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Separation Axioms Using Soft Turing Point of a Soft Ideal in Soft Topological Space

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Abstract – In this paper, we define new separation axioms in soft topological space via the concept of the Soft Truing Point and study the most important properties and results of it.

Keywords – *Soft Turing Point, Separation Axioms*.

1. Introduction and Preliminaries

In 1999, Molodtsov [2] begins whit the soft set theory as mathematical tool to solve many complicated problems in economics, engineering, and environment which we cannot successfully use classical methods because of various uncertainties typical of those problems. Maji et al. [13] studied a soft set theory and they introduced many of new concepts of this theory as an inclusion relation between the soft set, the formula of the empty set in this theory, the equality of two soft sets, the complement of a soft set also the soft intersection and the soft union with some of important results and properties. Cagman et al. [12] introduced a new type of a soft set and a new definition of a soft intersection and a soft union with many results and properties. Irfran et al. [11] discussed new operations in the soft set theory such as De Morgan laws in the soft set theory in addition to some anew views of the soft union and the soft intersection. S. Hussain and B. Ahmad [15] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. The notion of soft ideal is initiated for the first time by Kandil et al. [1]. Al-Swidi, and Al-Amri [10] studied the soft sets theory as an analytical study and dividing the kinds to four families every family is different from other in

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some adjectives and properties and correspond to other. In the same year, [9] defined the separation axioms in soft topological space and practically in certain point of the parameters, and to study the most important properties and results of it. Recently, a new separation axiom in soft topological space is introduced by using the concept of the " ∗Soft turing point·" as explained by [21]

Al-Swidi and Al-Fathly classified the soft points in soft topological spaces to three types. In this paper, we choose one of these types to be the focus of our work and which is defined in [19] as: let $F_A \in SS(X;A) = \{F_{i_A} : A \subseteq E_X \text{ and } F_i : A \longrightarrow IP(X)\}$, then F_A is called a soft point in $\tilde{X}_A$ denoted by F_a^x, where

$$F(P) = \begin{cases} \{x\} & \text{if } P = a \\ \varphi & P \in A \setminus \{a\} \end{cases}$$

So, a soft point F_a^x belong to the soft set F_A , denoted by $F_a^x \tilde{\in} F_A$ iff $x \in F(a)$. $F_a^x \tilde{\notin} F_A$ if $x \notin F(a)$

Definition 1.1 [2] A pair (F,A) denoted by F_A is called a soft set over X, where F is a mapping given by F: A → P(X). In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular a ∈ A, F(a) may be considered the set of a−approximate elements of the soft set (F,A) and if $a \notin A$, then $F(a)=\Phi$ i.e. $F_A=\{F(a):a\in A\subset E_X,\ F:A\longrightarrow P(X)\}$. We denote the family of all soft sets over X by SS(X;A).

A soft set F_A over X is said to be the null soft set , denoted by $\tilde{\Phi}_A$ if $\forall a \in A, F(a) = \varphi$. A soft set F_A over X is said to be the absolute soft set and denoted by $\tilde{X}_A$, if $\forall a \in A\ F(a) = X$.

Definition 1.2 [11] The complement of the soft set F_A is denoted by $(F_A)^C$ is defined by:

$$(F_A)^C = F^C{}_A = \tilde{X}_A \tilde{-} F_A ,$$

where $F^C : A \longrightarrow IP(X)$ is a mapping given by: $F^C(a) = X - F(a)\ \forall a \in A$. F^C is called the soft complement function of F

Definition 1.3 [15], [4] Let $\tilde{\tau}$ be the collection of soft sets over X, then $\tilde{\tau}$ is said to be a soft topology on X if

1- $\tilde{\Phi}_A$, $\tilde{X}_A$ belong to $\tilde{\tau}$
2- The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$
3- The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

- The members of $\tilde{\tau}$ are called soft open sets.
- A soft set F_A is called soft closed set if the complement $(\tilde{\chi}_A - F_A)$ is soft open set (belong to $\tilde{\tau}$) .
- The family of all soft closed sets are denoted by $C(\tilde{\chi})$ and defined as follows: $C(\tilde{\chi}_A) = \{\tilde{\chi}_A - F_A , F_A \tilde{\in} \tilde{\tau}\}$.

