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Serdar Koçkanat[a*]

[a] Assoc. Prof., Cumhuriyet University, skockanat@cumhuriyet.edu.tr

[*] Corresponding Author

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Harmonic Estimation in Power Systems with Kalman Filters under Varying Noise Conditions: Performance Analysis and Evaluation

Abstract

In this study, a test signal proposed in the literature for amplitude and phase estimation of harmonics in power systems is utilized. The test signal is applied to the proposed Kalman-based approach under noise conditions of 10 dB, 20 dB, and 40 dB. Performance is evaluated by comparing the obtained estimation results with those reported in the literature. For 10 dB Gaussian noise, the mean squared error (MSE) was 0.006649, and the normalized mean squared error (NMSE) was 0.014542; for 20 dB Gaussian noise, the MSE was 0.000613, and the NMSE was 0.001340; and for 40 dB Gaussian noise, the MSE was 0.000004, and the NMSE was 0.000009. The average computational time across all three cases is 0.06 seconds. For each noise level, the original, noisy, and estimated signals are analyzed in both the time and frequency domains, with visual assessments consistent with the numerical results.

Keywords: Estimation, Harmonic, Kalman, Noise, Power.

Değişen Gürültü Koşullarında Kalman Filtreleri ile Güç Sistemlerinde Harmonik Tahmini: Performans Analizi ve Değerlendirmesi

Öz

Bu çalışmada, güç sistemlerinde harmoniklerin genlik ve faz tahmini için literatürde önerilen bir test sinyali kullanılmıştır. Test sinyali, 10 dB, 20 dB ve 40 dB gürültü koşulları altında önerilen Kalman tabanlı yaklaşıma uygulanmıştır. Performans, elde edilen tahmin sonuçları ile literatürde bildirilen sonuçlar karşılaştırılarak değerlendirilmiştir. 10 dB Gauss gürültüsü için, ortalama karesel hata (OKH) 0.006649 ve normalleştirilmiş ortalama karesel hata (NOKH) 0.014542 idi; 20 dB Gauss gürültüsü için OKH 0.000613 ve NOKH 0.001340; 40 dB Gauss gürültüsü için OKH 0.000004 ve NOKH 0.000009 olmuştur. Her üç gürültü durumu için de ortalama hesaplama süresi 0.06 saniyedir. Her gürültü seviyesi için, orijinal, gürültülü ve tahmini sinyaller hem zaman hem de frekans alanlarında analiz edilmiş ve görsel değerlendirmeler sayısal sonuçlarla tutarlıdır.

Anahtar kelimeler: Kestirim, Harmonik, Kalman, Gürültü, Güç.

1. Introduction

Harmonics are frequency components in electrical systems that are integer multiples of the fundamental grid frequency (50 Hz or 60 Hz). These components are generated by harmonic sources commonly found in power systems, such as non-linear loads, nonlinear electronic-based devices, electronic frequency drives, and renewable energy integration systems. The emergence of presence distorts sinusoidal voltage and current waveforms, significantly affecting system impacting and causing power quality issues [1, 2]. Among the adverse harmonics are interference include communication with overheating of electrical equipment, mechanical resonance, maloperation of malfunction systems, and reduced power transmission efficiency [3, 4].

Harmonic estimation plays a vital role in monitoring and improving power quality in modern electrical systems [5, 6]. The accurate detection of harmonic components generated by sources such as non-linear loads and renewable energy converters enables the effective design of filtering and active compensation systems. The limitations of traditional methods in dynamic and noisy environments have made the use of advanced algorithms such as Kalman filter variants and swarm optimization approaches [7, 8]. This allows harmonic parameters to be estimated with high accuracy and in real time, thereby enhancing system reliability and minimizing energy losses. The Kalman Filter is an iterative mathematical algorithm used to optimally estimate the state variables of a dynamic system under noisy measurements [9]. In harmonic estimation, Kalman Filter-based approaches are of great importance due to their ability to estimate parameters such as amplitude, phase, and frequency of fundamental and harmonic components in real time and with high accuracy, especially under Gaussian noise and time-varying conditions. Key advantages include linearity, robustness, computational efficiency, and resistance to noise. However, traditional Kalman Filter (KF) performance has significant drawbacks, such as dependence on accurate initial values for process and measurement noise covariance matrices (Q and R) and the linearity of the system model; additionally, computational load can increase in high-dimensional systems. To overcome these limitations, various advanced variants have been proposed in the literature. These include the Ensemble Kalman Filter (EnKF), which uses sample covariance; the Local Ensemble Transform Kalman Filter (LET-KF), which offers computational efficiency with fewer multiplication operations and reduced matrix storage requirements; the Unscented Kalman Filter (UKF) capable of handling nonlinear models; and hybrid methods such as the Improved Sage-Husa UKF (ISHUKF), which incorporates a Sage-Husa noise estimator to adapt to time-varying noise. These enhanced approaches provide a significant performance improvement over traditional methods in terms of accuracy, convergence speed, and robustness for harmonic estimation [10, 11].

