

Analytical Optimization of BLDC Motor Magnetic Parameters for Electric Vehicles Using the Whale Optimization Algorithm: PSO Comparison and Cost Analysis

Elektrikli Araçlar İçin BLDC Motor Manyetik Parametrelerinin Balina Optimizasyon Algoritması ile Analitik Optimizasyonu: PSO Karşılaştırması ve Maliyet Analizi

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Abstract

In this study, the magnetic design parameters of brushless DC (BLDC) motors for electric vehicle applications remanent flux density (Br), magnet coverage ratio (α), and air-gap length (g) are optimized using an analytical framework with the Whale Optimization Algorithm (WOA) and Particle Swarm Optimization (PSO), within a search space defined by manufacturing and material feasibility limits reported in the literature. For each objective function (total harmonic distortion (THD), electromagnetic torque, and back electromotive force (back-EMF)), 100 independent runs are performed and evaluated using mean, variance, and standard deviation, supported by the Wilcoxon Signed-Rank test. The THD objective possesses a genuine interior optimum, where PSO converges more stably (variance 1.0420×10^{-5}) while WOA shows stronger exploration (variance 3.2643×10^{-5}); the Wilcoxon test ($p = 1.86 \times 10^{-9}$) confirms a statistically significant difference in favour of WOA. For torque and back-EMF, which are monotonic functions of the air-gap flux density, both algorithms converge to the same boundary-constrained optimum. From a practical perspective, the optimized parameters improve efficiency by about 2.5% and reduce NdFeB magnet usage by approximately 18% (~101 g), corresponding to an estimated saving of about \$25 per motor.

Keywords: BLDC motor, Electric vehicle, Particle Swarm Optimization (PSO), Whale Optimization Algorithm (WOA).

Öz

Bu çalışmada, elektrikli araç uygulamalarında kullanılan fırçasız doğru akım (BLDC) motorlarının manyetik tasarım parametreleri; artık akı yoğunluğu (Br), mıknatıs kaplama oranı (α) ve hava aralığı uzunluğu (g) literatürde bildirilen üretim ve malzeme uygulanabilirlik sınırlarıyla tanımlanan bir arama uzayında, analitik bir çerçeve ile Balina Optimizasyon Algoritması (WOA) ve Parçacık Sürü Optimizasyonu (PSO) kullanılarak optimize edilmiştir. Her bir amaç fonksiyonu (toplam harmonik bozulma THD, tork ve geri elektromotor kuvveti) için 100 bağımsız koşum gerçekleştirilmiş; sonuçlar ortalama, varyans ve standart sapma ile değerlendirilmiş ve Wilcoxon İşaretli Sıra testiyle desteklenmiştir. THD amaç fonksiyonu gerçek bir iç optimuma sahiptir; bu durumda PSO daha kararlı yakınsama (varyans 1.0420×10^{-5}), WOA ise daha güçlü keşif kabiliyeti (varyans 3.2643×10^{-5}) göstermiştir. Wilcoxon testi ($p = 1.86 \times 10^{-9}$) farkın WOA lehine istatistiksel olarak anlamlı olduğunu doğrulamaktadır. Hava aralığı akı yoğunluğunun monoton fonksiyonları olan tork ve geri elektromotor kuvvetinde ise her iki algoritma da aynı sınır-kısıtlı optimuma yakınsamıştır. Pratik açıdan, optimize edilen parametreler verimliliği yaklaşık %2.5 artırmış ve NdFeB mıknatıs kullanımını yaklaşık %18 (~101 g) azaltarak motor başına yaklaşık 25 USD tasarruf sağlamıştır.

Anahtar Kelimeler: BLDC motor, Elektrikli araç, Parçacık sürü optimizasyonu (PSO), Balina optimizasyon algoritması (WOA).

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1. Introduction

Brushless DC motors (BLDC) have recently become indispensable parts of modern motor drives due to the emerging need for high efficiency and precise dynamic performance. Some of their advantages over other types of electric motors include higher torque density, higher efficiency due to the absence of field excitation induced losses, almost unit power factor and virtually maintenance-free construction. Permanent Magnet (PM) motors have become a preferred choice in industry due to their adaptability to new sophisticated control systems such as direct torque control. They are used in many areas such as electric vehicles, industrial machinery, robotic systems, medical equipment and even household appliances. Many studies have been carried out on the design and development of these widely used motors.

Studies on BLDC motor design focus on areas such as efficiency, power density, weight reduction, vibration, and noise reduction (Khamari *et al.*, 2023; Toren and Mollahasanoğlu, 2023a). Studies on magnetic field structures (Khlifi *et al.*, 2022), winding designs (Sun *et al.*, 2022), and control strategies (Yao *et al.*, 2023) have been carried out to improve motor efficiency. At the same time, studies have been carried out on the use of lightweight materials and more compact designs to reduce the weight of the motor (Payza *et al.*, 2018).

However, in recent years, design studies using heuristic algorithms are becoming more popular. Heuristic algorithms are heuristic approaches designed to solve or optimize complex problems. These algorithms do not aim to guarantee the best solution to a given problem, but rather to reach an acceptable solution, usually within a reasonable amount of time (Karabaş *et al.*, 2023). Swarm intelligence algorithms are an important algorithmic approach inspired by nature and used to solve optimization problems (Naranjo-Mora *et al.*, 2023). In the last two decades, the proliferation of these algorithms has motivated researchers to compare them with other traditional optimization methods and to study their performance (Toren and Mollahasanoglu, 2023). The applicability of these algorithms to a variety of complex optimization problems continues to make them attractive. Swarm intelligence algorithms typically work on two different search features, exploitation, and exploration. Exploration scans the entire search space and prevents the algorithm from falling into a local optimum, while exploitation increases the efficiency of the search and allows the algorithm to converge to the optimal solution (Ariyaratne *et al.*, 2019). There are many popular optimization algorithms such as Ant Colony Optimization (Ren *et al.*, 2023), Artificial Bee Colony (ABC) (Zhang *et al.*, 2023), Particle Swarm Optimization (PSO) (Han *et al.*, 2023), Cuckoo Search (CS) (Li *et al.*, 2023), Bat Algorithm (BA) (Cao *et al.*, 2020), Grey Wolf Optimizer (GWO) (Çavdar and Öztürk, 2022), Firefly Algorithm (Gedikli and Tuğcu, 2024), and many others (Huang *et al.*, 2024; Kahveci *et al.*, 2025; Toren and Mollahasanoğlu, 2023b). These algorithms stand out in the literature for their

effectiveness and applicability to various optimization problems. While this diversity emphasizes the suitability of each algorithm for a specific problem domain, swarm intelligence algorithms are generally flexible enough to be successfully used in multiple domains.

