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Expansions and Reductions on Neutrosophic Classical Soft Set

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Abstract: In the paper, we first proposed a new notation is called expansion and reduction of the neutrosophic classical soft sets that are based on the linguistic modifiers. By using this new notions, we then developed a neutrosophic classical soft reduction method and present a reel example for the method.

Neutrosophik Klasik Esnek Kümeler Üzerine Genişlemeler ve İndirgemeler

Anahtar Kelimeler
Esnek kümeler,
Neutrosophik klasik esnek kümeler,
Genişleme,
Neutrosophik klasik esnek indirgeme

Özet: Bu makale, ilk olarak dilsel düzenleyicilere bağlı olarak neutrosophik klasik esnek kümeler üzerine genişleme ve indirgeme olarak adlandırılan yeni bir notasyon ileri sürdük. Bu yeni notasyonları kullanarak, biz daha sonra bir neutrosophik klasik esnek indirgeme metodu geliştirdik ve bu metot için bir reel örnek sunduk.

1. Introduction

Uncertainty takes place almost everywhere in our daily life. There are a number of theories have been presented tackle these uncertainty such as fuzzy set theory [46], intuitionistic fuzzy set theory [4], interval valued intuitionistic fuzzy sets [3], rough set theory [34], vague set theory [22], neutrosophic set theory [2, 5–8, 14–16, 30, 32, 36, 38, 41, 42], interval neutrosophic sets [39, 48] etc. After these theories, Molodtsov [27] presented the notations of soft set theory. Then, different studies on soft sets have been introduced such as [1, 9, 10, 18, 19, 28, 29, 37, 49, 50]. After Molodtsov, the theory of soft sets has been extended in a number of directions in [11–13, 20, 21, 24, 26, 31, 33, 35, 40, 47].

In recent years, by using the concept of linguistic variables by given Zadeh [44, 45], Eraslan [17] alternatively proposed soft reduction method that reduces a number of alternatives. As a generalization of soft set theory, neutrosophic classical soft set theory [25] is defined. In this paper, our objective is to developed the concept of extension and reduction on neutrosophic classical soft and apply to decision making problems. Therefore, we proposed expansion and reduction of the neutrosophic classical soft sets that are based on the linguistic modifiers. By using this new notions, we then developed a neutrosophic classical soft reduction method and present a reel example for the method.

2. Preliminaries

In this section, we will give some definitions and properties of neutrosophic set [36], neutrosophic classical set [23] and neutrosophic classical soft set [25] that are used in the remaining parts of the paper.

Definition 2.1. [36] A neutrosophic set A is characterized by a three membership function is called truth T_A, indeterminacy I_A and falsity F_A on the universe U as;

$$A = \left\{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in U \right\}$$

where $T_A, I_A, F_A : U \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.
From now on, set of all neutrosophic sets over U is denoted by $NS(U)$.

Definition 2.2. [23] Let U$\neq \emptyset$. A neutrosophic classical set K is presented by;

$$K = \left\{ \langle x, A_1, A_2, A_3 \rangle : x \in U \right\}$$

where $A_1, A_2, A_3 \subseteq U$. The set A_1, A_2 and A_3 is called the set of member, indeterminacy and non-members of K, respectively.
From now on, set of all neutrosophic classical sets over U is denoted by $NCS(U)$.

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Definition 2.3. [25] Let X be a parameter set and U be an initial universe. Then, a neutrosophic classical soft set (*ncs*-set) f over U is defined by

$$f = \{(x, \langle f_t(x), f_i(x), f_f(x)\rangle) : x \in X\}$$

where $f_t, f_i, f_f : X \to \mathscr{P}(U)$ and $\emptyset \subseteq f_t(x) \cup f_i(x) \cup f_f(x) \subseteq P(U)$.
From now on, set of all ncs-sets over U is denoted by $NCSS_X(U)$.

Example 2.4. In here we give a neutrosophic classical soft set f on universe $U = \{u_1, u_2, ..., u_{10}\}$ and set of parameters $X = \{x_1, x_2, x_3, x_4\}$ as;

$$\begin{aligned} f = \; & \{(x_1, \langle \{u_5, u_7, u_9, u_{10}\}, \{u_1, u_2, u_4\}, \\ & \{u_3, u_6, u_8\}\rangle), (x_2, \langle \{u_2, u_4, u_9, u_{10}\}, \\ & \{u_3, u_5, u_6\}, \{u_1, u_7, u_8\}\rangle), (x_3, \langle \{u_2, u_3, u_7, u_8\}, \\ & \{u_1, u_{10}\}, \{u_4, u_5, u_6, u_9\}\rangle), \\ & (x_4, \langle \{u_6, u_7, u_8, u_9, u_{10}\}, \{u_5\}, \{u_1, u_2, u_3, u_4\}\rangle)\} \end{aligned}$$

Definition 2.5. [25] Let $f, g \in NCSS_X(U)$. Then,

1. if, for all $x \in X$, $f_t(x) = \emptyset$, $f_i(x) = U$ and $f_f(x) = U$, then f is called null neutrosophic classical soft set and denoted by $\tilde{\phi}$.

2. if, for all $x \in X$, $f_t(x) = U$, $f_i(x) = \emptyset$ and $f_f(x) = \emptyset$, then f is called absolute neutrosophic classical soft set and denoted by $\tilde{U}$.

3. f is soft neutrosophic classical subset of g, denoted by $f \tilde{\subseteq} g$, if for all $x \in X$,
   $f_t(x) \subseteq g_t(x)$, $f_i(x) \supseteq g_i(x)$ and $f_f(x) \supseteq g_f(x)$.

4. f and g are equal, denoted by $f = g$, if $f \tilde{\subseteq} g$ and $g \tilde{\subseteq} f$.

5. soft neutrosophic classical union of f and g, denoted by $f \tilde{\cup} g$, and is defined as follow
$$f \tilde{\cup} g = \Big\{(x, \langle f_t(x) \cup g_t(x), f_i(x) \cap g_i(x), f_f(x) \cap g_f(x)\rangle) : x \in X\Big\}$$

6. soft neutrosophic classical intersection of f and g is, denoted by $f \tilde{\cap} g$, defined as follow
$$f \tilde{\cap} g = \Big\{(x, \langle f_t(x) \cap g_t(x), f_i(x) \cup g_i(x), f_f(x) \cup g_f(x)\rangle) : x \in X\Big\}$$

7. complement of f, denoted by $f^{\tilde{c}}$, is defined as follow
$$f^{\tilde{c}} = \Big\{(x, \langle f_f(x), (f_i(x))^c, f_t(x)\rangle) : x \in X\Big\}$$

8. OR-product of f and g, denoted by $f \vee g$, is defined by
$$f \vee g = \Big\{((x_1, x_2), \langle f_t(x_1) \cup g_t(x_2), f_i(x_1) \cap g_i(x_2), f_f(x_1) \cap g_f(x_2)\rangle) : x_1, x_2 \in X\Big\}$$

9. AND-product of f and g, denoted by $f \wedge g$, is defined by
$$f \wedge g = \Big\{((x_1, x_2), \langle f_t(x_1) \cap g_t(x_2), f_i(x_1) \cup g_i(x_2), f_f(x_1) \cup g_f(x_2)\rangle) : x_1, x_2 \in X\Big\}$$

Definition 2.6. [17] Linguistic modifiers are words like "not very", "more or less", "very", "very very", "extremely" which modify the meaning of parameters of a soft set. For example, "beautiful house" becomes "very beautiful house".
In this case, if x is a parameter and m is a linguistic modifier, then modified parameter x by m is denoted by $m(x)$.
In general, unless otherwise specified, the linguistic modifiers will be shown by the symbols m_k for all $k \in I_n = \{1, 2, ..., n\}$.

