

Design and Development of AR System Combining Physical Cards and Digital Visualization Technologies in Geometry Education

Geometri Eğitiminde Fiziksel Kartlar ve Dijital Görselleştirme Teknolojilerini Birleştiren AR Sistem Tasarımı ve Geliştirilmesi

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Özet

Bu çalışmada, geometri eğitiminde karşılaşılan uzamsal görselleştirme zorluklarını ve öğrenci motivasyon eksikliğini gidermek amacıyla AR destekli hibrit geometri öğrenme sistemi tasarlanmış ve geliştirilmiştir. Design Science Research metodolojisi kullanılarak yürütülen araştırmada, fiziksel kartlar ile mobil AR uygulamasının entegrasyonu yoluyla bütünleşik bir öğrenme sistemi oluşturulmuştur. Sistem, Türkiye Cumhuriyeti Millî Eğitim Bakanlığı 6-12. sınıf matematik müfredatına uygun olarak seçilen 18 geometrik şekil (9 adet 2D, 9 adet 3D) için tasarlanmıştır. Adobe Illustrator kullanılarak 300 DPI çözünürlükte fiziksel kartlar tasarlanmış, Autodesk 3ds Max ile optimize edilmiş 3D modeller oluşturulmuş ve Unity - Vuforia teknolojileri ile Android platformu için AR uygulaması geliştirilmiştir. Performans testleri 10 farklı Android cihazda gerçekleştirilmiş, AR takip doğruluğu ortalama %93,2 olarak elde edilmiştir. Sistem kararlılığı testlerinde 120 dakikalık kesintisiz kullanım boyunca hiçbir çökme yaşanmamış, işlemci yükü %28-34 aralığında stabil kalmıştır. İçerik uygunluk analizi sonucunda sistemin müfredat ile %83,3 uyumluluk düzeyi gösterdiği tespit edilmiştir. Literatür karşılaştırması, geliştirilen sistemin hibrit öğrenme yaklaşımı, çevrim dışı kullanım imkânı ve müfredat uyumluluğu açısından mevcut çözümlere kıyasla önemli avantajlar sağladığını ortaya koymuştur. Sistem, geometri eğitiminde teknolojik yenilik ile eğitsel değerini başarılı şekilde birleştirdiği yenilikçi bir çözüm sunmaktadır.

Anahtar kelimeler: Artırılmış Gerçeklik, Eğitim Teknolojisi, Geometri Eğitimi, Hibrit Öğrenme, Mobil Uygulamalar, Uzamsal Görselleştirme

Abstract

This study designed and developed an augmented reality (AR)-supported hybrid geometry learning system to address spatial visualization difficulties and a lack of student motivation in geometry education. Using the Design Science Research methodology, we created an integrated learning system that combines physical cards with a mobile AR application. The system was designed for 18 geometric shapes (nine 2D and nine 3D) selected according to the mathematics curriculum for grades six through twelve from the Ministry of National Education of the Republic of Turkey. The physical cards were designed in Adobe Illustrator at 300 DPI, and the optimized 3D models were created in Autodesk 3ds Max. The AR application was developed for the Android platform using Unity and Vuforia technologies. Performance tests were conducted on ten different Android devices, revealing an average AR tracking accuracy of 93.2%. System stability tests revealed no crashes during 120 minutes of uninterrupted use, with a stable processor load ranging from 28% to 34%. Content relevance analysis showed that the system is 83.3% compatible with the curriculum. A literature comparison revealed that the developed system offers significant advantages over existing solutions due to its hybrid learning approach, offline functionality, and curriculum compatibility. This system provides an innovative approach that successfully combines technological innovation and educational value in geometry education.

Keywords: Augmented Reality, Educational Technology, Geometry Education, Hybrid Learning, Mobile Applications, Spatial Visualization

1. Introduction

The difficulty students have in understanding geometric concepts is a longstanding issue that has attracted the attention of educational researchers. The primary challenge among these difficulties is the inadequacy of spatial visualization skills. Spatial visualization is the ability to mentally manipulate objects and understand spatial relationships. As Medina Herrera et al. (2024) noted, spatial visualization is not only critical for understanding the relationships between two- and three-dimensional geometric shapes but also plays a central role in mathematics education overall [1].

However, traditional geometry teaching methods, which are mainly limited to static visuals, are insufficient for helping students understand dynamic spatial relationships. This makes it difficult for students to visualize geometric concepts, and the lack of support for abstract geometric concepts with concrete experiences reduces the effectiveness of the learning process.

Another significant challenge in mathematics education is students' lack of motivation toward the subject. Traditional teaching methods are unidirectional and passive, limiting students' active participation and leading to boredom during the learning process. This issue is particularly pronounced in areas that require visual and spatial skills, such as geometry. AR technology has recently attracted attention for its potential to revolutionize education. AR combines 2D and 3D objects from the virtual world with the real world in real time [2]. AR enriches traditional learning experiences by adding digital content to students' physical environments, making them more interactive.

