Perception of Nano Generalized $t^{\#}$-Closed Sets

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Abstaract − In this paper, we introduce a new class of sets called $nt_g^{\#}$-closed sets, which is stronger than ng-closed sets and weaker than n-closed sets.

1 Introduction

Thivagar et al. [6] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space.

Bhuvaneswari et al. [4] introduced and investigated nano g-closed sets in nano topological spaces. Recently, Parvathy and Bhuvaneswari the notions of nano gpr-closed sets which are implied both that of nano rg-closed sets. In 2017, Rajasekaran et al. [9, 10] introduced the notion of nano πgp-closed sets and nano πgs-closed sets in nano topological spaces. In this paper, we introduce a new class of sets called nano $t_g^{\#}$-closed sets, which is stronger than nano g-closed sets and weaker than nano closed sets.

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2 Preliminaries

Definition 2.1. *[11] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Property 2.2. *[6] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. *$L_R(X) \subseteq X \subseteq U_R(X)$;*

2. *$L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;*

3. *$U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;*

4. *$U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;*

5. *$L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;*

6. *$L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;*

7. *$L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;*

8. *$U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;*

9. *$U_R U_R(X) = L_R U_R(X) = U_R(X)$;*

10. *$L_R L_R(X) = U_R L_R(X) = L_R(X)$.*

Definition 2.3. *[6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*

2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by n-$int(A)$ and n-$cl(A)$, respectively.

Definition 2.4. *A subset H of a space $(U, \mathcal{N})$ is called;*

1. *nano regular-open (briefly nr-open) set [6] if $H = n$-$int(n$-$cl(H))$.*

2. *nano pre open (briefly np-open) set [6] if $H \subseteq n$-$int(n$-$cl(H))$.*

3. *nano semi open (briefly ns-open) set [6] if $H \subseteq n$-$cl(n$-$int(H))$.*

4. *nano π-open (briefly nπ-open) set [1] if the finite union of nr-open sets.*

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.5. *[7] A subset H of a space $(U, \mathcal{N})$ is called a $nt^{\#}$-set if n-$int(H) = n$-$cl(n$-$int(H))$.*

Definition 2.6. *A subset H of a space $(U, \mathcal{N})$ is called;*

1. *nano g-closed (briefly ng-closed) [2] if n-$cl(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.*

2. *nano πg-closed (briefly nπg-closed) [8] if n-$cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nπ-open.*

3. *nano gp-closed set (briefly ngp-closed) [4] if n-$pcl(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.*

4. *nano gs-closed (briefly ngs-closed) [3] if n-$scl(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.*

5. *nano πgp-closed (briefly nπgp-closed) [9] if n-$pcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nπ-open.*

6. *nano πgs-closed (briefly nπgs-closed) [10] if n-$scl(H) \subseteq G$, whenever $H \subseteq G$ and G is nπ-open.*

7. *nano $g^{\star}$-closed (briefly n$g^{\star}$-closed) [12] if n-$cl(H) \subseteq G$ whenever $H \subseteq G$ and G is ng-open.*

8. *nano LC-set (briefly nLC-set) [5] if $H = G \cap K$, where G is n-open and K is n-closed.*

3 On Nano $t_g^{\#}$-Closed Sets, Nano $t_{sg}^{\#}$-Closed Sets and Nano $t_{pg}^{\#}$-Closed Sets

Definition 3.1. *A subset H of a space $(U, \mathcal{N})$ is called;*

1. *nano $t_g^{\#}$-closed (briefly $nt_g^{\#}$-closed) if $n\text{-}cl(H) \subseteq G$ whenever $H \subseteq G$ and G is $nt^{\#}$-set.*

2. *nano $t_{sg}^{\#}$-closed (briefly $nt_{sg}^{\#}$-closed) if $n\text{-}scl(H) \subseteq G$ whenever $H \subseteq G$ and G is $nt^{\#}$-set.*

3. *nano $t_{pg}^{\#}$-closed (briefly $nt_{pg}^{\#}$-closed) if $n\text{-}pcl(H) \subseteq G$ whenever $H \subseteq G$ and G is $nt^{\#}$-set.*

The complements of the above mentioned sets are called their respective n-open sets.