Example 2.7. [17] Let x_1 ="expensive", x_2 ="large", x_3 ="good" be three parameters of a neutrosophic classical soft set and m_1 ="very", m_2 ="very very", m_3 ="extremely" be three linguistic modifiers. Then, some modified parameters will be as follows;

$$\begin{aligned} m_1(x_1) &= \text{"very expensive"} \\ m_1(x_3) &= \text{"very good"} \\ m_2(x_3) &= \text{"very very good"} \\ m_3(x_2) &= \text{"extremely large"} \end{aligned}$$

Definition 2.8. [17] Let $m_1, m_2, ..., m_n$ be linguistic modifiers. Then,

$$M^n = \{m_i : i \in I_n\}$$

is called an *n-level linguistic modifier set*.

Definition 2.9. [17] Let X be a set of parameters and M^n be an n-level linguistic modifiers set. Then, an n-level modified set of set of parameter x is a set defined by

$$M_x^n = \{m(x) : m \in M^n\}, \text{ for all } x \in X.$$

Example 2.10. Assume that we have a 4-level linguistic modifier set as

$$M^4 = \{not\ very,\ very,\ very\ very, extremly\}$$

Then, for a parameter "expensive", the 4-level modified set of "expensive" will be written by

$$\begin{aligned} M^4_{expensive} = \; & \{not\ very\ expensive,\ very\ expensive, \\ & very\ very\ expensive, extremly\ expensive\} \end{aligned}$$

Definition 2.11. [17] Let X be a set of parameters and M^n be an n-level linguistic modifier set. Then, n-level modified set of X is a set defined by

$$M_X^n = \{m(x) : x \in X, m \in M^n\}$$

Note that if $M^n = \{m_j : j \in I_n\}$ and $X = \{x_k : k \in I_k\}$, then when no confusion arises we will briefly use x_{kj} instead of $m_j(x_k)$, that is

$$m_j(x_k) = x_{kj}$$

3. Expansions and reductions of neutrosophic classical soft sets

In this section, we apply the linguistic modifiers to neutrosophic classical soft set theory to make applications of neutrosophic classical soft sets more suitable.
Therefore, we give an expansion and reduction of a neutrosophic classical soft set by using linguistic modifiers. The definitions and applications on soft set defined in [17], we are extended the definitions and applications to the case of neutrosophic classical soft sets.

Definition 3.1. Let $f \in NCSS_X(U)$ and M_x^n be an n-level modified set of x for $x \in X$. Then, an n-level expansion of each element $(x, \langle f_t(x), f_i(x), f_f(x)\rangle)$ of f is a neutrosophic classical soft set over U is defined by

$$f^x : M_x^n \to NCS(U), \text{ for all } x \in X,$$

if following conditions hold;

1. $f_t^x(m_k(x)) \cap f_t^x(m_j(x)) = f_i^x(m_k(x)) \cap f_i^x(m_j(x)) = f_f^x(m_k(x)) \cap f_f^x(m_j(x)) = \emptyset$ for all $j, k \in I_n$, $k \neq j$,
2. $$\begin{array}{ll} \bigcup_{k\in I_n} f_t^x(m_k(x)) & = f_t(x), \\ \bigcup_{k\in I_n} f_i^x(m_k(x)) & = f_i(x), \\ \bigcup_{k\in I_n} f_f^x(m_k(x)) & = f_f(x) \end{array}$$

It is clear that $f^x \in NCSS_{M_x^n}(U)$, for every $x \in X$.
Note that from now on, we may not use "n-level" if does not cause confusion.

Example 3.2. If $U = \{u_1, u_2, ..., u_{12}\}$ is a set of houses and $X = \{x_1, x_2, x_3, x_4\}$ is a set of parameters where x_i, $(i = 1, 2, 3, 4)$, stand for the parameters "expensive", "large", "beautiful", and "in green surroundings" respectively. Then, we can consider a neutrosophic classical soft set f to describe the attractiveness of houses as follows:

$$\begin{array}{rcl} f & = & \{(x_1, \langle\{u_5, u_7, u_9, u_{10}, u_{11}\}, \{u_1, u_4, u_2\}, \{u_3, u_6, u_8, u_{12}\}\rangle), \\ & & (x_2, \langle\{u_2, u_4, u_9, u_{10}, u_{11}\}, \{u_3, u_5, u_6, u_{12}\}, \{u_1, u_7, u_8\}\rangle), \\ & & (x_3, \langle\{u_2, u_3, u_7, u_8\}, \{u_1, u_{10}\}, \{u_4, u_5, u_6, u_9, u_{11}, u_{12}\}\rangle), \\ & & (x_4, \langle\{u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}, \{u_5\}, \{u_1, u_2, u_3, u_4\}\rangle)\} \end{array}$$

Let us consider an element $(x_1, \langle\{u_5, u_7, u_9, u_{10}, u_{11}\}, \{u_1, u_4, u_2\}, \{u_3, u_6, u_8, u_{12}\}\rangle)$ of f and 3-level linguistic modifier set

$$M^3 = \{m_1, m_2, m_3\}$$

where m_j, $(j = 1, 2, 3)$, stand for the linguistic modifiers "less", "very" and "extremely" respectively. Then, 3-level modified set of x_1 = "expensive" can be written by

$$M_{x_1}^3 = \{x_{11}, x_{12}, x_{13}\}$$

where x_{11}="less expensive", x_{12}="very expensive", x_{13}="extremely expensive". Assume that

$$\begin{array}{rcl} f^{x_1}(x_{11}) & = & \langle\{u_5, u_{10}\}, \{u_2\}, \emptyset\rangle\} \\ f^{x_1}(x_{12}) & = & \langle\{u_7, u_9\}, \{u_4\}, \{u_3\}\rangle \\ f^{x_1}(x_{13}) & = & \langle\{u_{11}\}, \{u_1\}, \{u_6, u_8, u_{12}\}\rangle \end{array}$$

then, a 3-level expansion of $(x_1, \langle f_t(x_1), f_i(x_1), f_f(x_1)\rangle)$ can be written by

$$\begin{array}{rcl} f^{x_1} & = & \Big\{(x_{11}, \langle\{u_5, u_{10}\}, \{u_2\}, \emptyset\rangle), (x_{12}, \langle\{u_7, u_9\}, \{u_4\}, \\ & & \{u_3\}\rangle), (x_{13}, \{\langle\{u_{11}\}, \{u_1\}, \{u_6, u_8, u_{12}\}\rangle)\Big\} \end{array}$$