Definition 1.4 [16] Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space , and let $Y \subseteq X$, the relative soft topology for $\tilde{Y}_A$ is the collection $\tilde{\tau}_Y$ given by: $\tilde{\tau}_Y = \{\tilde{Y}_A \tilde{\cap} F_A , F_A \tilde{\in} \tilde{\tau}\}$. Note that $\tilde{Y}_A$ means that $Y(a) = Y$, $\forall a \in A$. The soft topological space $(\tilde{Y}_A, \tilde{\tau}_Y, A)$ is called soft subspace of $(\tilde{X}_A, \tilde{\tau}, A)$. The soft topology $\tilde{\tau}_Y$ is called induced by $\tilde{\tau}$.

Theorem 1.5 [17] Let $(\tilde{Y}_A, \tilde{\tau}_Y, A)$ be a soft subspace of a soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ and F_A is a soft set over X .Then :F_A is soft open in $(\tilde{Y}_A, \tilde{\tau}_Y, A)$ iff $F_A = \tilde{Y}_A \tilde{\cap} G_A$, for some soft open set G_A in $(\tilde{X}_A, \tilde{\tau}, A)$ F_A is soft closed in $(\tilde{Y}_A, \tilde{\tau}_Y, A)$ iff $F_A = \tilde{Y}_A \tilde{\cap} K_A$, for some soft closed set K_A in $(\tilde{X}_A, \tilde{\tau}, A)$.

Definition 1.6 [20] Let χ and Y be two initial universal sets and A, B be sets of parameters, $u: \chi \longrightarrow Y$ and $p: A \longrightarrow B$, then the mapping $f: (\chi, A) \longrightarrow (Y, A)$ (i.e. $f: SS_{.}(\chi) \longrightarrow SS_{.}(Y)$) on A and B respectively is denoted by f_{pu} and can be shown as:

$$f_{pu} = \left\{\left(f_{pu}(\ F_A), p(A)\right), p(A) \subseteq B\right)\} .$$

where

$$f_{pu}(F_A)(\beta) = \begin{cases} u\left(\bigcup\limits_{\alpha \in p^{-1}(\beta) \cap A \neq \varphi} \left(F(\alpha)\right)\right), & \text{if } \ p^{-1}\,(\beta) \neq \varphi \\ \varphi & \text{other wise} \end{cases}$$

For $\beta \in B$ $\exists$ $a \in p(A)$ such that $p(a) = \beta$ that is $p^{-1}\,(\beta) \neq \varphi$. Since $p^{-1}(\beta) \subseteq A$, hence $p^{-1}(\beta) \cap A \neq p^{-1}(\beta)$, hence we get that $f_{pu}(F_A)(\beta) = u\left(\cup_{\alpha \in p^{-1}(\beta)} F(\alpha)\right)$

Constructing: Since p is a mapping, so $p(A) \neq \varphi$, $\forall A \neq \varphi$, that is $\forall \beta \in p(A) \exists\, a \in A$ such that $p(a) = \beta$ and $p^{-1}\,(\beta) \neq \varphi$ since $a \in p^{-1}\left(p(a)\right)$ so:

$$f_{pu}(F_A)(\beta) = u\left\{\cup_{\alpha \in p^{-1}(\beta)} \left(F(\alpha)\right) \right\} \forall\ \beta \in p(A)$$

- If p is a one to one (1-1), then $p^{-1}\left(p(A)\right) = A$, that is $\forall\ \beta \in p(A) \exists$ a A such that $p(a) = \beta$ and $f_{pu}(F_A)(\beta) = u\left(F(a)\right)$.
- If G_B $\in SS(Y)$ then the inverse image of G_B under f_{pu} is denoted by ${f_{pu}}^{-1}(G_B)$ is a soft set $(F_A) \in SS_{.}(X)$ such that P(a)= $u^{-1}(G\left(p(\alpha)\right)$,for each $a \in A$.

Remark 1.7 [9, 20] For each $a \in A$ and $x \in X$,then we can define the soft mapping f_{pu} on soft point x_a ,as follows:

1- $\left(f_{pu}(x_a)\right)_{p(a)} = \{(p(a), \{u(x)\})\}$

2- Now, for $b \in B$ and $y \in Y$, ${f_{pu}}^{-1}(y_b)(a)$= $u_{.}^{-1}(y)$,for b=f(a).