All these optimization algorithms are used to solve many problems as given above. In the field of electrical machines, swarm optimization algorithms are also preferred. A recent study applied multi-objective genetic algorithms (MOGA) to optimize a 4-kW synchronous reluctance motor using ANSYS Maxwell, where rotor geometric parameters were tuned to improve output power, torque ripple, and efficiency. Among the tested configurations, the four-barrier design achieved a 0.31% increase in output power and a 31.71% reduction in torque ripple compared to the reference motor (Çamcı *et al.*, 2022). Azari *et al.* dealt with the optimal design of a brushless DC motor using collective intelligence-based optimization algorithms. Three algorithms, namely cuckoo, GA and particle swarm, were investigated in this study. In particular, the cuckoo optimization algorithm was used for the first time for brushless DC motor design optimization. A comparative study between these optimization approaches showed that the cuckoo optimization algorithm converged to the optimal solution in less than 250 iterations and had a standard deviation (Niaz Azari *et al.*, 2017). In another study, the recently developed Harmony Search (HS) algorithm was used to maximize the efficiency of a brushless DC wheel motor. The numerical results obtained showed that the efficiency of the brushless DC wheel motor obtained with the IHS algorithm was higher than the HS algorithm and GA (Yadav *et al.*, 2010).

Recent studies on BLDC motor design have increasingly focused on the optimization of magnetic circuit parameters, geometric configurations, and material utilization in order to improve overall motor performance. In particular, parameters such as air-gap length, magnet thickness, and magnet coverage ratio have been identified as critical factors influencing torque production, flux distribution, efficiency, and harmonic behavior (Bin *et al.*, 2023). Magnetic design-oriented approaches have been widely adopted to enhance the electromagnetic performance of BLDC motors. For instance, several studies have investigated the impact of permanent magnet characteristics, including remanent flux density (B_r) and magnet topology, on torque density and back-EMF waveform quality (Usman and Rajpurohit, 2020). Similarly, geometric optimization techniques have been applied to improve flux linkage and reduce torque ripple by adjusting stator and rotor configurations (Zhu *et al.*, 2017). In addition to geometric and magnetic design, recent works have also explored material-based optimization strategies, particularly focusing on reducing the usage of rare-earth permanent magnets while maintaining performance levels. This has become increasingly important due to the high cost and limited availability of NdFeB materials (Toren and Mollahasanoglu, 2023). As a result, design-oriented optimization methods that balance performance

and material usage have gained significant attention in the literature (Barbieri and Muzzupappa, 2022).

Compared to control-oriented studies, which primarily focus on dynamic response and system stability, design-based optimization approaches aim to improve the intrinsic electromagnetic characteristics of the motor at the early design stage. This distinction is crucial, as improvements at the design level can lead to more efficient, cost-effective, and robust motor systems without requiring additional control complexity (Tušar *et al.*, 2007).

These studies confirm the increasing prevalence and effectiveness of metaheuristic approaches in BLDC motor optimization, addressing challenges such as nonlinear dynamics, real-time adaptability, and control precision.

In this study, the Whale Optimization Algorithm (WOA), originally proposed by Mirjalili and Lewis in 2016 and successfully applied to a wide variety of engineering optimization problems (Mirjalili and Lewis, 2016), is explored as a design optimization tool for BLDC motors. Unlike many existing studies that primarily focus on control strategies or rely on computationally intensive finite element method (FEM)-based approaches, this work introduces a computationally efficient analytical framework for the optimization of magnetic design parameters. Within this framework, three key design variables remanent flux density (B_r), magnet coverage ratio (α), and air-gap length (g) are optimized using analytically defined objective functions, as these parameters directly govern the air-gap flux density and, consequently, the electromagnetic performance of the motor, including torque production, total harmonic distortion (THD), and back-EMF characteristics.

To ensure a reliable and statistically meaningful comparison, both WOA and the widely used PSO algorithm are evaluated through multiple independent runs, and their performances are assessed using statistical metrics such as mean, variance, and standard deviation, along with the Wilcoxon Signed-Rank test. Furthermore, beyond purely electromagnetic performance, the study incorporates a practical evaluation based on NdFeB magnet volume and material cost, linking the optimization results to real-world engineering constraints. This integrated approach demonstrates that the proposed method not only improves motor performance but also provides measurable advantages in terms of material utilization and cost-effectiveness, offering a balanced and practically applicable design optimization strategy.

2. Materials and Methods

2.1. Whale Optimization Algorithm (WOA)

The Whale Optimization Algorithm was proposed by Seyedali Mirjalili and Andrew Lewis in 2016 (Mirjalili and Lewis, 2016). This algorithm is a metaheuristic algorithm that mimics the hunting behavior of humpback whales. The Whale Optimization Algorithm is inspired by schools of krill and small fish, which are the most preferred prey of humpback whales. An interesting feature of humpback whales, the "bubble-net" hunting method, which is only seen in them, is the inspiration for this algorithm (Watkins and Schevill, 1979). Humpback whales can identify their prey and move towards them. The Whale Optimization Algorithm is used to mimic this behaviour. The algorithm recognizes the best available solution as the prey or the closest solution to the prey. The working principle of the algorithm is as follows: First, the best search agent is determined. Then, the other search agents are updated according to the position of the best search agent. This update process is performed using certain equations. These equations, shown in Equation 1, are used to determine the movements and position updates of the search agents. In this way, the algorithm searches in regions close to the best solution and aims to obtain better results. This algorithm is used to solve optimization problems by simulating the hunting strategies of whales. The Whale Optimization Algorithm uses certain movement rules and balancing mechanisms to enable exploration and propagation among individuals within the population. In this way, it aims to achieve a result close to the best solution.

$$\vec{D} = \left| \vec{C} \cdot \vec{X}^*(t) - \vec{X}(t) \right|$$

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D} \quad (1)$$

In these equations, t is the number of iterations, $\vec{A}$ and $\vec{C}$ are the coefficient vector, X^* is the best available location vector and $\vec{X}$ is the location vector of the corresponding search agent. At the end of each iteration, it is checked whether there is a better solution than X^* . If there is a better solution, the X^* value is updated.

The bubble-net behavior of humpback whales is mathematically modelled by two approaches: The shrinking encirclement mechanism and spiral motion. The shrinking encirclement mechanism is due to the decreasing value of $\vec{A}$ in Equation 1. At the same time, the fluctuation in the range of values that $\vec{A}$ can take is also reduced by a as a decreases from 2 to 0 during the iteration and $\vec{A}$ can take values in the range $[-a, a]$. In spiral motion, as shown in Figure 1, the distance between the whale at

(X, Y) and the prey at (X^*, Y^*) is first calculated. Then Equation 2 is used to model the spiral motion of humpback whales.

$$\vec{D} = |\vec{X}^*(t) - \vec{X}(t)|$$

$$\vec{X}(t+1) = \vec{D}e^{bl} \cos(2\pi l) + \vec{X}^*(t) \quad (2)$$

Here, b is a constant used to determine the model of the spiral; l is a random number in the range $[-1, 1]$; $\vec{D}$ is the distance between the whale and the prey (the best solution obtained so far).

