In-Plane Free Vibration Frequencies of Stepped Circular Beams

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Abstract

In this study, in-plane natural frequencies of stepped circular beams are presented. The material of the beam is considered to be isotropic, homogeneous and elastic. The effects of shear deformation and rotary inertia are considered in the necessary assumptions and formulations. The obtained canonical form of governing equations is solved numerically by the complementary functions method (CFM) in the Laplace domain. To obtain the natural frequencies of the considered structures the Laplace parameter is replaced with the parameter of free vibration. The 5th order Runge Kutta (RK5) algorithm is applied to solve the initial-value problem based on the CFM. To examine the in-plane free vibration of Timoshenko beams with stepped circular cross-section a program is coded in FORTRAN. In order to demonstrate the accuracy of the current scheme, present results are compared with those of literature. The accuracy and efficiency of the presented results are observed.

Keywords: Free vibration, Stepped beams, Circular Cross-section, Laplace transform.

1. INTRODUCTION

Rods are used in many man-made structures such as bridges, roof structures and aerospace structures. In modern architectural design, curved beams with stepped cross sections are widely used in applications. These important practical applications of, curved beams have attracted the interest of engineers and scientist. Aktan [1] carried out natural frequencies of in-plane circular beams. The Timoshenko beam theory was used and the effect of the rotary inertia was considered. Çoban [2] studied the dynamic response of curved beams using mixed finite element method with Gâteaux derivative. Free in-plane vibrations of circular arches with variable cross-sections were investigate by Tong et al. [3]. Huang et al. [4] combined the dynamic stiffness method with the Laplace transformation to study the forced vibration of an arch with variable curvature. A new approach for free vibration analysis of arches with effects of shear deformation and rotary inertia considered was presented by Wu and Chiang [5]. An exact solution of free in-plane vibrations of circular arches of uniform cross-section is presented by Tüfekci and Arpaci [6]. Tüfekci and Özdemirci [7] obtained the free in-plane vibration of stepped circular arches. Free vibration analysis using the transfer-matrix method on a tapered Bernoulli-Euler beam was presented by Lee and Lee [8]. Temel et al. [9] studied the natural frequencies of isotropic-elastic planar curved rods in Laplace domain with the CFM. The forced vibration of linear elastic stepped circular arches under various in-plane dynamic loads was examined by Noori et al. [10] in the Laplace domain. Aslan et al. [11] examined, out of plane forced vibration of elastic out-of-plane loaded stepped circular rods subjected to various time depended loads in the Laplace domain.

This paper is as special case of Noori [12]. In this study, the natural frequencies are obtained by the CFM in the Laplace domain. CFM is a numerical solution method which transforms a two point boundary value problem to a system of initial value problems. The Butcher's RK5 algorithm [13], which is almost coincide with 7th order Taylor series solution, is applied for the solution of initial value problems. Materials of the beam are assumed to be linear and elastic. To demonstrate the efficiency of the proposed procedure, results of the present method are compared with those obtained by ANSYS [14]. The advantages of the present method are high accuracy and computational time savings.

2. MATERIAL AND METHOD

The governing equations of the free vibration behavior of stepped curved beams are obtained and given as:

$$\frac{\partial U_t}{\partial \phi} = U_n + r_0 \frac{T_t}{EA(\phi)} \tag{1}$$

$$\frac{\partial U_n}{\partial \phi} = -U_t + r_0 \Omega_b + r_0 \frac{T_n \alpha_n}{GA(\phi)} \tag{2}$$

$$\frac{\partial \Omega_b}{\partial \phi} = \frac{M_b}{EI_b(\phi)}\, r_0 \tag{3}$$

$$\frac{\partial T_t}{\partial \phi} = r_0 \rho A(\phi) \frac{\partial^2 U_t}{\partial t^2} + T_n - p_t\, r_0 \tag{4}$$

$$\frac{\partial T_n}{\partial \phi} = r_0 \rho A(\phi) \frac{\partial^2 U_n}{\partial t^2} - T_t - p_n\, r_0 \tag{5}$$

$$\frac{\partial M_b}{\partial \phi} = r_0 \rho I_b(\phi) \frac{\partial^2 \Omega_b}{\partial t^2} - r_0 T_n - m_b\, r_0 \tag{6}$$

where T_t and T_n are the tangential and normal components of the internal forces, M_b is the internal bending moment, U_t and U_n are the tangential and normal components of the displacement, Ω_b is the component of related rotation, E, ρ, $h(\phi)$,$A(\phi)$, $I_b(\phi)$, α_n and r_0, indicate the modulus of elasticity, mass density, height of the cross-section, cross-sectional area, the moment of inertia, shear correction factor, radius of curvature of the midpoint of the rod.

