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# Factors Revealed while Posing Mathematical Modelling Problems by Mathematics Student Teachers

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**Abstract:** The purpose of this study is to reveal factors considered by mathematics student teachers while posing modelling problems. The participants were twenty-seven mathematics student teachers and posed their modelling problems within their groups. The data were obtained from the modelling problems posed by the participants, their solutions on these problems and the groups' reflective diaries regarding their problem posing and solution processes. The data were analyzed by using content analysis and the codes were constructed according to the problems' contents. The participants' diaries were examined in terms of generated codes and the expressions supporting/relating the codes were determined. While designing the problems, the participants considered the factors such as being interesting, understandable, appropriateness to real life and modelling process, model construction, and usability of different mathematical concepts. Their solutions were generally handled in terms of usage of the mathematical statements, appropriateness to the modelling process and being meaningful for real life. Modelling training should be provided to enable the student teachers to develop modelling problems and their designs should be examined and the feedbacks should be given.

**Keywords:** *Mathematical modelling; mathematics student teacher; modelling problem posing.*

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## Introduction

Singer and Voica (2013) stated that many teachers are not active problem posers because of not being have the required skills (cited in Paolucci & Wessels, 2017). Therefore, it is important that teachers acquire these skills before they start their professional lives. In a way that supports this idea, different researchers (eg. Ellerton, 2015; Grundmeier, 2015; Hošpesovà & Tichà, 2015; Lavy & Shriki, 2007; Osana & Pelczer, 2015; Rosli et al., 2015) suggested that student teachers should be included and supported in problem posing experiences during their initial training (cited in Paolucci & Wessels, 2017). It is important to encourage posing real-life contextual, non-routine and open-ended problems rather than routine problems while examining problem-posing activities for mathematics student teachers. Crespo (2003) pointed out that during teacher training the teacher educators suggested two ways: gaining experience on nontraditional mathematical problems and encouraging collaborative problem posing. This study examines the experiences of mathematics student teachers in posing real-life problems. The real life problems considered in this study are discussed in the context of mathematical modelling.

Mathematical modelling is defined by different researchers in line with their perspectives. While Pollak (1979) defined mathematical modelling as interaction between mathematics and the world out of mathematics, Blum (2002) stated it contains the whole transmission process from real life to mathematical life. Peter Koop (2004) also expressed mathematical modelling as a complex process which requires constructing a mathematical model about real life problems and transferring the model's results to the real life situation. Mathematical modelling process starts with a real life problem. Blum and Niss (1989) defined modelling problem as a tool which requires creative and critical thinking, benefits from mathematical knowledge and competences, includes solutions appropriate to real life, and facilitates the process of learning mathematical concepts by relating them to real life. A modelling problem requires multiple solution approaches based on different assumptions for constructing models about real-life situations (Fox, 2006) and also enables students a better way to learn mathematical subjects (Yoon, Dreyfus & Thomas, 2010). In addition to not having strict criteria for the basic features of modelling problems, there are criteria/principles that

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different researchers have identified for modelling problems or modelling activities. A modelling problem has main properties such as; (a) having open-ended structure, (b) basing on real life situations, (c) triggering students' experiences, (d) enabling the development of modelling competencies, (e) making assumptions appropriate to real life, (f) comprising multiple solutions based on assumptions, (g) constructing mathematical models, and (h) having generalizable solution (Blum & Niss, 1991; Borromeo Ferri, 2006; English, 2003; Fox, 2006; Lesh & Doerr, 2003; Mousoulides, 2009; Peter Koop, 2004). Additionally, Lesh, Hoover, Hole, Kelly and Post (2000) determined the six principles to develop a model eliciting activity. These principles are reality, model construction, self-assessment, construct documentation, construct shareability and reusability, and effective prototype. As seen, researchers have found some properties for modelling problems. However, the meaning that a mathematics teacher or a mathematics student teacher puts on a modelling problem could change. Posing a modelling problem is influenced by both the person's modelling perspective and the experience of problem posing. In this context, studies examined modelling and problem posing are also important. In addition, what individuals take into consideration while posing modelling problem and what they focus on while solving the modelling problems are some of the key factors in modelling studies.

