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Oyun Teorisi ve Tabu Araması Yoluyla Tehlikeli Maddelerin Araç Rotalama Problemine Risk Kaçınımlı Metasezgisel Hibrit Çözüm Yaklaşımı

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Öz

Günlük yaşamımızın ayrılmaz bir parçası haline gelmelerinin bir sonucu olarak, her yıl büyük miktarlarda tehlikeli madde üretilmekte ve taşınmaktadır. Sanayi toplumlarının çoğunda, tehlikeli maddeler olmadan bir yaşam neredeyse tahayyül edilemez hâle gelmiştir. Tehlikeli maddeler; taşınma sürecinde halk sağlığı, kamu güvenliği ya da taşınmaz mallar üzerinde olumsuz etkiler veya riskler yaratma potansiyeline sahip miktar ve formda bulunan maddeler olarak tanımlanır. Bu bağlamda, tehlikeli maddeler veya ürünler; patlayıcı, gaz hâlinde, yanıcı, oksitleyici, toksik, bulaşıcı, radyoaktif ya da aşındırıcı özellik gösteren maddeler ile bunlara ait tehlikeli atıkları kapsamına almaktadır. Tehlikeli maddelerin güvenli taşınması; sürece etki eden yasal ve fiziksel faktörler ile taşıma araçlarının maruz kalabileceği çeşitli riskler nedeniyle kapsamlı ve çok boyutlu bir sorun alanı olarak değerlendirilmektedir. Tehlikeli madde kazalarının halk sağlığına yönelik olası etkilerine karşı artan çevresel duyarlılık, tehlikeli madde taşımacılığı konusuna yönelik akademik ve kurumsal ilgiyi önemli ölçüde artırmaktadır. Bu makale, oyun teorisi temelli bir yaklaşımla tabu arama algoritmasının entegrasyonu sonucu geliştirilen model üzerinden, tehlikeli madde taşımacılığı problemine yönelik riskten kaçınan bir çözüm önermektedir. Model kapsamında, dağıtım görevlisi, sevkiyat ağındaki herhangi bir bağlantıda problem oluşması durumunda, olası en olumsuz koşullar altında ortaya çıkabilecek beklenen zararı minimize etmeyi hedeflemektedir. Bu çerçevede, Nash dengesi aracılığıyla belirlenen beklenen maliyet fonksiyonu, tehlikeli madde taşımacılığında güvenli güzergâh seçimine yönelik stratejik kararların analizinde etkili ve pratik bir çözüm aracı olarak değerlendirilmektedir.

Anahtar kelimeler: Tehlikeli madde taşımacılığı, Araç rotalama problemi, Oyun teorisi, Tabu arama, Nash-dengesi

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Risk Averse Metaheuristic Hybrid Solution Approach to Vehicle Routing Problem of Hazardous Materials Through the Combination of Game Theory and Tabu Search

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Abstract
Because they have become an integral part of our daily lives, large quantities of hazardous materials are produced and transported each year. In most industrial societies, life without hazardous materials has become almost unimaginable. Hazardous materials are defined as substances that, during transportation, have the potential to pose adverse effects or risks to public health, safety, or property due to their quantity or form. In this context, hazardous materials or products include explosives, gases, flammable and oxidizing substances, toxic and infectious materials, as well as radioactive, corrosive substances and their associated hazardous wastes. The safe transportation of hazardous materials is considered a comprehensive and multidimensional issue, influenced by various legal and physical factors, as well as the numerous risks that vehicles may encounter during transit. Increasing environmental awareness of the potential impacts of hazardous material accidents on public health has significantly heightened both academic and institutional interest in this field. This study proposes a risk-averse solution to the hazardous material transportation problem through a model developed by integrating the Tabu Search algorithm with a game theory–based approach. Within the model, the dispatcher aims to minimize the expected loss under the worst possible conditions in the event of a disruption in any link of the distribution network. In this framework, the expected cost determined through Nash equilibrium is evaluated as an effective and practical analytical tool for strategic decision-making in the selection of safe routes for hazardous material transportation.

Keywords: Transportation of hazardous materials, Vehicle routing problem, Game theory, Tabu search, Nash-equilibrium

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1. Introduction

The release of hazardous materials poses short- and long-term toxicological risks to humans, flora, fauna, and entire ecosystems. Such incidents can result in contamination of air, water, and soil, as well as pose significant threats to public health and environmental stability. The adverse results may be classified as,

- Injury and/or loss of life
- Displacement of affected populations
- Damage to water and sewerage systems
- Blockage of roads, streets, and bridges
- Disruption of emergency response services (e.g., police, fire departments, highway rescue teams)
- Interruption of public and private transportation networks
- Disruption of delivery and supply chain operations
- Closure of educational institutions
- Temporary or permanent shutdown of businesses and industries
- Contamination of air, water, and soil

The safe transportation of hazardous materials within large urban areas can be significantly improved through the implementation of effective routing strategies. A minimum-risk routing approach helps to reduce the potential damages associated with hazardous material spills while generating net economic benefits for society. In classical terms, risk is characterized as the interplay between the likelihood that a specific hazardous event will occur within a given timeframe or under certain conditions. and the magnitude of its potential consequences. However, estimating the consequences of a hazardous release is inherently complex. Evaluating the consequence domain requires determining both the extent and characteristics of the population at risk. This assessment must also account for the type of hazardous material being transported and its specific hazard properties - such as toxicity, effects on human health and safety, and environmental impacts - alongside factors like population density, potential evacuation zones by material type, and local geographical conditions.

For the reasons outlined above, estimating the number of fatalities resulting from an incident involving a vehicle transporting hazardous materials is inherently challenging. To address this issue, a risk model based on the traditional definition of risk was proposed for assessing the hazards associated with hazardous materials transportation [1]. However, this model has been criticized for neglecting variance in risk distribution. For instance, a route passing through two small towns might be deemed equally risky as another passing through one large town if only expected consequences are considered. Moreover, the traditional model focuses on minimizing a priori risk assuming that hazardous material shipments continue along the same route even after an incident occurs. To address this limitation, a conditional risk model was later proposed to identify routes that minimize risk given that an incident has occurred [2].

A key area of research in hazardous materials transportation focuses on developing strategies to mitigate the risks associated with Low-Probability, High-Consequence (LPHC) incidents. As the term suggests, these events occur infrequently but can result in severe consequences when they do. Consequently, decision-makers (such as route planners) tend to adopt a risk-averse stance when addressing such scenarios. Previous models have incorporated the decision-maker's risk aversion under the assumption that the probabilities of incident occurrence are known in advance [3–4]. However, there is a need for a modelling approach capable of generating risk-averse routing strategies even when these probabilities are unknown, beyond the general understanding that they are small. The model proposed in this research provides an effective optimization framework for hazardous materials transportation, enabling decision-making in contexts where the likelihood of incidents is minimal but the potential consequences are highly significant.

Another limitation of existing models is their focus on transporting hazardous materials from a single origin to a single destination along a defined path. In practice, however, many real-world logistics problems require determining optimal routes from a main depot to multiple delivery-consumption points (customers) using a fleet of vehicles, an instance of the widely accepted Vehicle Routing Problem (VRP). To illustrate, consider a petroleum distributor operating a central refinery that serves as a single depot. A group of tanker trucks based at this depot must deliver fuel to a number of geographically dispersed service stations. The objective of the routing plan is to determine paths that minimize the expected total cost, ensuring that each route departs from the depot and, when needed, returns to it. The routing constraints require that each service station be served exactly once by one vehicle, while the cumulative demand along any route must not surpass the vehicle's maximum carrying capacity. This formulation characterises a single-depot capacitated VRP (potentially with stochastic costs). The algorithm proposed in this study addresses this problem by combining a game-theoretic framework with a Tabu Search heuristic to generate efficient and robust routing solutions. Through this integration, the study provides an analysis of hazardous materials transportation within the context of a VRP and derives corresponding equilibrium transportation costs, as well as link usage and failure probabilities (i.e., the likelihood of an incident occurring).

The chief aim of this study is to evolve, design, and analyse a reliability model that integrates the savings algorithm, Tabu Search, and game theory to provide a risk-averse solution framework for routing extremely hazardous materials. The proposed model aims to formulate a routing strategy that ensures the safe transportation of hazardous materials under uncertain conditions, particularly in situations where the likelihoods of link failures are unknown.

To address the problem described above, a game-theoretic approach is employed to determine a risk-averse routing solution. In this framework, the route planner (dispatcher) and a computer modelled system spoiler (evil entity) engage in a strategic two-agent, zero-sum, non-cooperative scenario. Given the substantial potential losses associated with low-probability, high-consequence hazardous material routing, the optimal strategy for the route planner is one that minimizes the maximum expected loss.

The route construction process is carried out using a combination of the conventional savings algorithm and a novel Tabu Search procedure. The savings algorithm generates the initial set of routes, which are subsequently refined and optimized through the Tabu Search. The method of successive averages is employed to determine the Nash equilibrium values for the expected trip cost and link usage or failure probabilities. The Nash equilibrium's expected cost serves as an effective indicator for evaluating the system's performance and overall reliability.

2. Vehicle Routing Problem (VRP)

VRP is a complex management challenge in the field of physical distribution and logistics. It involves creating a set of vehicle routes that begin and end at a main depot, aiming to deliver a specific product to a group of geographically distributed customers with known locations and demand levels. Each customer must be served exactly once by a single vehicle.

In the light of the above explanation, the classical VRP can be defined on a graph $G = (V, L)$; where, $V = \{v_0, v_1, v_2, v_3, \ldots, v_k\}$ is a vertex set representing k demand points, with demands $d_1, d_2, d_3, \ldots, d_k$ respectively. Here, v_o, is the depot. The demands of each customer are provided by a set of "*m*" alike vehicles with capacity "ρ" located at depot. $L = \{(v_i, v_{i`}): i \# i`, v_i, v_{i`} \in V\}$ is a link set. A non-negative cost is defined over the link $(v_i, v_{i`})$ for any $i, i' \in \{0,1,2,3,\ldots.k\}$ representing a distance, a travel cost or a travel time.

VRP is simple to describe but challenging to solve. A problem of size Y is considered solvable in polynomial time if the number of computational steps required grows as a polynomial function of Y. Non-deterministic polynomial-time (NP) problems are those for which a proposed solution can be verified in polynomial time, even if finding the solution may take exponential time. NP-hard problems, on the other hand, are not solvable in polynomial time. Most variants of the VRP are NP-hard and, as a result, are unlikely to be solvable efficiently within polynomial time [5].