Definition 1.8 [5] For a topological space (X,T), $x\in X$, $Y\subseteq X$, we define an ideal ${}^{Y}I_x$ respect to subspace (Y,T_Y), as follows

$${}^{Y}I_x=\{G\subseteq Y: x\in (X-G)\}.$$

Definition 1.9 [1] Let $\tilde{I}_A$ be a non-null collection of soft sets over a universe X with the same set of parameters A. Then $\tilde{I}_A\in SS(X)$ is called a soft ideal on X with the same set A if

1- $F_A\tilde{\in}\tilde{I}_A$ and $G_A\tilde{\in}\tilde{I}_A$ then $F_A\tilde{\cup}G_A\tilde{\in}\tilde{I}_A$
2- $F_A\tilde{\in}\tilde{I}_A$ and $F_A\tilde{\subseteq}G_A$ then $G_A\tilde{\in}\tilde{I}_A$

Definition 1.10 [16] Let $(\tilde{X}_A,\tilde{\tau},A)$, be a soft topological space, and let G_A be a soft set over the universe X, then the soft closure of G_A is a soft closed set defined as

$$ClG_A=\tilde{\cap}\{S_A, S_A \text{ is soft closed and } G_A\tilde{\subseteq}S_A\}$$

Proposition 1.11 [6] Let $(\tilde{X}_A,\tilde{\tau},A)$, be a soft topological space, and let F_A, G_A be soft set over X , then G_A is soft closed iff $Cl(G_A)=G_A$.

2. Separation Axioms Using Soft Turing Point

Definition 2.1 Let SI be a soft ideal in a soft topological space $(\tilde{X}_A,\tilde{\tau},A)$, A soft point F_a^x in SS(X;A) is called "soft Turing point" of SI, if $(F_A)^C\tilde{\in}SI$ for each $F_A\tilde{\in}N_{\tilde{\tau}(F_a^x)}$ where $N_{\tilde{\tau}(F_a^x)}$ is collection of all soft open nhd of soft point F_a^x.

Example 1.2 Let $((\tilde{X}_A,\tilde{\tau},A)$ be a soft topological space ,for some $a\in A$, $x\in X$, we define soft ideal $SI(F_a^x)$, as follows: $SI(F_a^x)=\{F_A\tilde{\subseteq}N_{\tilde{\tau}(F_a^x)}: F_a^x\tilde{\in}(F_A)^C\}$. Then a soft point F_a^x is called "Soft Turing point" of $SI(F_a^x)$

Example 2.3 Let E_X be the set of all parameters and let X be the initial universe consisting of: $X=\{x,y\}$ and $A\tilde{\subseteq}E_X$ such that $A=\{a_1,a_2\}$.

$$\tilde{\tau}=\{\tilde{\varphi}_A,\tilde{\chi}_A,F_{1_A},F_{2_A},F_{3_A},F_{4_A},F_{5_A},F_{6_A},F_{7_A},F_{8_A},F_{9_A},F_{10_A},F_{11_A},F_{12_A}F_{13_A},F_{14_A}\},$$

where:

$$F_{1_A}(p)=\begin{cases}\{x\} & \text{if } p=a_1\\ \varphi & \text{if } p=a_2\end{cases}\qquad F_{2_A}(p)=\begin{cases}\{x\} & \text{if } p=a_2\\ \varphi & \text{if } p=a_1\end{cases}$$

$$F_{3_A}(p)=\begin{cases}\{y\} & \text{if } p=a_2\\ \varphi & \text{if } p=a_1\end{cases}\qquad F_{4_A}(p)=\begin{cases}\{y\} & \text{if } p=a_1\\ \varphi & \text{if } p=a_2\end{cases}$$

$$F_{5_A}(p)=\begin{cases}\{x\} & \text{if } p=a_1\\ \{y\} & \text{if } p=a_2\end{cases}\qquad F_{6_A}(p)=\begin{cases}\{x\} & \text{if } p=a_2\\ \{y\} & \text{if } p=a_1\end{cases}$$