This study introduces an adaptive Kalman filter–based framework for the accurate estimation of harmonic amplitudes and phases in power signals, addressing the limitations of conventional fixed-parameter estimation methods reported in the literature. Unlike existing approaches, the proposed method explicitly adapts to varying measurement noise conditions and is systematically evaluated under Gaussian noise levels of 10 dB, 20 dB, and 40 dB SNR. The results demonstrate that this adaptive structure enables more reliable and precise harmonic characterization, thereby contributing to improved monitoring and control capabilities in power system applications and extending the current body of research on harmonic estimation techniques.

2. Material and Method

The proposed approach uses an adaptive Kalman filter to estimate the amplitudes and phases of multiple harmonics in a power signal. The observed signal *x(n)* is modeled as

$$x(n) = \sum_{k=1}^{N_h} A_k \sin(2\pi n_k f_0 t + \phi_k) + v(t) \quad (1)$$

where N_h is the number of harmonics, A_k and ϕ_k are the amplitude and phase of the *k-th* harmonic, n_k is the harmonic order, and $v(t)$ represents Gaussian measurement noise. Each harmonic is represented in a sine-cose state-space form by defining the state vector

$$\mathbf{x}_k = [a_1 \quad b_1 \quad a_5 \quad b_5 \quad \dots \quad a_{13} \quad b_{13}]^T \quad (2)$$

where a_k and b_k are the sine and cosine coefficients corresponding to each harmonic. The state-space model can then be written as

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{w}_k, \quad \mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k) \tag{3}$$

$$z_k = \mathbf{H}_k \mathbf{x}_k + v_k, \quad v_k \sim \mathcal{N}(0, R_k) \tag{4}$$

where H_k is the measurement matrix at time step k, containing $sin(2\pi n_k f_0 t)$ and $cos(2\pi n_k f_0 t)$ terms for each harmonic. Q_k and R_k are the process and measurement noise covariance matrices, respectively, and are adaptively updated during the filtering process.

The Kalman filter recursively performs the following steps (prediction, measurement update, adaptive noise update):

$$\hat{\mathbf{x}}_{k|k-1} = \hat{\mathbf{x}}_{k-1|k-1}, \quad \mathbf{P}_{k|k-1} = \mathbf{P}_{k-1|k-1} + \mathbf{Q}_{k-1} \tag{5}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^{\mathrm{T}}\left(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^{\mathrm{T}} + R_k\right)^{-1} \tag{6}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\left(z_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1}\right) \tag{7}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k|k-1} \tag{8}$$

$$R_k = \beta R_{k-1} + (1-\beta)\left[\left(z_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k}\right)^2 + \mathbf{H}_k\mathbf{P}_{k|k}\mathbf{H}_k^{\mathrm{T}}\right] \tag{9}$$

$$\mathbf{Q}_k = \alpha\mathbf{Q}_{k-1} + (1-\alpha)\left(\mathbf{K}_k\left(z_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k}\right)\right)\left(\mathbf{K}_k\left(z_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k}\right)\right)^{\mathrm{T}} \tag{10}$$

where α and β are forgetting factors controlling the adaptation speed of Q_k and R_k, respectively. The forgetting factors α and β were selected to provide a balanced trade-off between estimation stability and adaptation speed, ensuring robust tracking performance under varying noise conditions without causing excessive fluctuations in the covariance matrices. Once the state vector is estimated, the amplitude and phase of each harmonic are recovered as

$$\hat{A}_k = \sqrt{\hat{a}_k^2 + \hat{b}_k^2}, \quad \hat{\phi}_k = \arctan2\left(\hat{b}_k, \hat{a}_k\right) \tag{11}$$