Note: It is well known that construction of a neutrosophic classical soft set may depend on experts. Therefore, each element of a neutrosophic classical soft set may have different expansions. If it is necessary to use more than one expansion of an element of a neutrosophic classical soft set, then to avoid the confusion different expansions will be indicated by a, b, c, ... as a superscript of the neutrosophic classical soft sets. In this case, the expansions of an element $(x, \langle f_t(x), f_i(x), f_f(x)\rangle)$ of neutrosophic classical soft set f can be denoted by f_a^x, f_b^x, f_c^x, ... and then they called a-expansion, b-expansion, c-expansion, ... , respectively. For example, let us consider Example 3.2, we can construct different expansions of $(x_1, \langle f_t(x_1), f_i(x_1), f_f(x_1)\rangle)$ as follows;

$$\begin{array}{rcl} f_a^{x_1} & = & \Big\{(x_{11}, \langle\{u_5, u_{10}\}, \{u_2\}, \emptyset\rangle), (x_{12}, \langle\{u_7\}, \{u_4\}, \\ & & \{u_3\}\rangle), (x_{13}, \{\langle\{u_9, u_{11}\}, \{u_1\}, \{u_6, u_8, u_{12}\}\rangle)\Big\} \end{array}$$

$$\begin{array}{rcl} f_b^{x_1} & = & \Big\{(x_{11}, \langle\emptyset, \{u_2\}, \{u_5, u_{10}\}\rangle), (x_{12}, \langle\{u_4\}, \{u_7\}, \\ & & \{u_3\}\rangle), (x_{13}, \{\langle\{u_8, u_{11}\}, \{u_1\}, \{u_6, u_9, u_{12}\}\rangle)\Big\} \end{array}$$

$$\begin{array}{rcl} f_c^{x_1} & = & \Big\{(x_{11}, \langle\{u_2, u_{10}\}, \{u_5\}, \emptyset\rangle), (x_{12}, \langle\{u_3, u_9\}, \{u_4\}, \\ & & \{u_7\}\rangle), (x_{13}, \{\langle\{u_{11}\}, \{u_1\}, \{u_6, u_8, u_{12}\}\rangle)\Big\} \end{array}$$

Definition 3.3. Let $f \in NCSS_X(U)$, $f^x \in NCSS_{M_x^n}(U)$. Then,

$$E_n(f) = \{f^x : x \in X\}$$

is called an n-level expansion family of f.

Now we give an example for n-level expansion family of f.

Example 3.4. Let us consider Example 3.2 where f^{x_1} is given and assume that in a similar way 3-level expansion of $(x_2, \langle f_t(x_2), f_i(x_2, f_f(x_2)\rangle)$, $(x_3, \langle f_t(x_3), f_i(x_3), f_f(x_3)\rangle)$ and $(x_4, \langle f_t(x_4), f_i(x_4), f_f(x_4)\rangle)$ can be written respectively by

$$\begin{array}{rcl} f^{x_2} & = & \Big\{(x_{21}, \langle\{u_9, u_{10}\}, \{u_3\}, \emptyset\rangle), (x_{22}, \langle\{u_2, u_{11}\}, \\ & & \{u_5\}, \{u_1\}\rangle), (x_{23}, \{\langle\{u_4\}, \{u_6, u_{12}\}, \{u_7, u_8\}\rangle)\rangle\Big\} \end{array}$$

$$\begin{array}{rcl} f^{x_3} & = & \Big\{(x_{31}, \langle\{u_2\}, \emptyset, \{u_4, u_5, u_{12}\}\rangle), (x_{32}, \langle\{u_3, u_7\}, \\ & & \{u_{10}\}, \{u_6, u_{11}\}\rangle), (x_{33}, \{\langle\{u_8\}, \{u_1\}, \{u_9\}\rangle)\Big\} \end{array}$$

$$f^{x_4} = \Big\{(x_{41}, \langle\{u_6,u_7\},\emptyset,\{u_2\}\rangle),(x_{42},\langle\{u_8,u_9\},\emptyset,\{u_3\}\rangle),(x_{43},\{\langle\{u_{10},u_{11},u_{12}\},\{u_5\},\{u_1,u_4\}\rangle)\Big\}$$

Then,

$$\begin{aligned} E_3(f) &= \Big\{f^{x_1}, f^{x_2}, f^{x_3}, f^{x_4}\Big\} \\ &= \Big\{\{(x_{11},\langle\{u_5,u_{10}\},\{u_2\},\emptyset\rangle),(x_{12},\langle\{u_7,u_9\},\{u_4\},\{u_3\}\rangle),(x_{13},\{\langle\{u_{11}\},\{u_1\},\{u_6,u_8,u_{12}\}\rangle)\Big\} \\ &\quad \{(x_{21},\langle\{u_9,u_{10}\},\{u_3\},\emptyset\rangle),(x_{22},\langle\{u_2,u_{11}\},\{u_5\},\{u_1\}\rangle),(x_{23},\{\langle\{u_4\},\{u_6,u_{12}\},\{u_7,u_8\}\rangle)\Big\}, \\ &\quad \{(x_{31},\langle\{u_2\},\emptyset,\{u_4,u_5,u_{12}\}\rangle),(x_{32},\langle\{u_3,u_7\},\{u_{10}\},\{u_6,u_{11}\}\rangle),(x_{33},\{\langle\{u_8\},\{u_1\},\{u_9\}\rangle)\Big\}, \\ &\quad \{(x_{41},\langle\{u_6,u_7\},\emptyset,\{u_2\}\rangle),(x_{42},\langle\{u_8,u_9\},\emptyset,\{u_3\}\rangle),(x_{43},\{\langle\{u_{10},u_{11},u_{12}\},\{u_5\},\{u_1,u_4\}\rangle)\Big\}\Big\} \end{aligned}$$

is a 3-level expansion family of f.