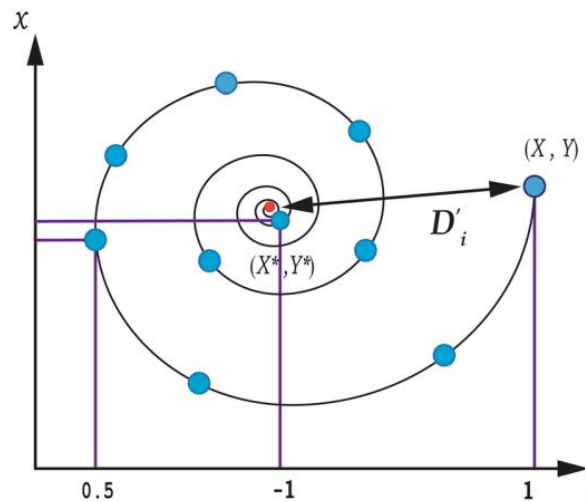


Figure 1. Spiral movement (Rana et al., 2020).

Humpback whales swim around their prey simultaneously according to both the shrinking encirclement mechanism and the spiral motion. To model this simultaneous movement in the Whale Optimization Algorithm, it is assumed that either the constriction mechanism or the spiral motion can be selected with a probability of 50% and the mathematical model is given in Equation 3.

$$\vec{X}(t+1) = \begin{cases} \vec{X}^*(t) - \vec{A} \cdot \vec{D} & p \leq 0.5 \\ \vec{D}e^{bl} \cos(2\pi l) + \vec{X}^*(t), & p \geq 0.5 \end{cases} \quad (3)$$

Here p is a random number in the range $[0-1]$. Humpback whales can also hunt their prey randomly, except for the Bubble-Net method. In random prey search, random values of the vector $\vec{A}$ greater than 1 or less than -1 are utilized to move away from the reference whale. In the Hunt Search mechanism, unlike the Bubble-Net mechanism, a randomly selected search agent is used instead of the best search agent ever found when updating the position of the search agent. $|\vec{A}| > 1$ and the use of

a random search agent causes the Whale Optimization Algorithm to perform a global search. The mathematical model used in this mechanism is given in Equation 4.

$$\vec{D} = |\vec{C} \cdot \vec{X}_{rand} - \vec{X}|$$

$$\vec{X}(t+1) = \vec{X}_{rand} - \vec{A} \cdot \vec{D} \quad (4)$$

Figure 2 shows the basic flow chart of the WOA used in this study.

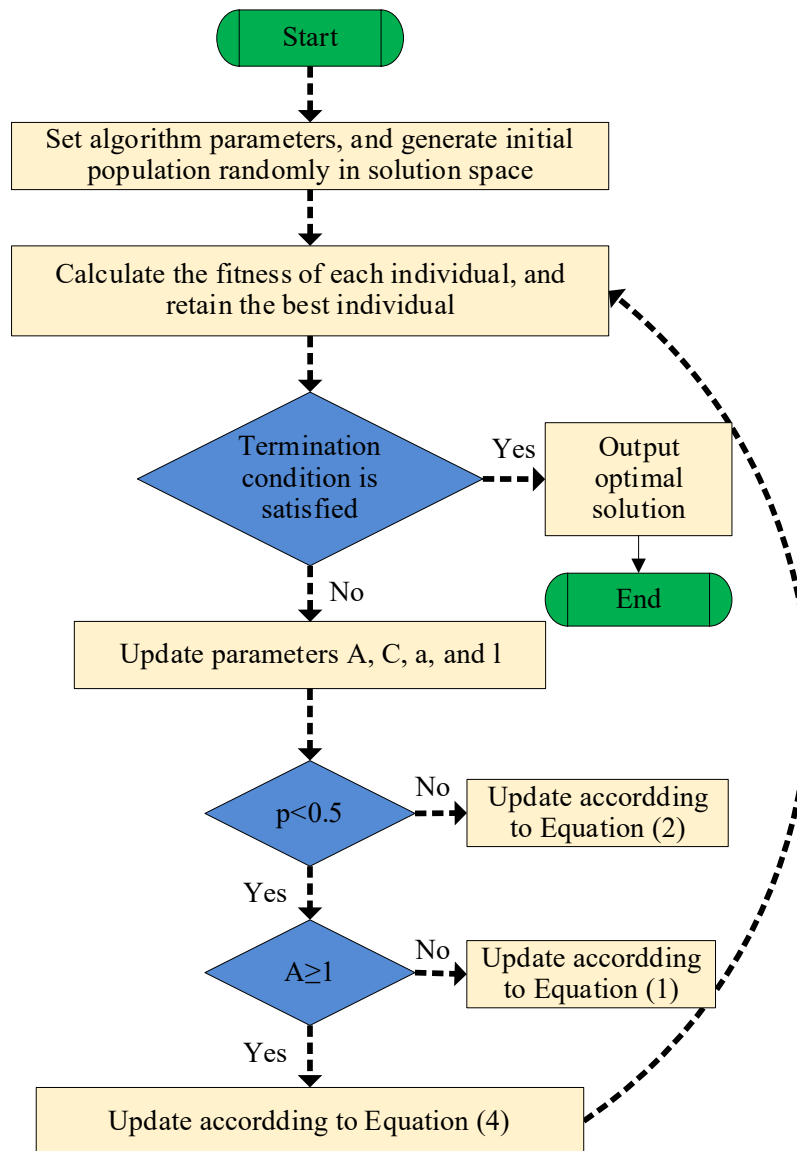


Figure 2. Flowchart of standard WOA.

The exploration and exploitation balance in WOA is crucial for its convergence behavior. In the algorithm, this balance is governed by parameter a , which linearly decreases from 2 to 0 during iterations. When the absolute value of the coefficient vector A exceeds 1, the algorithm promotes

exploration by pushing the search agents away from a randomly selected agent, thus covering a broader search space and avoiding premature convergence. Conversely, when $|A^+| < 1$, the agents are drawn toward the best solution found so far, reinforcing exploitation. Additionally, the spiral movement strategy adds a local search capability. The algorithm randomly alternates between these two movement mechanisms with a 50% probability, thereby achieving a dynamic balance. This balance allows WOA to efficiently switch between global and local search modes, which is one of the reasons behind its superior convergence properties compared to conventional algorithms like PSO.

2.2. Particle Swarm Optimization (PSO)

Particle swarm optimization is one of the most popular heuristic algorithms, developed by Kennedy and Eberhart in 1995 (Kennedy and Eberhart, 1995). The main advantages of PSO are simplicity, robustness, and good convergence. Other advantages are that it is written with little code and can be used to solve different problems with the same parameters. This algorithm represents the speed of communication with each other in swarming populations such as fish, birds, etc. depending on various adaptations, and basically each particle utilizes its previous position experiences to set the best position in the swarm (Wang *et al.*, 2015). PSO has been successfully used in many optimization problems such as order quantity determination, scheduling problems, power and voltage control, motor parameterization, supply selection and ranking problems. Figure 3 shows the basic flow chart of PSO.