To quantitatively evaluate the estimation performance of the proposed algorithm, error metrics based on the difference between the original signal $x(n)$ and its estimate $\hat{x}(n)$ are employed. The mean squared error (MSE) measures the average squared deviation and is defined as

$$MSE = \frac{1}{N}\sum_{n=1}^{N}(x(n) - \hat{x}(n))^2 \tag{12}$$

where N is the number of samples. While MSE provides a direct indication of the estimation accuracy, it does not consider signal magnitude, potentially leading to biased results when comparing signals with different power levels. To address this, the normalized mean squared error (NMSE) is used, which normalizes the squared error by the signal energy, expressed as

$$NMSE = \frac{\sum_{n=1}^{N}(x(n) - \hat{x}(n))^2}{\sum_{n=1}^{N}(x(n))^2} \tag{13}$$

NMSE enables fair comparison across scenarios with different signal amplitudes [12].

3. Results

To evaluate the estimation performance of the Kalman filter approach developed in this study, a test problem commonly used by researchers in the literature for harmonic estimation in power systems was employed [12]. The amplitude, frequency, phase, and harmonic information for this problem are presented in Table 1. The sampling frequency for this signal was set at 2 kHz. A time interval of 0.1 seconds was selected to better illustrate the differences between the curves in the graphs.

Table 1. Literature Test Signal

Harmonics	Frequency	Amplitude (p.u)	Phase (degree)
Fundamental	50 Hz	0.95	-2.02
5th	250 Hz	0.09	82.1
7th	350 Hz	0.043	7.9
11th	550 Hz	0.03	-147.1
13th	650 Hz	0.033	162.6

The harmonic test signal synthesized according to the parameters given in Table 1 is shown in Figure 1. Additionally, noisy harmonic signals with 10 dB, 20 dB, and 40 dB Gaussian noise are shown in Figures 2, 3, and 4, respectively, to evaluate the performance of the designed approach under noisy conditions. When examining the figures, it is evident that the highest noise intensity occurs at 10 dB, while the lowest noise intensity occurs at 40 dB.

The proposed Kalman filter-based estimation approach was evaluated using noisy harmonic signals with signal-to-noise ratios of 10 dB, 20 dB, and 40 dB. For each noisy signal, the amplitude and phase values of the fundamental frequency and the 5th, 7th, 11th, and 13th harmonics were estimated. Subsequently, the MSE and NMSE between the estimated and actual test signals were calculated. To assess the performance of the proposed method, these results were compared with those from a recent study that analyzed the estimation of the same test signal. The results are presented in Table 2.

Using the proposed Kalman-based approach, it was found that for 10 dB Gaussian noise, the MSE was 0.006649 and the NMSE was 0.014542; for 20 dB Gaussian noise, the MSE was 0.000613 and the NMSE was 0.001340; and for 40 dB Gaussian noise, the MSE was 0.000004 and the NMSE was 0.000009. Additionally, the calculation process takes an average of 0.06 seconds across three different noise levels.

Table 2. NMSE Error Comparison for 10 dB, 20 dB and 40 dB Gaussian noise

Algorithm	10 dB	20 dB	40 dB
MGO-LS [13]	0.013551	0.002892	0.000417
AVOA-LS [13]	0.019084	0.002856	0.000854
SWO-LS [13]	0.017824	0.005690	0.004404
ARO-LS [13]	0.043213	0.026807	0.034095
AO-LS [13]	0.016828	0.002404	0.000734
Proposed	**0.014542**	**0.001340**	**0.000009**