*Problem Posing for Mathematical Modelling*

Problem posing is an effective way to teach mathematical concepts and develop mathematical thinking (Bonotto, 2010; English, 2003). Problem posing process is constructed with verbal statements, objects, real life situations, questions, definitions or diagrams to reveal deeper mathematical thinking (Brown & Walter 2005; English, 1998; Whitin, 2004). Hansen and Hana (2015) stated that problem posing was an essential part of mathematical modelling and effective learning could be supported with the environments in which mathematical modelling and problem designing integrate. Downton (2013) stated that problem posing enables rich environments for the development of modelling skills. Stillman (2015) also emphasized that students should be given the chance to pose their own problems.

Silver and Cai (1996) noted that problem posing occurs in three different ways: (a) the individual can try to produce problems from the remarkable situations presented to him, (b) the individual can reshape a problem solved based on the previous experiences, (c) after the individual solves a problem, she/he can create new problems by changing the situations in the problem. Christou, Mousoulides, Pittalis, Pitta-Pantazi and Sriraman (2005) proposed model for enabling students' problem posing thinking in the four basic processes named editing, selecting, comprehending and translating. While engaging in problem posing, the process means editing quantitative information, selecting quantitative information, comprehending and organizing quantitative information, and translating quantitative information from one form to another. These processes refer to the posing of problems based on a given quantitative information, processes/operations, pictures/scenarios and diagrams/tables. We did not gave any of these triggers for the modelling problem-posing process actually, we enabled the participants to pose their problems without depending on any subject, information, picture, etc. In another study, English, Fox and Watters (2005) emphasized that the designed modelling problems can be handled in two groups as problems involving/requiring too much data and involving/requiring very little data.

In the literature, the characteristics of the posed problems was handled, the strategies occurring in posing process were explained and the evaluation criteria of such problems were tried to be explained. This study will contribute to the literature by examining what factors are considered while posing mathematical modelling problems. For teachers to utilize mathematical modelling in their teaching, they need to have knowledge on mathematical modelling and modelling problems. Additionally, how to pose these problems and how to implement them in their lessons should be investigated. One way of ensuring that teachers have knowledge about modelling is that they are trained in such a way before graduating. The mathematics student teachers should be well-trained in terms of modelling knowledge and effective modelling applications. It was seen that there was a wide range of modelling problems developed by researchers in the literature. Besides, some researchers carried out their studies by developing their own modelling problems or selecting the developed problems from the literature (e.g. Deniz & Akgun, 2016; Delice & Kertil, 2013; Eraslan & Kant, 2015; Tekin Dede, Hidiroglu & Bukova Guzel, 2017; Sahin & Eraslan, 2016). Particularly, the modelling problems should be convenient to the students' cultures, levels and the necessities in the curriculum. For this reason, it is important for mathematics student teachers to design their own modelling problems for different concepts, purposes, students' levels and grades. In this study it is aimed to present the modelling problems posed by the mathematics student teachers and reveal the factors in the posing and solution process.

## Methodology

The study was conducted with a basic qualitative research approach. The aim of the qualitative research approach is to determine how individuals or groups understand the problems (Creswell, 2013), how they construct the world, and how they construct the truth (Merriam, 2013).

*Participants*

The study was carried out with twenty-seven mathematics student teachers. They were in the fourth year of a secondary mathematics education department of a state university and attended the Mathematical Modelling Course. The participants formed their groups according to their own wishes and $G_1$, $G_2$, ..., $G_8$ abbreviations were used for the groups while presenting the results. $G_3$ was a two-people group $G_2$, $G_6$ and $G_8$ were three-people groups, and $G_1$, $G_4$, $G_5$ and $G_7$ were four-people groups.

*Procedure*

The participants were instructed in mathematical modelling, modelling process, and the structure of modelling problems throughout the Mathematical Modelling Course during a semester. Various modelling problems from the literature were solved by the participants and their solutions were discussed in class. Then the problem posing process started and the participants posed three mathematical modelling problems with their groups. After posing process, what they considered while posing problems and how they solved them were discussed. The aim of these posing and discussion processes was to provide experience for the participants and encourage them to pose modelling problems. They finally posed the fourth modelling problems which are also the focus of this study. The participants were asked to pose their problems appropriate for the secondary school mathematics curriculum and there were no limitation regarding the problems' content.

*Instruments*

The data were obtained from the eight mathematical modelling problems posed by the participants, their solutions, and the participants' reflective diaries about posing and solving the problems. The names of the posed modelling problems were the Pulley Problem ($G_1$), Battery Problem ($G_2$), Earthquake Problem ($G_3$), Olive Problem ($G_4$), Box Problem ($G_5$), Soap Problem ($G_6$), Play-Off Problem ($G_7$) and Lantern Problem ($G_8$).