Definition 3.5. Let $f,g \in NCSS_X(U)$ and $f^x, g^x \in NCSS_{M_x^n}(U)$ for $x \in X$. Then, the expansion of each element of the union of neutrosophic classical soft sets are defined by

$$(f\tilde{\cup}g)^x = \{(m(x)),\langle(f\tilde{\cup}g)_t^x(m(x)),(f\tilde{\cap}g)_i^x(m(x)),(f\tilde{\cap}g)_f^x(m(x))\rangle : m(x) \in M_x^n\}$$

where

$$\begin{aligned} (f\tilde{\cup}g)_t^x(m(x)) &= f_t^x(m(x)) \cup g_t^x(m(x)), \quad x \in X, \\ (f\tilde{\cup}g)_i^x(m(x)) &= f_i^x(m(x)) \cap g_i^x(m(x)), \quad x \in X, \\ (f\tilde{\cup}g)_f^x(m(x)) &= f_f^x(m(x)) \cap g_f^x(m(x)), \quad x \in X, \end{aligned}$$

Definition 3.6. Let $f,g \in NCSS_X(U)$ and $f^x, g^x \in NCSS_{M_x^n}(U)$ for $x \in X$. Then, the expansion of each element of the intersection of neutrosophic classical soft sets are defined by

$$(f\tilde{\cap}g)^x = \{(m(x)),\langle(f\tilde{\cap}g)_t^x(m(x)),(f\tilde{\cup}g)_i^x(m(x)),(f\tilde{\cup}g)_f^x(m(x))\rangle : m(x) \in M_x^n\}$$

where

$$\begin{aligned} (f\tilde{\cap}g)_t^x(m(x)) &= f_t^x(m(x)) \cap g_t^x(m(x)), \quad x \in X, \\ (f\tilde{\cap}g)_i^x(m(x)) &= f_i^x(m(x)) \cup g_i^x(m(x)), \quad x \in X, \\ (f\tilde{\cap}g)_f^x(m(x)) &= f_f^x(m(x)) \cup g_f^x(m(x)), \quad x \in X, \end{aligned}$$

Definition 3.7. Let $f \in NCSS_X(U)$ and $f^x \in NCSS_{M_x^n}(U)$ for $x \in X$. Then, the expansion of each element of the complement of neutrosophic classical soft sets are defined by

$$(f^{\tilde{c}})^x = \{(m(x)),\langle(f^{\tilde{c}})_t^x(m(x)),(f^{\tilde{c}})_i^x(m(x)),(f^{\tilde{c}})_f^x(m(x))\rangle : m(x) \in M_x^n\}$$

where

$$\begin{aligned} (f^{\tilde{c}})_t^x(m(x)) &= f_f^x(m(x)), \quad x \in X. \\ (f^{\tilde{c}})_i^x(m(x)) &= (f_i^x)^c(m(x)), \quad x \in X. \\ (f^{\tilde{c}})_f^x(m(x)) &= f_t^x(m(x)), \quad x \in X. \end{aligned}$$

Example 3.8. Let us consider Example 3.2 where the 3-level expansion of $(x_1, \langle f_t(x_1), f_i(x_1), f_f(x_1)\rangle)$ is given as

$$f^{x_1} = \Big\{(x_{11},\langle\{u_5,u_{10}\},\{u_2\},\emptyset\rangle),(x_{12},\langle\{u_7,u_9\},\{u_4\},\{u_3\}\rangle),(x_{13},\{\langle\{u_{11}\},\{u_1\},\{u_6,u_8,u_{12}\}\rangle)\Big\}$$

and assume that another neutrosophic classical soft set g is given as follows

$$\begin{aligned} g = \ &\{(x_1,\langle\{u_1,u_6,u_9,u_7,u_{10},u_{11}\},\{u_2,u_4\},\{u_3,u_5,u_8,u_{12}\}\rangle),(x_2,\langle\{u_1,u_7,u_9,u_{10},u_{11}\},\{u_3,u_5,u_6,u_{12}\},\{u_2,u_4,u_8\}\rangle), \\ &(x_3,\langle\{u_2,u_3,u_7,u_9\},\{u_1,u_8,u_{11}\},\{u_4,u_5,u_6,u_{10},u_{12}\}\rangle),(x_4,\langle\{u_6,u_7,u_8,u_9,u_{10},u_{11},u_{12}\},\{u_5\},\{u_1,u_2,u_3,u_4\}\rangle)\} \end{aligned}$$

If 3-level expansion of $(x_1, \langle g_t(x_1), g_i(x_1), g_f(x_1)\rangle)$ is given as

$$g^{x_1} = \Big\{(x_{11},\langle\{u_{10},u_{11}\},\{u_2\},\emptyset\rangle),(x_{12},\langle\{u_1,u_6\},\{u_4\},\{u_3\}\rangle),(x_{13},\{\langle\{u_9,u_7\},\emptyset,\{u_5,u_8,u_{12}\}\rangle)\Big\}$$

then, we can construct $(f\tilde{\cup}g)^{x_1}$. We know that

$$(f\tilde{\cup}g)^{x_1}(m(x_1)) = f^{x_1}(m(x_1))\tilde{\cup}g^{x_1}(m(x_1)), \text{ for all } m(x_1) \in M_{x_1}^3$$

which gives

$$\begin{aligned} (f\tilde{\cup}g)^{x_1}(x_{11}) &= f^{x_1}(x_{11})\tilde{\cup}g^{x_1}(x_{11}) \\ &= \langle\{u_5,u_{10},u_{11}\},\{u_2\},\emptyset\rangle \end{aligned}$$

$$\begin{aligned} (f\tilde{\cup}g)^{x_1}(x_{12}) &= f^{x_1}(x_{12})\tilde{\cup}g^{x_1}(x_{12}) \\ &= \langle\{u_1,u_6,u_7,u_9\},\{u_4\},\{u_3\}\rangle \end{aligned}$$

$$\begin{aligned} (f\tilde{\cup}g)^{x_1}(x_{13}) &= f^{x_1}(x_{13})\tilde{\cup}g^{x_1}(x_{13}) \\ &= \langle\{u_7,u_9,u_{11}\},\emptyset,\{u_8,u_{12}\}\rangle \end{aligned}$$

Hence, we get

$$(f\tilde{\cup}g)^{x_1} = \Big\{(x_{11},\langle\{u_5,u_{10},u_{11}\},\{u_2\},\emptyset\rangle),(x_{12},\langle\{u_1,u_6,u_7,u_9\},\{u_4\},\{u_3\}\rangle),(x_{13},\{\langle\{u_7,u_9,u_{11}\},\emptyset,\{u_8,u_{12}\}\rangle)\Big\}$$