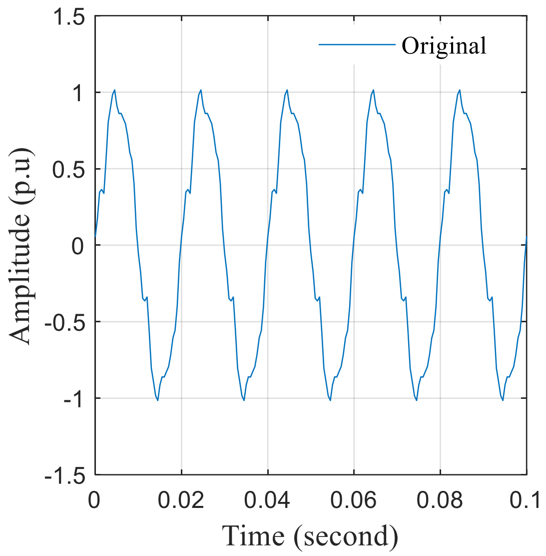

Figure 1. Original Harmonic Signal

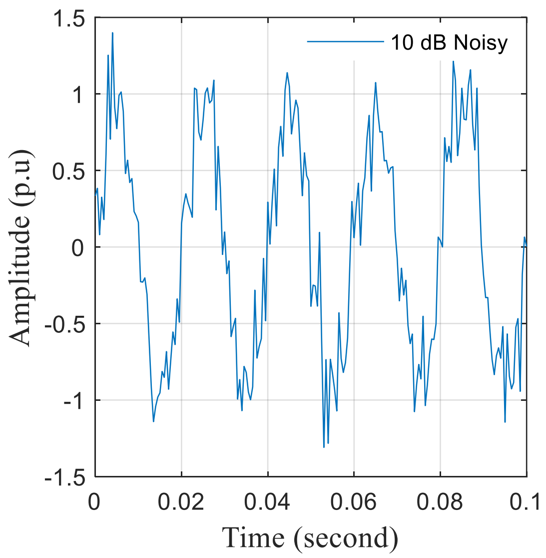

Figure 2. 10 dB Noisy Harmonic Signal

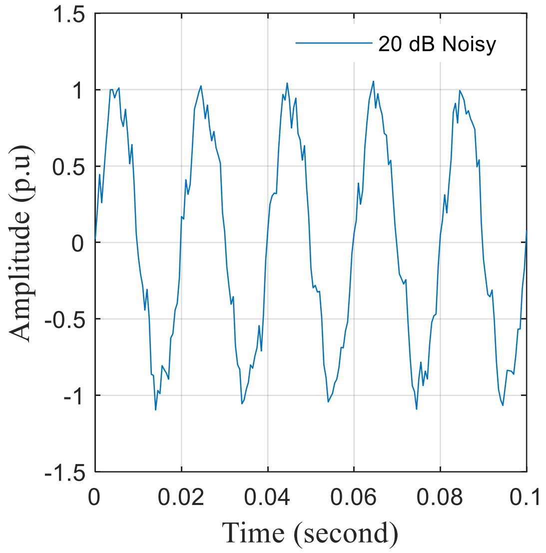

Figure 3. 20 dB Noisy Harmonic Signal

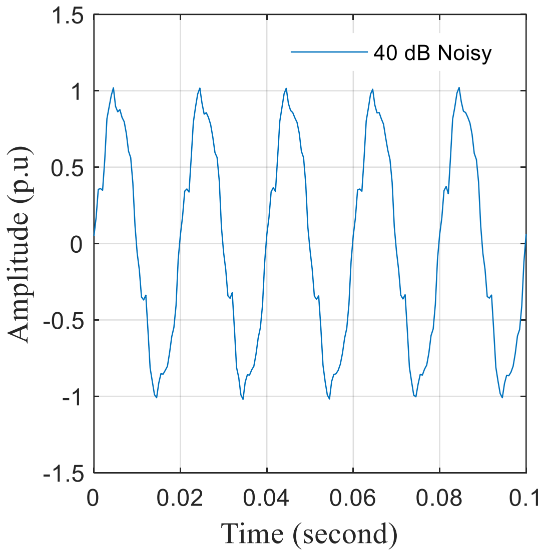

Figure 4. 40 dB Noisy Harmonic Signal

When examining the NMSE results in Table 2, the proposed approach demonstrates robust harmonic estimation performance compared to the Mountain Gazelle Optimization-Least Squares (MGO-LS) [13], African Vulture Optimization Algorithm-Least Squares (AVOA-LS) [13], Spider Wasp Optimization-Least Squares (SWO-LS) [13], Artificial Rabbit Optimization-Least Squares (ARO-LS) [13], and Aquila Optimization-Least Squares (AO-LS) [13] algorithms previously reported in the literature. Furthermore, it is evident that noise significantly limits the harmonic estimation accuracy across all proposed methods.

