration constrained by manufacturability, wearable form-factor considerations, and safety limits. For instance, the 0.25 mm copper wire and magnet type were chosen thanks to their high availability in the market, mechanical robustness, and ease of winding. Yet, it might have directly limited the thermal resistivity. Different magnets were tested to limit the overall heating as much as possible for the given haptic use. The number of turns was decided upon as a trade-off between various requirements. Designing the coil with 60 turns resulted in sufficiently high output forces and a compact design, at the cost of a thermal load and power dissipation within an acceptable limit. The cylindrical base diameter and height have been kept large enough without limiting the wearability of the proposed wrist-worn device concept. Finally, the magnets were reported to have approximately 1-2 mm spacing between them. This spacing was not defined or arranged specifically with a particular motivation, but rather based on an observation from the actual implementation. The implementation goal was to attach them as closely as possible.

Furthermore, CoWrHap was controlled using a Raspberry Pi Pico Microcontroller due to its dual-core Arm Cortex-M0 processor, which enables threading to collect data from the environment and send control signals to the motor driver simultaneously, as previously reported (Adeyemi et al., 2024). L298N motor driver runs the voice coil actuator with a 1 A output current capacity per channel, even though its peak current can reach up to 2 A. An external heat sink was used to prevent the motor driver from overheating. These limitations of the motor driver result in $I_{max} \cong 1$ A.

Experimental Collection of the Force Measurements

After its implementation, the levels of interaction forces that can be rendered by CoWrHap were validated through experimental measurements. A Force Sensitive Resistor (FSR) sensor was placed between the CoWrHap tactor and the user's skin to capture the normal interaction forces produced at different duty-cycle levels (Adeyemi et al., 2024). Prior to data collection, the FSR was calibrated using a series of known weights (0.5–5 N range), enabling the voltage–force relationship to be fitted with a standard logarithmic curve recommended by the manufacturer. This calibration ensured that all forces produced by CoWrHap during testing were kept within the FSR's usable operating range (approximately 0–10 N), well below its saturation point. Although FSRs are known to exhibit measurement noise and drift, repeating each measurement seven times for duty cycles between 50% and 100% and averaging the results helped stabilize the characterization. Duty cycles below 50% were excluded because the combination of static friction and the preload from the wearable strap prevented the actuator from producing measurable displacement on the sensor, resulting in inconsistent and near-zero readings.

Figure 2 (a) details the measured interaction forces: blue bars represent the average of measured forces, while the orange line shows the overall behavioral model. The collected data reveal that CoWrHap's output forces saturate after 80% duty cycle, possibly due to the limitations imposed by the motor driver.

Comparison between Theoretical and Experimental Insights

Figure 2 (b) provides a systematic comparison across the experimental data (i.e., the data presented in Figure 2 (a)) and theoretical predictions derived from Equation (3) using the design parameters listed above, under the assumption of ideal implementation of the device, which has been depicted as the blue line. While the theoretically predicted force profile exhibits the expected linear relationship between duty cycle and force, a consistent discrepancy is observed between the model and the measured values. This gap is likely to correspond to the constant force loss term, F_{loss}, which was introduced to the mathematical model with Equation (4). When fitting Equation (4) to the experimental data, the loss term is estimated to be approximately $F_{loss} = 0.35N$. The resulting loss-adjusted theoretical curve is shown in green.

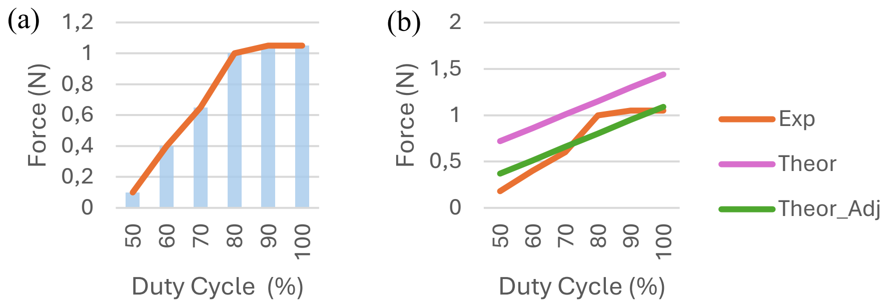

Figure 2. Output force Validation for CoWrHap: (a) Force measurements collected experimentally via FSR sensors (Adeyemi et al., 2024). (b) Experimentally collected force models were compared to theoretical calculations under the assumption of ideal conditions, as well as the effective theoretical calculations that also account for constant force losses.