Koparan et al. (2023) conducted a thorough examination of the potential of AR technology in mathematics education [2]. They revealed that AR technology offers significant advantages in geometry teaching by enabling visualization, interaction, and scenario creation. The researchers noted that AR helps students easily grasp basic geometry concepts and supports sensory perception by adding computer-generated content to the student environment in real time. One of the most significant advantages of AR technology in education is its ability to capture students' attention and boost motivation. Jesionkowska et al. (2020) demonstrated that AR technology helps students develop their learning abilities in STEAM (science, technology, engineering, arts, and mathematics) fields [3].

With the widespread adoption of mobile devices, access to AR technology has become easier, and mobile AR applications can display marked pages as three-dimensional models through webcams, providing augmented teaching environments. This development allows AR applications to be practically used in classroom environments and makes them accessible to large student populations. Experimental studies also support the effectiveness of using AR technology in geometry education. Uriarte-Portillo et al. (2023) developed ARGeoITS, a geometry learning tool that combines augmented reality with intelligent tutoring systems [4]. In their study with 106 middle school students, they

showed that students using ARGeoITS achieved higher scores in learning outcomes compared to students using only AR (7.47 versus 6.83). Similarly, Su et al. (2022) investigated the effect of immersive virtual reality technology on learning geometry, demonstrating that their developed system improved students' motivation and performance [5].

In addition to these technological developments, hybrid learning approaches play an important role. Hybrid learning refers to the integration of traditional face-to-face teaching methods with digital technologies. This approach aims to preserve the strengths of conventional teaching methods while utilizing the advantages of modern technology. It provides students with greater flexibility in the learning process and allows teachers to implement a range of strategies.

Hybrid learning approaches offer significant advantages in mathematics education, particularly in areas where visual and spatial skills are used extensively, such as geometry. It enables the combined use of physical manipulatives and digital tools, providing students with tactile and visual experiences. Integrating AR technology into hybrid learning environments further enriches the learning experience, creating deeper, more meaningful learning by enabling students to experience real-world objects and digital content simultaneously.

In this context, spatial visualization skills are recognized as a fundamental component of mathematics education. These skills play a critical role in students' understanding of geometric concepts and problem-solving processes. The OECD's Programme for International Student Assessment (PISA) defines problem-solving skills as "cognitive processing capacity in situations where the solution method is not immediately apparent" [6]. This definition highlights the strong connection between spatial visualization and problem-solving skills. AR and VR technologies allow students to interact with three-dimensional objects and examine them from different angles. This develops their spatial understanding, helps them grasp mathematical concepts, and increases their participation.

The current literature indicates that AR technology improves student motivation, learning performance, and spatial visualization skills in geometry education. However, significant research gaps remain in exploring the potential of hybrid approaches and developing curriculum-compatible systems. The study's main objective is to create an integrated learning system that combines the strengths of traditional and digital learning approaches by pairing physical cards with mobile AR applications.

2. Literature Review

A comprehensive literature review was conducted to determine the place of the developed AR-supported hybrid geometry system in the existing literature and to examine related studies. The evaluation aimed to understand how AR technology is used in mathematics and geometry education and to identify research gaps that this study addresses.

Several AR-based systems have been developed to support geometry education, each with distinct approaches and features.

Construct3D, developed in 2000, is a three-dimensional geometric construction tool specially designed for mathematics and geometry education. Built on the "Studierstube" collaborative augmented reality system, Construct3D uses a stereoscopic head-mounted display (HMD) and a two-handed 3D interaction tool called the Personal Interaction Panel (PIP) [9].

ARGeoITS is a comprehensive geometry learning tool that combines augmented reality with intelligent tutoring systems. Built with Unity 3D and Vuforia, the system employs a fuzzy-logic-based adaptive learning approach. A study involving 106 middle school students found that those using ARGeoITS achieved significantly higher learning outcomes than those using only AR (7.47 versus 6.83) [4].

Geo+ is an AR application designed to enhance elementary students' understanding of solid geometry. The system uses a human-centered design approach and was developed with Unity and Vuforia technologies. A user study involving 96 elementary school students demonstrated that Geo+ effectively increased student engagement and was user-friendly [10].

The GeoGebra 3D Calculator AR is a comprehensive mathematics learning platform incorporating the AR features of the GeoGebra 3D Calculator. The system uses an active, student-centered learning approach. When tested with 78 students using a quasi-experimental method, 70% of the experimental group achieved superior performance levels [11].

CleverBooks Geometry is an innovative, immersive AR application designed to support geometry learning. It provides 3D models of the five basic 2D geometric shapes and allows teachers to explain abstract geometry concepts using 3D visualization. It is used in more than 20 countries [12].