$F_{7_A}(p) = \begin{cases} \{x\} & \text{if } p = a_1 \\ X & \text{if } p = a_2 \end{cases}$ $F_{8_A}(p) = \begin{cases} \{x\} & \text{if } p = a_2 \\ X & \text{if } p = a_1 \end{cases}$

$F_{9_A}(p) = \begin{cases} X & \text{if } p = a_2 \\ \{y\} & \text{if } p = a_1 \end{cases}$ $F_{10_A}(p) = \begin{cases} X & \text{if } p = a_1 \\ \varphi & \text{if } p = a_2 \end{cases}$

$F_{11_A}(p) = \begin{cases} X & \text{if } p = a_2 \\ \varphi & \text{if } p = a_1 \end{cases}$ $F_{12_A}(p) = \begin{cases} X & \text{if } p = a_1 \\ \{y\} & \text{if } p = a_2 \end{cases}$

$F_{13_A}(p) = \begin{cases} \{x\} & \text{if } p = a_2 \\ \{x\} & \text{if } p = a_1 \end{cases}$ $F_{14_A}(p) = \begin{cases} \{y\} & \text{if } p = a_1 \\ \{y\} & \text{if } p = a_2 \end{cases}$

Then

$SI(F^x_{a_1}) = \{\tilde{\varphi}_A, F_{2_A}(p), F_{3_A}(p), F_{4_A}(p), F_{6_A}(p), F_{9_A}(p), F_{11_A}(p), F_{14_A}(p)\}$.
$SI(F^y_{a_1}) = \{\tilde{\varphi}_A, F_{1_A}(p), F_{2_A}(p), F_{3_A}(p), F_{5_A}(p), F_{7_A}(p), F_{11_A}(p), F_{13_A}(p)\}$
$SI(F^y_{a_2}) = \{\tilde{\varphi}_A, F_{1_A}(p), F_{2_A}(p), F_{4_A}(p), F_{6_A}(p), F_{8_A}(p), F_{10_A}(p)\ F_{13_A}(p)\}$

$F^x_{a_1}$ is soft Turing point of $SI(F^x_{a_1})$,but $F^y_{a_1}$ is not soft Turing point of $SI(F^x_{a_1})$

Note: ((F^y_a is not soft Turing point of $SI(F^x_a)$ i.e $F^y_a \,\tilde{\notin}\, SI(F^x_a)$)).

Definition 2.4 Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, the space $\tilde{X}_A$ is called SI-T$_0$-space if and only if, for any pair of distinct points F^x_a and F^y_a of $\tilde{X}_A$, $F^x_a \,\tilde{\notin} SI(F^y_a)$ or $F^y_a \,\tilde{\notin}$ $SI(F^x_a)$.

Example 2.5 Consider [Example 2.3] Let $\tilde{\tau} = \{\tilde{\varphi}_A, \tilde{X}_A, F_{10_A}, F_{8_A}\}$ be a soft topology on $\tilde{X}_A$ .Then $(\tilde{X}_A, \tilde{\tau}, A)$ is SI-T$_0$-space because, for any pair of distinct points F^x_a and F^y_a of X, F^x_a $\tilde{\notin} SI(F^y_a)$ or $F^y_a \,\tilde{\notin}\, SI(F^x_a)$.

Lemma 2.6 Let $(\tilde{X}_A, \tilde{\tau}, A)$ be soft topological space ,for some $a \in A$ and $F^{x_1}_a \neq F^{x_2}_a$ in X, $F^{x_2}_a$ is soft closed set in $\tilde{X}_A$ if and only if $F^{x_1}_a \,\tilde{\notin}\, SI(F^{x_2}_a)$.

Proof: Let $F^{x_1}_a, F^{x_2}_a$ in $\tilde{X}_A$ such that $F^{x_1}_a \neq F^{x_2}_a$. Assume that $F^{x_1}_a$ is a soft closed set in $\tilde{X}_A$, so that $F^{x_2}_a = cl(F^{x_2}_a)$.But $F^{x_1}_a \neq F^{x_2}_a$, we get that $F^{x_1}_a \,\tilde{\notin} cl(F^{x_2}_a)$.Therefore , there exists U $N_{\tilde{\tau}(F^{x_1}_a)}$ such that, $F^{x_1}_a \,\tilde{\in}\, U$, $U \cap F^{x_2}_a = \varnothing$ .So that $F^{x_1}_a \,\tilde{\in}\, U$, $U^c \tilde{\notin} SI(F^{x_2}_a)$,because if $U^c \tilde{\in}\, SI(F^{x_2}_a)$, then $F^{x_2}_a \tilde{\in}$ U, that means $U \cap F^{x_2}_a \neq \varnothing$,this a contradiction. Hence $F^{x_1}_a \,\tilde{\notin}\, SI(F^{x_2}_a)$.