A problem is considered easy if there exists an algorithm that solves it in time complexity $O(Y^t)$ for some constant t, where Y represents the size of the problem. Here, $O(Y^t)$ denotes a function such that:

$$O(Y^l) = a_l Y^l + a_{l-1} Y^{l-1} + \ldots + a_l Y + a_0 \tag{1}$$

where, a_i are constants.

This type of complexity is known as polynomial order, and an algorithm with such complexity is referred to as a polynomial-time algorithm. In contrast, if a problem can only be solved by an algorithm whose time complexity cannot be bounded by any polynomial function of Y, the problem is considered difficult or intractable. Computational complexity theory provides strong evidence that, for certain optimization problems within the NP class, the computation time required to solve them grows exponentially with the size of the problem.

Since VRP is an NP-hard problem, exact algorithms can only handle instances of relatively small size [6]. In practice, however, it is often necessary to address large and complex instances. Even the most sophisticated exact methods may become computationally infeasible in such cases. Consequently, approximate (heuristic) algorithms are crucial, as they can produce near-best solutions for large-scale problems within realistic computation times and memory constraints.
The following expressions present a standard mathematical formulation of the classical VRP.

$$\text{Minimise} \quad \sum_{i}\sum_{i'}\sum_{m} c_{ii'} x_{ii'}^{m} \tag{2}$$

subject to

$$\sum_{i}\sum_{m} x_{ii'}^{m} = 1 \quad for \quad all \quad i' \quad (i \neq i') \tag{3}$$

$$\sum_{i'}\sum_{m} x_{ii'}^{m} = 1 \quad for \quad all \quad i \quad (i \neq i') \tag{4}$$

$$\sum_{i} x_{ip}^{m} - \sum_{i'} x_{pi'}^{m} = 0 \quad for \quad all \quad p, m \tag{5}$$

$$\sum_{i} d_i (\sum_{i'} x_{ii'}^{m}) \leq \rho \quad for \quad all \quad m \tag{6}$$

$$\sum_{i'=1} x_{0i'}^{m} \leq 1 \quad for \quad all \quad m \tag{7}$$

$$\sum_{i=1}^{k} x_{i0}^{m} \leq 1 \quad for \quad all \quad m \tag{8}$$

$$x_{ii}^{m} \in Z \quad for \quad all \quad i, i', m \tag{9}$$

$$Z = \{(x_{ii'}^{m}) : \sum_{i \in B}\sum_{i' \in B} x_{ii'}^{m} \leq |B| - 1 \quad for \quad B \subseteq V/(0); |B| \geq 2\} \tag{10}$$

where;

$x_{ii'}^{m}$ is a binary variable indicating if link *i-i`* is traversed by vehicle *m*.

The objective of minimizing distance or travel time is expressed in Equation (2). Equations (3) and (4) together ensure that each customer is visited exactly once by a single vehicle. Equation (5) enforces that a vehicle leaves every customer it visits. Equation (6) guarantees that vehicle capacity constraints are not violated, while Equations (7) and (8) ensure that the number of available vehicles is not exceeded. Finally, Equation (9) is used to eliminate sub-tours.

As shown in the expressions above, the VRP may involve numerous constraints, which can generally be classified as relating either to vehicles or customers. It should be noted, however, that a specific VRP instance may not include all of these constraints.

2.1. Why do heuristic (approximate) algorithms work effectively

As noted, since VRP is an NP-hard problem and it cannot be solved optimally within polynomial time. Consequently, heuristic algorithms are commonly employed to obtain near-optimal solutions within a rational computational time frame.In the broadest sense, a heuristic can be defined as a problem-solving procedure that relies on intelligent interpretation and exploration of the problem structure to derive a reasonable solution. This definition highlights the importance of understanding and utilizing the underlying characteristics of a problem when designing a heuristic method. Furthermore, it emphasizes that heuristics aim to produce good though not necessarily optimal solutions within practical computational limits.

Solving NP-hard combinatorial problems of practical size would require exploring all possible combinations of decisions and variables, which is computationally unworkable. Heuristic methods play a crucial role in addressing such problems by intelligently guiding the search process, reducing the number of evaluations, and enabling the discovery of satisfactory solutions within reasonable time limits. Comprehensive combinatorial optimization problems, such as the Traveling Salesman Problem, VRPs and various network flow problems, can often be successfully addressed using heuristic methods.

2.2. Metaheuristics for VRP

Heuristic methods are designed to explore selected portions of the search space, concentrating on regions that appear most promising for improving solution quality. This targeted exploration significantly reduces computation time, typically yielding solutions that, while not optimal, represent substantial improvements over the initial state. A heuristic generally capitalizes on the specific characteristics of a problem to construct efficient solutions. In contrast, broader empirical approaches draw not only on problem-specific knowledge but also on analogies with natural optimization processes. These more general, problem-independent heuristic strategies are referred as "*metaheuristics*".

While heuristic methods typically conduct a relatively limited exploration of the search space, metaheuristics engage in a more extensive and intensive search within the most promising regions of the solution space. Consequently, metaheuristics integrate advanced neighbourhood search strategies, adaptive memory structures, and mechanisms for recombining solutions to enhance search efficiency and solution quality [7].

The heuristic methods proposed for solving VRPs between 1960 and 1990 are commonly referred to as classical heuristics. Algorithms belonging to this category include the Savings Algorithm, the Sweep Algorithm, and the Cluster-First, Route-Second approaches [8–10]. In contrast, metaheuristic methods (such as Simulated Annealing, Tabu Search, Granular Tabu Search, and Genetic Algorithms) have been widely applied to VRPs, providing more flexible and controlling solution strategies [11–14].

A solution procedure was proposed that employed 22 low-level heuristics, each contributing to the development of higher-level heuristics. In this approach, the selection probabilities of the low-level heuristics were dynamically updated based on whether the objective function improved, thereby increasing the likelihood of choosing more effective heuristic sequences and ultimately enhancing solution quality [15].

One of the suggested hybrid approaches joining two metaheuristics is known as the Unified Hybrid Genetic Search (UHGS). This method incorporates advanced mechanisms for diversity preservation, feasibility control, and a restart strategy to enhance search performance [16]. The algorithm is applied to tackle a complex optimization problem referred as the Heterogeneous Site-Dependent Multi-Depot Multi-Trip Periodic VRP (HSDMDMTPVRP).

Another metaheuristic algorithm was investigated using a column generation approach to assess solution quality [17]. In this method, switch point options were enumerated to reduce the overall complexity of the problem and improve computational efficiency.

The formulations are assessed by comparing the quality of their Linear Programming (LP) lower bounds to those obtained from a compact formulation. In this context, the LP lower bounds from the compact model function as a benchmark for evaluating the effectiveness and quality of the suggested solutions.

An alternative metaheuristic approach was proposed with a twofold objective: first, to categorize VRP-related variants addressed by metaheuristic algorithms suggested between 2009 and 2017, and second, to evaluate the effectiveness of these algorithms in obtaining reasonable solutions [18].

The Simulated Annealing (SA) metaheuristic originates from statistical mechanics [19]. As an iterative improvement and approximate optimization method, SA seeks to overcome the limitations of poor local optima by incorporating random acceptance criteria, allowing the algorithm to occasionally accept worse solutions in order to explore the solution space more broadly.

Ant Colony Optimization (ACO) is a biologically inspired, population-grounded metaheuristic developed to answer combinatorial optimization problems [20]. The core concept of ACO is that a large number of simple artificial agents can collaboratively construct high-quality solutions to complex combinatorial problems through indirect, low-level communication, analogous to the pheromone-based foraging behaviour of real ants.

One of the most effective strategies for escaping local minima, points where no neighbouring solution improves the objective function, is Tabu Search, initially introduced by Glover in 1986 [21]. Tabu Search enables the exploration of solutions that do not immediately improve the objective function, permitting moves that would otherwise be restricted, provided they are not classified as forbidden according to the algorithm's tabu criteria.

Kumari et al. [22] proposed a novel hybrid metaheuristic, referred to as GA-RR, to address the Capacitated Vehicle Routing Problem (CVRP). The method integrates two prominent optimization strategies, the Genetic Algorithm (GA) and the Ruin-and-Recreate (RR) heuristic, thereby leveraging the global search capabilities of evolutionary computation and the intensive neighbourhood restructuring characteristic of RR. The objective of the hybrid approach is to generate high-quality solutions across a diverse set of VRP benchmark instances. To assess its performance, the GA-RR algorithm is evaluated on 34 standard benchmark datasets and its results are compared against those produced by several state-of-the-art algorithms. Empirical findings demonstrate that GA-RR delivers superior performance on the majority of test instances. This enhanced effectiveness is attributed to the refinement of GA-generated candidate solutions during the ruin-and-recreate phase, which strengthens the algorithm's capacity to escape local optima. Furthermore, the study provides an analysis of the exploration–exploitation dynamics inherent in the proposed hybrid framework.

Niyomphon and Nakkiew [23] developed a model to identify the optimal daily routing plan minimising total transportation costs, encompassing both fixed vehicle-rental expenses and variable costs associated with travel distance, fuel prices, and fuel consumption. They further exacerbated the complexity of the problem through high customer demand levels that frequently surpass the weight and volume capacities of individual vehicles. Furthermore, all deliveries are subject to strict time-window constraints, necessitating precise scheduling. As a result, the problem is formally modelled as a Multi-Trip Capacitated Vehicle Routing Problem with Time Windows (MTCVRPTW). Owing to its NP-hard nature, the application of metaheuristic optimization techniques represents a pragmatic and effective strategy for generating high-quality solutions within acceptable computational limits. In this investigation, Particle Swarm Optimization (PSO) and Differential Evolution (DE) are implemented to address the MTCVRPTW. The computational results demonstrate that DE consistently outperforms both PSO and the company's existing routing procedures in terms of solution quality.

Zuhanda et al. [24] suggested an approach by utilizing two prominent metaheuristic optimization frameworks, the Artificial Immune System (AIS) and the GA. Whereas GA is situated within the broader domain of evolutionary computation, AIS is rooted in principles of swarm intelligence. The research formulates a vehicle routing model that explicitly accounts for product consolidation and heterogeneous product dimensions, both of which markedly intensify the computational complexity inherent in the Vehicle Routing Problem with Simultaneous Pickup and Delivery and Product Consolidation (VRPSC). To assess the efficacy of these metaheuristic strategies, their performance is systematically compared with that of exact optimization methods, thereby enabling a rigorous evaluation of alternative solution paradigms for the VRPSC.