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$. Then the nano topology $\mathcal{N} = \{\phi, \{d\}, \{b, c\}, \{b, c, d\}, U\}$.*

1. *$H = \{a\}$ is $nt^{\#}$-closed set.*

2. *$H = \{d\}$ is $nt_{sg}^{\#}$-closed set.*

3. *$H = \{a, c\}$ is $nt_{pg}^{\#}$-closed set.*

Remark 3.3. *The family of nLC-set and the family of ng-closed sets are independent.*

Example 3.4. *In Example 3.2,*

1. *$H = \{b, c\}$ is nLC-set but not ng-closed set.*

2. *$H = \{a, b\}$ is ng-closed set but not nLC-set.*

Theorem 3.5. *For a subset H of a space $(U, \mathcal{N})$, the following are equivalent:*

1. *H is nLC-set.*

2. *$H = G \cap n\text{-}cl(H)$ for some G n-open set.*

Proof. $(1) \Rightarrow (2)$. Since H is a nLC-set, then $H = G \cap K$, where G is n-open and K is n-closed. So, $H \subseteq G$ and $H \subseteq K$. Hence, $n\text{-}cl(H) \subseteq n\text{-}cl(K)$. Therefore, $H \subseteq G \cap n\text{-}cl(H) \subseteq G \cap n\text{-}cl(K) = G \cap K = H$. Thus, $H = G \cap n\text{-}cl(H)$.

$(2) \Rightarrow (1)$. It is obvious because $n\text{-}cl(H)$ is n-closed.

Theorem 3.6. *For a subset H of a space $(U, \mathcal{N})$, the following are equivalent:*

1. *H is n-closed.*

2. *H is nLC-set and ng-closed.*

Proof. $(1) \Rightarrow (2)$. This is obvious.

$(2) \Rightarrow (1)$. Since H is nLC-set, then $H = G \cap n\text{-}cl(H)$, where G is n-open set in U. So, $H \subseteq G$ and since H is ng-closed, then $n\text{-}cl(H) \subseteq G$. Therefore, $n\text{-}cl(H) \subseteq G \cap n\text{-}cl(H) = H$. Hence, H is n-closed.

Theorem 3.7. *In a space $(U, \mathcal{N})$,*

1. *If H is n-closed set, then H is $nt_g^{\#}$-closed set.*

2. *If is $nt_g^{\#}$-closed set, then H is ng-closed set.*

Proof. (1). Obvious.

(2). Let H be a $nt_g^{\#}$-closed set and $H \subseteq G$ where $G \in \mathcal{N}$. Since each n-open set is $nt^{\#}$-set, so G is $nt^{\#}$-set. Since H is $nt_g^{\#}$-closed set, we obtain that $n\text{-}cl(H) \subseteq G$, hence H is ng-closed set

Remark 3.8. *The converses of Theorem 3.7 are not true as seen from the following Example.*

Example 3.9. *In Example 3.2,*

1. *$H = \{d\}$ is $nt_g^{\#}$-closed set but not n-closed.*

2. *$H = \{a, b\}$ is ng-closed set but not $nt_g^{\#}$-closed set.*

Theorem 3.10. *In a space $(U, \mathcal{N})$,*

1. *If H is $nt_g^{\#}$-closed set, then H is $nt_{pg}^{\#}$-closed set.*

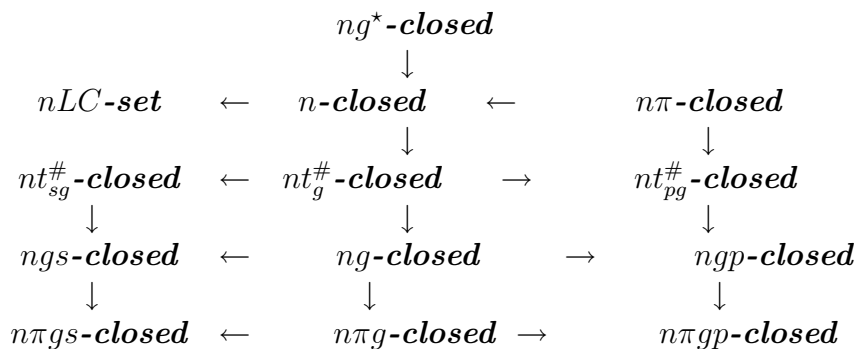2. *If H is $nt_g^{\#}$-closed set, then H is $nt_{sg}^{\#}$-closed set.*

Proof. Obvious.

Remark 3.11. *The converses of Theorem 3.10 are not true as seen from the following Example.*

Example 3.12. 1. *In Example 3.2, $H = \{b\}$ is $nt_{pg}^{\#}$-closed set but not $nt_g^{\#}$-closed set.*

2. *Let $U = \{a, b, c\}$ with $U/R = \{\{a, b\}, \{c\}\}$ and $X = \{c\}$. Then the nano topology $\mathcal{N} = \{\phi, \{c\}, U\}$, $H = \{a\}$ is $nt_{sg}^{\#}$-closed set but not $nt_g^{\#}$-closed set.*

Remark 3.13. *We obtain the following diagram, where $A \longrightarrow B$ represents A implies B but not conversely.*

$$
\begin{array}{ccccc}
 & & ng^{\star}\textbf{-closed} & & \\
 & & \downarrow & & \\
nLC\textbf{-set} & \leftarrow & n\textbf{-closed} & \leftarrow & n\pi\textbf{-closed} \\
 & & \downarrow & & \downarrow \\
nt_{sg}^{\#}\textbf{-closed} & \leftarrow & nt_g^{\#}\textbf{-closed} & \rightarrow & nt_{pg}^{\#}\textbf{-closed} \\
\downarrow & & \downarrow & & \downarrow \\
ngs\textbf{-closed} & \leftarrow & ng\textbf{-closed} & \rightarrow & ngp\textbf{-closed} \\
\downarrow & & \downarrow & & \downarrow \\
n\pi gs\textbf{-closed} & \leftarrow & n\pi g\textbf{-closed} & \rightarrow & n\pi gp\textbf{-closed}
\end{array}
$$

None of the above implications are reversible

Example 3.14. *In Example 3.2,*

    1. $H = \{b\}$ is $nt_{pg}^{\#}$-closed but not $n\pi$-closed.