Conversely, Let $F^{x_1}_a, F^{x_2}_a$ in $\tilde{X}_A$ such that $F^{x_1}_a \neq F^{x_2}_a$. Since $F^{x_1}_a \,\tilde{\notin}\, SI(F^{x_2}_a)$, there exists $U \,\tilde{\in}\, N_{\tilde{\tau}(F^{x_1}_a)}$such that, $F^{x_1}_a \,\tilde{\in}\, U$, $U^c \tilde{\notin} SI(F^{x_2}_a)$, so $F^{x_2}_a \,\tilde{\notin}\, U$. Thus $F^{x_1}_a \,\tilde{\in}\, U$, $U \cap F^{x_2}_a = \varnothing$ implies $F^{x_1}_a \,\tilde{\notin}\, cl(F^{x_2}_a)$.Hence $F^{x_2}_a = cl(F^{x_2}_a)$ .Thus, $F^{x_2}_a$ is a soft closed set in $\tilde{X}_A$.

Theorem 2.7 For every point F_{a}^{x} is a soft closed set, $a \in A$, then $(\tilde{X}_A, \tilde{\tau}, A)$ is SI-T_0-space.

Proof: Directly by Lemma 2.6.

Theorem 2.8 A soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ is SI-T_0-space, $a \in A$ iff for every distinct points $F_a^{x_1}, F_a^{x_2}$ in $\tilde{X}_A$ we have : $cl(F_a^{x_1}) \neq cl(F_a^{x_2})$

Proof: Suppose that $(\tilde{X}_A, \tilde{\tau}, A)$ is SI-T_0-space , $a \in A$ and $F_a^{x_1} \neq F_a^{x_2}$, then $F_a^{x_1} \tilde{\notin} SI(F_a^{x_2})$ or $F_a^{x_2} \tilde{\notin} SI(F_a^{x_1})$,so there exists $U \tilde{\in} N_{\tilde{\tau}(F_a^{x_1})}$ such that, $F_a^{x_1} \tilde{\in} U$, $U^c \tilde{\notin} SI(F_a^{x_2})$, or there exists $G \tilde{\in} N_{\tilde{\tau}(F_a^{x_2})}$ such that, $F_a^{x_2} \tilde{\in} G$, $G^c \tilde{\notin} SI(F_a^{x_1})$.Then by lemma 2.6, $cl(x_{1_a}) = x_{1_a}$ or $cl(x_{2_a}) = x_{2_a}$ That means $x_{1_a} \tilde{\in} cl(x_{1_a})$ and $x_{2_a} \tilde{\notin} cl(x_{1_a})$ or $x_{2_a} \tilde{\in} cl(x_{2_a})$ and $x_{1_a} \tilde{\notin} cl(x_{2_a})$.Thus, $x_{1_a} \tilde{\in} cl(x_{1_a})$ but $x_{1_a} \tilde{\notin} cl(x_{2_a})$.Hence,$cl(x_{1_a}) \neq cl(x_{2_a})$.

Conversely: Let $a \in A$ and $F_a^{x_1} \neq F_a^{x_2}$ in X, with $cl(F_a^{x_1}) \neq cl(F_a^{x_2})$, then there exist $F_a^{Z} \tilde{\in} cl(F_a^{x_1})$, but $F_a^{Z} \tilde{\notin} cl(F_a^{x_2})$, then $F_a^{x_1} \tilde{\notin} cl(F_a^{x_2})$ because, if $F_a^{x_1} \tilde{\in} cl(F_a^{x_2}$, then $cl(F_a^{x_1}) \tilde{\subseteq} cl(cl(F_a^{x_2}) = cl(F_a^{x_2})$, but $F_a^{Z} \tilde{\in} cl(F_a^{x_1}) \tilde{\subseteq} cl(F_a^{x_2})$ which is a contradiction, thus $F_a^{x_1} \tilde{\notin} cl(F_a^{x_2})$, that is, $F_a^{x_1} \tilde{\in} (\tilde{X}_A - cl(F_a^{x_2}))$ is a soft open nhf and $F_a^{x_2} \tilde{\notin} (\tilde{X}_A - cl(F_a^{x_2}))$, so $cl(x_{2_a}) \tilde{\notin} SI(x_{2_a})$. Hence $(\tilde{X}_A, \tilde{\tau}, A)$ is SI-T_0-space.