*Data Analysis*

The data were analyzed by two researchers with the purpose of identifying the factors considered by the participants while problem posing process. Content analysis was used in this process and two researchers evaluated the data and identified the codes individually. Then they came together, compared their identified codes and reached consensus on codes by discussing on differences. The posed modelling problems and their solutions were examined in terms of their contents and the codes were generated. Then the participants' diaries were also examined in terms of generated codes and the expressions supporting/relating the codes were determined. For the inter reliability of coding, the data were evaluated by two researchers again and their final evaluations were compared. The codes which were determined same or different by the two researchers are combined, and then the percentage of agreement was calculated by proportioning the same codes to all the identified codes. So the inter-coder reliability was calculated as 88% by using the inter-coder reliability formula (Miles & Huberman, 1994). The findings were presented in tables. The factors considered while posing modelling problems and which group considered each factors were included in the tables. Additionally, the extracts from modelling problems and diaries were given to support the findings; however, due to the fact that the original writings were in Turkish, the extracts were presented in English.

## Findings

When the problems posed by the groups and their solutions were examined, two different factors arose for the content and solution of the problem. Table 1 and Table 2 are presented as a result of the coding within these factors.

*Table 1. The factors related to problem content*

| The Factors | Groups | Frequency |
|---|---|---|
| Problem content | | |
| to direct to construct mathematical model | 1-2-3-4-5-6-7-8 | 8 |
| to be appropriate to real life | 1-2-3-4-6-7-8 | 7 |
| to be understandable | 1-2-3-4-5-6-8 | 7 |
| to have interesting content | 1-2-3-4-6-8 | 6 |
| to use real life data | 1-2-3-4-6-7 | 6 |
| to be visualized | 1-3-7 | 3 |

When the participants posed their problems, they paid attention to its content and the fact that the problem was interesting, understandable and appropriate for real life. They considered that the problem needed to be contained real life data and visualized with figure and tables. They also emphasized the necessity of constructing mathematical model for solving problem. All groups except one was evaluated as enabling the appropriateness to real life. The problems had the scenarios which related to real life or the possibility of being taken from real life. All groups tried to focus on solving the problem by translating to mathematics by taking place model construction while posing the problems. The groups gave the statement about constructing model in their problems. It was considered that the groups except $G_7$ had the problem statements which should be understandable to everyone.

After the brief information about the problems of all groups, the problems of each group and the factors they consider when posing this problem are discussed with their samples in the following section. $G_1$ posed the Pulley Problem, which required constructing and solving mathematical models indicating how many rope layers were used from inside to outside (see Figure 1).

Rope pulleys are essential materials of the textile industry used to stitch shirts. Construct a mathematical model showing how many rope layers are used to form these pulleys, inside and outside, and solve this model.

*Figure 1. Pulley Problem ($G_1$)*

The Pulley Problem is a problem that can be of interest to students because it is about an object that students can meet in daily life. The problem written in understandable language requires the use of real life data. To find out how many layers of the pulley, it is necessary to use tea glass and pen. Glass and pen images help to measure, determine variables and assign mathematical values. Therefore, the images used by $G_1$ are essential for solving this problem situation. They stated that they used visualization because they asked students to determine the sizes of the pulleys by taking advantage of the size of tea glass.

> We thought that they could find the length of the pulleys given in the picture from the dimensions of the tea glass and the pen. Thanks to the picture, approximate measurements of a pulley can be found. ($G_1$-reflective diary)

$G_2$ posed the Battery Problem aimed at finding out how many years later the land of Turkey would become unusable by drawing attention to issues of waste batteries which cause environmental pollution (see Figure 2).

In these days when we face major environmental problems, one of the biggest problems is the disposal of waste batteries randomly into the environment. A battery makes the 4 $m^2$ soil unusable. In our country, 5 batteries per capita are used per year. If it is thought that all of these batteries put into the environment and each battery is thought to be thrown into each separate territory of 4$m^2$, after how many years Turkey's territory will become unusable?