The unknown column matrix, $\{\mathbf{Y}(\phi, t)\}$, for the free vibration of in-plane loaded beams is given as:

$$\{\mathbf{Y}(\phi, t)\} = \{U_t, U_n, \Omega_b, T_t, T_n, M_b\}^T \tag{7}$$

Applying the Laplace transform to Eqs. (1 – 6) converts these partial differential equations to variable-coefficient ordinary differential equations. Thereby, the governing ordinary differential equations of the dynamic behavior of stepped circular beams can be obtained in the Laplace domain as follows:

$$\frac{d\bar{U}_t}{d\phi} = \bar{U}_n + r_0 \frac{\bar{T}_t}{EA(\phi)} \tag{8}$$

$$\frac{d\bar{U}_n}{d\phi} = -\bar{U}_t + \bar{\Omega}_b r_0 + r_0 \frac{\bar{T}_n \alpha_n}{GA(\phi)} \tag{9}$$

$$\frac{d\bar{\Omega}_b}{d\phi} = r_0 \frac{\bar{M}_b}{EI_b(\phi)} \tag{10}$$

$$\frac{d\bar{T}_t}{d\phi} = r_0 s^2 \rho A(\phi) \bar{U}_t + \bar{T}_n - p_t\, r_0 \tag{11}$$

$$\frac{d\bar{T}_n}{d\phi} = r_0 s^2 \rho A(\phi) \bar{U}_n - \bar{T}_t - p_n\, r_0 \tag{12}$$

$$\frac{d\bar{M}_b}{d\phi} = r_0 s^2 \rho I_b(\phi) \bar{\Omega}_b - r_0 \bar{T}_n - m_b\, r_0 \tag{13}$$

where the terms shown by $(\bar{\bullet})$ indicates the Laplace transform of the quantities. The matrix notation of the ODEs (8-13) obtained in the Laplace domain is given as:

$$\frac{d\{\bar{\mathbf{Y}}(\phi,s)\}}{d\phi} = [\bar{\mathbf{A}}(\phi,s)]\{\bar{\mathbf{Y}}(\phi,s)\} + \{\bar{\mathbf{F}}(\phi,s)\} \tag{14}$$

here ϕ is the variable and s is the Laplace parameter. The RK5 algorithm is used for the solution of initial-value problem based on the CFM.

The general solution of the differential equation (15) which governs the in- plane free vibration response of the beam is given as follows:

$$\{\bar{\mathbf{Y}}(\phi,s)\} = \sum_{m=1}^{6} C_m [\bar{\mathbf{U}}^{(m)}(\phi,s)] + \{\bar{\mathbf{V}}(\phi,s)\} \tag{15}$$

In order to investigate the free vibration of the problem Laplace parameter “*s*” is replaced with “*iω*”. Also the inhomogeneous solution $\{\bar{\mathbf{V}}(\phi,s)\}$ is equal to zero. To determine the integration constants “C_m” of homogeneous solution from the boundary conditions, simultaneous equations are obtained and the matrix of the coefficients of those equations are performed. Since the mass and stiffness matrix of the system are not obtained separately by the presented procedure the eigenvalues and eigenvectors of the problem are not calculated thus the values of ω which make the determinant of coefficient's matrix zero are the natural frequencies of the structure.

3. NUMERICAL EXAMPLES AND DISCUSSION

A fix-ended and a pin-ended isotropic stepped circular beam, shown in Figure 1, is considered. Material properties are: mass density, $\rho=7850\text{x}10^{-6}$ kgf/cm^3, Poisson's ratio, $\nu = 0.3$ and modulus of elasticity, $E=2.1\text{x}10^6$ kgf/cm^2.