By the similar way we get

$$(f\tilde{\cap}g)^{x_1} = \Big\{(x_{11},\langle\{u_{10}\},\{u_2\},\emptyset\rangle),(x_{12},\langle\emptyset,\{u_4\},\{u_3\}\rangle),(x_{13},\{\langle\emptyset,\{u_1\},\{u_5,u_6,u_8,u_{12}\}\rangle)\Big\}$$

and

$$(f^{\tilde{c}})^{x_1} = \Big\{(x_{11},\langle\emptyset,\{u_1,u_3,u_4,u_5,u_6,u_7,u_8,u_9,u_{10},u_{11},u_{12}\},\{u_5,u_{10}\}\rangle),(x_{12},\langle\{u_3\},\{u_1,u_2,u_3,u_5,u_6,u_7,u_8,u_9,u_{10},u_{11},u_{12}\},\{u_7,u_9\}\rangle),(x_{13},\{\langle\{u_6,u_8,u_{12}\},\{u_2,u_3,u_4,u_5,u_6,u_7,u_8,u_9,u_{10},u_{11},u_{12}\},\{u_{11}\}\rangle)\Big\}$$

Theorem 3.9. *Let $f,g \in NCSS_X(U)$ and $f^x, g^x \in NCSS_{M_x^n}(U)$ for $x \in X$. Then, for every $x \in X$, followings hold;*

1. $(f\tilde{\cup}g)^x = f^x\tilde{\cup}g^x$
2. $(f\tilde{\cap}g)^x = f^x\tilde{\cap}g^x$
3. $(f^{\tilde{c}})^x = (f^x)^{\tilde{c}}$

Definition 3.10. Let $f \in NCSS_X(U)$, $f^x \in NCSS_{M_x^n}(U)$ and M_X^n be an n-level modified set of X. Then, an n-level expansion of f is a neutrosophic classical soft set over U, is denoted by $f \circledR f^X$, is defined by

$$f^X : M_X^n \longrightarrow NCS(U), \quad f^X(m(x)) = f^x(m(x))$$

It is clear that $f^X \in NCSS_{M_X^n}(U)$.
From now on, n-level expansions of neutrosophic classical soft sets $f,g,h,...$ will be denoted by $f^X, g^X, h^X, ...$, respectively.

Proposition 3.11. *Let $f^x \in NCSS_{M_x^n}(U)$ and $f^X \in NCSS_{M_X^n}(U)$. Then,*

$$f^X = \bigcup_{x\in X} f^x$$

Proof: It is easy from Definition 3.10.

Example 3.12. Let us consider Example 3.2 and 3.4. Then, a 3-level expansion of f can be written by

$$f^{x_1} = \Big\{(x_{11},\langle\{u_5,u_{10}\},\{u_2\},\emptyset\rangle),(x_{12},\langle\{u_7,u_9\},\{u_4\},\{u_3\}\rangle),(x_{13},\{\langle\{u_{11}\},\{u_1\},\{u_6,u_8,u_{12}\}\rangle)\Big\}$$

and

$$f^{x_2} = \Big\{(x_{21},\langle\{u_9,u_{10}\},\{u_3\},\emptyset\rangle),(x_{22},\langle\{u_2,u_{11}\},\{u_5\},\{u_1\}\rangle),(x_{23},\{\langle\{u_4\},\{u_6,u_{12}\},\{u_7,u_8\}\rangle)\rangle\Big\}$$

$$f^{x_3} = \Big\{(x_{31},\langle\{u_2\},\emptyset,\{u_4,u_5,u_{12}\}\rangle),(x_{32},\langle\{u_3,u_7\},\{u_{10}\},\{u_6,u_{11}\}\rangle),(x_{33},\{\langle\{u_8\},\{u_1\},\{u_9\}\rangle)\Big\}$$

$$f^{x_4} = \Big\{(x_{41},\langle\{u_6,u_7\},\emptyset,\{u_2\}\rangle),(x_{42},\langle\{u_8,u_9\},\emptyset,\{u_3\}\rangle),(x_{43},\{\langle\{u_{10},u_{11},u_{12}\},\{u_5\},\{u_1,u_4\}\rangle)\Big\}$$

It can also be obtained easily by using above the Proposition 3.11 as

$$f^X = f^{x_1} \cup f^{x_2} \cup f^{x_3} \cup f^{x_4}$$

Theorem 3.13. *Let $f,g \in NCSS_X(U)$ and $f^X, g^X \in NCSS_{M_x^n}(U)$. If $f \circledR f^X$ and $g \circledR g^X$, then followings hold;*

1. $(f\tilde{\cup}g)\circledR(f^X\tilde{\cup}g^X)$
2. $(f\tilde{\cap}g)\circledR(f^X\tilde{\cap}g^X)$
3. $f^{\tilde{c}}\circledR(f^X)^{\tilde{c}}$

Definition 3.14. Let U be a universal set. Then, for $n \in N$, n-level choice set is a set defined by

$$L_n = \{l_i^n : i, \in I_n\}$$

where $l_i^n = (c_{i1}, c_{i2}, ..., c_{in})$ is an n-tuple such that for $j \in I_n$

$$c_{ij} = \begin{cases} \tilde{U}, & j = i \\ \tilde{\phi}, & j \neq i \end{cases}$$

such that $\tilde{U} = \langle U, \emptyset, \emptyset\rangle$ and $\tilde{\phi} = \langle\emptyset, U, U\rangle$.

Therefore, the 1, 2,..., n-level choice set can be written as, respectively,

$$\begin{array}{lcl} L_1 & = & \{(\tilde{U})\} \\ L_2 & = & \{(\tilde{U},\tilde{\phi}),(\tilde{\phi},\tilde{U})\} \\ \vdots & = & \vdots \\ L_n & = & \{(\tilde{U},\tilde{\phi},...,\tilde{\phi}),(\tilde{\phi},\tilde{U},...,\tilde{\phi}),(\tilde{\phi},\tilde{\phi},...,\tilde{U})\} \end{array}$$

Definition 3.15. Let X be a set of parameters, L_n be the n-level choice set. Then, n-level choice function is defined by

$$\alpha^n : X \to L_n, \quad \alpha^n(x) = l_k^n$$

where l_k^n is one of $l_1^n, l_2^n, ..., l_n^n$ chosen by a decision maker according to the situation of $x \in X$. The $\alpha^n(x)$ is called n-level choice value of $x \in X$.

Example 3.16. Let us consider $X = \{x_1, x_2, x_3, x_4\}$ as a set of parameters and L_3 be the 3-level choice set. Then, 3-level choice values for x_i, $i \in I_4$, may be chosen respectively by

$$\begin{array}{l} \alpha^3(x_1) = l_1^3 = (c_{11},c_{12},c_{13}) = (\tilde{U},\tilde{\phi},\tilde{\phi}) \\ \alpha^3(x_2) = l_2^3 = (c_{21},c_{22},c_{23}) = (\tilde{\phi},\tilde{U},\tilde{\phi}) \\ \alpha^3(x_4) = l_3^3 = (c_{31},c_{32},c_{33}) = (\tilde{\phi},\tilde{\phi},\tilde{U}) \\ \alpha^3(x_4) = l_2^3 = (c_{21},c_{22},c_{23}) = (\tilde{\phi},\tilde{U},\tilde{\phi}) \end{array}$$

Definition 3.17. Let $f \in NCSS_X(U)$, $f^x \in NCSS_{M_x^n}(U)$, $E_n(f)$ be an n-level expansion family of f and α^{nt} and t. n-level choice function. Then, a reduction function of f, for $x \in X$, is defined by

$$\alpha^n(f^x) = \{(m_j(x),(\alpha^n(x)\tilde{\cap}f^x(m_j(x))) : j \in I_n, x \in X\}$$

where the value $\alpha^n(f^x)$ is called α^n-reduction of $(x,\langle f_t(x), f_i(x), f_f(x)\rangle)$ for $x \in X$.

