RESEARCH ON EDUCATION AND PSYCHOLOGY (REP)

Received: September 28, 2017
Revision received: November 12, 2017
Accepted: November 25, 2017
OnlineFirst: December 4, 2017

e-ISSN: 2602-3733

http://journalrep.com
December 2017 ◆ 1(1) ◆ 21-30

Research Article

The Ability of Mathematics Teacher Candidates to Use Algebraic Representation and Geometric Representation

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Abstract

Mathematical concepts can be represented in multiple ways. Teaching a concept together with the relationships between its multiple representations is regarded as one of the most important components of mathematics teaching. This study aims to examine the relationship between the ability to use geometric representation and algebraic representation bidirectionally on subject of conic sections. Sample of the study comprised 200 teacher candidates studying at Necmettin Erbakan University, Ahmet Keleşoğlu Education Faculty, Department of Mathematics Education. The questions were addressed to these mathematics teacher candidates at the end of the spring term 2016-2017, when they took the analytic geometry lesson. Quantitative research methods were used in the study. The problem sentence of this study is the success of bidirectional shift between the skill of ability of using algebraic representation and geometric representation on the subject of conic sections. In conclucion of the study, it was found that the mathematics teacher candidates were more successful in the geometrical representations than the algebraic representations and there was a meaningful difference in favor of in the geometrical representation of algebraic representations.

Key Words

Geometric representation • Algebraic representations • Conic section • Math teacher candidates

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Citation: Çenberci, S. I., Yavuz, A., & Yüca, G. (2017). The ability of mathematics teacher candidates to use algebraic representation and geometric representation. *Research on Education and Psychology (REP)*, *1*(1), 21-30.

Geometry is a discipline that brings in vision to the individual, facilitates thinking, and ensures arriving at solution by visualizing the figures (Hızarcı, 2004). Analytic geometry is expressed as a branch of mathematics that enables examination of an algebraic problem with the aid of geometry and also examination of the geometry problems with the aid of algebra (Gözen, 2001). Euclid and Al-Khwarizmi have concentrated on geometrization of algebra, Descartes has taken the first steps toward algebraic of geometry by expressing the geometric objects, concepts and relationships with algebraic equations (Baki, 2006). Those studies conducted by Descartes in the 1600s have resulted in emergence of analytic geometry, a new form of geometry (Baki, 2014). This ensures finding multiple representations and solutions to the given problems in analytic geometry.

Multiple representation approach is based on expression of a concept or situation in different ways (Ministry of National Education [MEB], 2013). At this point, various classifications have been made for different representations. For example, Adu-Gyamfi (1993) has classified the representation methods as oral representation (written words), graphical representation, algebraic or symbolic representation (equations showing the relationships between the variables), figure representation (diagrams, drawings) and chart representation (value tables). In addition, classifications have been made with regard to the representation types specific to subject fields. Özhan-Turan (2011) have taken into consideration statuses of the straight lines with respect to each other with three representation types including status (verbal) representation, formula (algebraic) representation and figure (visual) representation. In this study, taking into consideration the definition of analytic geometry that emphasizes the shift from algebraic representations to geometric representations and from geometric representations to algebraic representations, it was concentrated on algebraic and geometric representations. Studies concerning the expressions containing multiple representations are encountered not only in analytic geometry, but also in the lessons such as numeric analysis. For example, Schultz and Waters (2000) and Arslan (2008) have mentioned of different approaches that can be used in solution of linear equation systems and solution of differential equations, respectively.

State of the literature

In many researches, considering teaching of multiple representations, many studies have shown that the individuals who are able to use different representations and the shift between the representations are more successful in the respective area (Delice & Sevimli, 2010).

According to Neria and Amit (2004), we can say that the representation type selected for problem solution affects the success.

Considering that use of multiple representation is teachable (Keller & Hirsch, 1998), multiple representations have acquired a place in the teaching programs, and it has been emphasized in the teaching programs that individuals who have the ability of illustrating the mathematical concepts with multiple representation forms and the shift between representations (National Council of Teachers of Mathematics [NCTM], 2000).

