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A Unified Framework for Threshold Unit Root Testing with Asymmetric ESTAR Behavior[1]

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Abstract

In the context of the alternative hypothesis proposed by the exponential smooth transition autoregressive (ESTAR)nonlinear model, unit root tests serve similar objectives but differ in underlying assumptions across the literature. Various tests have been formulated to interpret the ESTAR framework under conditions of asymmetric and threshold effects. This study introduces a straightforward unit root test designed to assess the alternative hypothesis of asymmetric ESTAR nonlinearity with a threshold effect ($AESTAR_C$). The asymptotic properties of the test statistics are established, followed by an evaluation of critical values, size, and power characteristics using Monte Carlo simulations. Based on the findings pertaining to size and power properties, the developed test demonstrates greater suitability than previous tests in scenarios involving asymmetric reversion and threshold effects. An empirical application of the proposed test is illustrated using methodologies described by Sollis (2009) and Kruse (2011).

Keywords: ESTAR, Nonlinear Time Series, Unit Root Test

Asimetrik ESTAR Yapısını Temel Alan Eşikli Birim Kök Testi için Bütünleşik Bir Yaklaşım

Öz

ESTAR (Exponential Smooth Transition Autoregressive) doğrusal olmayan model tarafından önerilen alternatif hipotez bağlamında, birim kök testleri benzer amaçlara hizmet etmekle birlikte, literatürde ki temel varsayımlar açısından farklılık göstermektedir. Literatürde, asimetrik ve eşik etkileri koşulları altında ESTAR çerçevesini yorumlamak üzere çeşitli testler geliştirilmiştir. Bu çalışma, asimetrik ESTAR doğrusal olmayanlığı ve eşik etkisini ($AESTAR_C$) içeren alternatif hipotezi değerlendirmeye yönelik basit bir birim kök testi önermektedir. Önerilen testin istatistiklerine ilişkin asimptotik özellikler teorik olarak ortaya konulmuş, ardından Monte Carlo benzetimleri aracılığıyla kritik değerler, boyut ve güç özellikleri incelenmiştir. Elde edilen bulgular, geliştirilen testin özellikle asimetrik geri dönüş dinamiklerinin ve eşik etkilerinin var olduğu durumlarda, önceki testlere kıyasla daha uygun performans sergilediğini göstermektedir. Son olarak, önerilen testin ampirik bir uygulaması Sollis (2009) ve Kruse (2011) tarafından tanımlanan metodolojiler kullanılarak sunulmuştur.

Anahtar Kelimeler: ESTAR, Doğrusaldışı zaman serileri, Birim Kök Testleri

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Introduction

Many economic time series exhibit behavior within natural bounds due to their economic structures. Consequently, modeling these series under a random walk framework is inappropriate. Examples include unemployment rates and nominal interest rates, which range between zero and one, and additionally, variables such as exchange rates do not take negative values. Given this constraint, it is unsuitable to evaluate these series under the random walk model. In practice, time series with bounded values, such as these variables, can be assessed as exhibiting local nonstationarity. For modeling such series, structures that allow for unit root behavior (while not permitting persistent mean reversion) are employed, allowing for different value transitions as they approach certain limits or fall within specific value ranges. As a result, series that operate within certain value ranges or have bounded values are considered nonstationary in the context of linear time series analysis.

In nonlinear time series analysis, the presence of asymmetry and threshold effects can lead to several significant implications for both the modeling and interpretation of data. Firstly, asymmetry in a time series may indicate that the response of the series to shocks or innovations is not uniform; that is, positive and negative shocks can have differing effects on the level or volatility of the series. This phenomenon complicates the assumptions of linear models, which typically presume symmetry in the response to disturbances. Consequently, models that account for asymmetry, such as threshold autoregressive (TAR) models or asymmetric $GARCH$ models, are often employed to capture these dynamics more accurately. Secondly, the presence of threshold effects implies that the relationship between variables may change when certain critical values are crossed. This means that the impact of an independent variable on the dependent variable can vary significantly depending on the level of the independent variable. Such nonlinear relationships can lead to differing policy implications or forecasting strategies, as the effect of interventions may differ based on the state of the system. Moreover, these features can also affect the stability and persistence of shocks within the series. Asymmetric responses and threshold effects may result in a situation where shocks can either amplify or dampen over time, leading to complex dynamics that are not easily predictable.

The analysis of nonlinear time series, particularly in the presence of threshold effects and asymmetry, poses significant challenges in econometrics and statistics. Conventional unit root tests often rely on linear assumptions that may not adequately capture the complexities inherent in such data structures. As nonlinearities and regime shifts can lead to misleading inferences if ignored, there exists a pressing need for a robust methodological framework that integrates both threshold effects and asymmetry into unit root testing.

The absence of a dedicated test methodology that simultaneously addresses these dimensions limits the analytical capacity of researchers. The potential for spurious results increases when conventional linear tests are applied to nonlinear processes, which may ultimately misguide policy decisions and economic forecasting. By developing a comprehensive testing framework that accounts for both threshold behavior (Ulusoy,2017) and asymmetry, we can enhance the accuracy of unit root analysis, thereby improving our understanding of the underlying processes governing economic time series.

Among these models, the most popular one is Exponential Smooth Transition Autoregressive $(ESTAR)$ model that was first defined by Haggan and Ozaki (1981), and subsequently re-evaluated by Granger and Terasvirta (1993) as well as Terasvirta (1994). Recent developments in $ESTAR$ modeling were presented by van Dijk et al. (2002). Due to its incorporation of symmetric/asymmetric behavior and nonlinearity, $ESTAR$ models have increasingly gained popularity in the analysis of real exchange rates. The phenomenon of transaction costs leads to significant deviations from equilibrium prices, which are corrected through arbitrage. However, smaller deviations may remain uncorrected. In examining the structure of the series under these small deviations, the use of the $ESTAR$ model, which accounts for nonlinearity, explores global stationarity and investigates nonlinearity in the central regime. Alternatively, despite the stationary nature of the $ESTAR$ model, numerous unit root tests have been developed.