In the PSO algorithm, the position of each particle is a vector containing the parameters to be optimized. It is updated at each iteration from the start and is expected to converge to a reasonable performance after a certain period. Each particle has a velocity. This velocity determines the instantaneous and next position. At each iteration, the instantaneous velocity of the particle and the velocity calculated in the previous iteration are summed to obtain the new position. This is expressed in Equation 5.

$$x_n = X_{n-1} + v_n \quad (5)$$

Here, x_n is the position of the particle and v_n is the velocity of the particle. To calculate the particle's velocity, the velocity of the particle at the last iteration (v_{n-1}), the position of the best instantaneous point of the particle (p_{n-1}), and the position of the best instantaneous point of all

particles (g_{n-1}) are used. The new velocity of the particle is calculated by Equation 6, where c_1 , c_2 , c_3 are the specific coefficients.

$$v_n = c_1 v_{n-1} + c_2 (X_{n-1} - p_{n-1}) + c_3 (X_{n-1} - g_{n-1}) \quad (6)$$

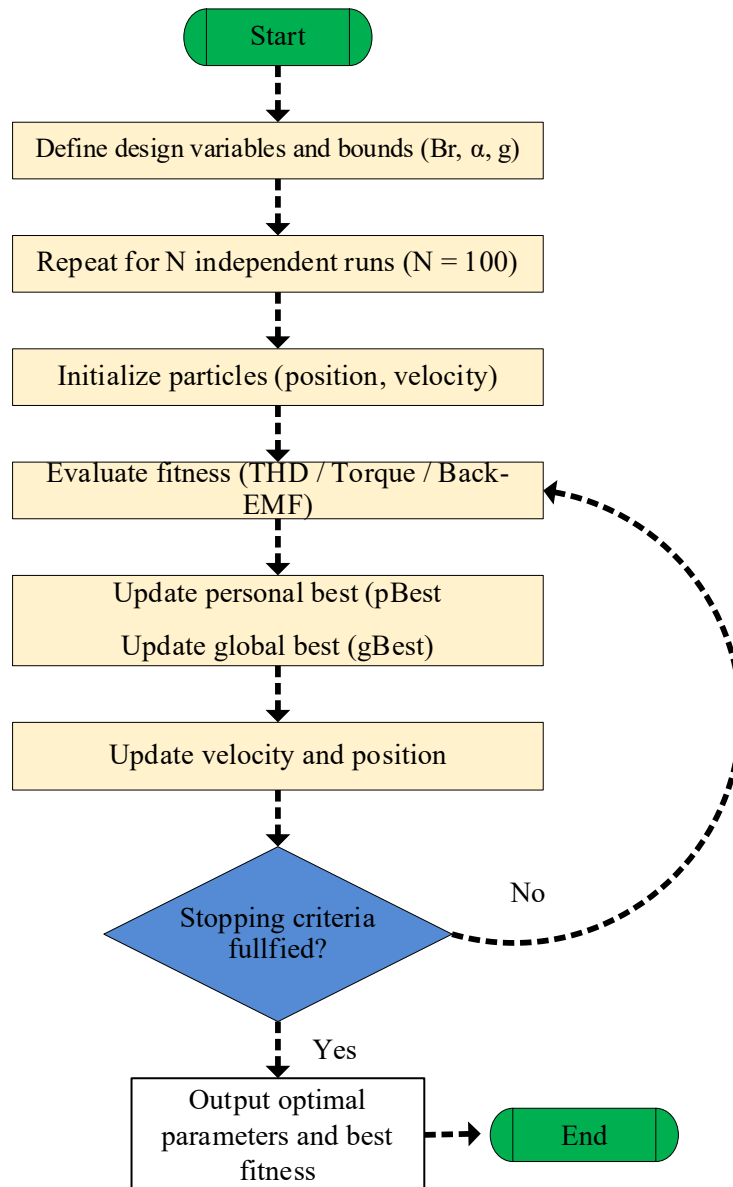


Figure 3. Flowchart of standard PSO.

2.3. Brushless DC Motor Parameters

Brushless DC (BLDC) motors are electric motors that have a more advanced technology compared to conventional brushed DC motors. BLDC motors use an electronically controlled commutator instead of brushes. Therefore, they require less friction, longer life, higher efficiency,

and less maintenance. The design of brushless BLDC motors has a major impact on the performance, efficiency, reliability, and availability of the motor.

In the performance evaluation of BLDC motors, the magnetic equivalent electrical circuit of the motor is utilized. Figure 4 shows the BLDC motor structure with reluctances (Şahin, 2010).

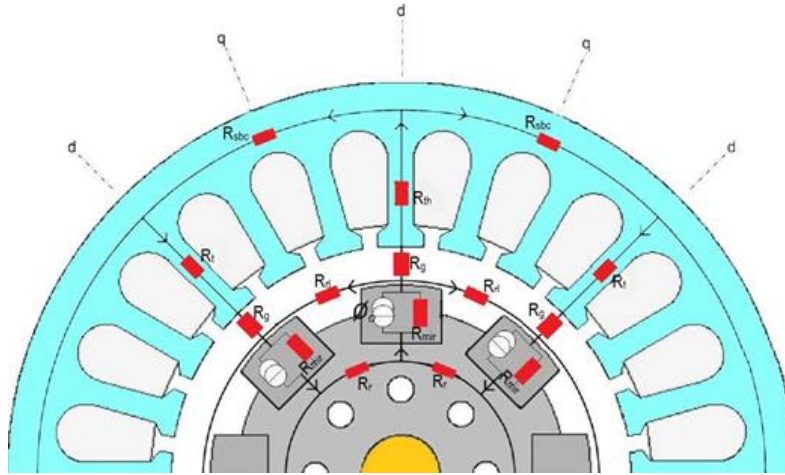


Figure 4. BLDC motor structure with reluctances.

In here, Φ_0 is flux generated by magnet, R_{mir} is magnet internal reluctance, R_{rl} is reluctance of flux path in the rotor between successive poles, R_g is equivalent air gap magnetic reluctance, R_{th} is reluctance of tooth path, R_{sbc} is reluctance of stator back core, and R_r is reluctance of rotor yoke.

In Figure 5, magnetic equivalent circuit model of BLDC motor is shown. The arrows on lines in the figure show direction of flux. These fluxes are called Φ_m flux leaving magnet and passing to air gap, Φ_{rl} rotor leakage flux and Φ_g flux at air gap passing from rotor to stator side.

Thanks to the equations that can be obtained using this equivalent circuit, the effect of permanent magnets and other motor parts on the performance parameters can be determined (Nory, 2018).