The identification of this systematic loss is crucial: it confirms that the discrepancy is not random noise, but a repeatable and quantifiable mechanical and electromagnetic limitation of CoWrHap. This loss serves as a key motivation for the redesign effort presented in the following sections, where FEM analyses are employed to identify and mitigate the sources of inefficiency.

2.1.3. Finite Element Methods (FEM) Analysis

Output forces modeled as Equations (3) and (4) are assumed to be directly proportional to the magnetic flux density. However, these forces are also directly affected by the position of the coil with respect to the magnets: as the moving coil changes its position with respect to the magnet, the effective flux density within the interaction volume *also* varies. This variance might result in deviations in the rendered force outputs.

The spatial variations of the coil around the magnet should be analyzed with a deeper focus to understand how they affect the output forces. Finite Element Method (FEM) analyses enable a precise evaluation of the magnetic field distribution within the coil–magnet interaction volume, which analytical expressions cannot accurately capture. A

FEM simulation model was created, including a permanent magnet and a cylindrical coil, mimicking the physical assembly of CoWrHap as represented in Figure 1. CoWrHap was supplied with a maximum current of 1 A, limited by the physical capabilities of the L298N motor driver, as discussed in the previous section. In this simulation, the magnetic field was analyzed to determine the distribution of magnetic flux density for various displacement positions of the coil relative to the magnet position. The variations in the magnetic field directly imply the variations of the output forces, as influenced by Equation (2).

Figure 3 shows the results of the FEM analysis, which illustrates the relationship between the coil displacements and the output forces. This plot verifies that the magnetic density might lead to positional variations in the achievable force outputs, even under similar current supplies and physical design specifications.

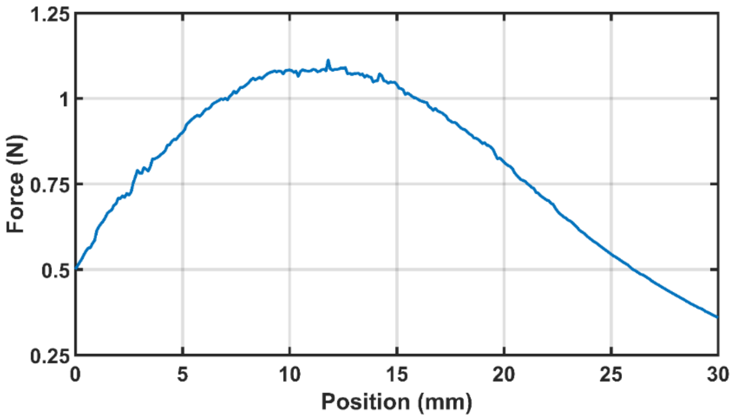

Figure 3. FEM Simulation Results for CoWrHap: The Output Force Varies as the Coil Position Changes with Respect to the Magnet Nonlinearly

Figure 3 shows that the output force of the CoWrHap is directly affected by the position of the coil with respect to the magnet. While this output might be acceptable for certain applications, it has critical implications when it comes to rendering haptic feedback in the context of human-robot interaction. It is expected that the same CoWrHap device should be effective across different users with anatomical differences, such as variations in skin thickness, muscle volume, etc. Such anatomical properties might impose disruptions on the location or displacements of the tactor, leading to unexpected behavior of the output forces. Providing uneven interaction forces could lead different users to perceive the force feedback in completely different ways, which complicates the development of effective, robust, and consistent haptic devices that follow a one-fits-most design requirement.