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In summary, the motivation for this study arises from the need to develop a more comprehensive and robust testing methodology that can accurately capture the complex dynamics of nonlinear time series under realistic empirical conditions, where both asymmetry and threshold effects play a decisive role. The central aim of the study is to move beyond conventional model structures that consider only asymmetry or only threshold behavior in isolation, and instead to propose a broader and more powerful framework in which these two features are jointly taken into account. This unified perspective is particularly important for time series exhibiting $ESTAR$ dynamics such as interest rates, exchange rates, unemployment rates, inflation, and purchasing power indicators for which empirical evidence strongly suggests the coexistence of asymmetric adjustments and regime-dependent responses. By explicitly accommodating both effects within a single testing methodology, the study seeks to provide a more realistic representation of the underlying data-generating processes, thereby improving the interpretation of such series and enhancing the reliability of statistical inference in applied macroeconomic and financial analyses.

1.Unit Root Testing in The Presence of ESTAR Nonlinearity: Addressing Asymmetry and Threshold Effects

The $ESTAR$ model, initially presumed to exhibit symmetry, is considered acceptable under current specifications. To redefine the model structure, the first differences of the time series are taken, introducing an exponential function that imparts nonlinearity to the framework. The transition variable, denoted as y_{t-d}, plays a pivotal role, with empirical practices often assuming $d = 1$ to enhance clarity in model estimation and interpretation.

$$\boldsymbol{y_t = \beta y_{t-1} + \gamma y_{t-1}\left(1 - \exp(-\theta(y_{t-d} - c)^2)\right) + \varepsilon_t} \tag{1}$$

$$\triangle \boldsymbol{y_t = \phi y_{t-1} + \gamma y_{t-1}\left(1 - \exp(-\theta(y_{t-d} - c)^2)\right) + \varepsilon_t} \tag{1.1}$$

The first order of the autoregressive term becomes as $\phi = \beta - 1$. At the $ESTAR$ model θ term refers to the measure of reversion speed. Threshold parameter (c) is another term of $ESTAR$ model that can be explained as an exhibition of time series behavior like a repressed or broken appearance when time-series approach it. This can be interpreted as a change in the regime when approaching the threshold. While moving away from the threshold (c), the instability of the time series increases. If the stationary of the return series is being questioned, it can be assumed that this assumption is mostly correct. As such, these parameters in the $ESTAR$ model (the reversion speed and the threshold value) must be identified, thus stationary of time series can be accurately defined. The identification of these parameters the reversion speed and the threshold value is crucial for accurately determining the stationarity of the time series under consideration. This process ensures that any assumptions regarding the stationary nature of the return series are appropriately validated within the $ESTAR$ framework.

In the KSS(2003) test, the stationarity of a time series is examined under the assumption within $c = 0$ the $ESTAR$ framework. Under this assumption, a first-order Taylor expansion of the $ESTAR$ model leads to the derivation of a differential equation known as the auxiliary equation. The auxiliary regression is then formulated under the condition where $y = 0$. This issue of parameter identification of γ, as outlined by Davies (1987), has been mitigated. This is because the model under examination is non-stationary and linear under the fundamental hypothesis.

$$\boldsymbol{\Delta y_t = \delta y_{t-1}^3 + u_t} \tag{1.2}$$

Here, the unit root nature of the time series is assessed using a t-statistic to test the null hypothesis $\delta = 0$ against the alternative hypothesis $\delta<0$. The test statistic t_{nl} is computed as $\hat{\delta}\ /\ (\text{s.e.}(\hat{\delta}))$, where $\hat{\delta}$ represents the estimated coefficient of the parameter δ obtained through a least squares estimation of the auxiliary regression equation. The KSS (2003) test does not incorporate considerations for asymmetric reversion and

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threshold effects. To address these limitations, subsequent unit root tests have been introduced that account for asymmetry, such as those proposed by Sollis (2009), and threshold effects, as developed by Kruse (2011). However, these tests typically assume independence between the asymmetric reversion and threshold characteristics, treating them separately. Consequently, there has been an effort to develop a unified unit root test capable of jointly evaluating these two influences, which serves as a focal point in current research endeavors.

Solis (2009) introduced a unit root test tailored for ESTAR models that accommodates both symmetric and asymmetric reversion dynamics. A similar approach has been employed by Enders and Granger (1998) to determine whether the tendencies of threshold autoregressive (TAR) models are asymmetric or symmetric. This extended framework, known as the AESTAR model ($AESTAR$ model definition was also used by Anderson (1997) and Siliverstovs (2005)), incorporates asymmetry through the logistic function alongside $\mathrm{S}_t(\theta_2; y_{t-d})$ the exponential transition function $\mathrm{G}_t(\theta_1; y_{t-d})$. Below are the first-order difference equation and transition functions that characterize the $AESTAR$ model structure.

$$\Delta y_t = \mathrm{G}_t(\theta_1; y_{t-d})\{\mathrm{S}_t(\theta_2; y_{t-d})\gamma_1 + (1 - \mathrm{S}_t(\theta_2; y_{t-d}))\gamma_2\}y_{t-1} + \varepsilon_t \quad (2)$$

$\varepsilon_t \sim iid(0, \sigma^2)$ for exponential functions;

$$\mathrm{G}_t(\theta_1; y_{t-d}) = 1 - \exp(-\theta_1 y_{t-d}^2),\ \theta_1 \geq 0$$

$$\mathrm{S}_t(\theta_2; y_{t-d}) = [1 - \exp(-\theta_2 y_{t-d}^2)]^{-1},\ \theta_2 \geq 0$$