An adaptive hybrid metaheuristic that integrates GA with Local Search (LS) through incorporating a stochastic uncertainty model using probabilistic travel times was proposed by Reis [25]. The method automatically adjusts critical parameters, such as mutation rates and the likelihood of invoking local search based on ongoing search behaviour, resulting in a more resilient and efficient exploration of the solution space.

As the primary solution approach employed in the model developed for this study, the following section provides a detailed explanation of the method."

2.2.1. Tabu search

Tabu Search is established on the principle of intelligent problem solving and is somewhat stimulated by the observation that human behaviour often includes a random component, leading to diverse answers under alike conditions. Tabu Search is an iterative meta-strategy designed to enhance local steepest-descent searches by enabling a more extensive exploration of the solution space. It accomplishes this by generating extended neighbourhoods and focusing on preventing the search from becoming trapped in local optima. Here, the neighbourhood of a solution refers to all solutions reachable via a single transformation, and a move denotes the transition from one feasible solution to another.

Tabu Search operates on the principle of accepting new solutions primarily to prevent revisiting previously explored paths. This approach encourages the exploration of new regions within the solution space, reducing the risk of becoming trapped in local minima and enhancing the likelihood of approaching a global optimum. Similar to other metaheuristics, such as Simulated Annealing, Tabu Search effectively guides iterative improvement processes, providing a robust framework for exploring areas of the solution space where conventional heuristics might otherwise stall.

The core of the Tabu Search procedure lies in effective management of the tabu list. This list records prohibited moves, those that are not allowed in the current iteration, to prevent the search from backtracking. Tabu list management involves updating the list by determining how many moves, and which specific moves, should be designated as tabu during each iteration, as well as specifying the duration for which these moves remain prohibited.

One of the earliest applications of Tabu Search to the VRP was proposed by Willard [26]. In his approach, the solution is initially transformed into a giant tour through depot replication. Neighbourhoods are then defined as all feasible solutions reachable from the current solution via 2-opt or 3-opt exchanges. Another algorithm applies Tabu Search by either moving a vertex to a different route or swapping vertices between two routes to generate neighbours, subsequently selecting the best non-tabu feasible move at each iteration [27].

Since its introduction, Tabu Search has been employed to a wide range of VRPs and has emerged as one of the most effective methods for obtaining high-quality-often near-optimal-solutions for instances involving several hundred customers. This observation is supported by [28]. The success of Tabu Search can be attributed to several key features commonly incorporated in its implementations, including the allowance of infeasible solutions during the search, as well as the use of diversification and intensification strategies.

2.2.2. Game theoretic approach

A game can be defined as a conflict of gains and losses between two or more opponents who act according to a set of formal rules. Several characteristics are common to most games. First, games have rules that dictate the order of actions, specify the set of allowable actions, and determine how the outcomes are linked to the actions taken. Second, games involve two or more players, each consciously striving to maximize their own benefit. Third, the outcome for each player depends on the actions of the other players. Players are aware of this interdependence and recognize that selecting the best action requires careful evaluation of the actions likely to be taken by their opponents.

As a branch of mathematics and logic, game theory studies decision-making in situations where two or more rational and intelligent opponents interact under conditions of conflict or competition. It focuses particularly on the process of making decisions. Players take turns sequentially, after the first player moves, the next player(s) take their turn, and this continues until the game concludes. At the end of the game, each player receives a payoff based on the outcomes of their decisions.

A central question in game theory revolves around situations in which all decision makers (players) pursue their best possible strategies, taking into account the strategies chosen by others. The key issue is: what characteristics of certain game outcomes justify labelling them as equilibria? By analogy, in physical systems, an equilibrium refers to a stable state where all internal forces balance each other, leaving the system at rest unless disturbed by an external influence. Likewise, in game theory, an equilibrium is a condition where no player can achieve a better outcome by changing their strategy alone.

Keeping in mind that a player's best strategy (or best response) is the action that maximizes their payoff given the strategies of the other players, Nash (1951) proved that every finite non-cooperative game possesses at least one equilibrium point [29]. A Nash Equilibrium is a situation in which no player has an incentive to unilaterally deviate from their chosen strategy. This concept of self-reference can be made more explicit by describing a Nash equilibrium as a profile of strategies in which each strategy represents an optimal reaction to the other players' optimal reactions.

The following definition, provided by [30], can be used to further conceptualize the core idea of a Nash equilibrium."

In an n-player game V == $\{H_1,\ldots\ldots H_n; Z_1,\ldots\ldots Z_n\}$ a set of strategies $(h_1^*,\ldots\ldots\ldots, h_n^*)$ forms a Nash equilibrium if; for individual player i, h_i is player i 's best reaction to the strategies taken by the other n-l players, $(h_1^*,\ldots., h_{i-1}^*, h_{i+1}^*,\ldots., h_n^*\}$

$Z_i(h_1^*,\ldots.,h_i^*,\ldots.h_n^*)\ \ Z_i(h_1^*,\ldots.,h_i^*,\ldots.h_n^*)$,
$\forall h_i \in H_i$

That is, each player 's strategy h_i^* solves

$$\max_{h_i \in H_i} \quad Z_i(h_1^*,\ldots..h_i,\ldots.h_n^*) \tag{11}$$

where;
H_i is the set of strategies available to player i.
h_i is an arbitrary member of H_i set
Z_i is the payoff function for player *i"*

As possibly the most widely used solution concept in game theory, what makes the Nash Equilibrium unique is its general applicability: in typical mathematical fashion, it remains valid without being affected by specific examples when analysing a problem. The Nash Equilibrium asserts that every finite game possesses at least one equilibrium, which may exist in either pure or mixed strategies.

This research presents an innovative framework that integrates risk aversion into the routing process for hazardous material transportation. The routing challenge is modelled as a two-player, zero-sum, non-cooperative game that captures rational strategic behaviour between two entities: a route planner (dispatcher) and a hypothetical network attacker. The dispatcher's goal is to identify the most efficient routes for vehicles departing from a single depot, whereas the attacker seeks to disrupt the system by disabling one critical link, thereby maximizing operational losses [31]. The "non-cooperative" nature of the game indicates that the dispatcher has no information about which connection will be targeted, and the attacker, in turn, lacks knowledge of the specific routing decisions made for the hazardous cargo.

The mixed-strategy Nash equilibrium provides a robust framework for estimating the expected routing cost at equilibrium by accounting for both link usage and failure probabilities. This equilibrium reflects the result of a theoretical game in which the route organiser exhibits a highly risk-averse attitude toward potential link failures. At this equilibrium, the expected trip cost is minimized with respect to the link selection probabilities and maximized with respect to the link failure probabilities. The game is classified as a Nash game because neither player holds supremacy over the other, and neither can anticipate the opponent's next move.

2.3. Main concerns and risk averse decision-making models in hazmat shipment

The total number of fatalities, injuries, and damage to the environment and property constitute the consequences of an incident involving a vehicle transporting hazardous materials. In this context, it is important to distinguish between an accident and an incident. Events that cause the release of hazardous material, resulting in a spill, fire, explosion, or leakage, are classified as incidents. In contrast, events that do not lead to any of these undesirable outcomes are considered accidents. While a comprehensive risk model should account for all potential consequences of an incident, it is generally assumed that policymakers focus on mitigating catastrophic outcomes, with fatalities representing the most severe and undesirable category.

Risk is generally defined as the multiplication of the probability of an undesirable event occurring and the severity of its results. This interpretation of risk is commonly referred to as the expected cost. To this end, individual risk levels are determined as a function of each person's distance from the route on which hazardous materials are conveyed. As illustrated in Figure 1, the vehicle enters the inhabited area at point "a" and leaves at point "b." The effect area surrounding the segment "a–b" is referred as the threshold distance corresponding to the specific hazardous material being transported, represented by "t_i."

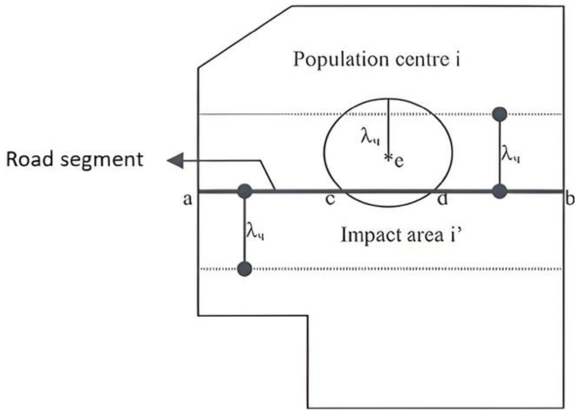

Figure 1. Enhanced risk assessment model

At point "e", within the impact area, people might be affected by a hazardous material incident if it happens within the threshold distance, symbolised as $\lambda_ч$. Hence, when the vehicle travels along the segment "c–d," the individual at point "e" is exposed to risk. As illustrated in the figure, the length of segment "c–d" increases as point "e" moves nearer to the road, leading to a corresponding increase in risk. The individual risk to be encountered at point "e" is expressed mathematically as follows:

$$IR_{esч} = P_{sч}(I)L(c,d) \quad (12)$$

where,
$IR_{esч}$ is the individual risk at point "e" on a "s" road segment due to single transportation of material "ч".
$P_{sч}(I)$ is the probability of incident on road segment "s"
$L(c,d)$ is the length of c-d.

The total public risk imposed on the population centre "*i*" due to a single shipment of material "*ч*" can be calculated by summing the individual risks in the impact area, which is,

$$R_{isч} = \int_{e \in i} (IR_{es\gamma})(POP_e) d_e \quad (13)$$

where,
POP_e denotes the population density at point "e"

The foundation of the risk-averse modelling approach is the assumption that the decision-maker (route planner) prefers to distribute hazardous material shipments across multiple routes rather than relying on a single route. This strategy aims to minimize the expected consequences of any potential incident during transportation.