In Figures 5, 6, and 7, the original signals (blue solid line), noisy signals at 10 dB, 20 dB, and 40 dB (red dashed line), and the reconstructed signals obtained using the estimated amplitudes and phase values of the harmonics via the Kalman filter (green dashed line) are compared. When all three figures are examined, the reduction in Gaussian noise has positively impacted the harmonic estimation capability of the Kalman-based approach, resulting in an excellent match between the original and estimated signals. Indeed, Table 2 suggests that, consistent with literature studies, estimation performance improves as noise levels decrease.

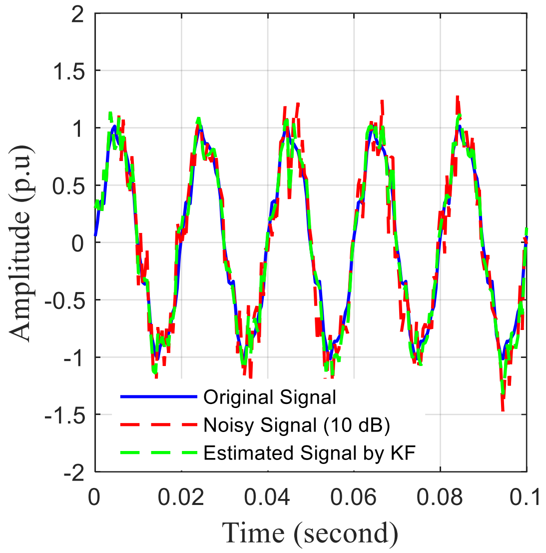

Figure 5. Performance Comparison of Signals (10 dB Noise Level)

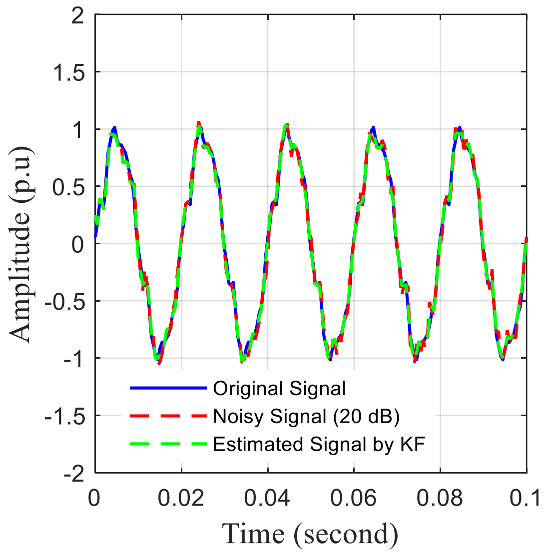

Figure 6. Performance Comparison of Signals (20 dB Noise Level)

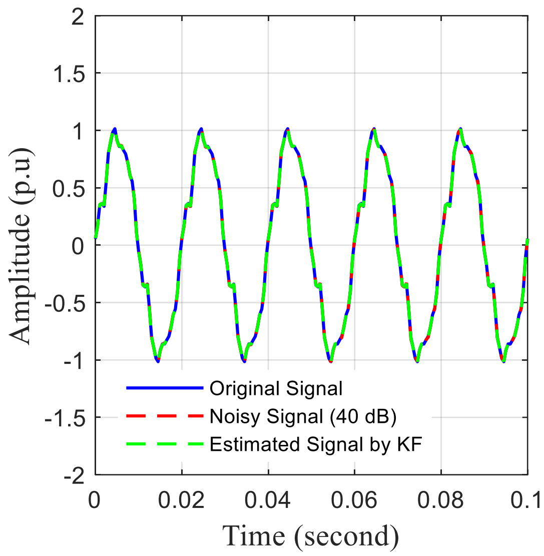

Figure 7. Performance Comparison of Signals (40 dB Noise Level)

