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Comparison of Tangent Curve Method and Reciprocal Slope Transformation Method for Describing Mechanical Behaviour of *Jatropha Curcas* L. Bulk Seeds Under Compression Loading

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Abstract: The study investigates the use of two different methods for the description of deformation characteristics of *Jatropha curcas* L. bulk seed in linear compression. These methods include the tangent curve and reciprocal slope transformation. The merits and limitations of both methods were also compared. The different methods were used for development of mathematical equations for fitting experimental data describing the mechanical behaviour of Jatropha bulk seed. Pressing vessel diameter 60 mm was used to compress the bulk Jatropha seed which was measured at pressing height 80 mm. For tangent curve method, the experimental data was fitted by using the Marquardt Levenberg process whiles the Least Square Method (LSM) satisfies the reciprocal slope transformation method. Statistical analysis of both experimental and fitted data coefficients of reciprocal slope transformation and tangent curve methods indicated significant results with high coefficient of determination which suggests the suitability of both methods for the description of deformation characteristic of *Jatropha curcas* L. bulk seed in linear compression.
Key words: Oil bearing bulk seed, mathematical model, force deformation curve, Marquardt Levenberg Process, Least Square Method

INTRODUCTION

Mathematical description of deformation characteristic is important to understand the mechanical behaviour of bulk oilseeds under compression loading which could be used for technological improvement. There has been some published information describing the mechanical behaviour and deformation characteristic curves of bulk Jatropha seeds (Kabutey et al., 2013) as well as different solution methods generally based on the Darcy's Law (Fasino and Ajibola, 1990) and fluid flow through porous media (Singh and Kulshreshtha, 1996). Darcy's law and rheological properties of deformable solid matrix of bulk oilseeds are fundamental for the development of mathematical models for mechanical behaviour description of bulk oilseeds (Petru at al., 2012). The mechanical behaviour of bulk oilseeds can be described also by methods based on the Terzagi's model (Willems, Kuipers and De Hann, 2008) or the energetic balance model (Zheng et al., 2005). In terms of the tangent curve method (Herak et al., 2013) and reciprocal slope transformation (Blahovec 2011; Herak et al., 2014), the bulk oilseeds are considered as a unit which is most relevant from construction engineering and economical point of view. The aim of this study is to compare the suitability of tangent curve and reciprocal slope transformation methods for mechanical behaviour of Jatropha bulk seeds under compression loading.

MATERIALS and METHODS

Sample

Samples of bulk *Jatropha curcas* L. seeds, variety IPB2, obtained from North Sumatra, Indonesia were used for the experiment. The general physical properties of the oilseed crop are given in Table 1.

The moisture content Mc (% d. b.) of the samples was determined using standard moisture measurement equipment (Farm Pro, model G, Czech Republic). The mass of sample m_s (g) was determined using an electronic balance (Kern 440–35, Kern & Sohn GmbH, Balingen, Germany). The porosity Pf (%) was calculated from the relationship between the bulk and true densities (Blahovec 2008). The bulk density was determined from the mass of the sample divided by initial pressing volume. The true seed density was determined gravimetrically (Blahovec, 2008). The results obtained were expressed as mean of three replicates.

Table 1. Physical properties of bulk seeds of Jatropha; data in the table are means ± SD

M_c (%)	m (g)	V (mm^3)	P_f (%)	ρ_b (kg m^{-3})	ρ_t (kg m^{-3})
8.5 ± 0.2	87.95 ± 1.19	226224 ± 6340	59.98 ± 1.26	388 ± 12	980 ± 12

M_c – moisture content of bulk seeds in dry basis, m – mass of bulk seeds, V – initial volume of bulk seeds, P_f – porosity of bulk seeds, ρ_b – bulk density, ρ_t – true density

Compression test

To determine the relationship between compressive force and deformation characteristic curves, a compression device (Labortech, model 50, Germany) was used to record the course of deformation function. A single pressing vessel diameter, 60 mm with plunger (Fig. 1) was used. Initial pressing height 80 mm of Jatropha bulk seeds were tested with a compression speed of 1 mm s-1 under temperature of 20 ºC. The compressive force was between 0 and 100 kN. The experiment was repeated three times.

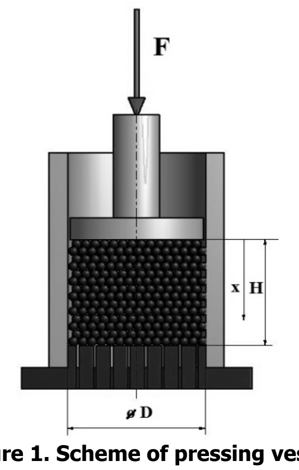

Figure 1. Scheme of pressing vessel

Reciprocal slope transformation method

Dependency between compressive force F_{RST} (N) and corresponding deformation x (mm) Eq. 2 was transformed using reciprocal slope transformation method (RST) (Herak et al., 2014) as described in Eq. 1, where T (mm N^{-1}) is amount of reciprocal transformation, a (N^{-1} mm^{-2}), b (N^{-1} mm^{-1}), c (N^{-1}) and d (N^{-1} mm) are coefficients of the RST method.

$$T(x) = \frac{x}{F_{RST}} = ax^3 + bx^2 + cx + d \qquad (1)$$

$$F_{RST}(x) = \frac{x}{ax^3 + bx^2 + cx + d} \qquad (2)$$

The coefficients were determined by the least squares method applicable in MathCAD software.