When path listing is feasible, i.e., in relatively small networks, a linear programming method can be employed to decide the best combined policy for the decision-maker [31]. Using the established notation, the linear programming formulation is expressed as follows;

$$\begin{aligned} &\sum_k a_{ijk} C_{ij} h_k - F \le 0 \quad for \quad all \quad links(i,j) \\ &\sum_k h_k = 1 \\ &h_k \le 0 \quad for \quad all \quad paths \quad k \end{aligned} \quad (14)$$

P_1: Minimise F with respect to h_k for all links (i,j)
The dual of the above problem is formulated as linear programming as follow
P_2: Maximise G with respect to q_{ij}
subject to,

$$\begin{aligned} &\sum_{ij \in A} a_{ijk} C_{ij} q_{ij} - G \ge 0 \quad for \quad all \quad path(k) \\ &\sum_{ij \in A} q_{ij} = 1 \\ &q_{ij} \ge 0 \quad for \quad all \quad links \quad i,j \end{aligned} \quad (15)$$

where;
q_{ij} is the probability that an incident occurs on link (*i*,*j*)
a_{ijk} is 1 if link (*i*,*j*) lies on path *k*, and 0 otherwise
h_k is the probability that path k is selected for shipment
C_{ij} is the population inside the danger circle with a given impact radius, centred at any points on the link (*i*,*j*)
F, G are the dual variables

Although solving P_1 and P_2 ensures that the values of F and G are unique (with F = G at the solution), the path usage probabilities, link usage probabilities and link incident probabilities are not necessarily unique.

Because the linear programming approach requires prior path enumeration, it is unsuitable for large networks with numerous nodes and links. To address this limitation, an alternative method was proposed that combines a shortest path algorithm with the method of successive averages to generate optimal path sets, usage probabilities, and incident probabilities [32]. This iterative solution procedure employs a game-theoretic framework. In each iteration of the process, the decision-maker identifies the route that minimizes the expected travel cost, whereas a simulated network disruptor (the adversarial entity) deactivates a single link to create a failure scenario, thereby maximizing the operator's expected cost. The interaction between these two opposing objectives forms a strategic balance, and solving this optimisation problem produces the optimal probability distributions for both players. These probabilities correspond to the mixed-strategy Nash equilibrium of a two-player, zero-sum, non-cooperative game between the route planner and the virtual disruptor. Since this approach does not require prior knowledge of accident probabilities or link reliabilities, it provides an efficient solution method while offering valuable insights into the nature of the optimal routing strategy.

Building on this general background, the following section presents the model developed in this research for the transportation of low-probability, high-consequence hazardous material shipments.

Taking into account the operator's strongly risk-averse stance and the assumption that link failure probabilities are exceedingly small, the routing scenario is reframed as a two-player, zero-sum, non-cooperative game to represent strategic decision-making under uncertainty. Within this game structure, one player, the route planner, seeks to determine the most reliable routing configuration for vehicles departing from a single depot, while the opposing player, system attacker, attempts to disrupt the network by disabling a single link, thereby causing the greatest possible increase in overall transportation cost. The "*non-cooperative*" characterization signifies that neither party possesses prior knowledge of the other's move: the planner is unaware of which link may be failed, and the spoiler cannot predict which specific paths will be chosen to satisfy customer deliveries. To capture the balance between these conflicting strategies, the mixed-strategy Nash equilibrium is utilized, providing a quantitative means to approximate both the equilibrium expected trip cost and the associated probabilities of link usage and failure.

The iterative nature of this decision process is effectively unbounded, as both the route planner and the disposer can continuously adjust their tactics regardless of the number of moves made. This necessitates formulating the problem as a maximin problem, in which the operator selects routes by considering link usage and failure probabilities that yield the most favourable outcome. Solving this problem produces the optimal link usage probabilities for the minimisation of the total projected system cost, alongside the optimal link failure probabilities that maximize it. This resulting equilibrium is stated as the mixed-strategy Nash equilibrium.

Hence, the goal function of this mathematical structure can be expressed as follows:

$$Min(Max\sum_{ij} p_i(c_{ij}^f q_j + c_{ij}^n(1-q_j))\ with\ respect\ to\quad q_j\quad)\quad with\quad respect\quad to\quad p_i$$

subject to;

$$\sum_j q_j = 1$$
$$p_i \geq 0 \quad \forall \quad all \quad links \quad i \qquad\qquad (16)$$
$$q_j \geq 0 \quad \forall \quad all \quad scenarios \quad j$$

where;
p_i is the probability of link *i* to be used
q_j is the likelihood of scenario *j* to be in effect
c_{ij}^f is the route cost on link *i* when scenario *j prevails if* link *i* is failed
c_{ij}^n represents the travel cost on link *i* under scenario *j* when link *i* is functioning normally.

This theoretical framework is founded on the premise that the route planner initially assumes all network links possess an equal probability of failure. The initial set of optimal routes is generated through the Savings Algorithm [8]. In certain situations, some customers may end up being assigned individual routes due to network or cost constraints. Unlike traditional implementations, this method calculates the saving values using expected costs that are derived from the likelihood of link usage and potential failure, rather than relying on actual travel expenses.

The links in the system are considered to have two distinct types of costs. The first represents the normal operating condition cost, which applies when a vehicle carrying hazardous materials travels safely along the link without being involved in any incident. This cost may correspond to factors such as total travel distance, travel time, or fuel consumption. The second cost type represents the failure condition cost of a link. In this case, it is assumed that the vehicle travelling on the link is involved in an incident that causes damage to the environment, property, and nearby populations. Any of the conventional risk assessment frameworks-such as conditional risk models, perceived risk models, or other established methodologies-can be employed to estimate the potential cost associated with such an event, depending on the chosen definition of risk.

Since different risk models may yield varying cost estimates for a given incident, the failed cost values assigned to each link can differ depending on the model used. In the context of this study, however, both normal and failed cost values for each link are treated as exogenous inputs, known a priori. This approach provides the route planner with the flexibility to define the failed cost of each link based on multiple weighted criteria, reflecting the relative importance of different risk dimensions. For example, if certain links pass through rural areas near environmentally sensitive zones-such as natural parks, forests, or historical sites-the failed cost can emphasize environmental impact. Conversely, if a link passes through or near densely populated areas, the failed cost can be based primarily on the number of people potentially affected by an incident. Thus, the operator can determine the failed cost of each link by assigning appropriate weights to societal consequences such as environmental pollution, economic loss, evacuations, injuries, or fatalities, allowing the model to reflect context-specific risk priorities.

The vehicle routes generated by the Savings Algorithm were subsequently optimised using Tabu Search.

3. Solution Algorithm

The assumptions underlying the proposed model can be summarized as follows:

- The network connecting the depot and the customers is modelled using straight-line links. In other words, all the customers and depot are connected through straight line although real case represents curvy connections. This is universal simplification process accepted to represent the transportation networks allowing for more tractable calculations of distances and costs.
- Each link is assumed to exhibit uniform characteristics throughout its entire length. Factors such as road condition, traffic flow, and surrounding environment are considered uniform across each individual link.
- Although the actual failure probabilities of the links are unknown, they are assumed to be relatively low. However, any failure that does occur is expected to have extremely high consequences, consistent with the nature of low-probability, high-consequence hazardous material events.
- The normal and failure costs of each link are assumed to be homogeneous along its entire length. In other words, the normal and failure costs of the links stay the same regardless of the position and the time of the vehicle on the links. Both the operational (normal) cost and the potential incident (failure) cost are evenly distributed, avoiding the need for spatially variable cost modelling within a single link.
- The model is built on the premise that hazardous materials are transported through a series of repeated shipments starting from a central depot and distributed to various customers. This repetitive transport structure enables the estimation of both expected costs and link usage probabilities across multiple delivery cycles, thereby facilitating the integration of game-theoretic equilibrium principles into the routing analysis.

The flowchart below, Figure 2, outlines the calculation procedure of the proposed algorithm.