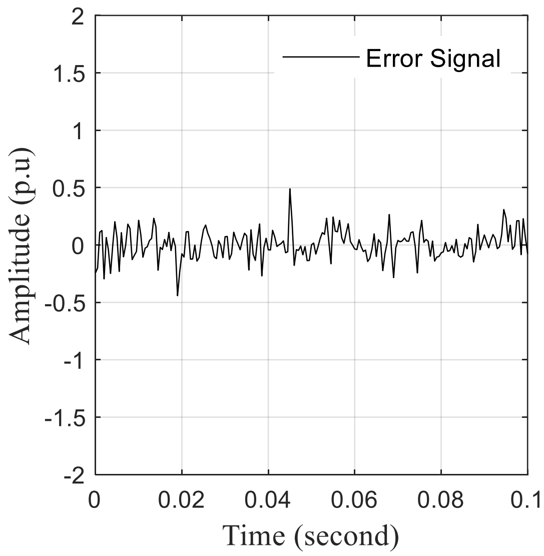

Figure 8. Error Signal (10 dB Noise Level)

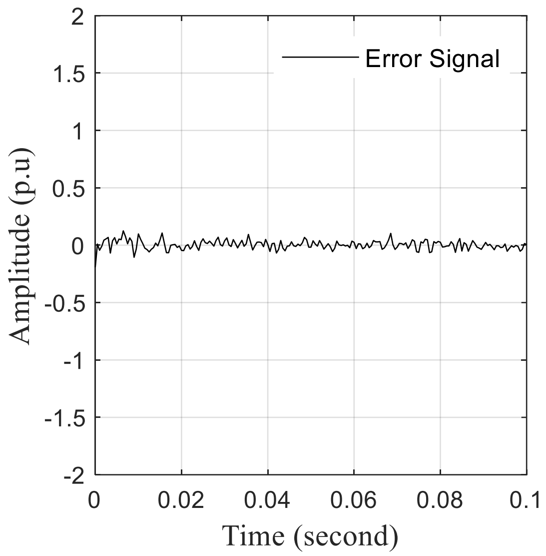

Figure 9. Error Signal (20 dB Noise Level)

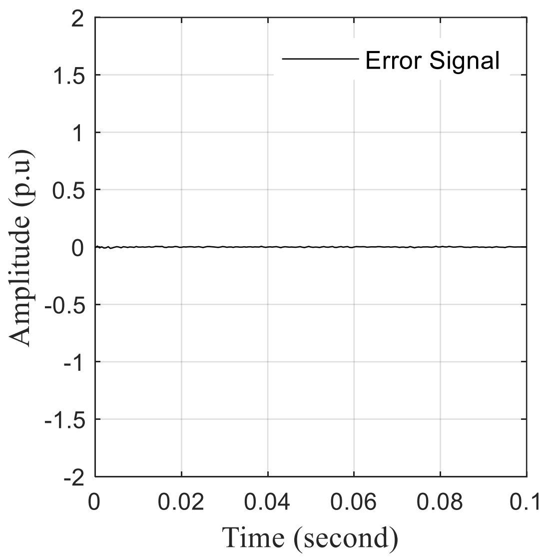

Figure 10. Error Signal (40 dB Noise Level)

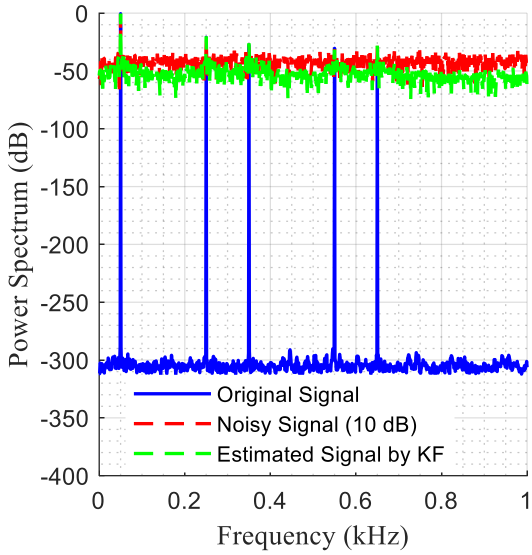

Figure 11. Power Spectrum Comparison (10 dB Noise Level)

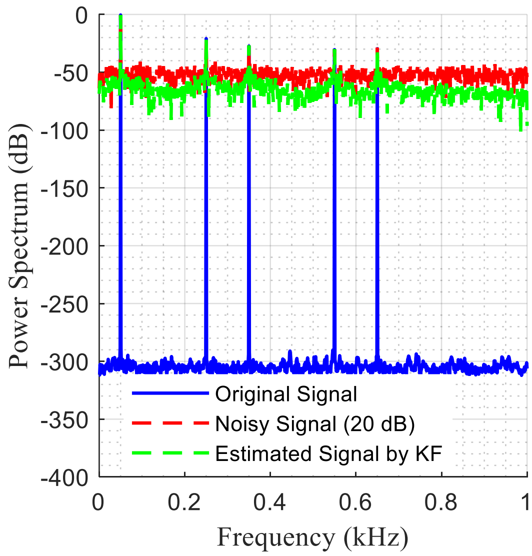

Figure 12. Power Spectrum Comparison (20 dB Noise Level)