Tangent curve method

Dependency between compressive force F_{TCM} (N) and corresponding deformation x (mm) was described by tangent curve method (TCM) (Herak, et al., 2013) which is given by Eq. 3.

$$F_{TCM}(x) = A \cdot [\tan(B \cdot x)]^n \qquad (3)$$

Where A (N) is force coefficient of mechanical behaviour, B (mm^{-1}) is coefficient of mechanical deformation behaviour and n (-) is curvature of deformation curve. These coefficients were determined by Marquardt Levenberg approximation process (Marquardt, 1963) using MathCAD software.

True deformation energy

True deformation energy W was measured as the area under the compressive force-deformation curve from the zero deformation to the maximum deformation. The energy was calculated using the software Engauge Digitizer 4.1 (Mark Mitchell, NY, USA) which measures all points on the curve with respect to compressive force and associated deformation values.

Theoretical deformation energy

Amount of deformation energy developed by reciprocal slope transformation W_{TCM} (J) (Eq. 4) was determined as integral of Eq. 2 and from tangent curve method W_{TCM} (J) (Eq. 5) was determined as integral of Eq. 3.

$$W_{RST} = \int_0^{x_{max}} \frac{x}{ax^3 + bx^2 + cx + d}\, dx \qquad (4)$$

$$W_{TCM} = \int_0^{x_{max}} A \cdot [\tan(B \cdot x)]^n\, dx \qquad (5)$$

Where x_{max} (mm) is maximal deformation of Jatropha bulk seeds.

RESULTS and DISCUSSION

Measured dependency between compressive force and deformation is presented in Fig. 2 showing standard deviation. Maximal deformation of Jatropha bulk seeds was determined as x_{max} = (55.3 ± 2.8) mm.

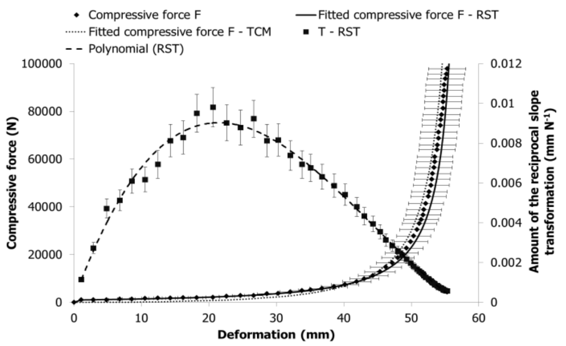

Figure 2. Measured amounts of mechanical characteristic of Jatropha bulk seeds with displayed amounts of standard deviation and their fitted functions by Eq. 2 and Eq.3.

Measured data were fitted by tangent curve method (Eq. 3) using Marguardt Levenberg process and by reciprocal slope transformation (Eq. 2) using least square method and their coefficients are shown in Table 2 and Table 3. The RST and TCM curves are presented in Fig. 2 as well as amount of reciprocal slope transformation (Eq. 1).

Table 2. Determined coefficients of RST method

a (N^{-1} mm^{-2})	***b*** (N^{-1} mm^{-1})	***c*** (N^{-1})	***d*** (N^{-1} mm)
$2.14 \cdot 10^{-7}$	$-2.85 \cdot 10^{-5}$	$0.93 \cdot 10^{-3}$	$16.34 \cdot 10^{-5}$

a, b, c, d - coefficients of reciprocal slope transformation

Table 3. Determined coefficients of TCM method

A (N)	***B*** (mm^{-1})	***n*** (-)
2320	0.026	2

A - force coefficient of mechanical behavior, B - deformation coefficient of mechanical behavior, n – tangent curve exponent

From statistical analysis ANOVA, it is evident that measured data from Jatropha bulk seeds were statistically significant with data determined by RST method (Eq. 2) as well as TCM method (Eq. 3) at significance level 0.05, that is, the values of Fcrit (critical value comparing a pair of models) being higher than the Frat values (value of the F – test) and Pvalue (significant level where it can be rejected the hypothesis of equality of models) (Table 4) also higher than 0.05 with very high coefficient of determination R2 (Table 4). This follows that both methods can be used for description of deformation curve characteristic of Jatropha bulk seeds under compression loading.

Table 4. ANOVA statistical analysis

	F_{crit} (-)	F_{rat} (-)	P_{value} (-)	R^2 (-)
RST	0.062	3.919	0.803	0.984
TCM	$5.404 \cdot 10^{-3}$	3.918	0.942	0.995

F_{rat} - value of the F test, F_{crit} - critical value that compares a pair of models, P_{value} - the significance level at which it can be rejected the hypothesis of equality of models, R^2 – coefficient of determination

Determined amounts of true deformation energy and energies calculated by Eq. 4 and Eq. 5 are presented in Table 5. From mutual comparison done by T test with significance level 0.05 (T_{crit} = 2.77 is greater than T_{value} = 2.24 for RST and also than T_{value} = 2.24 for TCM) it is clear that determined energies by Eq. 4 and Eq. 5 are significant. The RST (Eq. 4) and TCM (Eq. 5) models can be also applied for deformation energy determination.