Example 3.18. Let us consider $E_n(f)$ which is given in Example 3.4 and α^3 which is given in Example 3.16. Since the expansions are 3-level, we have to use the 3-level choice set L_3. Then, α^3-reduction of $(x_1,\langle f_t(x_1), f_i(x_1), f_f(x_1)\rangle)$ is computed by

$$\begin{array}{rcl} \alpha^3(f^{x_1}) & = & \{(m_j(x_1),(\alpha^3(x_1)\tilde{\cap}f^{x_1}(m_j(x_1))) : j \in I_3\} \\ & = & \{(x_{1j},(\alpha^3(x_1)\tilde{\cap}f^{x_1}(x_{1j})) : j \in I_3\}, \\ & & \text{since } m_j(x_i) = x_{ij} \\ & = & \{(x_{1j}, l_1^3\tilde{\cap}f^{x_1}(x_{1j})) : j \in I_3\}, \\ & & \text{since } \alpha^3(x_1) = l_1^3 \\ & = & \{(x_{1j}, c_{1j}\tilde{\cap}f^{x_1}(x_{1j})) : j \in I_3\}, \\ & & \text{since } l_1^3 = c_{1j} \\ & = & \{(x_{11}, c_{11}\tilde{\cap}f^{x_1}(x_{11})),(x_{12}, c_{12}\tilde{\cap} \\ & & f^{x_1}(x_{12})),(x_{13}, c_{13}\tilde{\cap}f^{x_1}(x_{13}))\} \\ & = & \{(x_{11},\tilde{U}\tilde{\cap}f^{x_1}(x_{11})),(x_{12},\tilde{\phi}\tilde{\cap} \\ & & f^{x_1}(x_{12})),(x_{13},\tilde{\phi}\tilde{\cap}f^{x_1}(x_{13}))\} \\ & = & \{(x_{11}, f^{x_1}(x_{11}))\} \\ & = & \{(x_{11},\langle\{u_5,u_{10}\},\{u_2\},\emptyset\rangle)\} \end{array}$$

and also by the similar way, α^3-reduction of $(x_i, \langle f_t(x_i), f_i(x_i), f_f(x_i)\rangle)$, $i = 2,3,4$, are computed as;

$$\alpha^3(f^{x_2}) = \{(x_{22}, \langle\{u_2, u_{11}\}, \{u_5\}, \{u_1\}\rangle)\}$$

$$\alpha^3(f^{x_3}) = \{(x_{32}, \langle\{u_8\}, \{u_1\}, \{u_9\}\rangle)\}$$

$$\alpha^3(f^{x_4}) = \{(x_{42}, \langle\{u_8, u_9\}, \emptyset, \{u_3\}\rangle)\}$$

where $\alpha^3(x_2) = l_2^3$, $\alpha^3(x_3) = l_3^3$, and $\alpha^3(x_4) = l_2^3$.

Proposition 3.19. *Let $f^x, g^x \in NCSS_{M_x^n}(U)$ and $\alpha^n{}_x$ be a reduction function for $x \in X$. Then, for $x \in X$,*

1. $\alpha^n(f^x \tilde{\cup} g^x) = \alpha^n(f^x) \tilde{\cup} \alpha^n(g^x)$
2. $\alpha^n(f^x \tilde{\cap} g^x) = \alpha^n(f^x) \tilde{\cap} \alpha^n(g^x)$
3. $\alpha^n((f^{\tilde{c}})^x) = \alpha^n((f^{\tilde{c}})^x)$

Proof: *1.* Since

$$f^x \tilde{\cup} g^x = \{(m(x)), \langle f_t^x(m(x)) \cup g_t^x(m(x)), f_i^x(m(x)) \cap g_i^x(m(x)), f_f^x(m(x)) \cap g_f^x(m(x))\rangle) : m(x) \in M_x^n\}$$

by using above Definition 3.17 we can write

$$\alpha^n(f^x \tilde{\cup} g^x) = \{(m_j(x), \alpha^n(x) \tilde{\cap} (f^x \tilde{\cup} g^x)(m_j(x))) : j \in I_n\}$$

Since

$$\alpha^n(f^x) = \{(m_j(x), \alpha^n(x) \tilde{\cap} f^x(m_j(x))) : j \in I_n\},$$
$$\alpha^n(g^x) = \{(m_j(x), \alpha^n(x) \tilde{\cap} g^x(m_j(x))) : j \in I_n\}$$

by using definition of union of neutrosophic classical soft sets we can write

$$\alpha^n(f^x) \tilde{\cup} \alpha^n(g^x) = \{(m_j(x), (\alpha^n(x) \tilde{\cap} f^x(m_j(x))) \tilde{\cup} (\alpha^n(x) \tilde{\cap} g^x(m_j(x))) : j \in I_n\}$$

From (3) and (3), we get that $\alpha^n(f^x \tilde{\cup} g^x) = \alpha^n(f^x) \tilde{\cup} \alpha^n(g^x)$

The proofs of *2.* and *3.* can be made by the similar way.

3. Since $(f^{\tilde{c}})^x = (f^x)^{\tilde{c}}$ we have
$\alpha^n((f^{\tilde{c}})^x) = \{(m(x)), \langle (f^{\tilde{c}})^x(m(x)), f_i^{\tilde{c}})^x(m(x)), f_f^{\tilde{c}})^x(m(x))\rangle) : m(x) \in M_x^n\}$
by using Theorem 3.9 we can write

$$\begin{aligned} \alpha^n((f^{\tilde{c}})^x) &= \{(m(x)), \langle (f^{\tilde{c}})^x(m(x)), f_i^{\tilde{c}})^x(m(x)) \\ &\quad , f_f^{\tilde{c}})^x(m(x))\rangle) : m(x) \in M_x^n\} \\ &= \{(m(x)), \langle (f^x)^{\tilde{c}}(m(x)), (f_i^x)^{\tilde{c}}(m(x)), \\ &\quad (f_f^x)^{\tilde{c}}(m(x))\rangle) : m(x) \in M_x^n\} \\ = \alpha^n((f^{\tilde{c}})^x) \end{aligned}$$

Definition 3.20. Let $f \in NCSS_X(U)$ and $\alpha^n(f^x)$ be an α^n-reduction of $(x, \langle f(x), f_i(x), f_f(x)\rangle)$ for $x \in X$. Then,

$$f_{\alpha^n} = \{\alpha^n(f^x) : x \in X\}$$

is called an α^n-reduction of neutrosophic classical soft set f.