(Turkey's land area: 780576 $km^2$, population: 75000000)

*Figure 2. Battery Problem ($G_2$)*

The Battery problem is written in an understandable language and addresses a problem from everyday life. It is thought to have an interesting content because it is aimed at creating environmental awareness. The problem leads to a mathematical model to find out how many years later the territory of Turkey would become unusable. The information that the students can have or reach to easily is included in the problem statement. The problem statement could be posed to require the students to achieve information such as the land area and population of Turkey. The information that a battery made 4 $m^2$ of soil unusable and approximately 5 batteries are consumed per person are included in the problem. In addition, the problem statement also leads to the selection of variables. Because the Battery Problem gives all the necessary information, the solution became a word problem in which the solution way can be easily predicted. For this reason, it is necessary to revise the problem by giving missing information and making students to decide solution steps by reaching required information.

The $G_3$ presented a table which contained the earthquake occurrences within a certain period of time in Turkey and asked to be found a solution by using these means (see Figure 3).

The earthquakes happening in Turkey's different cities from 19.12.2011, 00:38 to 20.12.2011, 14:33 in the table. By considering the table, construct a model enabling to estimate the intensity of the earthquake happening in 19.12.2011, 22:46.

| | Time | Intensity | | Time | Intensity | | Time | Intensity |
|---|---|---|---|---|---|---|---|---|
| 1. | 00.38 | 2.8 | 10. | 03.23 | 2.6 | 19. | 09.24 | 2.8 |
| 2. | 00.40 | 2.9 | 11. | 03.31 | 2.8 | 20. | 09.30 | 2.9 |
| 3. | 01.28 | 3.0 | 12. | 03.57 | 2.6 | 21. | 10.34 | 2.8 |
| 4. | 02.16 | 2.8 | 13. | 04.00 | 2.9 | 22. | 12.00 | 2.5 |
| 5. | 02.19 | 2.7 | 14. | 05.06 | 2.5 | 23. | 12.07 | 1.8 |
| 6. | 02.31 | 2.5 | 15. | 06.09 | 2.9 | 24. | 12.21 | 2.1 |
| 7. | 02.42 | 2.6 | 16. | 06.14 | 2.6 | 25. | 13.29 | 2.9 |
| 8. | 03.04 | 2.5 | 17. | 06.19 | 2.6 | 26. | 14.23 | 2.1 |
| 9. | 03.16 | 2.7 | 18. | 07.17 | 2.4 | | | |

*Figure 3. Earthquake Problem ($G_3$)*

$G_3$ visualizes the problem statement using the table. The given table is a visualization tool which must be used to solve the problem in the Earthquake Problem prepared using real life data obtained from the Kandilli Observatory and Earthquake Research Institute. In order to reach a solution, data should be arranged by making selections from a large number of data. It is asked to construct a mathematical model to estimate the magnitude of an aftershock that is likely to occur in an hour which is not appear in the table in the statement of the problem which is understandable. Since Turkey is located on the world's active seismic zone, the problem statement is considered to have an interesting content for the students.

The $G_4$'s Olive problem, based on a situation experienced in Turkish Airlines, is given in Figure 4.

Turkish Airlines offered breakfast to its customers in every flight. They decreased the olive portion from 5 to 3 according to a decision made in the last month. Please construct a model finding the monthly profit of Turkish Airlines as a consequence of this decision.

*Figure 4. Olive Problem ($G_4$)*

The $G_4$ read from the newspaper that Turkish Airlines changed its snacks and then they shaped their Olive Problem. $G_4$ brought the reality dimension of a modelling problem into the forefront by posing the problem considering a really real life problem situation. The group members used an understandable language and real life data in the problem which was thought to be interesting by the students due to the true-life situation. $G_4$ stated that the news they read in a newspaper was effective in determining the problem context and they solved this real life problem with the help of mathematical modeling as follows:

> We read a report on that Turkish Airlines wanted to change their snacks to save money. But we handled the snack as olive and modelled the problem with an example of olives. So we solved a problem from real life through mathematical modelling. ($G_4$-reflective diary)

$G_5$ handled an open box design as a modeling problem (see Figure 5).

An open-top can is wanted to be built up by removing similar squares from the corners of cardboard in the shape of square. Construct a mathematical model to calculate the volume of the open-top can. Calculate the biggest volume by using this mathematical model.

*Figure 5. Box Problem ($G_5$)*

The Box Problem is understandable and directive to construct mathematical model. But the $G_5$ constructed the Box Problem in a way that was like maximum-minimum textbook problems. Therefore, it is considered not to be of interest for students. It is not even possible to evaluate this problem as a modeling problem.

G6 posed a problem considering the stand which was opened by a soap factory in Edirne province that is famous for its scented soaps (see Figure 6).