GEMBI AR is a mobile educational game developed by Mandala and colleagues that focuses on developing spatial reasoning skills. An experimental study involving 18 eighth-grade students found that Wilcoxon test results ($Z = -3.578$, $p = 0.000 < 0.05$) showed a significant improvement in spatial reasoning skills [13].

McGraw Hill AR is a comprehensive educational platform for students. The system provides both an AR application experience and a 3D web-based experience, offering support in two languages: English and Spanish [14].

EDU-MAT COPERNICUS is an innovative educational platform integrating AR technology with satellite technologies to support mathematics and geoscience

learning. Interviews with ten mathematics educators and user tests with thirty students revealed that AR-supported tasks enhanced students' spatial reasoning, engagement, and motivation to learn mathematics [15].

The Augmented Reality Mathematics Learning System (ARMLS) is an augmented reality-based mathematics learning system developed for fifth-grade students. It was tested with 66 Taiwanese students, improving the performance of low- and medium-level learners while reducing their cognitive load. However, it did not affect motivation [16].

Plabook AR Math is an AR application that provides a game-based mathematics learning experience tailored to different grade levels [17].

A literature review shows that various approaches are adopted in AR technology applications in education. In terms of preferred platforms, mobile devices (iOS and Android) are widely used, with McGraw Hill AR offering broader access by providing a web-based experience too. ARGeoITS was found to adopt an intelligent tutoring system approach with fuzzy logic integration. Noteworthy in terms of educational content scope are the multidisciplinary approach of McGraw Hill AR and the in-depth mathematics content provided by GeoGebra AR.

Examining the literature on hybrid learning approaches reveals that most existing systems offer fully digital solutions. While the CleverBooks Geometry and ARMLS systems partially integrate physical components, few systems provide a comprehensive hybrid experience.

When evaluated on performance and effectiveness, the results obtained by ARGeoITS in a controlled experimental study (learning achievement: 7.47 versus 6.83) are remarkable. The statistically significant effect of GEMBI AR on spatial reasoning skills ($Z = -3.578$, $p < 0.05$) and the 70% higher performance rate with the GeoGebra 3D Calculator AR exemplify positive results in the literature.

However, when offline usage is considered, most systems in the literature require an internet connection, which can impose technical limitations. This can create barriers to usage, particularly in educational environments where infrastructure resources are limited.

When literature is evaluated for curriculum compatibility, it is found that the examined systems are generally designed in accordance with international education standards and that special solutions for local curriculum requirements are limited.

Table 1 presents a comprehensive comparison of AR-based geometry learning systems in the literature, highlighting the distinguishing features of the present study.

Table 1. Comparison of AR-based geometry learning systems in literature

| Study/System | Key Features/Performance | Limitations |
|---------------------------|---|-------------------------------|
| ARGeoITS | Learning achievement; Adaptive learning | Requires internet |
| GEMBI AR | Spatial reasoning improvement | No physical components |
| GeoGebra 3D Calculator AR | 70% superior performance | Limited offline |
| Construct3D | Collaborative learning; Advanced 3D interaction | Requires specialized hardware |
| Geo+ | High engagement | Elementary level only |

| | | |
|----------------------|---|------------------------|
| CleverBooks Geometry | 3D models of 5 basic 2D shapes | Limited shape coverage |
| McGraw Hill AR | Broad subject coverage; Bilingual | Not geometry-specific |
| EDU-MAT COPERNICUS | Enhanced spatial reasoning | Complex setup |
| ARMLS | Improved performance; Reduced cognitive load | No motivation effect |
| Plabook AR Math | Multi-grade support; Bilingual | Requires internet |
| This Study | Curriculum compatibility; 18 shapes; Stable performance | Android Only |

In conclusion, the AR-supported hybrid geometry learning system developed in this study offers significant advantages over existing systems, including its hybrid learning approach, offline use, and curriculum compatibility. It provides an innovative approach to integrating physical and digital learning materials and develops solutions tailored to the needs of geometry education in Turkey. The system addresses three critical gaps identified in the literature: (1) the lack of comprehensive hybrid learning solutions that combine physical and digital components, (2) the absence of offline-capable AR systems suitable for resource-limited educational environments, and (3) the limited availability of AR-based geometry systems designed specifically for local curriculum requirements.

3. Materials and Methods

This study adopted the Design Science Research (DSR) methodology to design and develop an AR-supported hybrid geometry learning system. DSR is a research approach that focuses on developing new technological solutions in the field of information systems and scientifically evaluating their effectiveness [7]. Proposed by Peffers et al. (2007), this methodology is implemented as a six-stage process consisting of problem definition and motivation, identification of solution objectives, design and development, demonstration/implementation, evaluation, and communication phases [8]. Through this systematic approach, the study aims to provide an innovative solution to overcome challenges in geometry education and make significant contributions to educational technology research and practical applications. The primary rationale for selecting the DSR methodology is its focus on developing artificial products designed to enhance performance.