Table 5. Deformation energy

W (J)	W_{RST} (J)	W_{TCM} (J)
525 ± 25	470 ± 24	539 ± 22

W – true deformation energy, W_{RST} - theoretical deformation energy based on RST, W_{TCM} – theoretical deformation energy based on TCM

It is evident that using derivation of Eq. 2 and Eq. 3 the dependency between modulus of elasticity and bulk deformation can be determined. This study shows that the developed mathematical equations Eq. 2 and Eq. 3 take into account the experimental boundary conditions of the linear compression of Jatropha bulk seeds for the mechanical behaviour description. The boundary conditions means that the origin of the deformation curve starts from zero force and zero deformation which is followed by an increasing function within the whole range of pressing process and when compressive force is approaching infinity then deformation reaches a maximum limit (Herak et al., 2013; Kabutey et al., 2013). It is important to mention here that both methods are suitable for mechanical behaviour description of bulk oilseeds of *Jatropha curcas* L. RST method using least square method for data fitting and its coefficients can be determined by standard computer software, however TCM method using Marquardt Levenberg approximation process is determined by the MathCAD software. On the other hand the derivation of RST energy equations (Eq. 4) using analytical methods form Eq. 2 is much more difficult than using TCM equation Eq. 3 and Eq. 5 respectively. However, this disadvantage fades into the background when using numerical method for solving Eq. 4 and Eq. 5. The results of our study are in accordance with already published theories related to the tangent curve method (Petru et al., 2012; Herak et al., 2013) and to the reciprocal slope transformation (Herak et al., 2014; Blahovec and Yanniotis, 2009; Blahovec 2011). The advantage of using both methods is that it is not conditional to resolve individual particles and their properties and relationships between particles since both methods use the bulk seeds as a unit which is affected by constrains between the pressing vessel and bulk oilseeds and the pressing process. These theories can be used for determining the mechanical behavior of different materials under compression loading.

CONCLUSIONS

- The reciprocal slope transformation method and tangential curve method were used for mathematical description of *Jatropha curcas* L. bulk oilseeds. From the statistical analysis, it is evident that both models can be significantly used for description of mechanical behaviour of Jatropha bulk seeds under compression loading as well as for deformation energy determination.
- Both methods consider bulk oilseeds as a unit which is important from technological and economical point of view.

REFERENCES

Blahovec J., 2008. Agromaterials. CULS Prague.

Blahovec J., 2011. Reciprocal slope transformation applied to physical problems. Applied Mathematics Letters, 24 (12): 2038 – 2041.

Blahovec J., S. Yanniotis, 2009. Modified classification of sorption isotherms. Journal of Food Engineering, 91: 72 – 77.

Herak D., J. Blahovec, A. Kabutey, 2014. Analysis of the axial pressing of bulk Jatropha curcas L. seeds using reciprocal slope transformation. Biosystems Engineering, 121: 67 – 76.

Herak D., A. Kabutey, M. Divisova, S. Simanjuntak, 2013. Mathematical model of mechanical behaviour of Jatropha curcas L. seeds under compression loading. Biosystems Engineering, 114 (3): 279-278.

Kabutey A., D. Herak, R. Choteborsky, O. Dajbych, M. Divisova, W. E. Boatri, 2013. Linear pressing analysis of Jatropha curcas L seeds using different pressing vessel diameters and seed pressing heights. Biosystems Engineering, 115 (1): 42–49.

Fasino O. O., O. O. Ajibola, 1990. Development of equations for the yield of oil expressed from conophor nut. Journal of Agricultural Engineering Research, 46: 45-53.

Marquardt D. W., 1963. An Algorithm for the Least-Squares Estimation of Nonlinear Parameters, SIAM. Journal of Applied Mathematics, 11 (2): 431-441.

Mrema, G. C., P. B. Mc Nulty, 1985. Mathematical model of mechanical oil expression from oilseeds. Journal of Agricultural Engineering Research: 31 (4), 361-370.

Petru M., O. Novak, D. Herak, S. Simanjuntak, 2012. Finite element method model of the mechanical behaviour of Jatropha curcas L. seed under compression loading. Biosystems Engineering, 111 (4): 412-421.

Willems P., N. J. M. Kuipers, A. B. De Haan, 2008. Hydraulic pressing of oilseeds: Experimental determination and modeling of yield and pressing rates. Journal of Food Engineering, 89 (1): 8-16.

Zheng Y., D. P. Wiesenborn, K. Tostensona, N. Kangasa, 2005. Energy analysis in the screw pressing of whole and dehulled flaxseed. Journal of Food Engineering, 66 (2): 193-202.