Similar to the approach in the KSS (2003) test, an alternative method involves employing an auxiliary regression derived from the $ESTAR$ model through Taylor expansion. However, this method introduces new challenges, particularly in the estimation of parameters such as $\theta_2, \gamma_1, \gamma_2$ which remain unidentified within the system. Notably, in the context of the first-order Taylor expansion, the assumption $\theta_1 = 0$ is applied to the exponential transition function $\mathrm{G}_t(\theta_1; y_{t-d})$.

$$\Delta y_t = \gamma_1 \theta_1 y_{t-1}^3 \mathrm{S}_t(\theta_2; y_{t-d})\gamma_1 + \gamma_2 \theta_1 y_{t-1}^3 (1 - \mathrm{S}_t(\theta_2; y_{t-d})) + \eta_t \quad (2.1)$$

In this context, the error term η_t incorporates the residuals R_t resulting from Taylor's expansion of the exponential transition function, impacting the original error term ε_t. This transformation proves inadequate due to the indeterminate nature of θ_2 , posing a systematic challenge. To address this issue, a transition is made from the logistic transition function $\mathrm{S}_t(\theta_2; y_{t-d})$ to $S_t^*(\theta_2; y_{t-d})$. The function obtained here becomes $S_t^*(\theta_2; y_{t-d}) = \mathrm{S}_t(\theta_2; y_{t-d}) - 0.5$, which is explained for the limit value of the logistic transition function. Further, $S_t^*(0; y_{t-d}) = 0$ will be obtained. This modification aims to redefine the AESTAR model, facilitating a more precise interpretation.

$$\Delta y_t = \gamma_1^* \theta_1 y_{t-1}^3 S_t^*(\theta_2; y_{t-d})\gamma_1 + \gamma_2^* \theta_1 y_{t-1}^3 (1 - S_t^*(\theta_2; y_{t-d})) + \eta_t \quad (2.2)$$

As the logistic function undergoes redefinition, the parameters γ_1^* and γ_2^* are adjusted to form linear combinations of the original parameters γ_1 and γ_2. When implementing the Taylor transformation for $S_t^*(\theta_2; y_{t-d})$ with $\theta_2 = 0$ on the revised logistic function, this adjustment aims to enhance the model's interpretability and analytical rigor.

$\Delta y_t = a(\gamma_2^* - \gamma_1^*)\theta_1 \theta_2 y_{t-1}^4 + \gamma_2^* \theta_1 y_{t-1}^3 + \eta_t$ is obtained. For $a = 1/4$ the difference equation becomes;

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$\Delta y_t = \phi_1 y_{t-1}^3 + \phi_2 y_{t-1}^4 + \eta_t$, the parameters of auxiliary regression $\phi_1 = \gamma_2^* \theta_1$ and $\phi_2 = a(\gamma_2^* - \gamma_1^*)\theta_1\theta_2$ contains. The updated form of the difference equation is established and regarded as the auxiliary regression in the analysis.

$$\Delta y_t = \phi_1 y_{t-1}^3 + \phi_2 y_{t-1}^4 + \sum_{j=1}^{\rho} \rho_j \Delta y_{t-j} + \eta_t \tag{2.3}$$

The analysis primarily focuses on the unit root regime and model architecture, which accommodates two symmetric and comparable average regimes. As the time series approaches the threshold parameter, it exhibits behavior akin to a random walk process. Sollis (2009) introduced asymmetric reversion within the $ESTAR$ framework. However, akin to KSS (2003), it assumes a threshold parameter of zero. However, when encountering statistically significant estimates of threshold parameters, the assumption $\mathrm{c} = 0$ becomes less tenable, necessitating a departure from standard testing procedures. In such scenarios, where one parameter is tested under a one-sided hypothesis and another parameter under a two-sided hypothesis, conventional integrated hypothesis testing methods prove inadequate. Abadir and Distaso (2007) proposed a novel approach to address such hypotheses.

The most popular structure utilized within the $ESTAR$ model framework is the KSS (2003) test statistic, which has a DF-type test structure. This test assumes the location parameter in the smooth transition function to be $\mathrm{c} = 0$. However, some studies have encountered statistically significant estimates for c. Micheal et al. (1997), Sarantis (1999), Taylor et al. (2001), and Rapach and Wohar (2006) have implied that the threshold parameter should be identified. Relaxing this assumption leads to an unconventional testing procedure. It is necessary to analyze under an integrated hypothesis where one parameter is one-sided and the others are two-sided. In this context, standard integrated hypothesis analyses may prove inadequate. Abadir and Distaso (2007) have developed a methodology for addressing such hypotheses with a novel approach. Accordingly, Kruse (2011) aimed to develop a testing mechanism for $ESTAR$ model structures with a non-zero threshold parameter c that would not lag behind the power and size properties of the KSS (2003) test. In a similar vein, Kruse (2011) endeavored to develop a testing framework for $ESTAR$ model structures that explicitly accounts for a non-zero threshold parameter c. The primary aim was to ensure that this adjustment does not compromise the statistical power and size characteristics of the KSS (2003) test. Under the assumption $\varepsilon_t \sim iid(0, \sigma^2)$, $ESTAR$ model is accepted with non-zero threshold or location parameter as

$$\triangle y_t = \phi y_{t-1} + \gamma y_{t-1}\big(1 - \exp(-\theta(y_{t-d} - c)^2)\big) + \varepsilon_t. \tag{1}$$