3.1. Problem Definition and Motivation

A comprehensive literature review was conducted to identify fundamental problems in geometry education and develop solutions systematically. The review encompassed academic studies published in the IEEE Xplore, ACM Digital Library, ScienceDirect, Springer Link, and Google Scholar databases. The search strategy employed keyword combinations such as "augmented reality AND geometry education," "spatial visualization AND educational technology," and "mobile learning AND geometry."

The main challenges in geometry education were identified through the literature review. One critical obstacle is the inadequacy of spatial visualization skills, which makes it difficult for students to comprehend the relationships between 2D and 3D geometric shapes. Traditional teaching methods are static and one-dimensional, making it difficult for students to visualize geometric concepts. The lack of support for abstract geometric concepts through concrete

experiences reduces the effectiveness of the learning process.

We examined the mathematics curriculum for grades 6–12 of the Ministry of National Education of the Republic of Turkey in detail. We analyzed the distribution of geometry topics across grade levels, learning objectives, and achievements. This analysis was conducted to ensure compatibility between the needs of the existing education system and proposed technological solutions.

3.2. Identification of Solution Objectives

Based on the results of the literature review and curriculum analysis, four main objectives were identified for the augmented reality (AR)-supported hybrid geometry learning system.

Objective 1: Provide a hybrid learning experience. Create a multi-sensory learning environment by integrating physical and digital learning materials. This objective preserves the strengths of traditional teaching methods while utilizing the advantages of modern technology.

Objective 2: Provide opportunities for 3D visualization. Developing students' spatial visualization skills using AR technology. This objective focuses on understanding the relationships between 2D and 3D geometric shapes.

Objective 3: To create an interactive learning environment. This objective supports active learning by providing interactive experiences that increase student participation. This objective aims to encourage students to move beyond passively receiving information and to become involved in active exploration.

Objective 4: Develop curriculum-appropriate content. The goal is to create educational material compatible with Turkey's secondary education geometry curriculum and applicable in practice.

These objectives include quantitative (improving learning performance) and qualitative (developing spatial visualization skills) dimensions.

3.3. Design and Development

The design and development of the AR-supported hybrid geometry learning system was carried out systematically in five stages. These stages encompass curriculum-compatible content selection, physical card design, 3D modeling, AR application development, and user experience optimization. Each stage builds on information obtained from the previous stage and follows best practices in educational technology development. During the design process, technical requirements and pedagogical objectives were considered to develop an integrated solution that maximizes the effectiveness of the hybrid learning experience.

To visually summarize the hybrid system development process, a flowchart describing the sequential stages of our

system was added (Figure 1). This diagram presents the main phases, including shape selection, physical card design, 3D model creation, AR application development, system integration, UX refinement, and system evaluation. The visual representation aims to improve the comprehensibility of the hybrid system architecture and provide a clear overview of the design workflow.

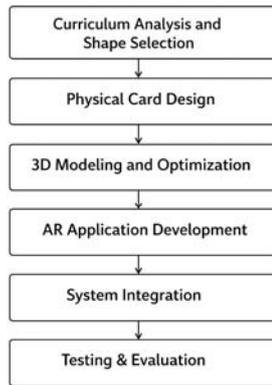


Figure 1. Flowchart illustrating the main stages of the hybrid learning system

3.3.1. Geometric Shape Selection and Curriculum Compatibility

The geometric shapes included in the system were selected using a systematic approach that considered curriculum compatibility and technical feasibility. When selecting two-dimensional shapes, priority was given to those fundamental to basic geometry education. Visual distinguishability was also considered to ensure that the shapes could function as image targets in the AR application. Triangle, square, rectangle, parallelogram, trapezoid, pentagon, hexagon, circle, and oval shapes were selected in this context.

For three-dimensional shapes, the main objective was to develop spatial visualization skills, and the selection criteria were usability in teaching surface area and volume calculations and effective visualization capability in AR environments. Accordingly, the following shapes were selected: cone, cube, sphere, cylinder, pyramid, rectangular prism, regular tetrahedron, triangular prism, and torus. Thus, 18 diverse geometric shapes (nine 2D and nine 3D) were selected to meet the needs of students at different grade levels.

3.3.2. Physical Card Design

The design of the physical cards was carried out in accordance with user experience (UX) principles and educational material design standards. A vector-based design approach was adopted using Adobe Illustrator, offering advantages in scalability and print quality. Principles of visual hierarchy, color harmony, typography, and space usage were applied to the design of each card, which was structured with the shape name at the top, the visual

representation in the middle, and the explanatory information at the bottom.