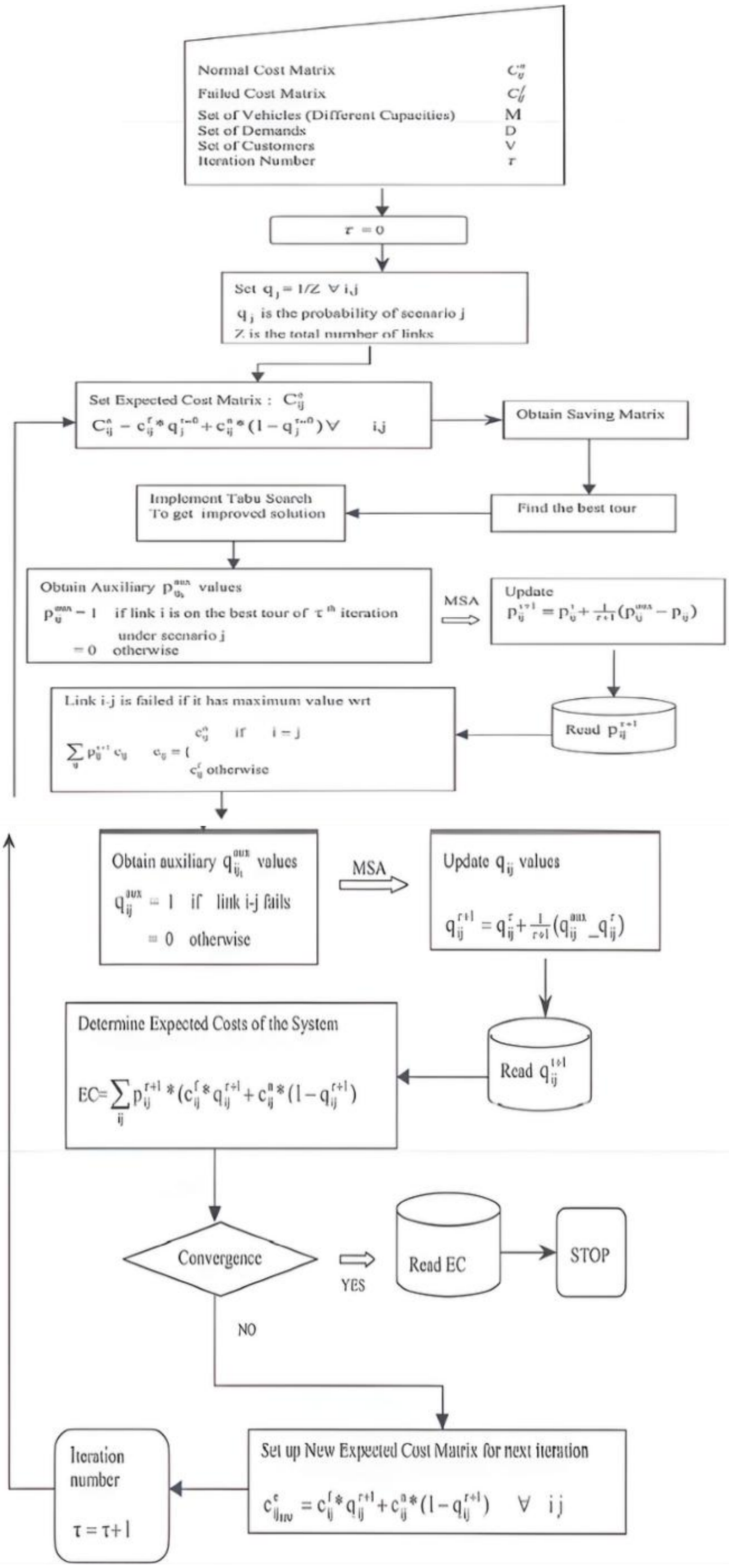

Figure 2. Algorithmic flow-diagram of the developed method

As demonstrated in this figure, the process begins with the planner identifying the optimal set of vehicle routes that minimise the total expected system trip cost, assuming that the link failure probabilities are known. Savings algorithm is the tool used for the identification of the initial set of routes with minimum total expected cost. At this stage Tabu Search is employed to attain the improved set of routes with regard to minimised total expected cost. The link usage probabilities are then updated by employing method of successive averages. Once these routes are established, the evil-entity responds by failing the link that results in the maximum possible cost to the operator. This action produces updated expected link costs, prompting the planner to generate a new best set of routes to be used based on the revised cost structure. Given these new routes, the evil entity again selects and fails the link whose disruption produces the greatest increase in the overall trip cost.

An achievable solution addressing the problem described above is presented by $S=(S_1,S_2,S_3,.....,S_m)$, where S_i outlines exclusively the subsection of customers of $V=(1,2,3,.....,k)$ used by a vehicle "*i*" from the set of $M=(1,2,3,....,m)$, such that;

$$\begin{aligned} &\bigcup_{i=1}^{m} S_i = V \\ &S_{i_1} \cap S_{i_2} = \phi \qquad \forall i_1, i_2 \in M \\ &\sum_{k \in S_i} d_k \le \rho_k \qquad \forall i \in M \\ &C(S) = \sum_{i \in M} C(S_i) \end{aligned} \tag{17}$$

As demonstrated by computational findings in the literature and the preceding sections, the initial solution derived from the savings algorithm often lacks sufficient optimality. To refine this solution, a Tabu Search procedure is applied. This method iteratively enhances feasible solutions by exploring their neighbourhoods. To intensify local improvements within each route, a generation mechanism is employed, defining how a current solution *S* can be transformed into a neighbouring solution *S′* within the developed model. The neighbourhood solutions are produced using the *I-interchange mechanism for generating solutions*, as introduced by [11]. After these neighbouring solutions are identified, they can be further refined through local improvement moves based on edge exchanges [33].

The algorithm described above maintains a list of previously made or visited moves to record a number of the most recent actions, thereby prohibiting repetitions or cycling. The duration for which moves and solutions remain on the tabu list is determined by the list size. A small tabu list may lead to cycling, whereas an excessively large list can divert the search away from regions near the global optimum. Ideally, the tabu list size should be sufficient to prevent cycling while still allowing continuous exploration of the solution space. If at iteration "τ" vertices (customers) $v_1 \in R_1$ and $v_2 \in R_2$ are exchanged, it is prohibited to use both v_1 in R_1 and v_2 in R_2 again through iterations (τ+1,…………., τ +TABLIST), where TABLIST is the tabu duration (tabu list size). The tabu list is enhanced with a frequency measure that monitors how often each tabu move or solution is revisited during the search, helping guide future decisions. Tracking this frequency information is central for understanding and guiding the evolution of the search process. A high tabu-hit frequency indicates that the process may be trapped in a suboptimal peak, signalling that the search should either be terminated or that a diversification strategy should be applied to explore regions near the global optimum.

After determining the final route configuration, the estimation of link usage and failure probabilities is further refined through the method of successive averages, an iterative optimization technique known for its stability and adaptability in converging toward equilibrium solutions. This method incrementally updates probability values at each iteration by averaging current and previous estimates, thereby enhancing overall solution accuracy. A detailed explanation of the theoretical foundations and implementation steps of method of successive averages can be found in [34], with a concise overview provided in the next section.

3.1. Method of successive averages

This is an iterative solution method for the optimisation problems of,

Minimise f(x) subject to b =Ax

As a descent search direction can be obtained if a negative of the gradient, -Δf(x) is provided at any point x, the solution of the linear programming problem below results in the maximum feasible descent from the point x'.

Minimise $x^T \Delta$ f(x') subject to b=Ax

This problem is classified to as an auxiliary problem, and its outcome, denoted as x^*, is called an auxiliary point. The method of successive averages calculates the average of a sequence of auxiliary points, where each auxiliary point is attained by answering an auxiliary problem in terms of the preceding average of auxiliary points. Thus, if x_n^* is the n^{th} auxiliary point, then an iteration N,

$$x' \leftarrow (1/N) \sum_{n=1toN} x_n^*$$

but,

$$(1/N) \sum_{n=1toN} x_n^* = (1/N) x_N^* + (1/N) \sum_{n=1toN-1} x_n^* \tag{18}$$

hence,

$$x'_{new} \leftarrow (1/N) x'_N + (1 - \frac{1}{N}) x'_{old}$$

where, x_N^* is obtained by solving the above auxiliary problem.

In order to update the link usage and link failure probabilities, so that the improved expected link travel costs are obtained, first the auxiliary link usage probabilities, denoted by p_i^{aux}, are determined for each link in the system, such that they are 1 if link i is on the one of the current best routes when scenario "*j*" prevails, "0" otherwise. By using these auxiliary values and employing method of successive averages algorithm, the updated link usage probabilities were found. After obtaining updated link usage probabilities, we can obtain the link which is likely to be intentionally disrupted by the hostile actor (evil entity) at the current iteration. Since we assume that the system operator is extremely pessimistic about the state of the system, the evil-entity fails the link which gives the highest value of the $\sum_i p_{ij} c_{ij}$. Here "c_{ij}" is the current travel cost of the link *i*, such that;

$$c_{ij} = \begin{bmatrix} c_i^n & if & i \neq j \\ c_i^f & if & i = j \end{bmatrix}$$

As can be seen, the evil entity has the power to fail one and only one link in the system to impose the highest possible operational cost on the operator. While the auxiliary link failure probability, denoted by $q_{ij}^{\max}$, of link *i* which is failed is 1, all the other remaining links' auxiliary failure probabilities, for that specific iteration, are set to 0. After this, link failure probabilities are updated through method of successive averages and denoted by q_{ij}^{updt}.

The stopping condition for the proposed algorithm is governed by a predefined maximum number of iterations. *MAXI*, counted from the iteration in which the best solution was found. The value of *MAXI* is provided as an input parameter to the model.

In the light of explanations above, the Figure 3 below provides a summary of the calculation steps involved in the main algorithm

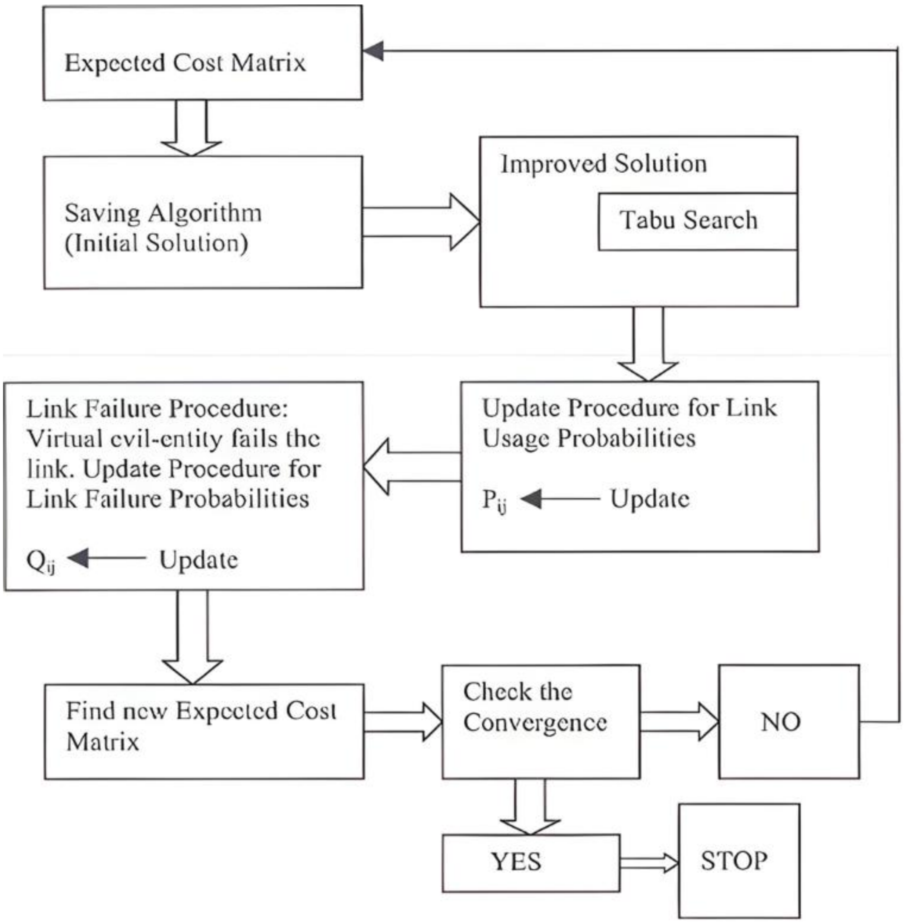

Figure 3. Computational procedure of the proposed method

3.2. Linear programming formulation

"As shown in Equation 16, the objective function is designed to simultaneously minimize and maximize the total expected routing cost, minimized with respect to the link usage probabilities and maximized with respect to the link failure probabilities. To simplify the formulation, Equation 16 can be re-expressed in terms of entire routes instead of individual network links, as follows;

$$
\begin{aligned}
& Max \quad \left(Min \quad C = \sum_{i,j} h_i c_{ij}\, q_j \quad with\, respect\, to \quad h_i\right) \quad with\, respect\, to \quad q_j \\
& subject\, to \\
& \sum_i h_i = 1 \\
& \sum_j q_j = 1 \\
& h_i \geq 0 \quad for\, all\, routes \quad i \\
& q_j \geq 0 \quad for\, all\, scenarios \quad j
\end{aligned}
\tag{19}
$$

where,

C is the total expected cost of assigning routes to vehicles
c_{ij} is the cost associated with route i when scenario j prevails
h_i is the likelihood that route i is used
q_j is the probability of j^{th} scenario to be in effect
The linear programming approach for solving this maximin problem is outlined below,
If "C^*" is the outcome of the problem. Then,

$$\sum_i c_{ij} h_i \leq C^* \qquad for\, every\, scenarios\ j \tag{20}$$

The linear programming model below yields the optimal mixed routing strategy for vehicles handling extremely hazardous materials.
Min M with regard to h_i depending on

$$\begin{aligned} &\sum_i c_{ij} h_i - M \leq 0 \qquad for\, every\, scenarios\ j \\ &\sum_i h_i = 1 \\ &h_i \geq 0 \qquad for\, every\, routes\, i \end{aligned} \tag{21}$$

Similarly, the optimal mixed link-failure strategy for the evil entity can be determined using the following linear programming model.