When first-order Taylor transformation is applied to the exponential transition function

$$\mathrm{G}_t(\theta_1; y_{t-d}, c) = \big(1 - \exp(-\theta(y_{t-d} - c)^2)\big) \text{ around } \theta = 0,$$

$$\Delta y_t = \beta_1 y_{t-1}^3 + \beta_2 y_{t-1}^2 + \beta_3 y_{t-1} + u_t \tag{2.4}$$

is obtained as auxiliary regression. Upon accepting the condition $\beta_3 = 0$, the term y_{t-1} is eliminated from the model, leading to an increase in the test's statistical power. The resulting structure of the auxiliary regression is as follows:

$$\Delta y_t = \beta_1 y_{t-1}^3 + \beta_2 y_{t-1}^2 + u_t \tag{2.4.1}$$

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This section provides a concise overview of unit root tests that incorporate ESTAR-type nonlinear dynamics through auxiliary regressions; however, an important limitation remains, as the existing literature does not offer a unit root test that is able to simultaneously and effectively accommodate both asymmetry and threshold effects. This methodological gap constitutes the primary motivation of the present study, which seeks to develop a unified testing framework that nests the asymmetric and threshold-based ESTAR models as special cases. By doing so, the proposed approach not only extends the existing tests in the literature but also yields an asymptotic distribution that coincides with those of Sollis (2009) and Kruse (2011) in the respective limiting cases where either threshold effects or asymmetry vanish, thereby providing a coherent and comprehensive framework for nonlinear unit root testing.

This study aims to develop an alternative unit root test that takes into account both threshold and asymmetry structures. The developed test provides similar results to previous tests, such as those by Sollis (2009) and Kruse (2011), while offering a more effective testing mechanism. The critical and size properties of the test have been determined using a simulation method, and its power has been evaluated across different structures. The results demonstrate that the developed test performs better in scenarios involving both threshold and asymmetry. The size and power analyses of the test provide suitable and comparable results, particularly for small sample sizes. This study contributes to the literature by developing a new and robust testing methodology. Additionally, it highlights that economic indicators with proportional and discrete period changes, such as inflation, interest rates, and unemployment rates, along with variables expected to be stationary according to purchasing power parity, and financial assets that should be stationary according to the Efficient Market Hypothesis, should not exhibit unit root behavior. It is crucial for policymakers to identify the unit root structure of series based on their data generation processes to enhance the predictability of economic variables and the effectiveness of policy decisions. This work introduces a significant innovation in the testing of stationarity in nonlinear time series, providing a powerful tool for economic analysis and offering a valuable resource for policymakers.

2. Examining Unit Root Tests For ESTAR Nonlinearity: Incorporating Asymmetric Reversions Alongsıde Threshold Effects

The Asymmetric Exponential Smooth Transition Autoregressive model with a threshold ($AESTAR_c$) is rooted in the methodological advancements of Sollis (2009) and Kruse (2011) in unit root testing (Eq. 3). Sollis (2009) introduced a unit root test specifically tailored for $ESTAR$ models, which considers both symmetrical and asymmetrical adjustment processes. This approach is essential for detecting non-linear characteristics in time series data, where deviations from equilibrium prompt adjustments that vary in speed and direction depending on the nature of the deviation.

The preceding tests have primarily focused on examining either asymmetry or threshold characteristics individually. However, the primary aim of this study is to develop a unit root test that incorporates both of these features simultaneously. Initially, a threshold was integrated into the $AESTAR$ structure as utilized in Sollis' research. In an expanded version known as the $AESTAR_c$ model, asymmetry is introduced through the logistic function $S_t(\theta_2; y_{t-d})$ alongside a threshold. Additionally, the exponential transition function with a threshold, $G_t(\theta_1; y_{t-d})$, is also employed. The first-order difference equation and transition functions are detailed below.

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$$\Delta y_t = \mathrm{G}_t(\theta_1; y_{t-d} - c)\{\mathrm{S}_t(\theta_2; y_{t-d} - c)\gamma_1 + (1 - \mathrm{S}_t(\theta_2; y_{t-d} - c))\gamma_2\}y_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim iid(0, \sigma^2)$$
$$\mathrm{G}_t(\theta_1; y_{t-d}) = 1 - \exp(-\theta_1(y_{t-d} - c)^2),\ \theta_1 \geq 0 \tag{3}$$
$$\mathrm{S}_t(\theta_2; y_{t-d}) = [1 - \exp(-\theta_2(y_{t-d} - c))]^{-1},\ \theta_2 \geq 0$$

$$\Delta y_t = y_{t-1}(1 - exp(-\theta_1(y_{t-1} - c)^2))\left\{[1 - exp(-\theta_2(y_{t-1} - c))]^{-1}\gamma_1 + \left(1 - [1 - exp(-\theta_2(y_{t-1} - c))]^{-1}\right)\gamma_2\right\} + \varepsilon_t \tag{3.1}$$

The $AESTAR_c$ model incorporates a first-order Taylor approximation around the equilibrium level $y \to c$. This approximation leads to an auxiliary regression of the form $\Delta y_t = \delta_1(y_{t-1} - c)^3 + \delta_2(y_{t-1} - c)^2 + u_t$. Upon expanding this equation, the difference equation transforms into $\beta_1 y_{t-1}^3 + \beta_2 y_{t-1}^2 + \beta_3 y_{t-1} + c^* + u_t$, where c^* represents a function of the threshold term. The coefficients β_1, β_2 encompass the speed of reversion, asymmetric components, and threshold parameters. The condition $c^* = \beta_3 = 0$ is imposed to enhance the test's statistical power, leading to the omission of y_{t-1} from the model. Thus, the auxiliary regression simplifies accordingly.

The restrictions $c^* = \beta_3 = 0$ are introduced deliberately rather than arising mechanically from the expansion. First, under the unit root null hypothesis, the process is assumed to have no linear mean-reverting component; including a linear term in y_{t-1}would therefore be inconsistent with the null and could distort the size and power of the test. Second, the constant term $c^*$is not separately identifiable from the threshold parameter c under the null, since the nonlinear adjustment vanishes in the vicinity of the threshold. Setting $c^* = 0$ effectively centers the process around the threshold and avoids estimating nuisance parameters that do not contribute to discrimination between the null and the nonlinear alternative.