The cards were prepared at 300 DPI and in CMYK color mode to maintain print quality and color accuracy. Each card design underwent a quality control process based on text readability, visual clarity, and color harmony.

3.3.3. 3D Modeling Process

Autodesk 3ds Max software was used to model and render three-dimensional shapes. This process was carried out to create images for card designs and 3D models for the AR application. To enhance the performance of the AR application and ensure smooth operation on mobile devices, all 3D models were optimized for educational use with low polygon counts.

The models featured matte surfaces and pastel colors, which are aesthetically suitable for educational materials. Shiny or distracting surface effects were avoided to help students focus on geometric form. All 3D models were exported in OBJ format for use in the AR application. This format was chosen for its advantages in terms of Unity compatibility and file size optimization.

3.3.4. AR Application Development

The augmented reality application was developed using the Unity 3D game engine and Vuforia technology. These technologies offer advantages in cross-device compatibility and ease of development. Special image targets were created for each card through the Vuforia Developer Portal to enable the AR application to recognize physical cards.

To create effective image targets, images with high contrast, rich detail, and low repetition were selected. The best tracking performance was achieved by considering these criteria when designing each card. Vuforia's quality scoring system was used for all targets, and a minimum quality score of 3 out of 5 was achieved.

The application was developed to work on Android API Level 23 and higher, enabling broad device compatibility and the use of modern AR features. Given the limited processing power and memory capacity of mobile devices, the number of polygons in 3D models was kept low, and texture sizes were optimized.

3.3.5. User Experience Design

The UX design of the hybrid system considered UX best practices in educational technology. A minimalist approach was adopted for the interface design, taking into account student age groups, technology usage skills, and learning objectives. The main functions were placed in positions that could be accessed with a single touch. A clear visual hierarchy was created for presenting information, and essential information was emphasized with large fonts. The system was designed so that it does not require an internet connection. All 3D models and content are stored locally to enable offline use.

3.4. Demonstration Strategy

A multidimensional test strategy was developed to demonstrate the system's functionality and performance

comprehensively. This strategy includes technical performance tests and evaluations of system stability.

3.4.1. Technical Performance Test Strategy

Systematic test protocols were developed to measure the AR system's fundamental performance indicators. The selected test metrics were frame rate (FPS), processor load, and AR tracking accuracy. Performance tests were conducted on 10 Android devices across three performance categories: low, medium, and high. To ensure the performance evaluation was conducted under realistic, diverse usage conditions, the AR-supported hybrid geometry learning system was tested on 10 Android devices spanning low-, mid-, and high-performance categories. All selected devices support Android API Level 23 or higher, which is the minimum required to run the developed application. The test devices included the Samsung Galaxy A14, Xiaomi Redmi 12C, and Huawei Nova 12i from the low-end category; the Samsung Galaxy A54, Xiaomi Redmi Note 12, and Xiaomi Poco X5 from the mid-range category; and the Samsung Galaxy S23, Xiaomi 13, Huawei P60 Pro, and Samsung Galaxy S23 FE from the high-end category. These devices were chosen to reflect a broad spectrum of hardware configurations available in the Turkish smartphone market, ensuring that the system's compatibility, tracking performance, and stability were assessed comprehensively across different performance tiers. The target for each test scenario was 30 FPS, and the goal was to keep the processor load below 50%. AR tracking performance tests were conducted to evaluate recognition accuracy under varying distances, angles, and lighting conditions.

3.4.2. System Stability Test Strategy

Comprehensive test scenarios were designed to ensure the system's stable operation under various conditions. Two-hour continuous-use tests, memory-leak control, and crash analysis were performed. Compatibility tests were conducted on 10 different Android device models from various manufacturers, operating systems, and hardware configurations. Scenarios such as multiple card scanning, rapid transitions, and maximum resource usage were tested under extreme load conditions.

3.5. Evaluation Strategy

A multidimensional strategy was adopted to evaluate the effectiveness of the AR-supported hybrid geometry learning system that was developed. This strategy included a technical performance evaluation, a literature-based comparison, and an analysis of the appropriateness of the educational content.

3.5.1. Technical Performance Evaluation

Objective evaluation of the system's technical performance was conducted using measurable metrics and standard test protocols. The evaluation process included frame rate (FPS) performance, processor load analysis, AR tracking accuracy, and system stability parameters. A target of at least 30 FPS was set for frame rate, and performance was continuously monitored. The processor load analysis included resource

usage measurements across different device categories. AR tracking accuracy was evaluated by calculating the success rate of image target recognition. System stability was measured through two-hour continuous use tests with memory leak and crash analysis. These metrics were systematically measured and analyzed on ten different Android device models.