Theorem 2.9 Every soft subspace of SI-T_0-space is SI-T_0-space.

Proof: Suppose that $\tilde{Y}_A$ is a soft subspace of the of the SI-T_0-space $(\tilde{X}_A, \tilde{\tau}, A)$. Let $F_a^{y_1}$ and $F_a^{y_2}$ be two distinct points of $\tilde{Y}_A$. Again, since $\tilde{X}_A$ is SI-T_0 –space and $\tilde{Y}_A \subseteq \tilde{X}_A$, then $F_a^{y_1} \tilde{\notin} SI(F_a^{y_2})$ or $F_a^{y_2} \tilde{\notin} SI(F_a^{y_1})$, for some a. Suppose, $F_a^{y_1} \tilde{\notin} SI(F_a^{y_2})$, then there exists $U \tilde{\in} N_{\tilde{\tau}(F_a^{y_1})}$ such that, $F_a^{y_1} \tilde{\in} U$, $U^C \tilde{\notin} SI(F_a^{y_2})$. Then $U' = U \tilde{\cap} \tilde{Y}_A$ is $\tilde{\tau}_{Y}$- soft open contains $F_a^{y_1}$ but not $F_a^{y_2}$. So that $F_a^{y_1} \tilde{\in} U'$ and $(U')^c \tilde{\notin} SI(F_a^{y_2})$, hence $\tilde{Y}_A$ is SI-T_0-space.

Theorem 2.10 Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces and let $\tilde{X}_A$ be SI-T_0-space, for some $a \in A$, if the map $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is a soft open and u, p are onto maps, then $\tilde{Y}_B$ is SI-T_0-space.

Proof: Let $b \in B$ and $F_a^{y_1} \neq F_a^{y_1}$ in Y, then there exist $a \in A$ and $F_a^{x_1} \neq F_a^{x_1}$ in X such that $p(a) = b$, $u(F_a^{x_1}) = F_a^{y_1}$ and $u(F_a^{x_2}) = F_a^{y_2}$ because p and u are onto maps . Now by assumption ,then $F_a^{x_1} \tilde{\notin} SI(F_a^{x_2})$ or $F_a^{x_2} \tilde{\notin} SI(F_a^{x_1})$, so there exists U_A, G_A a soft open sets in $\tilde{X}_A$, such that, $F_a^{x_1} \tilde{\in} G_A$, $(G_A)^C \tilde{\notin} SI(F_a^{x_2})$ or $F_a^{x_2} \tilde{\in} U_A$,$(U_A)^C \tilde{\notin} SI(F_a^{x_1})$. Now: $f_{pu}(F_a^{x_1}) \tilde{\in} f_{pu}(G_A)$, $f_{pu}((G_A)^C) \tilde{\notin} SI(f_{pu}(F_a^{x_2}))$ or $f_{pu}(F_a^{x_2}) \tilde{\in} f_{pu}(U_A)$,$f_{pu}((U_A)^C) \tilde{\notin} SI(f_{pu}(F_a^{x_1}))$, but f_{pu} is soft open , so $f_{pu}(G_A), f_{pu}(U_A)$ are be a soft open sets in $\tilde{Y}_B$, and $F_b^{y_1} = f_{pu}(F_a^{x_1})$ and $F_b^{y_2} = f_{pu}(F_a^{x_2})$, i.e $F_b^{y_1} \tilde{\notin} SI(F_b^{y_1})$ or $F_b^{y_2} \tilde{\notin} SI(F_b^{y_1})$ Therefore , $\tilde{Y}_B$ is a SI-T_0-space

Theorem 2.11 Let $(\widetilde{Y}_B,\widetilde{\sigma},B)$ be SI-T_0-spacefor $b\in B$ and let $(\widetilde{X}_A,\widetilde{\tau},A)$ be any soft topological space such that the mapping $u: X\longrightarrow Y$ be a one to one and $p: A\longrightarrow B$ be an onto map, then there exist $a\in A$ with $p(a)=b$ and $\widetilde{X}_A$ is SI-T_0-space, if $f_{pu}: (\widetilde{X}_A,\widetilde{\tau},A)\longrightarrow(\widetilde{Y}_B,\widetilde{\sigma},B)$ is soft continuous map.