Imposing these restrictions serves two key purposes. Econometrically, it simplifies the auxiliary regression to a form that is valid under the null hypothesis and avoids identification problems that arise in nonlinear unit root testing. Statistically, it concentrates the test's power on the nonlinear components captured by the cubic and quadratic terms, which jointly reflect the speed of mean reversion, asymmetry in adjustment, and threshold behavior. Consequently, the omission of the linear term y_{t-1} is not an arbitrary simplification but a deliberate assumption designed to enhance the test's ability to detect asymmetric ESTAR with threshold dynamics against the unit root null, while maintaining a well-defined limiting distribution.

$$\Delta y_t = \beta_1 y_{t-1}^3 + \beta_2 y_{t-1}^2 + \beta_0 + u_t \tag{3.2}$$

The null hypothesis of interest is $H_0\colon \beta_1 = \beta_2 = \beta_0 = 0$, against the alternative $H_0\colon \beta_1 \neq \beta_2 \neq \beta_0 \neq 0$. To derive critical values for this hypothesis test, the standard Wald test is deemed appropriate due to the presence of a unit root structure in the null hypothesis. Specifically, F_{AEc}, $F_{AEc,t}$ denote the test statistics for cases involving non-zero mean and deterministic trend. The random walk model shares identical critical values

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with the non-zero model. Its power characteristics suggest a low sensitivity. The extended test aims to discern unit root structure under conditions of asymmetry and threshold effects. Notably, the random walk model does not conform to the threshold effect due to the threshold parameter introducing a constant or drift term into the model.

In this case, the asymptotic distribution of the extended structure will behave similarly to the modified Sollis (2009) test, where a constant term is added. If it is ensured that the threshold value is set to zero during the data generation process, the auxiliary regression will have a zero constant term. Thus, by aligning with the conditions of the Sollis (2009) test, the same null and alternative hypotheses can be constructed, leading to the same asymptotic distribution. Therefore, the new distribution under the constant term will be as follows. If $\Delta y_t = \varepsilon_t$ and $y_0 = 0$ under $\varepsilon_t \approx iid(0, \sigma^2)$ condition. $Y^{-1}(\sum_t XX')Y^{-1} \Longrightarrow Q$ and $\{Y^{-1}(\sum_t X_t \varepsilon_t)\}' \Longrightarrow h'$ is defined with ⇒ de notes weak convergation

$F_{AEc} \Rightarrow {h'Q^{-1}h}/{2\sigma^2}$ can be determined with $W(r)$ standard Brownian motion on $r \in [0,1]$ is

$$h' = \begin{bmatrix} \sigma^3 \int_0^1 W(r)^2 d\, W(r) & \sigma^4 \int_0^1 W(r)^3 d\, W(r) & \sigma \end{bmatrix}$$

$$Q = \begin{bmatrix} \sigma^4 \int_0^1 W(r)^4 d(r) & \sigma^5 \int_0^1 W(r)^5 d(r) & \sigma^3 \int_0^1 W(r)^2 d(r) \\ \sigma^5 \int_0^1 W(r)^5 d(r) & \sigma^6 \int_0^1 W(r)^6 d(r) & \sigma^4 \int_0^1 W(r)^3 d(r) \\ \sigma^3 \int_0^1 W(r)^2 d(r) & \sigma^4 \int_0^1 W(r)^3 d(r) & \sigma^2 \end{bmatrix} \tag{3.3}$$

In the data generation process, a constant term is drawn from a uniform distribution over $\mu \sim [-1,1]$,which is added to $y_t = y_{t-1} + \varepsilon_t$. Critical values are obtained by simulating under a random walk with deterministic terms. For F_{AEc}, $F_{AEc,t}$ the test statistics are selected using least squares estimation on y_t after incorporating μ. Subsequently $F_{AEc,t}$, $\hat{y}_t = y_t - \hat{\alpha}_1 t$ is adjusted by $\hat{\alpha}_1$ is estimated.

To establish finite sample critical values, 50,000 simulated series are generated for each sample size $t = 1000$. Critical values from simulations at $t = 1000$ are accepted due to their convergence properties.

Table 1. The critical values of extended F_{AEc}, $F_{AEc,t}$ tests

$t = 1000$	F_{AEc}	$F_{AEc,t}$
1%	5.10	4.94
5%	3.83	3.62
10%	3.23	3.04

We investigate the size properties of the test with null data generation process of F_{AEc} and $F_{AEc,t}$ test. The data generation process is $y_t = y_{t-1} + \varepsilon_t$ for $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$ under assumption of $u_t \sim iid(0,1)$. Where $\rho = \{-0.5, 0, 0.5\}$ the sample size $t = 50, 100, 200, 1000, 10000$ replications are used. Test statistics are computed from the relevant model with one lag of Δy_t for $\rho = \{-0.5, 0.5\}$ and the nominal size is set 5%. It is accepted that models are correctly specified neither over nor less by in terms of lag. The results show that

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size distortion is getting smaller when the sample size increases. But $F_{AEc,t}$ cases have more size distortion than F_{AEc}.

Table 2. The size properties of extended F_{AEc}, $F_{AEc,t}$ tests

	F_{AEc}		$F_{AEc,t}$	
	$\rho = -0.5$	0.051	$\rho = -0.5$	0.053
$t = 50$	$\rho = 0$	0.060	$\rho = 0$	0.065
	$\rho = 0.5$	0.079	$\rho = 0.5$	0.086
	$\rho = -0.5$	0.051	$\rho = -0.5$	0.058
$t = 100$	$\rho = 0$	0.058	$\rho = 0$	0.067
	$\rho = 0.5$	0.066	$\rho = 0.5$	0.080
	$\rho = -0.5$	0.048	$\rho = -0.5$	0.060
$t = 200$	$\rho = 0$	0.050	$\rho = 0$	0.069
	$\rho = 0.5$	0.058	$\rho = 0.5$	0.078
	$\rho = -0.5$	0.048	$\rho = -0.5$	0.066
$t = 1000$	$\rho = 0$	0.051	$\rho = 0$	0.070
	$\rho = 0.5$	0.047	$\rho = 0.5$	0.067
	$\rho = -0.5$	0.047	$\rho = -0.5$	0.066
$t = 10000$	$\rho = 0$	0.052	$\rho = 0$	0.063
	$\rho = 0.5$	0.050	$\rho = 0.5$	0.067

The design for calculating finite sample power is influenced by Sollis (2009). The study computes results for sample sizes of $t = 100, 200, 1000, 10000$ with a nominal significance level of 5%, utilizing 5000 replications. The analysis focuses on the statistically most significant model structure, employing a maximum lag $k_{max} = 8$ determined through information criteria selection.