3.5.2. Literature-Based Comparative Evaluation

A comprehensive comparison of AR-based mathematics and geometry education systems was made in the existing literature to objectively determine the academic and practical value of the developed system. The comparison encompassed the following criteria: supported platform types (mobile, web, and desktop); geometric shape diversity and scope; AR tracking technology and quality; support for a hybrid learning approach; curriculum compatibility and localization status; offline usage capability; and system performance and usability. A systematic analysis was conducted according to these criteria to determine the position of the developed system in the literature.

3.5.3. Educational Content Suitability Evaluation

We systematically analyzed the compatibility of the developed content with the mathematics curriculum for grades 6-12 from the Ministry of National Education of the Republic of Turkey. The curriculum analysis process included determining the relevant grade levels for each geometric shape, matching curriculum achievements with geometric shapes, examining achievement codes and descriptions in detail, and determining the level of compatibility. The compatibility evaluation used the categories 'compatible' and 'incompatible'. As a result of this analysis, a compatibility table was created, and the percentage of compatibility between the system content and curriculum requirements was calculated.

4. Results

This section systematically presents the findings obtained during the design and development of the AR-supported hybrid geometry learning system. These findings are evaluated within four main categories: system demonstration and test results, performance evaluation results, comparative analysis results, and content appropriateness analysis results.

4.1. System Demonstration and Test Results

The AR-supported hybrid geometry learning system was successfully developed as an integrated solution consisting of physical cards and a mobile AR application. The system consists of 18 physical cards—nine 2D and nine 3D—and a mobile application that recognizes the cards and provides corresponding 3D visualizations.

4.1.1. Physical Card Design Results

The physical card design process was completed in Adobe Illustrator, and a professional layout for each card was produced. To ensure high print quality, the cards were prepared at 300 DPI and in CMYK mode (Figure 2).



Figure 2. Sphere physical card example

Visual hierarchy principles were applied, with the shape name placed at the top, the visual representation positioned centrally, and explanatory information placed at the bottom. A pastel color palette suitable for educational materials was selected, and high contrast ratios were maintained for readability.

4.1.2. 3D Modeling Results

All 3D models were created in Autodesk 3ds Max and were optimized for use in the AR application. Polygon counts were minimized to improve performance (Table 2).

Table 2. Technical specifications of 3D and 2D models

| Geometric Shape | Polygon Count |
|---------------------|---------------|
| Cube | 12 |
| Sphere | 2000 |
| Cylinder | 516 |
| Cone | 200 |
| Pyramid | 8 |
| Rectangular Prism | 12 |
| Triangular Prism | 12 |
| Regular Tetrahedron | 8 |
| Torus | 3072 |
| Pentagon | 3 |
| Hexagon | 4 |
| Circle | 26 |
| Rectangle | 32 |
| Square | 32 |
| Oval | 26 |
| Parallelogram | 2 |
| Triangle | 1 |
| Trapezoid | 2 |

The models were exported in OBJ format for Unity compatibility. Matte surfaces and non-distracting colors were applied to align with the aesthetics of educational materials (Figure 3).



Figure 3. Modeled torus

4.1.3. AR Application Development Results

The AR application was developed using Unity 3D and Vuforia and was optimized for Android devices. The resulting 59 MB application was made suitable for Google Play Store distribution requirements.

Image targets were generated on the Vuforia Developer Portal, and each card was assigned a minimum quality score of 3/5.

A minimalist interface design was implemented, and the card-scanning screen was set to open automatically to reduce distractions (Figure 4).



Figure 4. Scanning screen screenshot

4.1.4. System Integration Test Results

Integration tests were conducted under controlled conditions to evaluate the interaction between physical cards and the AR application.

Various lighting conditions, card angles, and distances were tested, and recognition was consistently achieved for all scenarios (Table 3).

Table 3. System integration test results specifications of 3D and 2D models

| Test Condition | Card Type | Card Recognition Status |
|-----------------|-----------|-------------------------|
| Normal Lighting | 2D | Successful |
| Normal Lighting | 3D | Successful |

| | | |
|---------------------|----|------------|
| Low Lighting | 2D | Successful |
| High Lighting | 3D | Successful |
| 45° Tilted Tracking | 2D | Successful |
| 45° Tilted Tracking | 3D | Successful |

Under normal usage conditions, the system was observed to perform reliably (Figure 5).



Figure 5. Scene where physical cards are scanned with mobile devices to obtain AR visualization

4.2. Performance Evaluation Results

System performance was evaluated in three main dimensions: technical metrics, AR tracking performance, and system stability. All tests were carried out on 10 Android device models.

4.2.1. Technical Performance Metrics

Technical performance across low-, medium-, and high-performance devices was measured, and the results are summarized in Table 4.

Although low-performance devices achieved an average FPS below the 30-FPS target, they still maintained an acceptable operation range.

Medium- and high-performance devices were found to meet or exceed the minimum FPS requirement, demonstrating efficient resource usage and stable performance.