Definition 2.12 Let $(\widetilde{X}_A,\widetilde{\tau},A)$ be a soft topological space and $a\in A$, the space $\widetilde{X}_A$ is called SI-T_1-space if and only if, for any pair of distinct points F_a^x and F_a^y of $\widetilde{X}_A$, $F_a^x \mathrel{\widetilde{\notin}} SI(F_a^y)$ and $F_a^y \mathrel{\widetilde{\notin}} SI(F_a^x)$.

Example 2.13 Consider [Example 2.3] Let $\widetilde{\tau}=\{\widetilde{\varphi}_A,\widetilde{X}_A,F_{10_A},F_{8_A},F_{12_A}\}$ be a soft topology on $\widetilde{X}_A$ .Then $(\widetilde{X}_A,\widetilde{\tau},A)$ is SI-T_1-space because, for any pair of distinct points F_a^x , F_a^y of $\widetilde{X}_A$, $F_a^x \mathrel{\widetilde{\notin}} SI(F_a^y)$ and $F_a^y \mathrel{\widetilde{\notin}} SI(F_a^x)$.

Remark 2.14 Every SI-T_1-space is SI-T_0-space.

Proof: Direct from [Def].

Remark 2.15 The converse, need not be true, as seen in.

Example 2.16 Consider [Example 2.3] et $\widetilde{\tau}=\{\widetilde{\varphi}_A,\widetilde{X}_A,F_{10_A},F_{8_A}\}$ be a soft topology on $\widetilde{X}_A$ .Then $(\widetilde{X}_A,\widetilde{\tau},A)$ is SI-T_0-space ,but not SI-T_1-space,because,there exist distinct points $F_{a_1}^y$ and $F_{a_2}^y$ of $\widetilde{X}_A$, $F_{a_1}^y \mathrel{\widetilde{\notin}} SI(F_{a_2}^y)$ and $F_{a_2}^y \mathrel{\widetilde{\in}} SI(F_{a_1}^y)$.

Theorem 2.17 Let $(\widetilde{X}_A,\tilde{\tau},A)$ be a soft topological space and $a\in A$,then the following properties are equivalent:

a) (X,T) is SI-T_1-space.
b) If the soft sets F_a^x and F_a^y such that $F_a^x\neq F_a^y$ are closed subsets in $\widetilde{X}_A$, $\forall\, a\in A$.

Proof: [Direct from definition and Lemma 2.6].

Theorem 2.18 Every soft subspace of SI-T_1-space is SI-T_1-space $\forall\, a\in A$

Proof: Suppose that $\widetilde{Y}_A$ is a soft subspace of the of the SI-T_1-space $(\widetilde{X}_A,\widetilde{\tau},A)$. Let $F_a^{y_1}$ and $F_a^{y_2}$ be two distinct points of $\widetilde{Y}_A$. Again, since $\widetilde{X}_A$ is SI-T_0 –space and $\widetilde{Y}_A\subseteq\widetilde{X}_A$, then $F_a^{y_1} \mathrel{\widetilde{\notin}} SI(F_a^{y_2})$ and $F_a^{y_2} \mathrel{\widetilde{\notin}} SI(F_a^{y_1})$, for some a. So that, there exists $U \mathrel{\tilde{\in}} N_{\widetilde{\tau}(F_a^{y_1})}$ such that, $F_a^{y_1}\mathrel{\tilde{\in}} U$, $U^C \mathrel{\widetilde{\notin}} SI(F_a^{y_2})$ and there exists $G \mathrel{\tilde{\in}} N_{\widetilde{\tau}(F_a^{y_2})}$ such that, $F_a^{y_2}\mathrel{\tilde{\in}} G$, $G^C \mathrel{\widetilde{\notin}} SI(F_a^{y_1})$.Then, $U'=U\widetilde{\cap}\widetilde{Y}_A$ is $\widetilde{\tau}_{Y}$- soft open contains $F_a^{y_1}$ but not $F_a^{y_2}$ and $G'=G\widetilde{\cap}\widetilde{Y}_A$ is $\widetilde{\tau}_{Y}$- soft open contains $F_a^{y_2}$ but not $F_a^{y_1}$. So that $F_a^{y_1}\mathrel{\tilde{\in}} U'$ and $(U')^c \mathrel{\widetilde{\notin}} SI(F_a^{y_2})$ and $F_a^{y_2}\mathrel{\tilde{\in}} G'$ and $(G')^c \mathrel{\widetilde{\notin}} SI(F_a^{y_1})$, hence $\widetilde{Y}_A$ is SI-T_0-space.