In constructing the model, parameters are chosen to create asymmetric reversion dynamics. Specifically, combinations of $\gamma_1 = \{-0.05, -0.10, -0.30\}$ and $\gamma_2 = \{-1.00, -0.90, -0.70\}$ are used. The threshold value ccc is drawn from a uniform distribution $c \sim [-1,1]$. Values $\theta_1 = \{0.1\}$ and $\theta_2 = \{0.1, 1\}$ are employed to further specify the model parameters.

For the test F_{AEc}, the $AESTAR$ with threshold model adjusts y_t by subtracting the estimated mean $\hat{\mu}$, resulting in $y_t^* = y_t - \hat{\mu}$. This transformation is employed to address non-zero mean conditions in the data. Similarly, for the $F_{AEc,t}$ test, the model y_t modifies by subtracting, $y_t^* = y_t - \hat{\alpha}_1 - \hat{\alpha}_2 t$ where $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are estimated using least squares regression. This adjustment accommodates non-zero mean and deterministic trend components in the data.

To assess the power characteristics under asymmetric dynamics with a threshold, comparisons are made among tests proposed by KSS (2003), Sollis (2009), Kruse (2011), and their extensions, namely F_{AEc} and $F_{AEc,t}$. These comparisons aim to evaluate how effectively each test captures the underlying asymmetries in the data, considering the specified model structures and parameterizations.

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Table 3. The comparison of power properties of KSS(2003), Sollis(2009), Kruse (2011) and F_{AEc} tests for non-zero mean models

T=100		$\theta_1 = 0.1$				$\theta_1 = 1$			
γ_1	γ_2	$t_{NL.\mu}$	$F_{AE.\mu}$	τ_μ	F_{AEc}	$t_{NL.\mu}$	$F_{AE.\mu}$	τ_μ	F_{AEc}
-0.05	-1.00	0.497	0.645	0.575	0.580	0.655	0.900	0.871	0.895
-0.05	-0.90	0.488	0.618	0.554	0.556	0.617	0.873	0.844	0.868
-0.05	-0.70	0.425	0.546	0.483	0.487	0.563	0.797	0.746	0.772
-0.10	-1.00	0.660	0.779	0.719	0.701	0.778	0.963	0.941	0.949
-0.10	-0.90	0.635	0.753	0.677	0.660	0.757	0.950	0.918	0.927
-0.10	-0.70	0.582	0.679	0.611	0.584	0.724	0.908	0.856	0.859
-0.30	-1.00	0.913	0.956	0.925	0.910	0.955	0.990	0.976	0.979
-0.30	-0.90	0.906	0.949	0.917	0.894	0.943	0.985	0.968	0.969
-0.30	-0.70	0.868	0.915	0.869	0.841	0.938	0.979	0.955	0.953
T=200		$\theta_1 = 0.1$				$\theta_1 = 1$			
γ_1	γ_2	$t_{NL.\mu}$	$F_{AE.\mu}$	τ_μ	F_{AEc}	$t_{NL.\mu}$	$F_{AE.\mu}$	τ_μ	F_{AEc}
-0.05	-1.00	0.725	0.943	0.914	0.915	0.704	0.992	0.992	0.997
-0.05	-0.90	0.716	0.940	0.906	0.903	0.694	0.991	0.995	0.997
-0.05	-0.70	0.695	0.905	0.860	0.855	0.692	0.966	0.981	0.987
-0.10	-1.00	0.990	0.994	0.983	0.976	0.868	0.999	0.998	0.999
-0.10	-0.90	0.902	0.993	0.980	0.977	0.867	0.999	0.997	0.998
-0.10	-0.70	0.885	0.987	0.962	0.955	0.863	0.996	0.994	0.995
-0.30	-1.00	0.997	1.000	0.999	0.999	0.984	0.999	0.996	0.997
-0.30	-0.90	0.996	0.999	0.998	0.999	0.986	0.999	0.996	0.996
-0.30	-0.70	0.996	1.000	0.999	0.999	0.990	0.999	0.997	0.997

Table 4. The comparison of power properties of KSS(2003), Sollis(2009), Kruse (2011) and $F_{AEc,t}$ tests for trended series