The radius of the cross section of the beam ($h\ (\phi)$) is considered to be;

$$h(\phi) = \ \rightarrow \begin{cases} 0.5, & -\frac{\pi}{6} \leq \phi \leq \frac{\pi}{6} \\ 1.0, & \frac{\pi}{6} \leq \phi \leq \frac{\pi}{4} \\ 1.0, & -\frac{\pi}{4} \leq \phi \leq -\frac{\pi}{6} \end{cases}$$

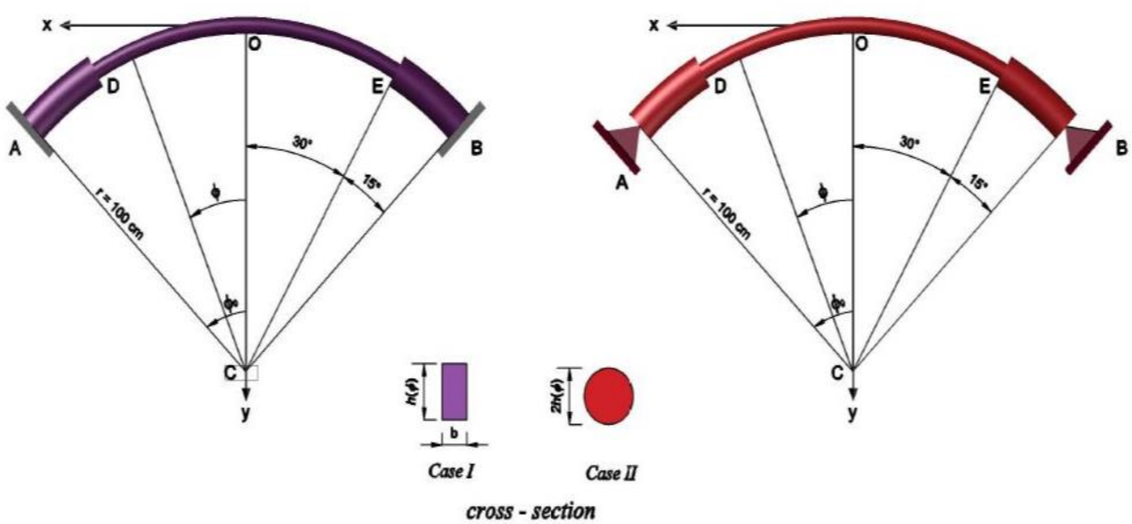

Figure 1. (a) Fix–ended circular stepped beam (b) Pin–ended circular stepped beam

Here, the flexural rigidity and cross section of rectangular and circular beam are given as,

$$EI_b(\phi) = E\frac{bh^3(\phi)}{12}, \ \ A(\phi) = b\ h(\phi) \quad ; \ EI_b(\phi) = E\frac{\pi h^4(\phi)}{4}\ , A(\phi) = \pi h^2(\phi)$$

The geometric properties of the rods are tabulated in Table 1.

Table 1. Geometric properties of the beams.

Cross-section	$b(0)$ **(cm)**	$h(0)$ **(cm)**	r_0**(cm)**	ϕ_0	α_n
Rectangular (Case I)	1	1	100	45	1.2
Circular (Case II)	-	0.5	100	45	1.11

At first the natural frequencies of a fix-ended circular stepped beam with circular cross section, shown in Figure 1, are obtained by the present scheme and compared with those of ANSYS. In the presented study the natural frequencies of the considered structure are listed in Table 2 for the fix-ended beams.

Table 2. The natural frequencies of the fix-ended rod with stepped cross-section (Hz).

Mode	**Present Study**	**ANSYS (100 Elements)**
1	8.0403	8.0417
2	17.574	17.559
3	25.580	25.588
4	35.384	35.323
5	58.537	58.532
6	78.004	78.106
7	105.810	105.703
8	126.250	126.310
9	144.280	144.860
10	180.700	181.110

Comparisons show that results of the present study are in a good agreement with the those of ANSYS. Then, the free vibration of fix-ended and pin-ended rods of rectangular and circular cross sections are examined by the present study and the results are given below in table 3 and 4. As expected, the in-plane free vibration responses of the stepped circular rods are significantly affected by the boundary and conditions geometry of the cross section of the structure. It can be clearly seen in Table 3 – 4 that a fix-ended circular stepped beam has the highest natural frequencies and the lowest oscillating period. While a pin ended rectangular stepped circular beam has the lowest frequency which corresponds to the highest period.