Min M* with regard to q_j depending on

$$\begin{aligned} &\sum_j c_{ij} q_i - M^* \geq 0 \qquad for\, all\, scenarios\ j \\ &\sum_j q_j = 1 \\ &q_j \geq 0 \qquad for\, all\, scenarios\ j \end{aligned} \tag{22}$$

It is important to emphasize that the route selection probabilities derived from Equation 21 and the link failure probabilities obtained from Equation 22 are *dual counterparts* within the optimization framework. In essence, the dual variables linked to the constraints in Equation 21 directly correspond to the scenario probabilities defined in Equation 22, reflecting the interdependent relationship between the planner's route-selection and the spoiler's link-failure strategies under the given constraint conditions. At the optimal solution, M attains the value M*. While this linear programming problem yields a unique solution for M, it does not generally produce unique values for "h_i" or "q_j".

The model developed in this research equips the route operator with valuable insights into the critical links that exhibit high failure probabilities within the network, an essential factor in determining the safest routing strategy. For low-probability, high-consequence hazardous material shipments, the safest (i.e., most reliable) strategy aligns with the one that yields the maximin expected routing cost. By focusing on minimizing the system's vulnerability to potential link failures, the proposed approach effectively maximizes the reliability and overall safety of the routing strategy.

4. Numerical Calculations and Results

We begin with how costs are encoded: the network is evaluated under two cost regimes (i) a baseline matrix that prices trips when every link operates safely, and (ii) a failed cost matrix that prices trips when an incident occurs on some links during transport. The latter reflects catastrophic consequences and is calibrated so that the implied social harm exceeds a predefined tolerability criterion.

On top of this cost structure, the logistics setting is a single-depot distribution problem with geographically dispersed customers of known demand. Service is provided by a homogenous fleet of fixed-capacity vehicles stationed at the depot, and all customer requirements must be met within those capacity limits. This coupling of dual cost regimes with the one-depot, fixed-capacity fleet formalizes routing decisions for extremely hazardous shipments under both normal operation and incident scenarios.

In the problem under consideration, the system consists of a single depot, 20 vehicles, and 20 customers. The demand of each customer is assumed to be known, and all vehicles are assigned equal capacities of 1,500 units. The failed costs associated with the links are set to values greater than or equal to a predetermined threshold of 1,400 units (refer to Appendix A and B for the Normal and Failure Cost Matrices, respectively). Using the developed model, the following equilibrium expected trip cost has been obtained. At this point it should be pointed out that the monetary units of the costs are not sensitive to the currency to be used. The distinctive feature of the normal and failure costs is that the failure costs are much higher than the normal costs regardless of the currency to illustrate the comparison of the two different situations for the analysis.

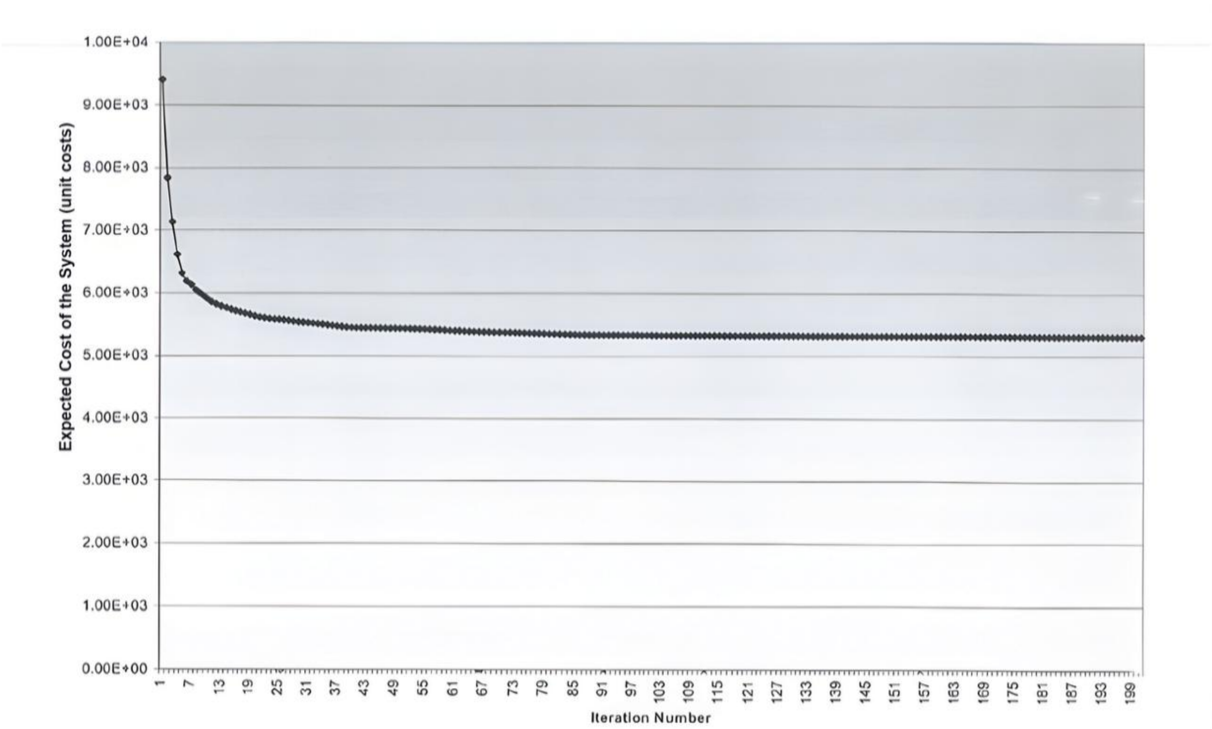

Figure 4. Convergence of trip cost: Tabu Search

Figure 4 illustrates that the equilibrium trip cost is 5318.104 units. This value signifies that, regardless of the specific link failure probabilities, the total expected trip cost will not surpass 5318.104 units when the equilibrium link usage probabilities are employed. Accordingly, the equilibrium value of the expected trip cost represents the outcome of the optimal routing strategy under the most adverse link failure conditions, assuming that an incident occurs. This strategy reflects a deliberately conservative, highly pessimistic approach that aligns with a risk-averse planner's perspective, prioritising safety and minimizing potential losses.

Figure 5. explains the convergence performance of the expected cost of the system for the same problem using Clark and Wright's savings algorithm. In this case, the equilibrium trip cost is 5393.185 units, which is higher than the one obtained using Tabu Search.

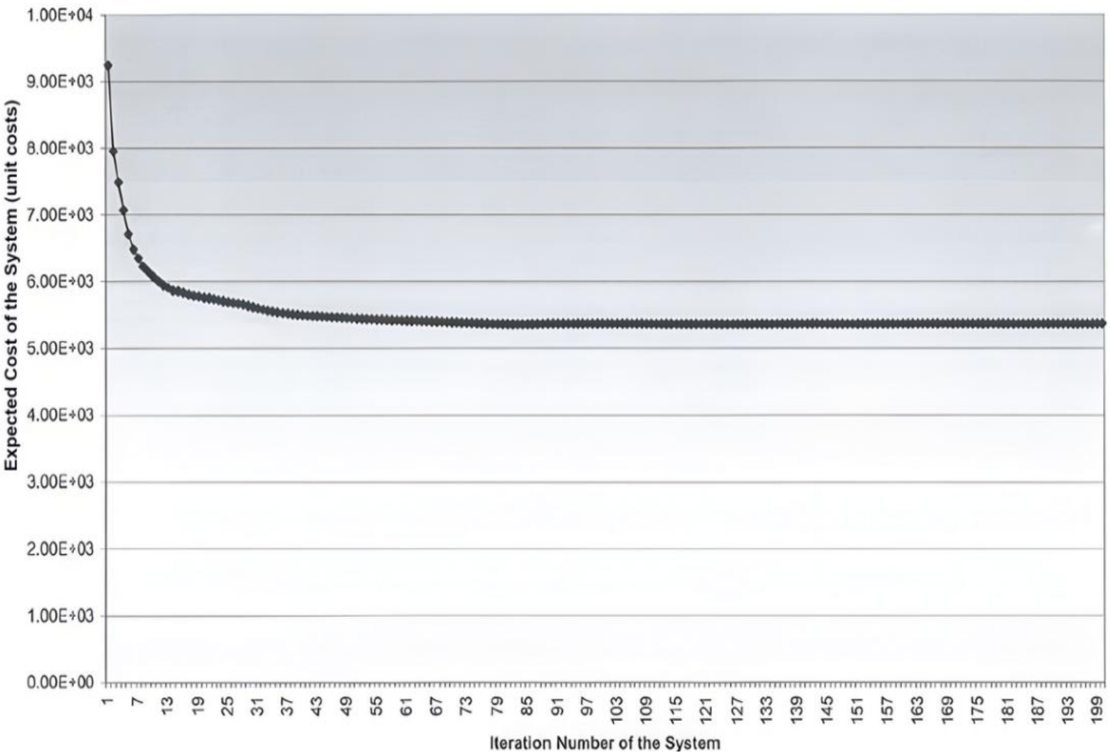

Figure 5. Convergence of expected trip cost: savings algorithm

The convergence behaviour of the link usage probabilities obtained by using Tabu Search for 20-customer problem is illustrated by Figure 6.