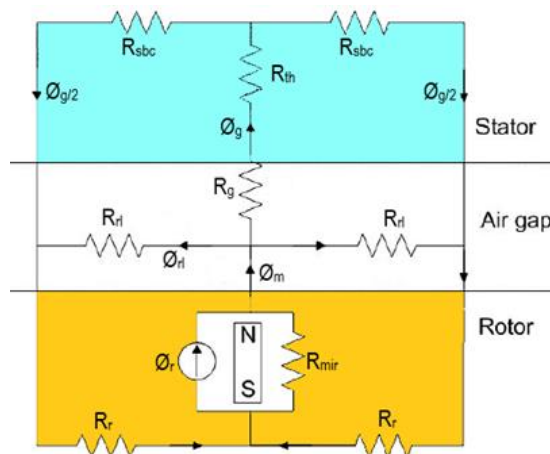


Figure 5. BLDC motor magnetic equivalent circuit.

As shown in Figure 5, the equivalent of the magnet in its position consists of a flux source and an internal reluctance value parallel to it. The magnetic flux value in this equivalent expression is expressed by Equation 7.

$$\Phi_r = B_r \times A_{mag} \quad (7)$$

Here, Φ_r is the magnetic flux generated by the magnet, B_r is the permanent flux density of the magnet and A_{mag} is the cross-sectional area of the magnet.

Magnet cross sectional area A_{mag} is calculated by multiplying magnet width W_{mag} with axial length L .

$$A_{mag} = L \times W_{mag} \quad (8)$$

The magnet volume (W_{mag}) is determined as follows, depending on the magnet surface diameter (D_{ms}), the number of poles (p), the magnet coverage ratio (α), and the axial length of the motor (L).

$$W_{mag} = D_{ms} \times \sin d \left(\frac{\alpha \times \frac{360}{p}}{2} \right) \quad (9)$$

The magnet coverage ratio α represents the ratio of the magnet arc to the pole pitch and directly determines the effective angular span of the magnet. Therefore, the magnet volume is directly proportional to α , since increasing α leads to a larger magnet arc length and consequently a higher magnet volume. Accordingly, the magnet volume (W_{mag}) can be expressed as a function of α , the magnet surface diameter (D_{ms}), the number of poles (p), and the axial length of the motor (L). This establishes a direct physical relationship between the optimization variable α and the material usage of the motor. Thus, any variation in α not only affects the electromagnetic performance but also has a direct impact on magnet volume and cost, which is considered in the practical evaluation of the optimized design.

Air gap reluctance is an important parameter of the equivalent magnetic circuit. This parameter is used to calculate the air gap flux and flux density. The air gap reluctance is generally expressed as in Equation 8.

$$R_g = \frac{g'}{\mu_0 A_g} \quad (10)$$

Since the gaps of the slots also affect the parameter R_g , the Karter coefficient will be used (Hanselman, 2003).

$$g' = g k_{cs}$$

$$k_{cs} = \frac{\frac{\pi D_{si}}{N_{slot}}}{\frac{\pi D_{si}}{N_{slot}} - \frac{w_0^2}{w_0 + 5g}} \quad (11)$$

Here, A_g , g , D_{si} , N_{slot} , w_0 are the air gaps cross-sectional area, air gap length, inner diameter of the stator, number of slots and tooth width of the slot, respectively.

As can be seen in the equations between Equations 7 -11, it is determined that the values affecting the BLDC Motor performance parameters are affected by the magnetic circuit parameters.

In this study, the values obtained by optimizing the design parameters of the BLDC motor are verified with the equations obtained from the magnetic and electrical equivalent circuits of the BLDC motor given in (Şahin, 2010).

3. Findings and Discussion

In this study, the key magnetic design parameters of a BLDC motor remanent flux density (Br), magnet coverage ratio (α), and air-gap length (g) were optimized using two widely adopted metaheuristic algorithms: WOA and PSO. The selected parameter ranges were determined based on both manufacturability constraints and values commonly reported in the literature. (Balkan Simsir and Bulent Ertan, 1999; Park *et al.*, 2015; Şahin, 2010). Accordingly, the design space was defined as $1.15 \leq Br \leq 1.5$ T, $0.6 \leq \alpha \leq 0.9$, and $0.5 \leq g \leq 0.75$ mm. These limits ensure a physically realizable and industrially applicable motor design. It should be emphasized that these ranges represent manufacturing and material feasibility limits rather than arbitrary numerical search bounds; they are taken directly from experimentally and industrially reported BLDC design data (Balkan Simsir and Bulent Ertan, 1999; Park *et al.*, 2015; Şahin, 2010). Consequently, the optimization is performed over a physically constrained design space, and any optimum located on the boundary of this space corresponds to a manufacturable limiting design rather than a numerical artefact. As discussed in the following subsections, the torque and back-EMF objectives are monotonic functions of the air-gap flux density, and hence of Br, α , and g, so their constrained optima are expected to lie on the feasible boundary. In contrast, the THD objective, being a ratio of harmonic to fundamental flux-density

components, possesses a genuine interior optimum within the same design space, where the exploration–exploitation behaviour of the two algorithms can be meaningfully distinguished.

Due to the stochastic nature of metaheuristic algorithms, a single run is not sufficient to evaluate algorithm performance. Therefore, in order to ensure a reliable and statistically meaningful comparison, both WOA and PSO were executed 100 independent runs for each objective function (THD, torque, and back-EMF). The convergence behaviors obtained from these repeated runs are illustrated in Figure 6, Figure 7, and Figure 8. In addition, statistical metrics such as mean, variance, and standard deviation were calculated. Furthermore, the Wilcoxon Signed-Rank test was applied using 30 independent runs for the THD objective function to evaluate the statistical significance of the observed differences.

3.1. Optimization of BLDC Motor Parameters

The objective functions considered in this study are analytically linked to the optimization variables through the air-gap flux density. Although B_r , α , and g are selected as the design variables, their influence on motor performance is evaluated through their effect on the air-gap flux density B_g . In particular, B_r determines the magnetic source strength, g affects the magnetic reluctance of the air gap, and α governs the spatial coverage of the magnet over the pole pitch. These parameters collectively shape the air-gap flux density distribution, which in turn determines the total harmonic distortion, electromagnetic torque, and back-EMF characteristics of the motor.

Therefore, the objective functions given in Equations 12-14 are not independent of the optimization variables; instead, they are indirectly governed by B_r , α , and g through their analytical influence on B_g . This establishes the physical basis of the optimization process and ensures that the selected objective functions are directly connected to the magnetic design parameters of the motor.