A factory in Edirne producing scented soap will open a stand in a fair. Baskets are prepared for the scented soaps to be presented at this stand. According to the wishes of customers, the soaps at least 1 of 3 soap types will be placed in these baskets. The basket is known to have a volume of 300 $cm^3$. Construct a model giving the number of soaps to be placed in these baskets.

The volumes of soaps are as follows:

For the rose soap $V_{rose}$=10 $cm^3$

For the heart soap $V_{heart}$=12,5 $cm^3$

For the pear soap $V_{pear}$=15 $cm^3$

*Figure 6. Soap Problem ($G_6$)*

The Soap Problem is a real life contextual problem and requires the use of real life data. The number of soaps to be placed in the basket was asked by constructing mathematical model in the problem. Since all the data needed to solve the problem was presented in the problem, it is no longer an open-ended problem. Therefore, the problem situation should be arranged in such a way that the students can make their own assumptions by reaching the data. Thus, it will have the characteristics of mathematical modeling problem. Since the problem inspired by the production of the scented soap in a province of Turkey, it would be remarkable for the students. $G_6$ emphasized that they focused on making their problem interesting, clear and understandable as follows:

> We gave more importance the selection of the problem's content, and we considered that the problem must be clear and understandable instead of being complicated… We gave more importance to the scenario of this problem. We wanted it to be more remarkable ($G_6$-reflective diary).

$G_7$ preferred to construct a scenario on a soccer game and focused on the Play-off system, which was a current situation in a soccer game (Figure 7).

Turkish Football Federation researched about the play-off system which the federation would use first time. According to the last seven years' statistics, construct a mathematical model showing the teams' minimum scores which was needed to join the play-off.

| | | Score | Average |
|---|---|---|---|
| 2010-2011 Season | 4. Gaziantep | 59 | +11 |
| 2009-2010 Season | 4. Besiktas | 64 | +22 |
| 2008-2009 Season | 4. Fenerbahce | 61 | +24 |
| 2007-2008 Season | 4. Sivasspor | 73 | +28 |
| 2006-2007 Season | 4. Trabzonspor | 52 | 10 |
| 2005-2006 Season | 4. Trabzonspor | 52 | 9 |
| 2004-2005 Season | 4. Besiktas | 69 | 31 |

*Figure 7. Play-Off Problem ($G_7$)*

Even though $G_7$ considered the scenario appropriate to daily life, they did not give importance to clarity in the design process. It may be problematic because all students may not know what a play-off is exactly and they may have trouble using the real life data given in the problem. Therefore, it was decided that the problem statement was inadequate in this sense. If they had explained what the play-off meant, the problem would have made more understandable.

The $G_8$ designed a problem that requires finding the distance between two lanterns (see in Figure 8).

Please construct a mathematical model indicating the distance between two lanterns. Calculate the lantern number in a 500-meters way.

*Figure 8. Lantern Problem ($G_8$)*

The real-life context Lantern problem is a problem that involves an understandable and interesting context, and requires the usage of real life data and construction of mathematical model. The participants of $G_8$ changed their problem situation when they were not convinced of their solutions. $G_8$ put into words their views as follows:

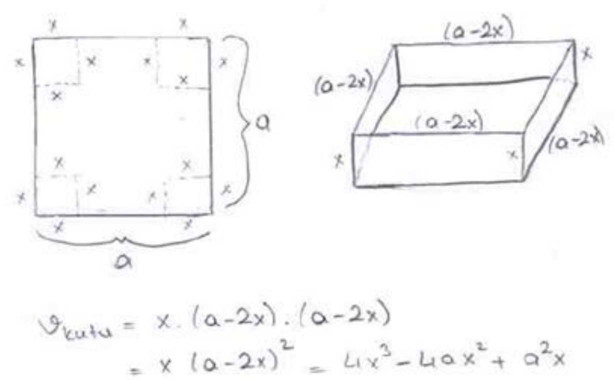
> We had difficulties in designing problems, so our designed problems included only one subject (mathematical concept). We thought first about the solution. After solving the problem, we then revised the problem ($G_8$-reflective diary).

As is also understood from the reflective diary of the $G_8$, ideal solution was an important factor in designing process. The factors considered by the groups in the problem solutions are given in Table 2.