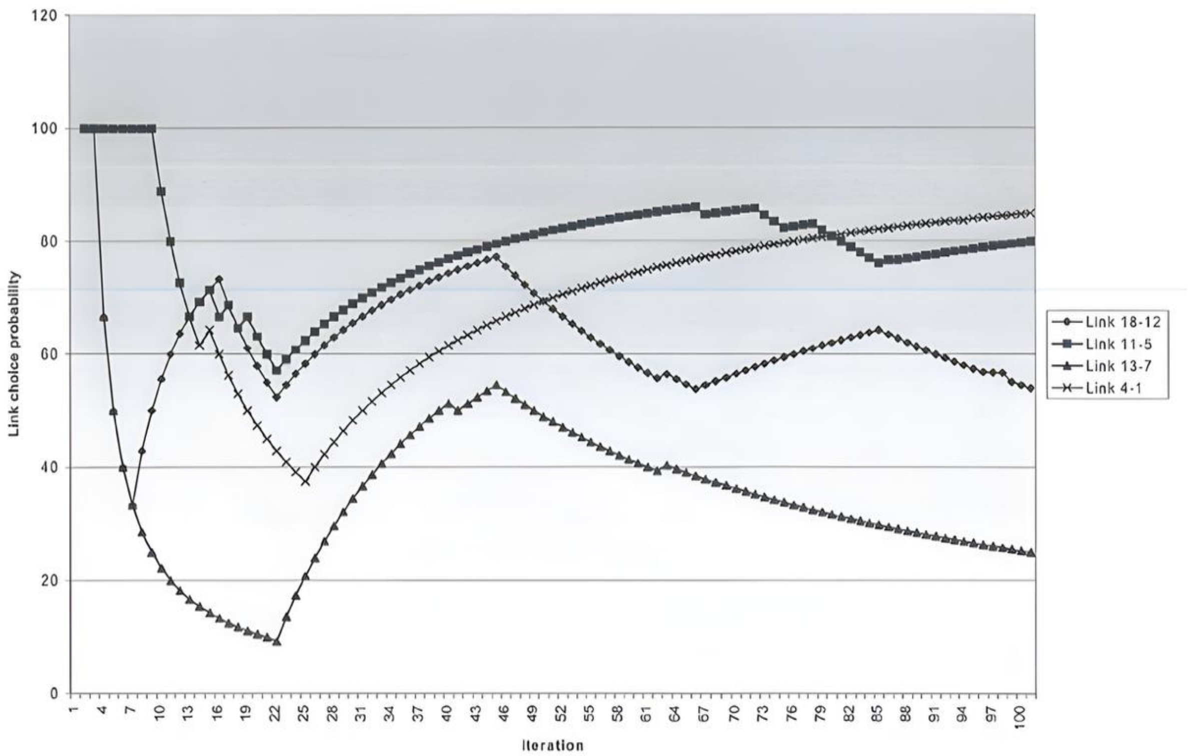

Figure 6. Convergence behaviour of the link usage probabilities: Tabu Search

To demonstrate the impact of vehicle capacities on the total expected trip cost, the same problem was resolved assuming that each vehicle has a capacity of 2.500 units instead of 1.500 units. The normal and failed cost matrices remain unchanged. Figure 7 below shows the convergence of the expected trip cost for this modified scenario.

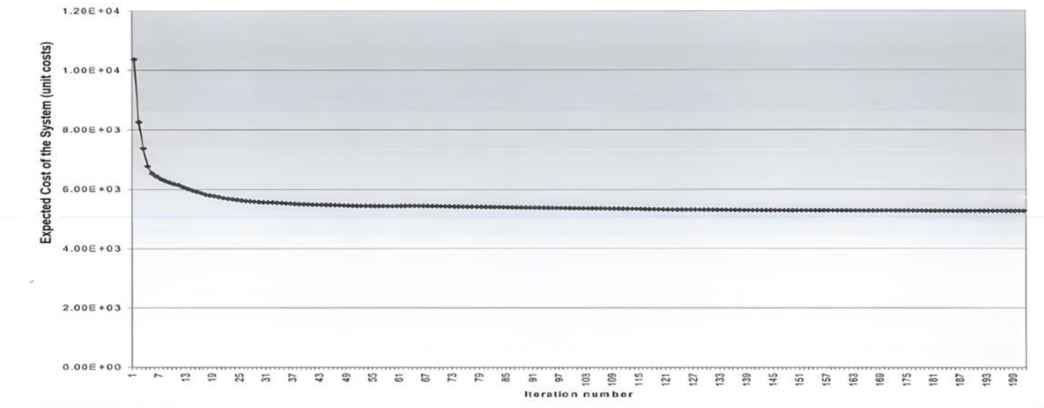

Figure 7. Convergence of expected trip cost: 20-customer, increased vehicle capacity problem

Figure 7 clearly states that increasing vehicle capacity leads to a lower expected trip cost at equilibrium for the same normal and failure costs used for lower capacity problem. In this way the effect of the capacity change only situation was analysed. Specifically, increasing vehicle capacity from 1.500 to 2.500 units reduces the system's expected trip cost from 5.393.185 to 5.275.21 units, while all other problem input values remain unchanged. This observation suggests that the optimal routing strategy may depend not only on the worst-case set of link failure probabilities but also on the number and capacities of vehicles stationed at the depot.

The developed model was tested on problems of varying sizes to evaluate both the computational effort required to determine the payoff and the improvements achieved by applying Tabu Search. The results of these tests, including computation times and solution enhancements, are summarized in Table 1.

Table 1. Test results in terms of required computational times

	Time required for each iteration when convergence is obtained (minute)		Total expected travel (routing) cost		Improvement in total expected cost (%)
	Savings Method	Tabu Search	Savings Method	Tabu Search	
10 Customers	10	7	2411.234	2400.866	0.43
11 Customers	10	7	2797.651	2785.062	0.45
12 Customers	10	6	3081.474	3066.067	0.50
13 Customers	10	6	3393.814	3374.809	0.56
14 Customers	11	5	3718.586	3695.159	0.63
15 Customers	11	5	4095.964	4067.292	0.70
16 Customers	11	4	4275.688	4241.910	0.79
17 Customers	12	4	4513.143	4472.524	0.9
18 Customers	12	3	4745.876	4696.993	1.03
19 Customers	12	3	4991.290	4932.393	1.18
20 Customers	13	2	5393.185	5318.104	1.39

The developed solution procedure is used to calculate the precise link usage probabilities in a 5-customer scenario, considering a total of 120 distinct routes. This analysis serves two purposes: to compare the results obtained using the iterative method of successive averages and to examine the potential existence of multiple optima. For this analysis, the following equation is introduced (see Appendix C and D for the normal and failure cost values, respectively.

$$\begin{aligned}
&Min \quad M\\
&CR_{1_1}h_1 + CR_{2_1}h_2 + CR_{3_1}h_3 + \dots\dots\dots\dots + CR_{120_1}h_{120} \le M\\
&CR_{1_2}h_1 + CR_{2_2}h_2 + CR_{3_2}h_3 + \dots\dots\dots\dots + CR_{120_2}h_{120} \le M\\
&CR_{1_3}h_1 + CR_{2_3}h_2 + CR_{3_3}h_3 + \dots\dots\dots\dots + CR_{120_3}h_{120} \le M\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
&CR_{1_{15}}h_1 + CR_{2_{15}}h_2 + CR_{3_{15}}h_3 + \dots\dots\dots\dots + CR_{120_{15}}h_{120} \le M
\end{aligned} \tag{23}$$

where;
CR_{k_i} is the cost of link *i* on route *k*
h_k is the usage probability of route *k*
subject to

$$\begin{aligned}
&h_1 + h_2 + h_3 + \dots\dots\dots\dots + h_{120} = 1\\
&h_1, h_2, h_3, \dots\dots\dots\dots\dots\dots, h_{120} \ge 0
\end{aligned} \tag{24}$$

In order to address this problem, the E04MFF/E04MFA computer program was utilized, specifically designed for linear problems of the form:

$$\underset{x \in F^N}{\text{minimise}} \quad c^T x, \quad subject\,to \quad l \le \left\{ \begin{matrix} x \\ Ax \end{matrix} \right\} \le u \tag{25}$$

where, "*c*" is "*n*" element vector and A is an m_L by "*n*" matrix."

A linear programming problem in its general form is called a type LP problem. The constraints that involve the matrix A are known as the main constraints. Each variable, as well as each general constraint, has specified lower and upper bounds. An equality restraint can be represented by setting its lower and upper bounds equal, i.e., *li =ui.* If a constraint or variable has no lower bound or no upper bound, the corresponding entry in the bound vectors *l* or *u* may be set to a special value that represents $-\infty$ or $+\infty$.

E04MFF/E04MFA operates in two phases. The first phase focuses on finding a feasible starting point by minimising the total constraint infeasibility. Once feasibility is achieved, the second phase minimises the linear objective function over the feasible region. Both phases use the same underlying computational routines; the only change is in the function being minimised from the sum of infeasibilities in phase one to the original objective function in phase two.

The resulting route choice probabilities obtained by solving Equation 23 are given by the following values.

Table 2. Linear programming results for route usage probabilities

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0. 24	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

From the complete set of 120 feasible routing configurations (from left to right), the optimization process ultimately converged on a remarkably compact solution. Only three of those routes are actually required to maintain the minimum overall expected transportation cost, even under the most disruptive conditions imaginable. In other words, no matter which network link the hypothetical system spoiler disables in an attempt to maximize the dispatcher's losses, the system's optimal response relies solely on this trio of resilient routes. As summarized in Table 2, the selected routes (Route 18, Route 34, and Route 42) are employed for transporting hazardous materials from the central depot to the designated customer locations. The detailed sequences of network links corresponding to these routes are listed below:

Route 18: 0-1-4-5-3-2-0 $h_{18} = 0.55$
Route 34: 0-2-3-4-5-1-0 $h_{34} = 0.24$
Route 42: 0-2-4-5-3-1-0 $h_{42} = 0.21$

The Table 3 below compares the link usage probabilities derived from the method of successive averages with those obtained through the linear programming model.