2.2. Improving CoWrHap with Topology Change and Parameter Tuning

FEM analyses presented in the previous section indicated that the original CoWrHap device could generate output forces with nonlinearities and variances, which were partially influenced by the coil displacements. With this motivation, a different voice coil topology can be designed to linearize the output forces regardless of the coil position with respect to the magnet during operation, i.e., CoWrHap-2. This topology requires certain design changes to be made from the first prototype, as summarized in Table 1.

Table 1. Changes of the Design Topology to Improve CoWrHap

CoWrHap	CoWrHap-2
It consists of a one-magnet and one-coil structure.	It consists of a one-magnet and two-coil structure.
The output forces are created by a single magnetic field.	The output forces are created by dual magnetic fields that supplement each other.
The magnet is grounded to the wrist as the coil moves to create output forces.	The coils are grounded to the wrist as the magnet moves to create output forces.

Figure 4 shows the dual coil structure of CoWrHap-2. The design features two identical cylindrical coils with wire wrapped around them. These two coils are grounded on the wrist-worn structure at the top and the bottom. The coils have a hole at their centers through which the single-structured magnet can pass via a shaft.

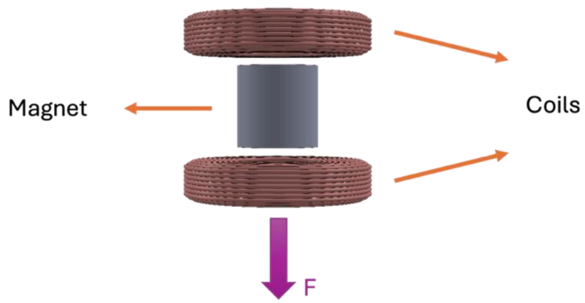

Figure 4. Dual Coil Voice Coil Topology

The principle of the force generation for each coil is identical to that of the original CoWrHap. Designing CoWrHap-2 with dual-coil topology allows the magnetic field to create two alternative sources of output forces, which then supplement each other. In other words, the ultimate forces applied by the moving magnet are the sum of the forces created by both coils individually.

For CoWrHap-2, FEM simulations were repeated to investigate whether the dual-coil design topology improved the linearity of the output forces in exchange for coil (in this scenario, magnet) displacements. Figure 5 represents the output forces: red and blue lines represent the forces created by each coil independently, while the yellow line represents the effective tactor force (i.e., haptic interaction forces in the case of CoWrHap-2).

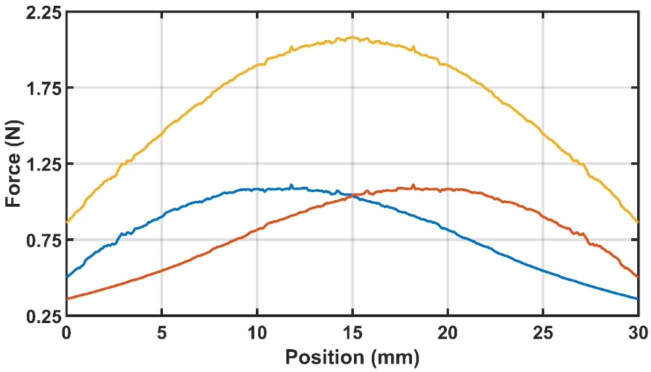

Figure 5. FEM Simulation results for CoWrHap-2: the output force varies as the coil position changes with respect to the magnet nonlinearly. The actual effective force is obtained as the sum of their contributions, which exhibits a more linear behavior than that of CoWrHap.

The CoWrHap-2 design parameters were tuned iteratively using FEM simulations such that the most linearized force output and the lowest mass were obtained. Considering that the overall output force of CoWrHap-2 is almost double that of CoWrHap, minimizing the device's weight is a relatively straightforward task. New design parameters were obtained by iteratively exploring until the smallest mechanical structure could be achieved that satisfies the output forces within the range of 1–2 N. These iterative analyses were performed through FEM simulations, modifying geometrical parameters for the given ranges. Table 2 presents the values assigned to the mechanical parameters and the possible ranges for parameters that may vary during operation.