Example 3.21. Let us consider Example 3.18. Then, an α^3-reduction of f can be written by

$$f_{\alpha^3} = \Big\{\{(x_{11}, \langle\{u_5, u_{10}\}, \{u_2\}, \emptyset\rangle)\}, \{(x_{22}, \langle \{u_2, u_{11}\}, \{u_5\}, \{u_1\}\rangle)\}, \{(x_{33}, \langle\{u_8\}, \{u_1\}, \{u_9\} \rangle)\}, \{(x_{42}, \langle\{u_8, u_9\}, \emptyset, \{u_3\}\rangle)\}\Big\}$$

Proposition 3.22. *Let $f \in NCSS_X(U)$, $\alpha^n(f^x)$ be an α^n-reduction of $(x, \langle f(x), f_i(x), f_f(x)\rangle)$ for $x \in X$ and f_{α^n} be an α^n-*reduction *of neutrosophic classical soft set f. Then,*

$$f_{\alpha^n} = \bigcup_{x \in X} \alpha^n(f^x)$$

Proof: It is easy from Definition 3.20.

Definition 3.23. Let $f \in NCSS_X(U)$. Then,

$$\mathbb{U}(f) = \tilde{\bigcup_{x \in X}} f(x)$$

is a set called *union set* of neutrosophic classical soft set f.

$$\mathbb{I}(f) = \tilde{\bigcap_{x \in X}} f(x)$$

is a set called *intersection set* of neutrosophic classical soft set f.

Example 3.24. Let

$$\begin{aligned} f = \{ & (x_1, \langle\{u_5, u_7, u_9, u_{10}, u_{11}\}, \{u_1, u_4, u_2\}, \{u_3, u_6, u_8, u_{12}\}\rangle), \\ & (x_2, \langle\{u_2, u_4, u_9, u_{10}, u_{11}\}, \{u_3, u_5, u_6, u_{12}\}, \{u_1, u_7, u_8\}\rangle), \\ & (x_3, \langle\{u_2, u_3, u_7, u_8\}, \{u_1, u_{10}\}, \{u_4, u_5, u_6, u_9, u_{11}, u_{12}\}\rangle), \\ & (x_4, \langle\{u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}, \{u_5\}, \{u_1, u_2, u_3, u_4\}\rangle)\} \end{aligned}$$

be a neutrosophic classical soft set over $U = \{u_1, u_2, ..., u_{12}\}$. Then, the union set of f is written as

$$\mathbb{U}(f) = \{\langle\{u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}, \emptyset, \emptyset\rangle\}$$

the intersection set of f is written as

$$\mathbb{I}(f) = \{\langle \emptyset, \{u_1, u_2, u_3, u_4, u_5, u_6, u_9, u_{12}\}, \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{11}, u_{12}\}\rangle\}$$

Proposition 3.25. *Let $f, g \in NCSS_X(U)$. Then,*

1. $\mathbb{I}(f \tilde{\cup} g) = \mathbb{I}(f) \tilde{\cup} \mathbb{I}(g)$
2. $\mathbb{I}(f \tilde{\cap} g) = \mathbb{I}(f) \tilde{\cap} \mathbb{I}(g)$
3. $\mathbb{U}(f \tilde{\cup} g) = \mathbb{U}(f) \tilde{\cup} \mathbb{U}(g)$
4. $\mathbb{U}(f \tilde{\cap} g) = \mathbb{U}(f) \tilde{\cap} \mathbb{U}(g)$
5. $\mathbb{U}(f) \tilde{\subseteq} \tilde{U}$
6. $\mathbb{I}(f) \tilde{\subseteq} \tilde{U}$
7. $\mathbb{I}(f) \tilde{\subseteq} \mathbb{U}(f)$

Proof: *1.* For all $x \in X$,

$$\begin{aligned}\mathbb{I}(f\tilde{\cup}g) &= \tilde{\bigcap_{x\in X}}(f\tilde{\cup}g)(x)\\ &= \tilde{\bigcap_{x\in X}}(f(x)\tilde{\cup}g(x))\\ &= (\tilde{\bigcap_{x\in X}}f(x))\tilde{\cup}(\tilde{\bigcap_{x\in X}}g(x))\\ &= \mathbb{I}(f)\tilde{\cup}\mathbb{I}(g)\end{aligned}$$

The other proofs can be made similarly.

Definition 3.26. Let $f \in NCSS_X(U)$ and f_{α^n} be an α^n-reduction of f. Then, the union set of f_{α^n} be $R = \{(\langle R(x_{ij}), R_i(x_{ij}), R_f(x_{ij})\rangle) : x_{ij} = m_j(x_i), x_i \in X\}$. Then,

$$Red(R) = \bigcup_{x_i\in X}\{R_i(x_{ij}) \cup (R_i(x_{ij}) \cap R_f(x_{ij}))\}$$

is a set called reduced set of neutrosophic classical soft set f.

Proposition 3.27. *Let $f \in NCSS_X(U)$ and $f_{\alpha^n} \in NCSS_{M_x^n}(U)$. Then, $\tilde{\emptyset}\tilde{\subseteq}Red(R)\tilde{\subseteq}\tilde{U}$.*

Now, we construct an algorithm of neutrosophic classical soft reduction method as follows:

Algorithm

1. Construct a neutrosophic classical soft set $f \in NCSS_X(U)$.
2. Input an n-level linguistic modifier set M^n.
3. Compute M_x^n for each $x \in X$.
4. Compute f^x for each $x \in X$ to obtain $E_4(f)$.
5. Input an n-level choice set C_n.
6. Compute $\alpha^n(x)$ for each $x \in X$
7. Compute $\alpha^n(f^x)$ for each $x \in X$.
8. Compute f_{α^n}.
9. Find the reduced set $Red(R)$.