Table 4. Technical performance metrics across device categories

| Device Category | Average FPS | Processor Load (%) |
|---------------------------|-------------|--------------------|
| Low-performance device | 25 FPS | 48% |
| Medium-performance device | 31 FPS | 30% |
| High-performance device | 47 FPS | 20% |

4.2.2. AR Tracking Performance Analysis

Tracking accuracy was analyzed in test scenarios involving different distances, angles, and lighting conditions.

An average tracking accuracy of 93.2% was achieved, indicating stable and reliable AR functionality.

4.2.3. System Stability and Reliability Tests

Two-hour continuous-use tests were performed to assess long-term stability.

Throughout the test period, processor load was recorded as 28% at 30 minutes, 32% at 60 minutes, 29% at 90 minutes, and 34% at 120 minutes, and no crashes were observed.

To represent these measurements visually, a line plot was generated (Figure 6).

The stable processor usage levels demonstrate that the system remains suitable for long-term mobile operation.

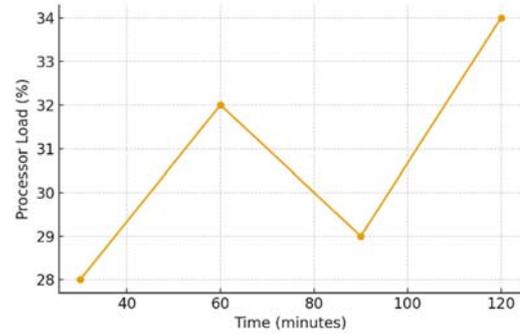


Figure 6. Processor load measurements during the two-hour long-term stability test

4.3. Content Compatibility Analysis Results

A detailed analysis of the alignment between the educational content and the Turkish Ministry of National Education curriculum for grades 6–12 was conducted.

Relevant achievement codes were matched to each geometric shape, and compatibility was evaluated (Table 5). As a result, an overall curriculum compatibility rate of 83.3% was calculated.

Shapes that did not align directly with curriculum outcomes were classified as enrichment materials.

Table 5. Curriculum achievement comparison of geometric shapes

| Geometric Shape | Achievements | Compatibility |
|---------------------|---------------------------------|---------------|
| Cube | M.5.2.5.1, M.6.3.4.4 | Compatible |
| Sphere | M.6.3.4.1, M.8.3.4.1 | Compatible |
| Cylinder | M.6.3.4.1, M.8.3.4.2, M.8.3.4.4 | Compatible |
| Cone | M.8.3.4.6 | Compatible |
| Pyramid | M.8.3.4.5 | Compatible |
| Rectangular Prism | M.5.2.5.1, M.6.3.4.4 | Compatible |
| Triangular Prism | M.8.3.4.1 | Compatible |
| Regular Tetrahedron | - | Incompatible |
| Torus | - | Incompatible |
| Pentagon | M.7.3.2.1 | Compatible |
| Hexagon | M.7.3.2.1 | Compatible |
| Circle | M.6.3.3.1, M.7.3.3.3 | Compatible |
| Rectangle | M.5.2.4.1, M.7.3.2.3 | Compatible |

| | | |
|---------------|---------------------------------------|--------------|
| Square | M.5.2.2.3, M.6.3.2.1 | Compatible |
| Oval | - | Incompatible |
| Parallelogram | M.5.2.2.3, M.6.3.2.2, M.7.3.2.3 | Compatible |
| Triangle | M.6.3.2.1, M.7.3.2.5, 9.4.1.1 | Compatible |
| Trapezoid | M.5.2.2.3, M.7.3.2.4 | Compatible |

5. Conclusions

This study involved designing and developing an AR-supported hybrid geometry learning system to address fundamental challenges in geometry education. This research, conducted using Design Science Research methodology, provided an innovative hybrid learning experience by integrating physical cards with mobile AR applications. The findings reveal that the developed system offers significant advantages over existing solutions in terms of technical performance and educational appropriateness.

The study's most important finding is that hybrid learning approaches can be successfully integrated with AR technology. The developed system provides an integrated solution comprising physical cards designed for 18 geometric shapes, as well as a mobile application that recognizes these cards and offers 3D visualization. This approach creates differentiating value compared to most existing AR-based geometry education systems, which provide fully digital solutions, as identified in the literature review. While the CleverBooks Geometry and ARMLS systems offer partial integration with physical tools, few systems provide a comprehensive hybrid experience.