The parameters are optimized by WOA and PSO, using the THD objective function given in Equation 12. Figure 6 shows the convergence graph obtained for THD.

$$THD = 100 \times \sqrt{\frac{B_{g3_peak}^2 + B_{g5_peak}^2 + B_{g7_peak}^2 + B_{g9_peak}^2}{B_{g1_peak}^2}} \quad (12)$$

For Equation (12), which represents the THD of the air-gap flux density, B_{g1_peak} denotes the peak value of the fundamental component of the air-gap flux density, while B_{g3_peak} , B_{g5_peak} , B_{g7_peak} , and B_{g9_peak} represent the peak values of the 3rd, 5th, 7th, and 9th harmonic components, respectively.

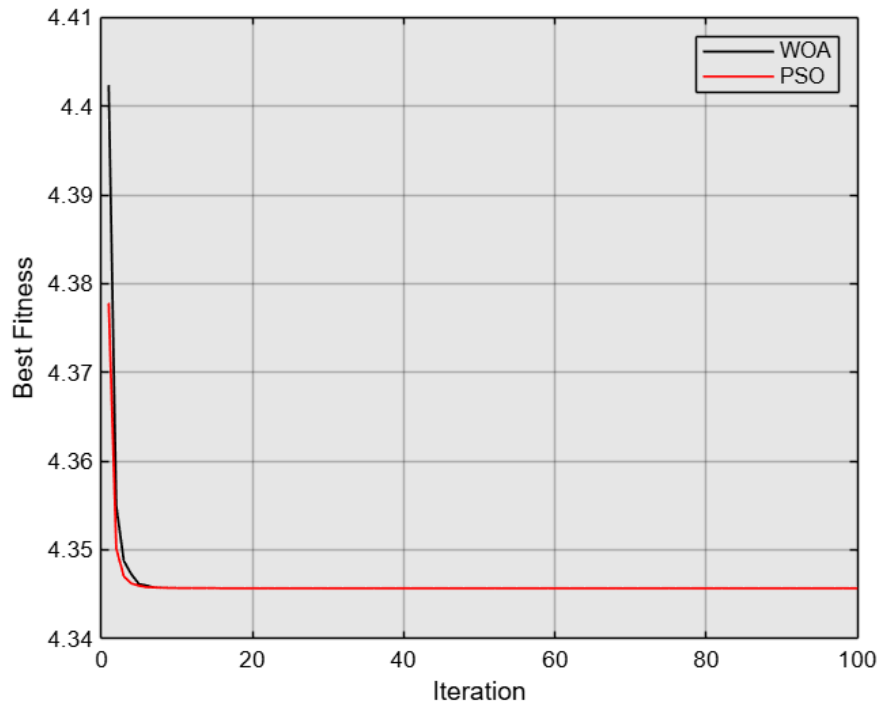


Figure 6. THD convergence graph.

As observed in Figure 6, both WOA and PSO converge rapidly within the first iterations and reach nearly identical optimum fitness values. WOA exhibits slightly faster initial convergence, whereas PSO demonstrates a smoother and more stable convergence behavior.

The optimal parameter values obtained for the THD objective function are given in Table 1.

Table 1. The parameter values optimized by WOA and PSO (THD).

Parameters	Value	
	WOA	PSO
Br (T)	1.176	1.1896
Alpha	0.61	0.6057
g (mm)	0.621	0.606

The statistical results obtained from 100 independent runs are presented in Table 2. Mean, variance, and standard deviation were used to evaluate convergence stability and repeatability. This multi-run analysis provides a statistically reliable comparison by accounting for the stochastic nature of the algorithms.

Table 2. Statistical results for THD objective function (100 runs)

Method	Mean	Variance	Std
PSO	4.3460498899	1.0420e-5	0.003228
WOA	4.3463752195	3.2643e-5	0.005713

Table 2 shows that PSO has lower variance and standard deviation values, indicating a more stable and consistent convergence behavior. In contrast, WOA exhibits higher variability, suggesting a broader exploration capability. Although WOA shows more fluctuations, both algorithms converge to similar optimal solutions. This indicates that PSO emphasizes local exploitation, while WOA provides stronger global exploration capability.

Using the torque objective function given in Equation 13, The convergence behavior for the torque objective function is presented in Figure 7.

$$k_{torq} = \frac{3}{\sqrt{2}} \cdot \frac{p}{2} \cdot (N_{turn} \cdot N_{coil} \cdot k_w) \cdot B_{g,mean} \cdot A_{pole} \quad (13)$$

For Equation (13), T represents the electromagnetic torque of the motor (Nm), B_g is the air-gap flux density (T), L is the axial (stack) length of the motor (m), r is the effective rotor radius (m), and p denotes the number of poles.

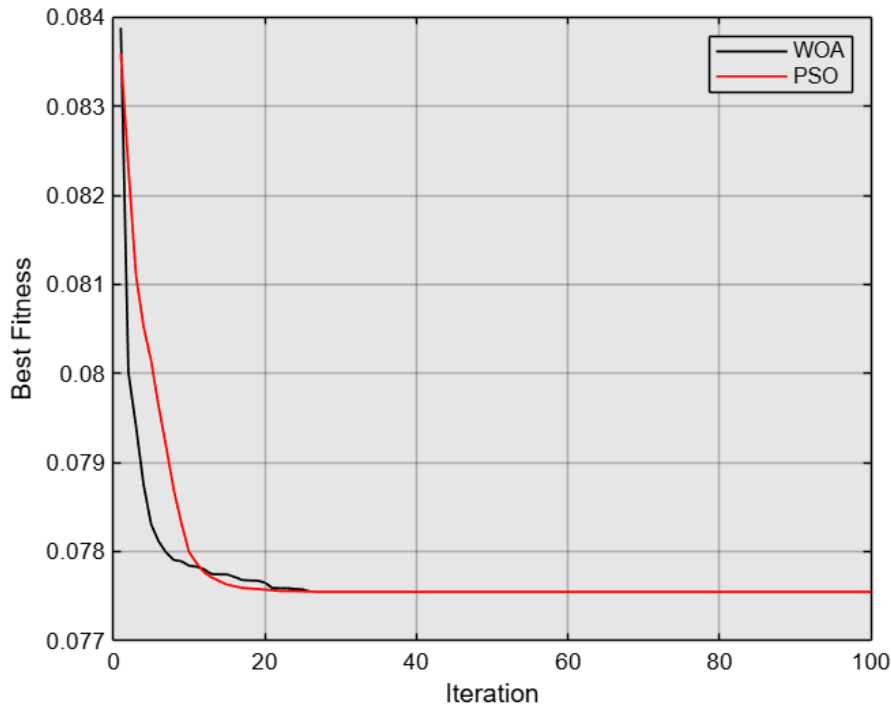


Figure 7. Torque convergence graph.

The parameters optimized using the torque objective function are given in Table 3. The statistical results are presented in Table 4.

Table 3. The parameter values optimized by WOA and PSO (torque).