T=100		$\theta_1 = 0.1$				$\theta_1 = 1$			
γ_1	γ_2	$t_{NL,t}$	$F_{AE,t}$	τ_t	$F_{AEc,t}$	$t_{NL,t}$	$F_{AE,t}$	τ_t	$F_{AEc,t}$
-0.05	-1.00	0.408	0.480	0.420	0.619	0.582	0.765	0.717	0.840
-0.05	-0.90	0.388	0.445	0.422	0.586	0.567	0.714	0.660	0.826
-0.05	-0.70	0.338	0.387	0.335	0.532	0.493	0.612	0.544	0.749
-0.10	-1.00	0.592	0.518	0.525	0.727	0.728	0.893	0.846	0.942
-0.10	-0.90	0.501	0.572	0.512	0.708	0.711	0.860	0.807	0.927
-0.10	-0.70	0.419	0.460	0.399	0.626	0.624	0.749	0.680	0.851
-0.30	-1.00	0.818	0.845	0.802	0.923	0.932	0.976	0.953	0.981
-0.30	-0.90	0.781	0.815	0.769	0.903	0.932	0.970	0.947	0.978
-0.30	-0.70	0.734	0.762	0.702	0.877	0.907	0.949	0.917	0.966
T=200		$\theta_1 = 0.1$				$\theta_1 = 1$			
γ_1	γ_2	$t_{NL.t}$	$F_{AE.t}$	τ_t	$F_{AEc.t}$	$t_{NL.t}$	$F_{AE.t}$	τ_t	$F_{AEc.t}$
-0.05	-1.00	0.627	0.816	0.782	0.900	0.686	0.957	0.952	0.986
-0.05	-0.90	0.618	0.782	0.748	0.883	0.665	0.940	0.933	0.977
-0.05	-0.70	0.585	0.723	0.686	0.848	0.617	0.900	0.886	0.956
-0.10	-1.00	0.838	0.946	0.927	0.977	0.837	0.994	0.990	0.998
-0.10	-0.90	0.824	0.943	0.920	0.975	0.834	0.991	0.983	0.996
-0.10	-0.70	0.805	0.899	0.871	0.953	0.829	0.981	0.970	0.995
-0.30	-1.00	0.993	0.999	0.997	0.999	0.978	0.997	0.991	0.997

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-0.30	-0.90	0.991	0.997	0.995	0.998	0.980	0.996	0.992	0.997
-0.30	-0.70	0.989	0.997	0.994	0.999	0.983	0.996	0.993	0.996

The analysis of power characteristics reveals several significant insights. Firstly, across all examined scenarios, increasing the sample size consistently enhances the statistical power of the tests. Secondly, as γ_1 approaches zero, indicating a reduction in asymmetry within the model, there is a uniform decrease in the power of all tests. Moreover, when γ_1 is held constant and γ_2 approaches zero, reflecting a decrease in asymmetry, the power of all tests similarly diminishes. An increase in θ_1, which denotes the speed of mean reversion in the $ESTAR$ function, consistently results in higher statistical power across all configurations of the tests. In most other scenarios examined, the extended tests F_{AEc} and $F_{AEc,t}$ demonstrate nearly the highest power among all considered tests.

It is crucial to note that these simulated series encompass both asymmetric reversion and threshold effects. The power of alternative tests is sufficiently high to be comparable with the extended tests F_{AEc} and $F_{AEc,t}$. KSS (2003) considers symmetric reversion without a threshold effect. Sollis (2009) focuses solely on asymmetric reversion, while Kruse (2011) incorporates a threshold effect but not asymmetrical reversion. Therefore, the power results of these tests are closely aligned due to the simulated series incorporating aspects relevant to each test. In most instances, the extended tests F_{AEc} and $F_{AEc,t}$ demonstrate slightly higher power compared to Sollis (2009). This situation illustrates that the developed test methodology is complementary to these two concepts and enhances their robustness.

3. Empirical Results

An empirical study has been conducted based on data where the properties of the developed test were evaluated against those of two benchmark tests. The initial segment of the empirical study employs novel tests to analyze quarterly time series data covering the period from 1973:1 to 1998:4. The data described in Sollis's (2009) study were downloaded and compiled from the same source. This dataset was sourced from the IMF's International Financial Statistics database (https://data.imf.org/IFS). Sollis considers these data as primary due to their pertinence. The study concludes its sampling period at 1998:4 to proactively address potential complexities arising from the introduction of the Euro. The data consists of natural logarithms of the real exchange rates (RER) between Denmark, Finland, Norway, and Sweden against the U.S. dollar. Specifically, the exchange rates are computed as the natural logarithm of the nominal dollar exchange rate, adjusted by the natural logarithm of both the U.S. and domestic consumer price indices. Micheal et al. (1997), Baum et al. (2001), and Taylor et al. (2001) advocate for the use of ESTAR models in their respective studies. Sercu et al. (1995) argue that a nonlinear adjustment is necessary for Purchasing Power Parity (PPP) due to transaction costs in real exchange rates. Micheal et al. (1997) contend that when the effects of STAR nonlinearity are not taken into account, the results obtained from applied cointegration and unit root tests regarding the long-term Purchasing Power Parity hypothesis may be inconsistent. Balke and Fomby (1997), along with Pipenger and Goering (1993) and Taylor (2001), have demonstrated through their experiments that traditional unit root tests exhibit weak power compared to their nonlinear alternatives.

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Figure 1. Natural Logarithms of the Real Exchange Rates of Nordic Countries

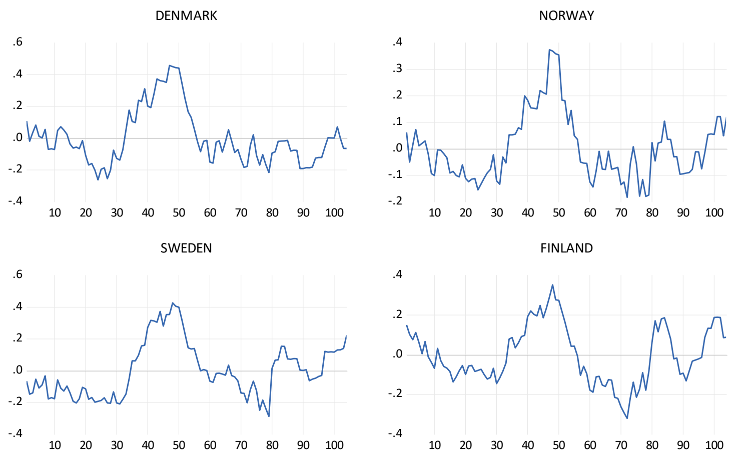

Table 5. Unit root test results of Nordic Countries RER

Variables	τ	t_{NL}	F_{AE}	τ_t	F_{AEc}
Denmark	-2.023 (0.276)	-1.024	0.973	1.776	0.836
Finland	-2.051 (0.264)	-1.688	1.505	3.022	1.043
Norway	-2.502 (0.117)	-1.931	5.697*	10.280*	4.816*
Sweden	-1.641 (0.457)	-1.762	4.762*	2.537	5.272**

Note: At the table ** and * indicates in order 1%, 5% statistical significance.