Table 3. Comparison of link usage probabilities

	Linear Programming	Method of Successive Averages
Link 0..1	1	0.97
Link 0..2	1	1
Link 0..5	0	0.03
Link 1..3	0.21	0.24
Link 1..4	0.55	0.54
Link 1..5	0.24	0.25
Link 2..3	0.79	0.78
Link 2..4	0.21	0.21
Link 2..5	0	0.009
Link 3..4	0.24	0.28
Link 3..5	0.76	0.70
Link 4..5	1	0.99

The analysis revealed that the gradual convergence of link-usage probabilities stems largely from the presence of several comparable near-optimal solutions rather than from any weakness in the numerical scheme itself.

Even so, the method of successive averages produced results that were virtually indistinguishable from the optimum, after roughly one hundred iterations, the total expected routing cost stabilized at 88.70 units, compared with 88.71 units obtained through the linear-programming formulation. This close agreement affirms the reliability and accuracy of method of successive averages. Nevertheless, adopting alternative optimization techniques with inherently faster convergence characteristics could further reduce computation time while yielding equally precise estimates of both link-usage probabilities and total expected routing costs.

5. Discussions and Recommendations

The main objective of this research was to formulate a model capable of assessing the reliability of transporting highly hazardous materials from a central depot to multiple geographically dispersed customers, using a fleet of vehicles with fixed capacity constraints. A key aspect addressed in this research is the uncertainty of incident probabilities on any given link during shipment, a challenge commonly encountered in real-world scenarios. Such situations arise when a route planner seeks optimal routing strategies for low-probability, high-consequence hazardous material shipments. In the proposed approach, initial routes are generated using the Savings Algorithm and subsequently refined through Tabu Search to improve solution quality. In this sense, the main contribution of this study is that it provides the best possible routing strategies for repeated low-probability high-consequence hazmat shipments determined by a route planner. The risk averse nature of the route planner was represented by game theoretic approach bu assuming that the route planner is extremely pessimistic about the link failure probabilities and tries to select his strategies by reacting against a virtual evil-entity's link failure strategies. The failed link by the evil-entity is the one which results in least favourable to route planner. The following sections summarize the main conclusions of the study and provide recommendations for future research.

- The findings indicate that the convergence of equilibrium point in terms of total routing cost is quick. However, the model converges more slowly as far as link usage and failure probabilities are concerned. This indicates that expected trip cost at the equilibrium can be achieved through multiple combinations of link usage and failure probabilities. In other words, the slower convergence highlights the non-uniqueness of the link usage and failure probability values.
- The convergence of the model could be significantly faster if the routes selected at each iteration are closer to the optimal solution. Since the model relies on heuristic routing strategies, employing a more effective heuristic algorithm would accelerate the convergence of the solution process.
- Although the method of successive averages is a simple and effective iterative procedure for guiding the solution toward the equilibrium point, it can lead to slow convergence of link usage and failure probabilities. Alternative iterative schemes might achieve faster convergence. Nevertheless, method of successive averages provides the advantage of allowing the route planner and the virtual evil entity to update their strategies based on the accumulated strategies of the other player.
- The failure probabilities indicate which links in the system are more vulnerable. These links help guide the route planner to design routes that avoid high-risk links whenever possible. It should be noted that while the equilibrium failure probabilities are generally not unique, the subset of links with zero failure probability is unique. This means the method effectively identifies a critical subset of links with failure probabilities greater than zero that warrant special attention.
- The expected trip cost provides a valuable measure of the reliability of hazardous materials shipments from a safety perspective. This is because the equilibrium expected trip cost reflects the expectations of a highly risk-averse (extremely pessimistic) route planner.
- The failure probabilities at equilibrium produced by the model do not correspond to the actual likelihood of link failures. Instead, they represent probabilities under the worst-case scenario, and the true link failure probabilities may differ from those generated by the model.
- The developed model offers a valuable tool for identifying the optimal routing strategies for the shipment of extremely hazardous materials when the actual link failure probabilities are unknown. By applying game theory, the model provides important insights into the problem and yields the optimal link usage and failure probabilities under a worst-case scenario framework.

5.1. Future work

The proposed model can be improved by allowing the evil entity to fail more than one link. The model presents the simple scenario in which only one link can be failed by the virtual evil-entity. However, batch failures may be included in the game as supplementary scenarios from which the system spoiler can choose. In addition to this, in real world cases there might be some link topologies in which some part of the same link can be shared by two or more customers. In these cases, the failure on those links will not only affect one element of the cost matrix, but also the other elements, too. This point may be paid attention to and included in further studies to increase the realism. Moreover, the model can be further enhanced by assuming that the demands of the customers are stochastic in nature. In real world problems, the amount of product required by the customers may not be known beforehand, unlike the assumption made in this study. Inclusion of the impact level of vehicle speed to the magnitude of the incident may be worth to investigate. For example, a link between any two customers through a rural area on which higher speeds are possible may be attractive if the population density is relatively low. However, another link passing through a densely populated area on which a lower speed limit applies may be even more attractive because the lower speed on this link means that the impact of an incident, should it occur, may be relatively limited.

6. Author Contribution Statement

In the realized study, Author 1 undertook the idea, design, literature review, methodology and writing of the article, Author 2 undertook supervision and investigation of the article.

7. Ethics Committee Approval and Conflict of Interest

“Ethics committee permission is not required for the prepared article”
“There is no conflict of interest with any person/institution in the prepared article.”

8. Ethical Statement Regarding the Use of Artificial Intelligence

No artificial intelligence-based tools or applications were used in the preparation of this study. The entire content of the study was produced by the author in accordance with scientific research methods and academic ethical principles.

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Appendix A. Normal cost matrix data for 20-customer 1-vehicle VRP

DATA FOR 20-CUSTOMER 20-VEHICLE PROBLEM

Normal Cost Matrix

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	110																			
2	80	170																		
3	100	150	110																	
4	130	100	170	150																
5	150	220	200	170	160															
6	80	170	120	120	140	170														
7	90	130	110	110	140	200	110													
8	110	120	100	160	180	210	140	170												
9	170	240	200	220	250	250	200	200	150											
10	190	250	220	250	270	240	150	250	150	220										
11	250	220	300	200	300	230	200	300	200	270	350									
12	220	200	250	180	250	280	250	190	170	300	480	400								
13	240	300	270	170	300	350	200	200	220	350	370	400	300							
14	280	300	300	200	350	320	230	220	190	330	400	400	450	410						
15	160	200	200	80	100	200	200	200	190	250	300	300	310	370	330					
16	150	250	200	200	250	250	140	200	170	270	300	320	300	280	370	270				
17	140	230	200	200	200	250	180	180	190	280	250	310	330	270	360	260	200			
18	190	250	210	220	240	300	240	250	200	260	250	300	200	300	320	250	200	220		
19	200	260	240	250	220	310	270	260	240	300	300	320	240	320	330	270	240	260	240	
20	210	230	250	280	260	320	220	270	330	320	360	260	280	360	300	280	280	270	380	200

Customer Demands: (units)

Customer1: 400 Customer2: 100 Customer3: 200 Customer4: 200 Customer5: 300 Customer6: 100 Customer7: 200 Customer8: 300 Customer9: 200 Customer10: 3

Customer11: 100 Customer12: 200 Customer13: 200 Customer14: 300 Customer15: 300 Customer16: 100 Customer17: 200 Customer18: 100 Customer19: 200 Customer20: 2

Vehicle capacities: 1500 units

Appendix B. Failed cost matrix data for 20-customer 1-vehicle problem

DATA FOR 20-CUSTOMER 20-VEHICLE PROBLEM

Failed Cost Matrix

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	2000																			
2	1400	2400																		
3	1900	3300	2500																	
4	1900	3000	2200	2500																
5	2400	3500	2700	3000	2700															
6	1400	3000	2100	2700	2000	3000														
7	1800	2800	3000	3100	2500	3500	2500													
8	1700	2500	2500	2000	2400	3000	2000	2400												
9	2500	4000	3000	3200	3700	4100	3200	3200	3200											
10	3000	4500	3500	4200	4200	4200	3300	3700	3500	4300										
11	4000	5000	4200	4500	4700	4300	4300	4500	4400	5100	6000									
12	3000	4500	4000	3700	3800	4400	3800	4000	3600	4500	4600	6200								
13	3500	4000	4200	4200	4300	4500	4500	4000	4200	4700	5200	6000	5500							
14	3700	5000	5000	4300	4200	4600	4600	4200	4300	4400	5500	6600	5200	5000						
15	3000	4000	3800	3800	3800	4700	3800	4300	4100	3700	5600	6200	4900	4500	5000					
16	2500	3500	3000	3700	3000	4000	3300	4000	3700	3600	4900	5800	4700	4500	5300	5000				
17	2600	4000	3100	4200	3700	3900	3200	3800	3400	3800	4800	5500	5000	4800	4800	4900	4000			
18	3000	4500	4000	4500	3800	4500	4000	4000	3500	4500	5000	5500	4700	4800	5000	4800	4500	4000		
19	3200	3900	3700	4400	4200	4400	4100	4200	4100	4600	6000	5700	5000	5000	4800	4600	5000	4500	5500	
20	2700	3800	3500	4000	4000	4500	3700	3600	3600	4200	4500	5000	4900	5200	5300	5000	4500	4000	4500	4600

Customer Demands: (units)

Customer1: 400 Customer2: 100 Customer3: 200 Customer4: 200 Customer5: 300 Customer6: 100 Customer7: 200 Customer8: 300 Customer9: 200 Customer10: 300

Customer11: 100 Customer12: 200 Customer13: 200 Customer14: 300 Customer15: 300 Customer16: 100 Customer17: 200 Customer18: 100 Customer19: 200 Customer20: 200

Vehicle capacities : 1500 units

Appendix C. Normal cost matrix data for 5-customer 1-vehicle problem (linear programming and method of successive averages)

Normal Cost Matrix

	0	1	2	3	4
1	11				
2	8	17			
3	10	15	11		
4	13	10	17	15	
5	15	16	18	17	16

Appendix D. Failed cost matrix data for 5-customer 1-vehicle problem (Linear programming and method of successive averages)

Failed Cost Matrix

	0	1	2	3	4
1	20				
2	21	24			
3	20	33	25		
4	22	33	22	25	
5	24	30	27	30	27