Table 2. CoWrHap-2 Design Parameters

Parameter	Value	Unit
Coil outer diameter	24	mm
Coil inner diameter	14	mm
Coil length	5	mm
Magnet diameter	10	mm
Magnet length	10	mm
Number of turns	200	
Excitation current	1	A
Magnet position	-10 to 20	mm
Gap between coils	0 to 20	mm

Using FEM simulations, 2D magnetostatic analyses were conducted using the parameters and ranges detailed in Table 2. Even though the required displacement of the moving part is defined as 10 mm, the analyses were conducted between -10 mm and 20 mm to observe the overall force output of the actuator. The position of "0 mm" refers to the position of the magnet at the edge of the coil, moving in given directions. The gap between coils is the main objective to be optimized, but the force output for the full range should also be observed to achieve the goal of a linear output force along the stroke.

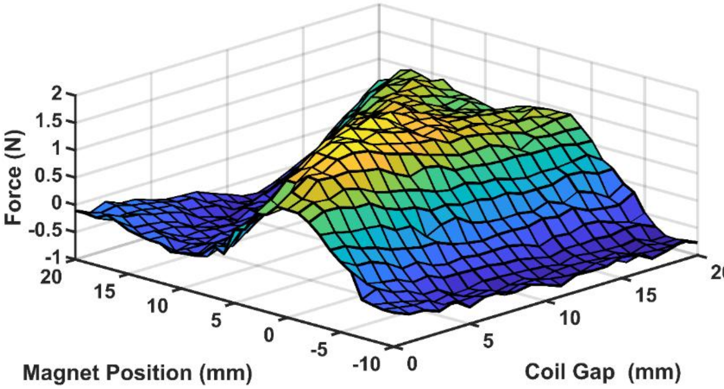

Figure 6. FEM Simulation Results for CoWrHap-2 as a Function of Magnet Position as well as the Coil Gap

Figure 6 shows the FEM simulation results for the output force of the dual-coil topology implemented for CoWrHap-2. This plot indicated that the assumption of dual-coil structures contributing to a more linearized output force was, in fact, correct since the forces depend on the position of the magnetic field source that generates them. Furthermore, it highlighted a critical relationship between the coil gap and the output forces: a smaller coil gap amplified the force output, at the cost of leading to a peak force region in which the output force was not distributed linearly along the stroke (i.e., as the magnet position changed). However, a larger coil gap led to saturated maximum force values, which followed a more constant (plateau) behavior as the magnet position changed. Ultimately, one could argue that even though having a higher range of motion is favorable, the linearity of the force is decreased. Therefore, the most efficient force outputs were observed at a 16 mm coil gap position.

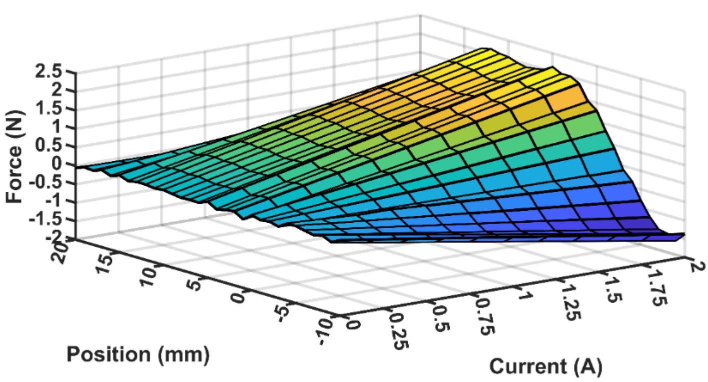

Figure 7. FEM Simulation Results for CoWrHap-2 as a Function of Magnet Position, as well as the Current Supply Provided to the Voice Coil

Even though the previous simulation that led to Figure 6 was performed under the assumption of a constant current supply (i.e., 1 A), the effective current that passes through CoWrHap-2 is intended to vary depending on the power supply or the duty cycle modulation. Figure 7 shows how CoWrHap-2 generates output forces for varying magnet positions and current input. Output forces are observed to be linear in both operational ranges. These results are in accordance with the output forces obtained from the previous design. Furthermore, the linearity of the output forces was maintained for a stroke of more than 10 mm, which was deemed sufficient for haptic applications.