According to Goldin and Kaput (1996), representation is defined as the name given to each of the different illustration forms of a mathematical concept. Based on this definition, use of multiple representations has been emphasized in the national and international teaching programs-standards. For example, as an indicator of the skill of association in mathematics teaching program, "association of different representation forms of mathematical concepts and rules with each other and their conversion into each other" is targeted to have been acquired (Ministry of National Education [MEB], 2013). Discovery by the students of the relationship between the graphical and algebraic illustration may contribute to their better understanding the mathematical concepts (Aspinwall & Shaw, 2002).

In mathematics teaching, conceptual understanding and conceptual learning occupies a very important place, and Baki (2006) expresses that conceptual learning regards mathematics as the network of inter-dependent concepts and thoughts, and that it requires the student to structure them by himself/herself. One of the most important factors in succeeding this effectively is the possibility of using multiple representations (Eroğlu & Tanışlı, 2015).

Tuluk (2014) has emphasized that teachers and teacher candidates should not regard each representation as an independent topic to be learnt while shifting from one representation to another, that for example, they should think and express a straight line passing through two points on the plane with attainment of the points through their placement one-to-one to the edge of a ruler together with its character of being an equation, i.e., they should think and express algebra, analytic and synthetic geometry as a whole.

Brenner et al., (1997) examined in their study whether prealgebra students can learn to be more effective in the ways they use symbols, words, and graphics to represent mathematical problems involving functional relations. They emphasized the multiple representations and in their study.

With regard to this, Goerdt (2007) has stated that presentation of the lessons by making use of different representations and allowing for such questions that measure the shift between representations in the evaluation process might affect this skill positively. No study has been encountered in regard to the topic of conic sections, which examines the skill of shifting between multiple representations.

Conics are often taught with the general conic equation or geometric locus definition. In analytic geometry, especially the topic of conic sections is regarded as a topic approached with prejudice by the students. Hence, using the shift between representations with a view to arriving at solution in regard to conic sections makes meaningful learning by the students effective. It also supports to eliminate the prejudice formed. Because the shift between representations assists students who have different learning styles to comprehend the concepts in more detail.

Boyer (1968) expresses that the conic concept first emerged through solution of Delos problem by Menaechmus using rectangular hyperbolas and parabolas in the 350s B.C. Works of Archimedes and Euclid contain information on cone sections, too. The one who named these cone sections as ellipsis, parabola, hyperbola was Apollonios (Topdemir, 2011).

Contribution of this paper to the literature

The main reason for our doing this examination is the fact that the topic of conic sections is not learnt permanently by the teacher candidates although they encounter it many times throughout their teaching lives. The shift between multiple representations has been selected as the examination tool in our study in order to make learning permanent for the students who have different learning styles.

Therefore answers have been sought in the study to the following sub problems in order to examine the relationship between the skill of ability of using geometric representation and algebraic representation of the subject of conic sections bidirectionally.

1- What is the bi-directional transition success scores of geometric and algebraic representations the topic of conic sections of the mathematics teacher candidates?

2- Is there a significant difference between two-way transition success scores of geometric and algebraic representations the topic of conic sections of mathematics teacher candidates?

3- What is the correlation between the two-way transition success scores of the geometric and algebraic representations on the topic of conic sections of the mathematics teacher candidates?

Method

Research Model

In the context of data, as statistical techniques for analyzing data in research; frequency, percentile, mean, standard deviation and correlation tests were used. The data obtained in the research process were processed using SPSS program in computer environment. Arithmetic mean and standard deviation values of the students' awareness points about the bi-directional relationship of geometric and algebraic representations of the teacher candidates on the conical sections were determined. The relation between geometric representation and algebraic representations of conic sections of teacher candidates was analyzed by Pearson moment product correlation. (Can, 2013).