*Table 2. The factors related to problem solution*

| The Factors | Groups | Frequency |
|---|---|---|
| Problem solution | | |
| to be appropriate to mathematical modelling process | 1-2-3-4-5-6-8 | 7 |
| to be supported by mathematical statements | 1-2-3-4-5-6-8 | 7 |
| to be meaningful for real life | 1-2-3-4-5-6-8 | 7 |
| to be understandable | 1-2-3-4-5-6 | 6 |
| to benefit from real-life experiences | 1-4-6-8 | 4 |
| to require utilizing different mathematical concepts | 1-3-5 | 3 |
| to use extra real-life data | 4-8 | 2 |
| to be easily validated | 1 | 1 |
| to be appropriate for using software specific to mathematics | 3 | 1 |

The groups displayed approaches regarding supporting problem solutions with mathematical statements, being appropriate to modelling process, being understandable, being meaningful to the real life, using real life experiences, using extra real-life data, requiring using different mathematical concepts, validating easily and being convenient for usage of mathematics software. The groups considered the situations which enable them to construct mathematical models and supporting solutions by mathematical statements. They constructed mathematical models by using variables, parameters and constants. An extract from $G_5$ supporting the solution with mathematical statements was given in Figure 9.




a: one side length of the big square

x: side length of cut square

$V_{box}$: the volume of the can

*Figure 9. An extract from the problem solution of the $G_5$.*

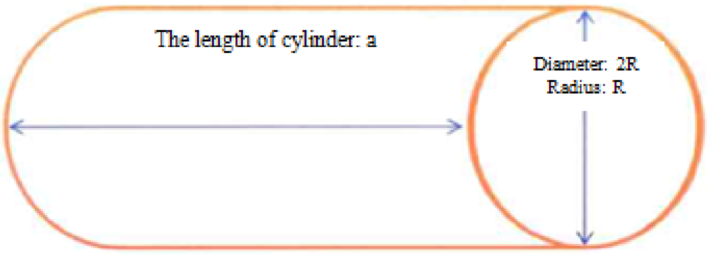
Although the problem of the Box Problem is not a modeling problem, $G_5$ tried to comply with the mathematical modeling process in the solution of the problem and construct a mathematical model by using the variables of a and x. The groups except $G_7$ came to a conclusion that problem solutions needed to be appropriate to the mathematical modelling process and that the solution was appropriate to the process was an important factor for them. The $G_5$ stated their opinions regarding this factor as follows:

> We solved the mathematical modelling problems which we prepared during previous weeks in an irregular way and without planning. In designing this problem we wrote our mathematical modelling problem by considering mathematical modelling process and solved it according to this process. ($G_5$-reflective diary)

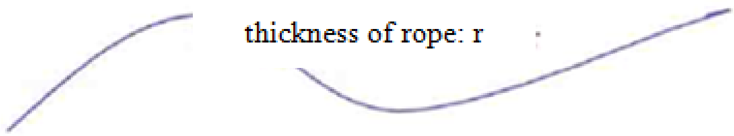
The $G_1$ solved the problem also according to modelling process by following the stages: understanding problem; associating data; stating problem mathematically; constructing mathematical models and correlating them; solving problem mathematically; deducing results from solution; interpreting and adapting the results to real life; and validating. They determined the assumptions by imagining the pulley as a cylinder and used mathematical symbols while stating problem mathematically (see Figure 10).

We expressed the required data with variables.

We described the pulley as cylinder. We gave "a" to the length of the pulley and "R" to the radius of the pulley.




We gave "r" to the thickness of shirt rope used as textile industry.




We gave "n" to the number of rope layers in the surface of the pulley.

*Figure 10. An extract from the problem solution of $G_1$*

The groups explained their assumptions and approaches in order to make it understandable, and took care to use mathematical language in an appropriate way. Moreover, they gave importance to clarity as an important factor. That the groups clarified their solutions in a detailed way was a clear indicator of this. The six of the groups (excluding $G_7$ and $G_8$) considered that the solutions should be understandable. Another important factor considered by the participants was that the solution of the problem needed to be meaningful for real life. For instance, the $G_4$ said the following regarding the appropriateness of their problem solution to real life:

> We are experiencing difficulties in building a scenario because we do not have enough experience in modelling yet. We can easily miss the various details. Thus, the appropriateness to real life of the model and solution decreases. We tried to reach a more realistic solution by using real data we found on the internet in our solution ($G_4$-reflective diary).