Theorem 2.19 Let $(\widetilde{X}_A,\tilde{\tau},A)$ and $(\tilde{Y}_B,\tilde{\sigma},B)$ be two soft topological spaces and let $\widetilde{X}_A$ be SI-T_1-space, for some $a\in A$, if the map $f_{pu}: (\widetilde{X}_A,\widetilde{\tau},A)\longrightarrow(\widetilde{Y}_B,\widetilde{\sigma},B)$ is a soft open and u , p are onto maps, then $\tilde{Y}_B$ is SI-T_1-space .

Proof: By the same way of proof of Theorem 2.10.

Definition 2.20 Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space and $a \in A$, the space $\tilde{X}_A$ is called SI-T_2-space if and only if, for any pair of distinct points F_a^x and F_a^y of $\tilde{X}_A$, $F_a^x \tilde{\notin} SI(F_a^y)$ and $F_a^y \tilde{\notin} SI(F_a^x)$, $SI(F_a^x) \tilde{\cap} SI(F_a^y) = \emptyset$.

Example 2.21 Let E_X be the set of all parameters and let X be the initial universe consisting of: $X = \{x, y\}$ and $A \tilde{\subseteq} E_X$ such that $A = \{a\}$. Let $\tilde{\tau} = \{\tilde{\varphi}_A, \tilde{X}_A, F_{10_A}\}$ be a soft topology on $\tilde{X}_A$. where $F_{1_A} = \{(a, \{x\})\}$, $F_{2_A} = \{(a, \{y\})\}$. Then $(\tilde{X}_A, \tilde{\tau}, A)$ is SI-T_2-space.

Remark 2.22 Every SI-T_2-space is SI-T_1-space.

Proof: Direct from [Def].

Remark 2.23 The converse, need not be true, as seen in [Example 2.3].

Example 2.24 Consider [Example 2.3] Let $\tilde{\tau} = \{\tilde{\varphi}_A, \tilde{X}_A, F_{10_A}, F_{8_A}, F_{12_A}\}$ be a soft topology on $\tilde{X}_A$ .Then $(\tilde{X}_A, \tilde{\tau}, A)$ is SI-T_1-space because, for any pair of distinct points F_a^x , F_a^y of $\tilde{X}_A$, $F_a^x \tilde{\notin} SI(F_a^y)$ and $F_a^y \tilde{\notin} SI(F_a^x)$,but not SI-$T_2$-space,because,there exist distinct points $F_{a_2}^x$ and $F_{a_2}^y$ of $\tilde{X}_A$, $F_{a_2}^x \tilde{\notin} SI(F_{a_2}^y)$ and $F_{a_2}^y \tilde{\notin} SI(F_{a_2}^x)$. $SI(F_{a_2}^x) \tilde{\cap} SI(F_{a_2}^y) \neq \emptyset$.

Theorem 2.25 Every soft subspace of SI-T_2-space is SI-T_2-space, $\forall\, a \in A$.

Proof: Similar to theorem 2.9.

Theorem 2.26 Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces and let $\tilde{X}_A$ be SI-T_2-space, for some $a \in A$, if the map $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is a soft open and u, p are onto maps, then $\tilde{Y}_B$ is SI-T_2-space.

Proof: By the same way of proof of Theorem 2.10.

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