Table 3. The natural frequencies of the fix-ended rod with stepped cross-section for different cases (Hz).

Mode	Case I	Case II
1	4.269	8.040
2	10.480	17.574
3	20.723	25.580
4	25.146	35.384
5	33.779	58.537
6	46.952	78.004
7	64.829	106.810
8	87.319	126.250
9	109.571	144.280
10	117.477	180.700

Table 4. The natural frequencies of the pin-ended rod with stepped cross-section for different cases (Hz).

Mode	Case I	Case II
1	2.738	4.349
2	9.036	14.977
3	18.657	25.065
4	25.143	32.972
5	29.475	50.803
6	42.826	70.596
7	65.606	102.495
8	82.624	126.246
9	102.266	139.622
10	117.230	167.622

4. CONCLUSION

In this study, the free vibration analysis of stepped circular beams made of linear elastic materials is investigated. This procedure combines with the Laplace transform and the CFM. The effects of shear deformation are considered in the analysis. The governing equations of motion of the problem are first obtained in the time domain. Laplace transform is then applied and the set of simultaneous linear algebraic equations are solved by the CFM in the transform domain. Several examples are presented and natural frequencies for various boundary conditions are presented in tabular form. The numerical examples have proved that the suggested procedure is highly accurate and efficient compared to with those of literature. The proposed scheme can be easily applied to the free vibration analysis of stepped circular rods.

REFERENCES

[1] Aktan H, 2008.In-Plane dynamic analysis of circular beams. MSc. Thesis, Istanbul Technical University, Istanbul.

[2] Çoban M, 2008.Out-Of-plane dynamic analysis of curved beams using mixed finite element method. MSc. Thesis, Istanbul Technical University, Istanbul.

[3] Tong X, Mrad N, Tabarrok B, 1998. In-Plane vibration of circular arches with variable cross-sections. J. Sound Vib. 212 (1) 121-140.

[4] Huang C S, Tseng Y P, and Lin C J, 1998. In-plane transient responses of arch with variable curvature using dynamic stiffness method. J. Eng. Mech. 124:826-835.

[5] Wu J S, and Chiang L K, 2004. A new approach for free vibration analysis of arches with effects of shear deformation and rotary inertia considered. J. Sound and Vib. 277 49-71.

[6] Tüfekci E, and Arpacı A, 1998. Exact solution of in-plane vibrations of circular arches with account taken of axial extension, transverse shear and rotatory inertia effects. J. Sound and Vibration, 209(5), 845-856 .

[7] Tüfekci E, Özdemirci Ö, 2006. Exact solution of free in-plane vibration of a stepped circular arch. Journal of Sound and Vibration, 295, 725-738.

[8] Lee W Y, Lee Y J, 2016. Free vibration analysis using the transfer-matrix method on a tapered beam. Computers and Structures 164 (2016) 75–82.

[9] Temel B, Aslan T A, and Noori A R., 2017. An efficient dynamic analysis of planar arches. European Mechanical Science, vol. 1(3), pp. 82-88.

[10] Noori A R, Temel B, and Aslan, T A, Transient analysis of in-plane loaded elastic stepped circular arches. International Conference on Advances and Innovations in Engineering (ICAIE), 2017 May 10-12, p.721-728.

[11] Aslan T A, Noori A R, Temel B, Forced vibration of out of plane loaded stepped circular rods. International Conference on Civil and Enviromental Engireening, May 8-10, 2017 , Nevsehir, Turkey, 2062-2074.

[12] Noori A R, Aslan T A, Temel B, Damped transient response of in-plane and out-of-plane loaded stepped curved rods. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 40 1-24, 2018.

[13] Chapra S C, and Canale, R P, 1998.Numerical Methods for Engineers With Programming and software Applications. McGraw-Hill Books.

[14] ANSYS Swanson Analysis System, Inc., 201 Johnson Road, Houston, PA15342-1300, USA.