Study Group and Sample

Necmettin Erbakan University Ahmet Keleşoğlu Faculty of Education Mathematics Education constitutes 200 teacher candidates who are studying in the field of education. This mathematics teacher candidates were asked at the end of the spring semester 2016-2017 when they took the analytic geometry course**.**

Data Collection Tools and Data Analysis

In the scope of the research, evaluation test consisting of 10 questions about conical sections was applied as data collection tool. In order to determine the questions to be used in the test, a related literature study was conducted and a pool of questions was created by examining the source books and studies. While selecting the questions, the classification of the conic sections and the observation of the transition as algebraic representations of the geometric representations are held on the front panel. It has been decided to use a total of ten questions for each of the representations discussed, with five questions to vary according to the conic sections, once the questions have been determined for the circle, ellipse, parabola and hyperbola topics with conic sections. While the evaluation test was presented and responded to the test, it was noted that the participants should be in the test environment and not affect each other. The necessary arrangements and examinations were made on the collected evaluation test and the analysis was made ready.

Findings and Interpretation

This section contains detailed about the study findings arrived as a result of the statistical analyses conducted in line with the answers of the mathematics teacher candidates participating the study, and the interpretations made regarding these results.

First sub-problem

The first sub-problem is, “What is the bi-directional transition success scores of geometric and algebraic representations the topic of conic sections of the mathematics teacher candidates?” Mathematics teacher candidates’ geometric and algebraic representations are given arithmetic mean and standard deviation values of bi-directional transition success points.

Table 1

Descriptive Statistical Data on Mathematics Teacher Candidates' Bi-Directional Transition Success Scores of Geometric and Algebraic Representations on the Topic of Conical Section

	N	Arithmetic Mean	Std. Deviation
Geometric representation transition point from algebraic representation	200	2.915	.976
Algebraic representation transition point from geometric representation	200	3.165	1.164
Sum of bi-directional transition point of geometric and algebraic representation	200	3.030	0.857

The average bi-directional transition point of geometric transition from algebraic representations of the 200 students who participated in our research was 2.91, and the average bi-directional transition point of algebraic representations from geometric transition was 3.16. When the average values obtained from the success tests that can be obtained with the highest five points and the lowest zero points are taken into consideration, sum of bi-directional transition point of Geometric and Algebraic representation is 3.03. It is seen that the bidirectional transitions of the geometric representations are higher than the bidirectional transition points average of the algebraic representations. According to this, we can say that students are more successful in geometric representation.

Second sub-problem

The second sub-problem of the study is "Is there a meaningful difference between the two-way transition success averages of geometric and algebraic representations about the topic of conic sections of the mathematics teacher candidates?"

We can also refer to this as the absence hypothesis that "there is no significant difference between the geometric and algebraic representation mean of points of the mathematics candidates on the topic of conic sections".

In order to test our hypothesis, the scores of the students with Wilcoxon test and those with normal distribution are given in table 2.

Table 2

Wilcoxon Test Results for Mathematics Teacher Candidates' Bi-Directional Transition Success Scores of Geometric and Algebraic Representations on the Topic of Conic Sections

Geometric Representations-Algebraic representation	N	Rank Average	Row Sum	z	*p*
Negative sequences	83	71.80	5959.00	-2.570	0.010
Positive sequences	55	66.04	3632.00		
Not different	62				

** *p<.01*

As can be seen in Table 3, according to regression analysis results, the values scale sub-dimensions explain the optimism at 25% level. In other words, subscales of the values scale’s explaining power of optimism is %25. Although this value is not considered to be high, it is meaningful at the level of 0.01 (F = 15.37). This situation can be interpreted as while %25 variability about optimism results from the sub-scales of values scale, the rest results from variables not included in the survey. In addition, when the subdimensions of the values scale are examined, subscales like responsibility, respect and honesty significantly explain optimism (t = 3.05; t = 3.25; t = 1.77) and other subdimensions such as friendship/fellowship, peace, tolerance do not explain optimism (t = -415, t = 15, t = 1.70).

According to the results of the Wilcoxon signed Ranks test showed that there is a significant difference between the two-way transition success point averages of the geometric and algebraic representations the topic of conic sections of the mathematics teacher candidates [z = -2.570, p <0.05]. Since the difference score is in favor of negative sequences (geometric representations), the mathematics teacher candidates' difference in the topic of conic sections is more successful in bi-directional transitions of geometric representations.