System performance test results show that AR tracking accuracy averages 93.2%. The performance analysis also reflects the challenges faced by mobile AR applications due to device diversity. The average performance of 25 FPS on low-performance devices, which falls below the targeted 30 FPS, reveals that mobile AR applications are subject to hardware constraints. This factor is essential in explaining why web-based solutions such as McGraw Hill AR and GeoGebra 3D Calculator AR are preferred. Conversely, the results obtained on medium- and high-performance devices (31 and 47 FPS, respectively) demonstrate the system's successful optimization and prove that it provides accessible technological solutions for students from different socioeconomic backgrounds. The system's reliability in educational environments is shown by the absence of crashes during 120 minutes of uninterrupted use and by the processor load remaining stable in the 28-34% range. This is a critical success factor for educational applications that require continuous use throughout the lesson.

The results of the content appropriateness analysis show that the developed system is 83.3% compatible with the mathematics curriculum for grades 6–12 of the Ministry of National Education of the Republic of Turkey. This reveals that the system is not only a technological innovation, but also a solution that addresses the real needs of the education system. A literature review shows that existing AR-based geometry education systems are generally designed in accordance with international standards, with limited

specialized solutions for local curriculum requirements. In this context, the developed system offers a significant advantage by providing solutions tailored to Turkey's geometry education needs. The three geometric shapes identified as incompatible (the regular tetrahedron, the torus, and the oval) account for 16.7% of incompatibility, material to expand students' geometric concept repertoire.

In conclusion, the AR-supported hybrid geometry learning system developed in this study offers a technologically innovative solution to the challenges of geometry education. It has achieved technical success, demonstrating 93.2% AR tracking accuracy and acceptable performance levels across different device categories, as well as 83.3% curriculum compatibility. Compared to systems in the literature, it offers significant advantages in its hybrid learning approach, offline use, and local curriculum compatibility. The system successfully combines technological innovation and educational value and has the potential to improve the quality of geometry education in Turkey.

6. Discussions

Compared to similar systems in the literature, the system's provision of offline usage capability is a critical advantage. This feature is essential given that internet infrastructure is not yet sufficient in all Turkish educational institutions and that mobile data costs constitute an economic burden for student families. The reliance on the internet for systems such as McGraw Hill AR and GeoGebra 3D Calculator AR poses a significant obstacle to their widespread adoption. Still, the developed system addresses this by storing all 3D models and content locally for offline use.

The theoretical basis of the hybrid approach is multi-sensory learning theory and the educational benefits of concrete manipulatives. The tactile experience of physical cards and the visual enrichment of AR technology activate various sensory channels, appealing to students' different learning styles. Compared to traditional AR applications that focus only on visual learning, this approach has the potential to provide a more comprehensive learning experience.

However, the potential drawbacks of using AR technology in education should also be considered. Some researchers argue that integrating technology may distract students from basic mathematical skills and create excessive dependence on technology. According to this perspective, the advantages of traditional teaching methods could be overshadowed by the use of technology, resulting in weaker paper-and-pencil calculation skills among students. Additionally, the digital divide is an essential point of criticism. The mobile devices and technical infrastructure required for AR technology could create access barriers for students from low-income backgrounds and deepen existing inequalities. In response to these criticisms, it should be emphasized that the developed hybrid system preserves the traditional learning experience with physical cards, with technology playing a complementary role.

An essential limitation of the study is that the system's educational effectiveness has not been tested experimentally. However, findings such as the significant increase in learning achievement obtained by the ARGeoITS system in

controlled experiments with 106 students (7.47 versus 6.83), the statistically significant effect of GEMBI AR on spatial reasoning skills ($Z = -3.578$, $p < 0.05$), and the 70% superior performance rate of GeoGebra 3D Calculator AR support the educational effectiveness of AR-based systems. It is critically important to test the developed hybrid system in similar experimental studies to assess its educational value objectively.

Other limitations include the system's development for only the Android platform, performance issues on low-spec devices, and the production and distribution costs of physical cards. In educational environments, for example, where iOS devices are widely used, the system's accessibility is limited. In large-scale applications, the cost of producing cards can also be a significant economic barrier.

7. Recommendations and Future Work

Various suggestions can be offered for future research. Firstly, the educational effectiveness of the developed system should be tested using an experimental design with control groups to measure its effects on student achievement, motivation, and spatial visualization skills. Investigating the system's impact on students of different ages and learning levels — particularly by adapting it for students with special learning needs — could yield valuable insights. Examining teachers' experiences with the system and their pedagogical approaches could provide useful insights to improve the development process.

Other important research areas include expanding the system to other mathematics topics, developing personalized learning experiences through the integration of artificial intelligence technologies, and investigating its applicability in different cultural environments. In terms of practical application, it is crucial to carefully plan pilot application processes, develop teacher training programs, establish technical support infrastructure, and develop sustainable financing models.

However, future studies are required to validate the system's educational effectiveness, increase platform diversity, and optimize costs.

Similarity Rate (Initial): 4%

AI Usage Rate: < 5%

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Researcher Contribution Rates

Author 1: Writing, original draft, software, validation.
Author 2: Writing, review, editing.

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