An Application of Soft Reduction Method

Assume that a high school received 12 teacher for a position. There is a decision maker who wants to interview some of the suitable candidates instead of all of the candidates. Therefore, by using the neutrosophic classical soft reduction method the number of candidates are reduced to a suitable subset of candidates. Let $U = \{u_1, u_2, ..., u_{12}\}$ be the set of teachers which may be characterized by a set of parameters $X = \{x_1, x_2, x_3, x_4\}$. For $i = 1,2,3,4$, the parameters x_i stand for "work experience", "computer knowledge", “young age" and “foreign language" respectively. Now, by using the algorithm of neutrosophic classical soft reduction method we can solve this problem step by step as follows:

Step 1: The decision maker constructs a neutrosophic classical soft set f over U according to the CV of teachers,

$$\begin{aligned} f = \;& \{(x_1, \langle\{u_5,u_7,u_9,u_{10},u_{11}\},\{u_1,u_4,u_2\},\\ & \{u_3,u_6,u_8,u_{12}\}\rangle),(x_2,\langle\{u_2,u_4,u_9,u_{10},u_{11}\},\\ & \{u_3,u_5,u_6,u_{12}\},\{u_1,u_7,u_8\}\rangle),(x_3,\langle\{u_2,u_3,u_7,u_8\},\\ & \{u_1,u_{10}\},\{u_4,u_5,u_6,u_9,u_{11},u_{12}\}\rangle),(x_4,\langle\\ & \{u_6,u_7,u_8,u_9,u_{10},u_{11},u_{12}\},\{u_5\},\{u_1,u_2,u_3,u_4\}\rangle)\}\end{aligned}$$

Step 2: The decision maker consider a 3-level linguistic modifier set as

$$M^3 = \{m_1 = \text{"not very"}, m_2 = \text{"very"}, m_3 = \text{“quite"}\}$$

Step 3: For all $i \in I_4$, 3-level linguistic modified set of x_i can be obtained respectively as

$$\begin{aligned} M_{x_1}^3 = \;& \{x_{11},x_{12},x_{13}\}, M_{x_2}^3 = \{x_{21},x_{22},x_{23}\}, M_{x_3}^3 =\\ & \{x_{31},x_{32},x_{33}\}, M_{x_4}^3 = \{x_{41},x_{42},x_{43}\}\end{aligned}$$

Step 4: Assume that the decision maker can construct 3-level expansion set of $(x_i, \langle f(x_i), f_i(x_i), f_f(x_i)\rangle)$, f^{x_i}, for $i \in I_4$, respectively as,

$$\begin{aligned} f^{x_1} = \;& \Big\{(x_{11},\langle\{u_5,u_{10}\},\{u_2\},\emptyset\rangle),(x_{12},\langle\{u_7,u_9\},\{u_4\},\\ & \{u_3\}\rangle),(x_{13},\{\langle\{u_{11}\},\{u_1\},\{u_6,u_8,u_{12}\}\rangle)\Big\}\\ f^{x_2} = \;& \Big\{(x_{21},\langle\{u_9,u_{10}\},\{u_3\},\emptyset\rangle),(x_{22},\langle\{u_2,u_{11}\},\\ & \{u_5\},\{u_1\}\rangle),(x_{23},\{\langle\{u_4\},\{u_6,u_{12}\},\{u_7,u_8\}\rangle)\rangle\Big\}\\ f^{x_3} = \;& \Big\{(x_{31},\langle\{u_2\},\emptyset,\{u_4,u_5,u_{12}\}\rangle),(x_{32},\langle\{u_3,u_7\},\\ & \{u_{10}\},\{u_6,u_{11}\}\rangle),(x_{33},\{\langle\{u_8\},\{u_1\},\{u_9\}\rangle)\Big\}\\ f^{x_4} = \;& \Big\{(x_{41},\langle\{u_6,u_7\},\emptyset,\{u_2\}\rangle),(x_{42},\langle\{u_8,u_9\},\\ & \emptyset,\{u_3\}\rangle),(x_{43},\{\langle\{u_{10},u_{11},u_{12}\},\\ & \{u_5\},\{u_1,u_4\}\rangle)\Big\}\end{aligned}$$

Now, an 3-level expansion family of f can be written by

$$E_4(f) = \Big\{f^{x_1}, f^{x_2}, f^{x_3}, f^{x_4}\Big\}$$

Step 5: The decision maker inputs a 3-level choice set L_3 as

$$L_3 = \{(\tilde{U},\tilde{\phi},\tilde{\phi}),(\tilde{\phi},\tilde{U},\tilde{\phi}),(\tilde{\phi},\tilde{\phi},\tilde{U})\}$$

Step 6: According to the 2. 3-level choice set a choice values $\alpha_2^3(x_i)$ can be computed for each x_i, $i \in I_4$, respectively,

$$\begin{aligned}\alpha_1^3(x_1) &= (\tilde{U},\tilde{\phi},\tilde{\phi}),\\ \alpha_3^3(x_2) &= (\tilde{\phi},\tilde{\phi},\tilde{U}),\\ \alpha_2^3(x_3) &= (\tilde{\phi},\tilde{U},\tilde{\phi}),\\ \alpha_1^3(x_4) &= (\tilde{U},\tilde{\phi},\tilde{\phi}),\end{aligned}$$

Step 7: By using the reduction function $\alpha^3(f^{x_i})$ can be computed for each x_i, $i \in I_4$, respectively,

$$\begin{aligned}\alpha^3(f^{x_1}) &= \{(x_{11},\langle\{u_5,u_{10}\},\{u_2\},\emptyset\rangle)\}\\ \alpha^3(f^{x_2}) &= \{(x_{23},\langle\{u_4\},\{u_6,u_{12}\},\{u_7,u_8\}\rangle)\}\\ \alpha^3(f^{x_3}) &= \{(x_{32},\langle\{u_3,u_7\},\{u_{10}\},\{u_6,u_{11}\}\rangle)\}\\ \alpha^3(f^{x_4}) &= \{(x_{41},\langle\{u_6,u_7\},\emptyset,\{u_2\}\rangle)\}\end{aligned}$$

Step 8: α^3-reduction of f can be computed by

$$f_{\alpha_2^3} = \Big\{\{(x_{11},\langle\{u_5,u_{10}\},\{u_2\},\emptyset\rangle)\},\{(x_{23},\langle\{u_4\},\{u_6,u_{12}\},\{u_7,u_8\}\rangle)\},\{(x_{32},\langle\{u_3,u_7\},\{u_{10}\},\{u_6,u_{11}\}\rangle)\},\{(x_{41},\langle\{u_6,u_7\},\emptyset,\{u_2\}\rangle)\}\Big\}$$

Step 9: Finally, the reduced set $Red(R)$ can be computed by

$$Red(R) = \mathbb{U}(f_{\alpha_2^3}) = \{u_3,u_4,u_5,u_6,u_7,u_{10}\}$$

which is a suitable subset of the set of alternatives U. In this problem, 12 applications is reduced to 6 applicants by the neutrosophic classical soft reduction method. So, decision maker interviews only 6 applicants instead of 12.

4. Conclusion

In this work, as a new notion on neutrosophic classical soft set theory, we first defined expansion and reduction of the neutrosophic classical soft sets based on linguistic modifiers. Using the expansion and reduction we then gave neutrosophic classical soft reduction method. The aim of this method is to obtain a subset of the set of alternatives through a decision maker. With this method, it is possible to reduce the number of alternatives significantly.

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