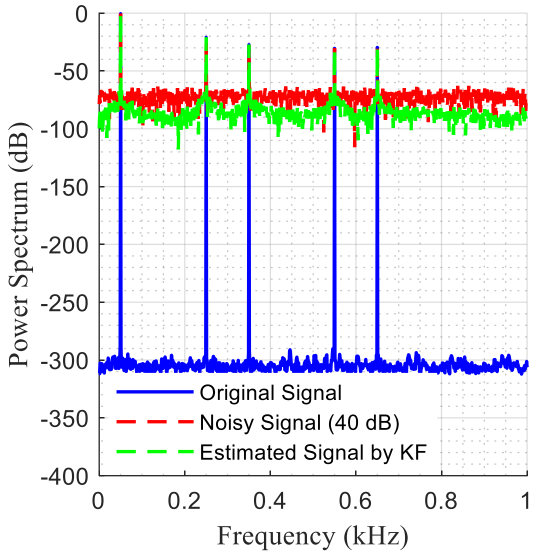

Figure 13. Power Spectrum Comparison (40 dB Noise Level)

Figures 8, 9, and 10 display the error signals—defined as the difference between the original and estimated signals—at noise levels of 10 dB, 20 dB, and 40 dB, respectively. Upon examining these curves, it is observed that, consistent with the results presented above, the amplitude values of the error curve for the Kalman-based approach decrease, indicating improved performance.

In Figures 11, 12, and 13, the power spectrum comparisons under varying noise conditions are presented to quantitatively assess the denoising capability of the proposed Kalman Filter (KF). At the 10 dB noise level, the estimated spectrum exhibits a significant reduction in the noise floor while preserving the dominant harmonic peaks, which is consistent with the achieved NMSE value of 0.014542. As the noise level decreases to 20 dB, the spectral leakage around the fundamental and higher-order harmonics is substantially reduced, resulting in a closer match to the original spectrum and a corresponding NMSE improvement to 0.001340. In the 40 dB case, the estimated spectrum almost completely overlaps the original signal spectrum, with negligible distortion and an NMSE as low as 0.000009, indicating near-perfect harmonic reconstruction. These results quantitatively confirm that the proposed KF maintains spectral peak accuracy and effectively suppresses broadband noise across a wide range of signal-to-noise ratios.

4. Discussion and Conclusion

In this study, a Kalman-based approach is proposed to estimate the amplitude and phase of harmonics in power systems. The analysis employs a test signal from the literature and considers three different noise levels. The estimation results are compared with those obtained using an optimization-based approach reported in the literature. Using the Kalman-based method, the MSE and NMSE are as follows: for 10 dB Gaussian noise, MSE = 0.006649 and NMSE = 0.014542; for 20 dB Gaussian noise, MSE = 0.000613 and NMSE = 0.001340; and for 40 dB Gaussian noise, MSE = 0.000004 and NMSE = 0.000009. The average computational time across all three cases is 0.06 seconds. Moreover, the average computation time of 0.06 seconds demonstrates that the proposed method is well suited for real-time and practical power system applications where fast and reliable harmonic estimation is required. For each noise level, the original, noisy, and estimated signals are analyzed in both the time and frequency domains, with visual assessments consistent with the numerical results. Although the proposed Kalman-based approach demonstrates high accuracy and robustness under Gaussian noise conditions, its performance may be limited in scenarios involving non-Gaussian noise distributions or rapidly time-varying harmonic components.

Future studies could focus on adapting the Kalman-based approach to accommodate time-varying harmonic components and non-Gaussian noise conditions. Another promising direction involves implementing the method on embedded systems or DSP-based platforms for real-time applications. Finally, evaluating the performance under various operating conditions and with large-scale datasets could provide a more comprehensive assessment of the method's capabilities.

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✵ ✵ ✵

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