It was seen that $G_7$ did not consider enough that their problem solutions were meaningful for real life. They displayed a simple solution approach by ignoring many assumptions (training of the teams, team values, the ability to be a team, the number of new-coming players, the conditions of the competitors, etc.) though these assumptions had a great influence on the problem solution, and they did not consider whether the result was appropriate to the real life or not.

If we ignore the squad values, we found $\frac{59+64+61+73+52+52+69}{7} = 61{,}42$ point by using arithmetic mean. Average is $\frac{11+22+24+28+10+9+31}{7} = 19{,}28$.

*Figure 11. The solution of $G_7$*

The participants regarded the problem situations which were appropriate to benefit from real life experiences. This situation became an important factor for them both to question the obtained solution and to make ideal assumptions. For example, $G_1$ made a prediction on the length of the pulley by using tea glass and pencil in the solution of the designed problem (Figure 12).

We can enter the data regarding the problem to the model we constructed. We can do with the help of the tea glass and pencil given in the figure. When we compare the sizes of the tea glass to the pencil, the diameter of the top of the glass is approximately 2 cm and the diameter of the bottom is approximately 4 cm. We think the glass is a cylinder and we found $2R = \frac{2+4}{2} R = 1{,}5\ cm$. When considering the length of the glass, we can say that the pulley's length is approximately 11 cm.

*Figure 12. An Extract from the solution of $G_1$*

$G_4$ and $G_8$ formed their solutions so as to include extra real-life data. These groups benefited from the real-life data which were not given in the problem statement by using them in their assumptions and solution approaches. While solving the Olive Problem, the $G_4$ used a table related to the plane types and the passenger capacity given on the website of Turkish Airlines (see Figure 13).

Plane types and passenger capacities of the planes in Turkish Airlines flights;

| Plane type | Passenger capacity | Plane type | Passenger capacity |
|---|---|---|---|
| Airbus A319-100 | 124 | Airbus A340-300 | 271 |
| Airbus A320-200 | 159 | Boeing 737-700 | 149 |
| Airbus A321-200 | 186 | Boeing 737-800 | 167 |
| Airbus A330-200 | 250 | Boeing 777-300 | 337 |
| Airbus A330-300 | 281 | | |

*Figure 13. Solution Extract of $G_4$*

The necessity of utilizing different mathematical concepts was another factor considered by the participants while designing the problem. The five groups preferred solving their problems through a main single concept. The equation was used in the solution of the Battery Problem and Olive Problem, the linear function was used in the solution of the Earthquake Problem, the probability was used in the solution the Soap Problem, arithmetic was used in the solution of the Play-Off Problem, and the trigonometry was used in the solution of the Lantern Problem. Three groups solved their problems by using different mathematical concepts. For example, the $G_1$ used the solid objects, plane geometry, sequences and series, and they stated that this situation increased the quality of the modelling problem as follows:

> We want students to use more than one concept like cylinder, diameter, radius and arithmetic sequence in this problem. Including a lot of mathematical concepts increased the quality of the modelling problem ($G_1$-reflective diary).

Although the groups tried to comply with the steps of the modeling process in solution, the groups except one had difficulties while validating the solutions/models/assumptions. For example, since the Box Problem ($G_5$) was a standard textbook problem or the Soap Problem ($G_6$) did not contain a realistic context, it was not possible to validate. Although the Earthquake Problem was suitable for validation due to its context, the $G_3$ completed their solution without validation. However, by using the model they created, they could be able to validate their solutions by comparing the intensity of earthquakes at different times with the actual earthquake intensities given in the table. $G_1$ exhibited approaches to enable validation of the solution which were appropriate to the real life. $G_1$ designed a problem in which they could easily compare the acquired solution with the measurements. They easily made measurements and validated their results by examining a rope pulley matching the dimensions.

We examined a pulley within these dimensions and measured the actual size using a ruler. At the stage of testing the constructed model, we found that the number of layers was 80 in our approximate measurements performed with a ruler. This value was larger than the value we found in the solution (67) because the diameter is decreased from the outside since the rope is wrapped tightly to the pulley by the machine.

*Figure 14. An Extract from the solution of $G_1$*

$G_3$ planned to use the GeoGebra software in solving the problem. They tried to express the relationship and the change in variables in the constructed mathematical model by assigning the values in the GeoGebra software.