Third sub-problem

The third sub-problem of the study is "What is the degree of correlation between the two-way transition success scores of the geometrical and algebraic representations of the mathematics teacher candidates about the topic of conic sections?". In order to reveal the correlation between the two-way success scores of the geometric and algebraic representations of the mathematics teacher candidates on the topic of conic sections, the Sperman Rank Difference Correlation Coefficient was calculated from the data with no normal distribution.

The scores of the mathematics teacher candidates obtained with Sperman Rank Difference Correlation Coefficient are given in Table 3

Table 3

Correlation Coefficient Results of the Mathematics Teacher Candidates on The Two-Way Transition Success Scores of Geometric and Algebraic Representations on the Topic of Conical Sections

			Algebraic Representation	Geometric Representation
Spearman's rho	Algebraic representation	Correlation coefficient	1.000	.230**
		Sig. (2-way)		.001
		N	200	200
	Geometric representation	Correlation coefficient	.230**	1.000
		Sig. (2-way)	.001	-
		N	200	200

** Correlation is significant at the 0.01 level (2-tailed).

In order to determine whether there is a relationship between geometric representation and algebraic representation by two-way transition success scores on the topic of conic sections of mathematics teacher candidates. There is a significantly low correlation between Sperman Rank Difference correlation processing, geometric representation and algebraic representation bidirectional transition success scores ($r=0.23$, $p<0.01$).

Conclusion and Recommendations

This study aims to examine the skill of ability of using the geometric representation and algebraic representation of conic sections birectionally. In this respect, the analysis results reached in line with the data obtained through the evaluation test results regarding the topic of conic sections applied to the mathematics teacher candidates were examined.

Teaching a mathematical concept using different forms of representation causes the concept to be learned in a meaningful way. Despite the fact that the use of multiple representations is one of the goals to be taught in mathematics curricula, the studies show that students are not successful in using different types of representation (Oktaç, 2008; Stewart & Thomas, 2004).

In a study by Gagatsis and Elia (2004), it was highlighted that there is strong support in the mathematics education community which students can grasp the meaning of mathematical concepts by experiencing multiple mathematical representations.

According to the results of the research, it was found that the mathematics teacher candidates were more successful in the geometrical representations than the algebraic representations and that there was a meaningful difference in favor of the geometrical representations in the geometrical representation of algebraic representations. However, there is a study shows different result from this research by Gagatsis and Shiakalli (2010), it is indicated that whenever the graphical representation is involved in the translation task the percentages of success are lower. It is also expressed Greek students and mathematics teachers avoid using and interpreting graphical representations and prefer using algebraic representations. We can say that different results can be seen not only in different countries, but also in different schools or students individually.

The individual differences of the students are the most important factors to be considered in the educational environment. Individual differences affect the learning of each student. Each student has a different learning style. Creating a learning environment that appeals to these learning styles is one of the components that are used to increase the success and motivation of teachers. Although it is difficult to prepare the course environment according to each student's learning style, facilitating this can be achieved by using different representations. The fact that the course presented to the students in the geometry courses is abstract, prevents the students to actively use spatial thinking skills. For this reason, as we have seen in our research, geometric representations increase the success level in geometry lessons.

A study conducted by Mousoulides and Gagatsis (2004) aimed to evaluate the relation between students' approach and their ability to solve geometric problems. Results provided support for students' intention to use the algebraic approach to solve simple function tasks. Students who were able to use geometric approach had better results in solving complex geometric problems.

Actually, geometric approach is about the visualization. In a study by Arcavi (2003), visualization is defined as both the product and the process of creation, interpretation and reflection upon pictures and images, is gaining increased visibility in mathematics and mathematics education. We can say that some students remember visual things than words easily. That reminds us the individualism in education again. It brings together the idea of multiple ways of learning, multiple learning styles and so multiple representations in mathematics.

According to the study result; it is recommended that the instructors be aware of the place of multiple representations in the process of mathematics teaching, too. Therefore, the curriculum is recommended to be arranged accordingly in order for being able to raise individuals who are able to use shift from multiple representations. Hence, use of multiple representations is one of the points which one should be aware of that it has an important place not only during the teaching process, but also in the evaluation process.

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