In the second segment of the empirical application, the monthly real effective exchange rate (REER) series for the European Union is analyzed using data that are comparable to those employed in Kruse (2011). While the original study relies on Datastream data spanning January 1993 to December 2007 with 180 observations, data limitations necessitate the use of the IMF's *International Financial Statistics* (https://data.imf.org/IFS) database for the same period and frequency. The natural logarithm of the series, illustrated in Figure 1, does not display a pronounced linear trend, although the mean is found to be statistically significant; accordingly, the series is demeaned prior to estimation. Test regressions are then estimated with the lag length selected by the Schwarz information criterion, yielding $p = 2$, whereas Kruse (2011) reports an optimal lag length of $p =$ 1under the same criterion. Within this empirical setting, the proposed test demonstrates its economic relevance by effectively capturing adjustment dynamics that are neither linear nor symmetric and that depend on the magnitude of deviations from equilibrium. Exchange rates, such as the REER, are well known to exhibit weak or delayed responses to small deviations due to transaction costs, policy inertia, and market frictions, while larger deviations tend to trigger stronger and nonlinear corrections; additionally, positive and negative deviations may generate asymmetric responses reflecting institutional constraints or asymmetric policy reactions. In this context, the empirical results indicate that the developed test successfully exploits these features, whereas conventional unit root tests—or nonlinear tests that consider only asymmetry or only threshold effects—are more likely to overlook nonlinear mean-reverting behavior and thus yield economically misleading inferences about persistence. The performance of the test in the REER application therefore

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provides empirical support for its ability to identify equilibrium adjustment under joint asymmetry and threshold effects, highlighting the practical effectiveness of its underlying theoretical properties.

Figure 2. Real Effective Exchange Rate for the European Union by Consuming Index

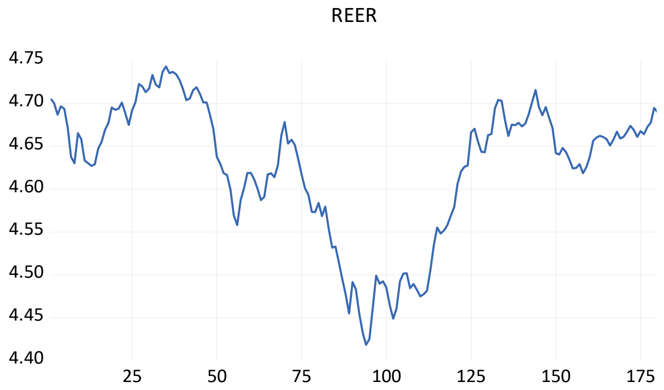

Table 6. Unit root test result of European Union REER

Variable	τ	t_{NL}	F_{AE}	τ_t	F_{AEc}
REER	-1.797 (0.380)	-1.949	1.878	2.290	1.284

The test developed by Kruse was deemed stationary only at the 10% level of statistical significance. However, in a repeated application using the same data, all unit root tests indicate non-stationarity. The consistency of these findings across all tests demonstrates the reliability of the developed test results. Perhaps this difference could stem from the varied determination of appropriate lag lengths, as we were unable to obtain the data from the same internet server.

4. Conclusion

The findings support the unit root hypothesis under the non-zero mean specification and indicate that similar conclusions are obtained when asymmetry and threshold effects are examined separately, thereby justifying the application of the proposed testing procedure and empirically strengthening the preference for the F_{AEcm}method over those of Sollis (2009) and Kruse (2011). Within this framework, purchasing power parity is interpreted as a currency conversion rate that enables standardized comparisons of purchasing power across countries by reflecting the relative amounts of currency required to purchase identical baskets of goods and services; since the validity of PPP hinges on the stationarity of the real exchange rate, testing the unit root hypothesis effectively corresponds to evaluating adherence to PPP, which forms the theoretical basis of the empirical analysis in this study. Accordingly, this study proposes a straightforward unit root test against the alternative hypothesis of asymmetric exponential smooth transition autoregressive nonlinearity with a threshold by integrating key elements of the approaches developed by Sollis (2009) and Kruse (2011), and, following the derivation of critical values, evaluates the combined test in terms of its size and power properties under extended assumptions. The results show that the power profile of the proposed test closely resembles those of its constituent tests and, under certain conditions, yields supportive outcomes; however, the empirical findings also point to important limitations, particularly the difficulty of identifying a unit root structure in the analyzed data, suggesting that the test may overlook regime shifts induced by structural breaks. Consequently, the proposed methodology should be regarded as an alternative and complementary framework for questioning the presence of unit root behavior under joint asymmetry and threshold effects within the ESTAR setting, rather than as a definitive solution, and future research is encouraged to extend this approach by incorporating structural breaks, multiple thresholds, or time-varying transition mechanisms to enhance its empirical applicability.

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Ethical Considerations of the Study

It is declared that the study was designed to realistically and ethically meet the needs, and that integrity was maintained in obtaining data, concluding the study, and publishing the results. Ethical committee approval was not required for this research. No research requiring ethics committee approval was conducted in this study.

Informed Consent

There was no need to obtain informed consent from individuals, as the study did not involve any procedures or interventions on human participants.

Author Contributions

Idea/Concept:A.H; Design: A.H; Supervision/Consultancy: A.H; Resources: A.H; Data Collection and/or Processing: A.H; Analysis and/or Interpretation: A.H; Literature Review: A.H; Writing: A.H; Critical Review: A.H

Conflict of Interest Statement

The author declares no conflict of interest

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Declarations

This study has not been presented at any congress.

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