> We solved a problem last week with the help of GeoGebra about the height and the foot length given availably. The line of approximation, which we used here, drew our attention. We wanted to design a problem, in which we could use it. ($G_3$-reflective diary)

## Discussion and Conclusion

We found that two key factors were taken into account while designing modelling problems. The groups' modeling problems were appropriate to real life, understandable and interesting. Their problems directed students to construct a mathematical model, use real-life data and visualize with the figures. These factors were parallel to Bukova Guzel's (2011) factors such as suitability to daily life, mathematizing, mathematical knowledge levels, the curriculum, and attractiveness.

The participants dealt with real life situations to enable the formation of relationships between the mathematical world and the real world, as stated by Berry and Houston (1995), Blum and Niss (1991), Blum (2002), Lesh, et al. (2000) and Pollak (1979). Similarly, where the modeling problem was posed, it was observed that the participants firstly tried to complete one of the priorities of modeling by giving importance to reality (Deniz & Akgun, 2016; Tekin Dede, Hidiroglu & Bukova Guzel, 2017; Ozaltun Celik, 2018). Ärlebäck (2009), Lesh and Doerr (2003) and Schoenfeld (1994) indicated that modelling problems should have interesting content, be presented in an environment understandable to the students, and examine different situations they may have encountered in real life. The six groups' problem contexts were considered as were interesting, different from routine problems in text books, and unlike those solved in lessons. However, the Box Problem was thought to be inadequate in attracting students because it was similar to other maximum-minimum problems in textbooks. In other words, the Box Problem refers to a constrained design process in terms of creativity, as Silver and Cai (2005) express in problem posing. Similarly, it was thought that the Play-Off Problem could not be interesting for the students who were not interested in soccer, as stated in the personal meaningfulness (Lesh et al., 2000).

The groups considered to what should be in the solution of modelling problems and displayed approaches that included supporting the problem statement with mathematical statements, being appropriate to the modelling process, using real life experiences (Borromeo Ferri, 2006; Lamberts, 2005), using extra real life data, requiring using different mathematical concepts (Schoenfeld, 1994), validating easily (Maaß, 2006) and being convenient for use in software (Hohenwarter, Hohenwarter, Kreis & Lavicza, 2008). All groups except one supported their solutions with mathematical statements and models, and solved them in accordance with the modelling process. When we consider the modelling as a complex process, it was suggested that such approaches of the participants were important. The three groups considered that they used different mathematical concepts in the solution process. In a similar way Isik and Kar (2012) stated that the pre-service teachers could not design the problems including different mathematical concepts while they designed problem.

The participants used verbal statements, real life situations, questions and diagrams to reveal deeper mathematical thinking, similar to cited in Baki (2002), Brown and Walter (2005), English (1998), and Whitin (2004). Only one group used the mathematical software in the problem solution. The group benefited from the real data on earthquakes in Turkey over a certain range of time, and used GeoGebra in the solution. Siller and Greefrad (2010) expressed the technology-based tools usable in the modelling process had an important role in identifying students' different strategies. This problem had a similar construction to experimental modelling problems in the modelling classification presented by Berry and Houston (1995). These types of problems are referred to as problems involving too much data (English, Fox & Watters, 2005). When thought from this point of view, this approach of the group seemed important.

Only one group gave an appropriate real-life statement for validating solution in an easy way. It is thought as important that the content of modelling problems should be organized such that students can display approaches in the validation stages. To that end, giving some instructions to enable interpretation and validation of the problem situation in the problem statements can motivate students to display approaches in this direction.

When we review the basic factors considered by student teachers in content and solution of a problem, it is seen that some groups were able to understand the meaning of mathematical modelling, how it is formed, as well as the structure of modelling problems, and were able to create problems capable of general representation of modelling problems. However, revision of problems, which are deemed insufficient from certain points of view, with the aid of feedback is thought to be important. For the participants to explain their statements in a detail way by considering understandability was related to the construct documentation principle of model eliciting activities. The construct documentation principle emphasizes that students need to reveal their own thinking styles in the solution process (Chamberlin & Moon, 2005).

In the light of the study's findings and results, the recommendations are as follows:

- Mathematics student teachers should be persuaded about the importance of modelling and believe in its necessity.
- For student teachers to display richer solution approaches to modelling problems, different modelling problems and their solutions should be implemented and discussed in lessons.
- Modelling training should be provided to enable the student teachers to develop modelling problems and their designs should be examined and the feedbacks should be given.
- For the mathematics student teachers to have new perspectives, they should be encountered the modelling problems related to the different disciplines.

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