Parameters	Value	
	WOA	PSO
Br (T)	1.15	1.15
Alpha	0.600	0.600
g (mm)	0.75	0.75

Table 4. Statistical results for torque objective function (100 runs)

Method	Mean	Variance	Std
PSO	0.07836968	2.26e-6	0.001503
WOA	0.078049	2.10e-6	0.001450

As shown in Table 3, both algorithms converge to the same parameter set located on the boundary of the feasible design space (Br and α at their lower limits and g at its upper limit). This behaviour is consistent with the monotonic dependence of the torque objective on the air-gap flux density: since the objective varies monotonically with Br, α , and g, its constrained optimum necessarily lies on the boundary defined by the manufacturing limits rather than at an interior point. The very low and nearly identical variance values in Table 4 (2.10×10^{-6} for WOA and 2.26×10^{-6} for PSO) confirm that both algorithms reliably and rapidly reach this boundary optimum in every run. In this case, therefore, the small variance difference does not indicate a meaningful performance gap between the algorithms but rather reflects the well-defined, boundary-constrained nature of the torque objective.

Finally, using the objective function of the back electromotive force given in Equation 14. Figure 8 shows the convergence graph obtained for back-EMF.

$$k_{emf} = \frac{1}{\sqrt{2}} \cdot \frac{pole}{2} \cdot (N_{turn} \cdot N_{coil} \cdot k_w) \cdot B_{g,mean} \cdot A_{pole} \quad (14)$$

For Equation (14), which defines the back electromotive force (back-EMF), E represents the induced voltage (V), N is the number of turns per phase, ω is the angular speed (rad/s), and Φ denotes the magnetic flux per pole (Wb).

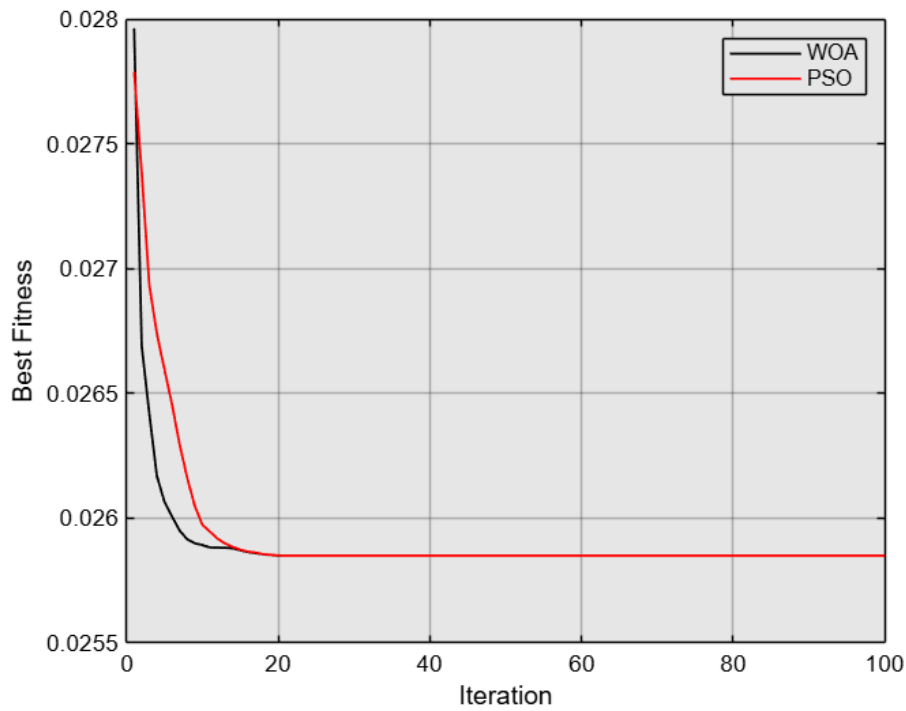


Figure 8. Back-EMF convergence graph.

The parameters optimized using the back-EMF objective function are given in Table 5. The statistical results are presented in Table 6.

Table 5. The parameter values optimized by WOA and PSO (back-EMF).

Parameters	Value	
	WOA	PSO
Br (T)	1.15	1.15
Alpha	0.600	0.600
g (mm)	0.75	0.75

Table 6. Statistical results for back-EMF objective function (100 runs).

Method	Mean	Variance	Std
PSO	0.0259750	1.2879e-7	0.0003589
WOA	0.0259750	1.3000e-7	0.0003600

As reported in Table 5, both algorithms again converge to the same boundary parameter set ($Br = 1.15$ T, $\alpha = 0.60$, $g = 0.75$ mm). This is expected, since the back-EMF objective, like the torque objective, is a monotonic function of the air-gap flux density and hence of the design variables; its constrained optimum therefore coincides with the feasible boundary. Accordingly, both algorithms exhibit nearly identical statistical performance, and the negligible difference in variability is not indicative of algorithmic superiority but of the boundary-constrained nature of the objective. These results highlight that, for monotonic objectives such as torque and back-EMF, the optimum is

governed primarily by the physically imposed manufacturing limits, whereas the comparative behaviour of WOA and PSO becomes meaningful for objectives with a genuine interior optimum, such as THD.

The Wilcoxon Signed-Rank test results for the THD objective function are presented in Table 7. The test was conducted using 30 independent runs to statistically evaluate the significance of the performance difference between the algorithms. The obtained p-value ($p = 1.86 \times 10^{-9}$) is significantly lower than the significance level ($\alpha = 0.05$), indicating that the null hypothesis is rejected. This result confirms that the performance difference between WOA and PSO is statistically significant, with WOA demonstrating superior convergence behavior.

Table 7. Wilcoxon Signed-Rank test results for THD objective function.

Comparison	Number of Runs (n)	Test Statistic (Z)	p-value	Significance ($\alpha = 0.05$)	Interpretation
WOA vs PSO	30	-6.01	1.86e-9	Significant	WOA outperforms PSO

The combined evaluation of convergence behaviors and statistical results shows that both WOA and PSO can reach similar optimal solutions. However, their search mechanisms differ significantly. PSO produces more stable and repeatable results, indicating strong local search (exploitation) capability. In contrast, WOA exhibits higher variability but stronger global search (exploration) capability, allowing it to explore a wider solution space.

These findings demonstrate that neither algorithm is universally superior. Instead, PSO is more suitable for problems requiring stability and consistency, while WOA is more advantageous in complex optimization problems where global search capability is critical.

3.2. Practical Evaluation of Efficiency and Cost Based on Optimized Parameters

To demonstrate the practical implications of the optimized parameters obtained through the WOA, a typical BLDC motor used in light electric vehicle applications (2.45 kW rated power) is selected as a reference. Based on commonly reported design data from literature (Patel et al., 2023; Hanselman, 2003; Şahin, 2020), the effects of changes in B_r , α (alpha), and g on efficiency and magnet cost are estimated. The nominal specifications and pre-/post-optimization parameter values of the reference BLDC motor are summarized in Table 8.

Table 8. Baseline motor specifications and optimized parameter set.