3. DISCUSSION AND FUTURE WORK

In this paper, a previously designed and presented wearable haptic device based on a custom-made voice coil actuator was mathematically modeled and studied (Adeyemi et al., 2024). Section 2.1. summarizes the well-known formulations derived from the Lorentz force principle that can mathematically model CoWrHap. Using these formulations, experimentally conducted force measurements were validated through theoretical calculations, which is potentially crucial for estimating force behavior and improving overall force generation.

The results in Figure 2 indicate disruptions between the practical and experimental assumptions. In practical actuator systems, the ideal electromagnetic force is often offset by constant mechanical losses, such as static friction, preload, or contact stiction. Prior studies in robotic actuation have consistently simulated these effects as constant bias forces that add an offset to the theoretical output following different models (Hynes & Hashtrudi-Zaad, 2013; Laffranchi & Caldwell, 2022). Based on these precedents, modeling a constant offset of approximately 0.35 N in the present CoWrHap design is both physically plausible and consistent with prior friction characterization methods in haptic and robotic literature. In addition to these mechanical losses, the electrical characteristics of the actuator introduce further departures from the analytical model. Specifically, Equation (3) assumes that the average coil current increases linearly with PWM duty cycle ($I = D_{PWM}\, I_{max}$), which does not strictly hold for inductive loads such as voice coils. Inductance slows the buildup of current within each PWM cycle, resulting in nonlinear current and duty-cycle behavior, and reduces the effective average current, particularly at high duty cycles where saturation has been experimentally observed. These combined mechanical and electrical nonlinearities help explain the systematic deviations between theory and measurement, underscoring the importance of incorporating potential losses into future analytical and FEM-based modeling efforts.

As a follow-up, simulation-based analysis performed with FEM expressed a relationship between the coil displacements and the output forces. Figure 3 verified that the magnetic density might lead to positional variations in the achievable force outputs, even under similar current supplies and physical design specifications. These insightful outcomes could justify some of the force behaviors observed with the experimental force measurements detailed in Figure 2. Figure 2 (a) shows that, around a 70% duty cycle, the experimental forces exhibit a more sudden change than the theoretically calculated forces. While the strict change could simply be caused by using simple FSR sensors for force measurements, it could also be related to the nonlinearity observed in Figure 3. To investigate this speculation any further, experimental data should be repeated to measure not only the output forces but also the displacement observed at the magnets (or the coils) in the future.

To better control the behavior of the output forces and achieve more linear forces, a new design topology was proposed as the baseline for CoWrHap-2. In this scenario, the idea behind the actuation remained majorly similar, except for creating a dual-coil structure that can accumulate the nonlinear forces generated by both coils and obtain a linear force at the end. Figure 5 represents the forces generated by both coils in addition to the overall device, and highlights two important take-away messages. Firstly, the variations of the output forces generated by each coil are still non-symmetrical and nonlinear. Secondly, it can be clearly addressed that the position of the coil directly affects the summing result more linearly and symmetrically. Ultimately, since the output force is nearly doubled, the mechanical parameters can be easily tuned to achieve a more compact design while still ensuring a 1 N output force is achieved.

Further FEM simulation analyses were conducted to decide how output forces are affected by important mechanical decisions, such as the coil gap. Figure 6 shows that decreasing the coil gap increases the force output, at the cost of decreased linearity. In other words, decreasing the coil gap increases the force output but negatively impact the linearity. However, considering the importance of controllability of such a custom-made actuator, it is safe to claim that the linearity of the output forces is more crucial than the levels of output forces, as long as it can reach a certain, noticeable range of force. In this sense, CoWrHap-2 has great potential for future applications of haptic scenarios.