    2. $H = \{c\}$ is ngs-closed but not $nt_{sg}^{\#}$-closed.

Theorem 3.15. *In a space $(U, \mathcal{N})$, if H is $ng^{\star}$-closed set, then H is $nt_{g}^{\#}$-closed.*

Proof. Obvious.

Remark 3.16. *The converses of Theorem 3.15 are not true as seen from the following Example.*

Example 3.17. *In Example 3.2, $H = \{a, b, d\}$ is $nt_{g}^{\#}$-closed set but not $ng^{\star}$-closed set.*

Remark 3.18. *The family of $nt_{g}^{\#}$-closed sets and the family of $nt^{\#}$-sets are independent.*

Example 3.19. *In Example 3.2,*

    1. $H = \{d\}$ is $nt_{g}^{\#}$-closed set but not $nt^{\#}$-set.

    2. $H = \{b\}$ is $nt^{\#}$-set but not $nt_{g}^{\#}$-closed set.

Theorem 3.20. *In a space $(U, \mathcal{N})$, if H is both $nt^{\#}$-set and $nt_{g}^{\#}$-closed set, then H is n-closed.*

Proof. Let H be both $nt^{\#}$-set and $nt_{g}^{\#}$-closed set. Then $n\text{-}cl(H) \subseteq H$, whenever H is a $nt^{\#}$-set and $H \subseteq H$. So we obtain that $H = n\text{-}cl(H)$ and hence H is n-closed.

Proposition 3.21. *In a space $(U, \mathcal{N}, I)$, the union of two $nt_{g}^{\#}$-closed sets is $nt_{g}^{\#}$-closed.*

Proof. Let $H \cup K \subseteq G$, where G is a $nt^{\#}$-set. Since H, K are $nt_{g}^{\#}$-closed sets, $n\text{-}cl(H) \subseteq G$ and $n\text{-}cl(K) \subseteq G$, whenever $H \subseteq G$, $K \subseteq G$ and G is a $nt^{\#}$-set. Therefore, $n\text{-}cl(H \cup K) = n\text{-}cl(H) \cup n\text{-}cl(K) \subseteq G$. Hence we obtain that $H \cup K$ is a $nt_{g}^{\#}$-closed set.

Theorem 3.22. *In a space $(U, \mathcal{N}, I)$, the intersection of two $nt_{g}^{\#}$-closed sets need not be $nt_{g}^{\#}$-closed as illustrated in the following Example.*

Example 3.23. *In Example 3.2, then $H = \{b, c\}$ and $Q = \{c, d\}$ is $nt_{g}^{\#}$-closed sets. Clearly $H \cap Q = \{c\}$ is not $nt_{g}^{\#}$-closed set.*

Theorem 3.24. *In a space $(U, \mathcal{N})$, If H is $nt_{g}^{\#}$-closed set such that $H \subseteq K \subseteq n\text{-}cl(H)$, then K is also $nt_{g}^{\#}$-closed set.*

Proof. Let G be a $nt^{\#}$-set such that $K \subseteq G$. Then $H \subseteq G$. Since H is $nt_{g}^{\#}$-closed, we have $n\text{-}cl(H) \subseteq G$. Now $n\text{-}cl(K) \subseteq n\text{-}cl(n\text{-}cl(H)) = n\text{-}cl(H) \subseteq G$. Therefore, K is also $nt_{g}^{\#}$-closed set.

Theorem 3.25. *In a space $(U, \mathcal{N})$, let H be $nt_{g}^{\#}$-closed. Then $n\text{-}cl(H) - H$ does not contain any non-empty complement of $nt^{\#}$-set.*

Proof. Let H be $nt_g^{\#}$-closed set. Suppose that P is the complement of $nt^{\#}$-set and $P \subseteq n\text{-}cl(H) - H$. Since $P \subseteq n\text{-}cl(H) - H \subseteq U - H$, $H \subseteq U - P$ and $U - P$ is $nt^{\#}$-set. Therefore, $n\text{-}cl(H) \subseteq U - P$ and $P \subseteq U - n\text{-}cl(H)$. However, since $P \subseteq n\text{-}cl(H) - H$, $P = \phi$.

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