Parameter	Value	Unit	Notes
Rated Power	2.45	kW	Typical for light EVs
Rated Voltage	48 – 72	V	Rated Voltage
Rated Torque	8.5	Nm	Approximate average
Rated Speed	2750	rpm	-
Nominal Efficiency	89	%	Estimated before optimization
Magnet Type	NdFeB (N35)	–	$B_r \approx 1.2 - 1.25$ T
Magnet Volume (pre-opt)	75	cm ³	Based on $\alpha = 0.85$
Magnet Volume (post-opt)	61.5	cm ³	Based on $\alpha = 0.70$
Air Gap (pre-opt)	1.0	mm	Typical design
Air Gap (post-opt)	0.75	mm	Optimized value

The efficiency rates presented in Table 9 are estimated based on analytical modeling under steady-state operating conditions. Efficiency is evaluated as the ratio of output mechanical power to input electrical power. For consistency, both pre-optimization and post-optimization cases are assessed under identical operating conditions, including the same speed, load, and supply parameters. It should be noted that the reported efficiency values represent estimated performance based on electromagnetic analysis and do not include detailed thermal or mechanical loss components.

Table 9. Estimated efficiency and cost improvements based on optimized parameters.

Metric	Before Optimization	After Optimization	Improvement
Efficiency	~89.0 %	~91.5 %	↑ ~2.5 %
NdFeB Magnet Mass	~562.5 g	~461.3 g	↓ ~101 g
Magnet Cost (\$250/kg)	~\$140.6	~\$115.3	↓ ~\$25.3

The optimized reduction in air gap from 1.0 mm to 0.75 mm improves magnetic coupling, thereby enhancing overall motor efficiency. As supported by experimental data (Şahin, 2020) and theoretical analysis (Hanselman, 2003), reducing the air gap by 0.25 mm can increase efficiency by approximately 2–3%, depending on saturation levels in the magnetic circuit.

Similarly, decreasing the magnet coverage ratio (α) from 0.85 to 0.70 results in an ~18% reduction in magnet volume, which corresponds to a material saving of ~101 grams of NdFeB (N35). Assuming a commercial magnet price of ~\$250/kg, this yields an estimated cost saving of ~\$25 per motor, as detailed in Table 9. In high-volume production, such savings can result in significant cost reductions at the system level, while also contributing to lighter motor design and improved energy efficiency in EV applications.

While these results are based on estimation models and standard motor data reported in literature, they provide a valuable link between the theoretical optimization framework and its real-world applicability. This analysis supports the potential of metaheuristic-based parameter optimization not only in improving motor performance but also in enhancing cost-efficiency trade-offs relevant for industrial deployment.

To further assess the scientific contribution and reliability of the proposed approach, the results obtained in this study are compared with representative metaheuristic-based BLDC and electrical-machine optimization studies reported in the literature. Table 10 summarizes the motor type, optimization method, optimized objective, and the main reported outcome of each study.

Table 10. Comparison of the present study with metaheuristic-based motor optimization studies in the literature.

References	Motor/Application	Optimization Method	Optimized Objective/Variables	Reported Outcome
(Niaz Azari <i>et al.</i> , 2017)	BLDC motor	Cuckoo, GA, PSO	Design parameters	Cuckoo converged in fewer than 250 iterations; first COA application to BLDC design
(Yadav <i>et al.</i> , 2010)	BLDC wheel motor	Improved Harmony Search (IHS)	Efficiency maximization	Higher efficiency than HS and GA
(Çamcı <i>et al.</i> , 2022)	4-kW synchronous reluctance motor	Multi-objective GA (MOGA)	Rotor geometry (power, torque ripple, efficiency)	0.31% output power increase; 31.71% torque ripple reduction
(Genc and Kalimbetova, 2024)	BLDC motor	Cuckoo + fuzzy logic	Speed control	Improved speed-control performance
(Rana <i>et al.</i> , 2020)	BLDC motor	Several metaheuristic algorithms	Speed control (statistical comparison)	Statistical performance comparison of algorithms
(Patel <i>et al.</i> , 2024)	BLDC-driven electric vehicle	Various metaheuristic techniques	Drive performance	Improved EV drive performance
This study	BLDC-driven	WOA and PSO	Magnetic parameters (Br, α , g): THD, torque, back-EMF + cost	~2.5% efficiency gain; ~101 g (~18%) NdFeB reduction; ~\$25/motor saving; statistically significant WOA advantage (Wilcoxon $p = 1.86 \times 10^{-9}$)

As summarized in Table 10, a large portion of the existing literature focuses either on control-oriented objectives, such as speed and position control (Genc and Kalimbetova, 2024; Patel *et al.*, 2024), or on single-method and geometry-oriented design optimization (Niaz Azari *et al.*, 2017; Yadav *et al.*, 2010; Çamcı *et al.*, 2022). In contrast, the present study addresses the magnetic design parameters (Br, α , and g) directly through an analytical framework and evaluates two algorithms (WOA and PSO) under a statistically rigorous procedure that includes 100 independent runs and the Wilcoxon Signed-Rank test. Moreover, unlike most previous works, which report performance gains primarily in electromagnetic terms, this study additionally quantifies the practical impact of optimization in terms of NdFeB magnet mass and cost. The obtained improvements approximately 2.5% in efficiency, an 18% (~101 g) reduction in magnet usage, and an estimated cost saving of about

\$25 per motor are consistent in order of magnitude with the performance and material gains reported in comparable studies, confirming both the validity and the practical relevance of the proposed optimization approach.

4. Conclusions and Recommendations

In this study, the magnetic design parameters of BLDC motors for electric vehicle applications (B_r , α , and g) were optimized using an analytical framework together with the WOA and PSO algorithms, based on 100 independent runs per objective and the Wilcoxon Signed-Rank test.

The THD objective exhibits a genuine interior optimum, where PSO converges more stably while WOA shows stronger exploration; the Wilcoxon test ($p = 1.86 \times 10^{-9}$) confirms a statistically significant difference in favour of WOA. The torque and back-EMF objectives, being monotonic functions of the air-gap flux density, converge to the same boundary-constrained optimum, so the algorithm choice is not decisive for them. Thus PSO is preferable when stability is required, whereas WOA is more advantageous for objectives with a non-trivial interior optimum.

From a practical perspective, the optimized parameters improve efficiency by about 2.5% and reduce NdFeB magnet usage by approximately 18% (~101 g), saving about \$25 per motor gains consistent with comparable studies in the literature. These results show that analytical model-based optimization of BLDC magnetic parameters can enhance both electromagnetic performance and material/cost efficiency, providing a practical design tool for EV applications.

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Authors' Contributions

The author contributed to all stages of the study, including conceptualization, methodology development, analysis, and manuscript preparation.

Statement of Conflicts of Interest

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The author declares that this study complies with Research and Publication Ethics.

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