Once the coil gap decision has been made, Figure 7 shows that the necessary output forces can be achieved linearly for an acceptable range of current supply and magnet position. These results also indicate that CoWrHap-2 has the potential to be operative with lower current requirements, which will potentially not saturate as CoWrHap due to the motor driver limitations. Working with lower ranges of current will potentially have two major benefits to CoWrHap-2, which has been specifically designed for haptic applications. Firstly, working with lower current supplies will ensure a more durable, robust, and sustainable prototype for the long run, preventing the electronic equipment from being damaged during operation. With this safety ensured, the magnet could be provided with a current supply for a longer time, which could potentially allow the forces to be rendered for users for longer times. This upgrade would overcome the most critical drawback that was reported related to CoWrHap in the previous work, which should be investigated further in the future by studying the power requirements and heat transfer.

The overall dimensions of CoWrHap-2 were reduced noticeably compared to CoWrHap. Designing dual coils increased the output forces to the levels that were deemed to be excessive and even potentially dangerous for human interactions. To keep the force levels in the range of 1 N, the height of the coils was minimized, in addition to using a smaller magnet. Using a smaller magnet allowed the coils to be grounded while leaving the magnet as the automated component. Obtaining a more compact mechanical design is crucial for haptic applications to promote portability, transparency, and pleasantness of the feedback during operation.

Overall, the identification of a consistent and repeatable loss term not only explains the gap between the idealized analytical force model and the measured forces of CoWrHap, but also serves as an important validation milestone. By demonstrating that the experimentally observed behavior can be reconciled with the Lorentz-force formulation through a physically meaningful correction, this study establishes that the analytical and numerical frameworks accurately capture the dominant electromagnetic behavior of the device. This alignment provides a scientifically grounded basis for trusting the FEM simulations used to explore topology modifications and compare alternative designs. In this sense, the simulation-based evaluation of CoWrHap-2 is not merely hypothetical, but is anchored in a validated physical model that reflects the real constraints and characteristics of the original prototype. While full experimental fabrication, benchmarking, and head-to-head comparison between CoWrHap and CoWrHap-2 remain essential components of future work, the initial experimental validation presented here enables these simulation-driven insights to be both meaningful and impactful. Accordingly, the results provide valuable guidance for the engineering community by revealing the performance trade-offs, dominant loss mechanisms, and promising design directions that warrant further investigation and refinement.

4. CONCLUSION

This study presented a systematic modeling and tuning framework for a voice-coil-based haptic actuator, integrating both analytical and numerical approaches to enhance its performance. A mathematical model based on the Lorentz force principle was first developed to describe the quantitative relationship between the input current, magnetic flux density, and generated force. Experimental characterization revealed that the effective magnetic coupling coefficient ranged between 0.6 and 0.7, while the motor driver exhibited a current saturation of around 1 A. These findings helped explain the nonlinearities observed in the actuator's force output.

To complement the analytical formulation, a three-dimensional Finite Element Method (FEM) analysis was conducted to evaluate the spatial variation of the magnetic flux along the actuator axis. The simulation results demonstrated significant flux nonuniformity across the interaction region, which was shown to influence the overall force generation. Insights obtained from the FEM results guided topological design modifications, leading to the development of an improved prototype, CoWrHap-2, featuring a more uniform magnetic field and enhanced force efficiency.

Future studies include developing a more comprehensive optimization algorithm of the actuator's design parameters and operating conditions. In particular, multi-objective optimization techniques can be employed to simultaneously maximize output force, linearity, and energy efficiency while minimizing mass and thermal losses. Further analysis of power transmission and heat dissipation within the coil–magnet assembly would enable a deeper understanding of performance limitations under continuous operation. Moreover, incorporating nonlinear magnetic material properties and dynamic excitation profiles into the FEM simulations could improve the predictive accuracy for time-varying interactions. Finally, the enhanced version of the wrist-worn haptic device should be validated for perceptual studies, whether during virtual interactions or teleoperation scenarios.

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