

Research Article / Araştırma Makalesi

STOCK MARKET VOLATILITY MODELING WITH MACRO-FINANCIAL FACTORS: THE AR-X-GARCH APPROACH

Sureyya TEMELLİ¹ 

ABSTRACT

The purpose of this study is to examine the dynamics of S&P 500 index volatility using daily data from January 1, 2020, to November 1, 2025. The analysis focuses on volatility persistence, clustering behavior, and the effects of the EUR/USD exchange rate and WTI crude oil prices on the stock market. For this purpose, daily return series for the S&P 500 index, the EUR/USD exchange rate, and WTI crude oil prices are employed. The stationarity properties of the series are examined using unit root tests, and the presence of conditional heteroskedasticity is confirmed prior to the estimation process. Volatility dynamics are modeled using the GARCH(1,1) framework, while cross-market interactions are analyzed through an AR-X-GARCH specification. Risk performance is further assessed through Value-at-Risk (VaR) backtesting at the 99% confidence level. The empirical findings indicate that all return series are stationary and exhibit significant volatility clustering. The GARCH(1,1) estimates reveal strong volatility persistence, confirming that market risk evolves over time. The AR-X-GARCH results show that EUR/USD exchange rate returns have a negative and statistically significant effect on S&P 500 returns, while WTI crude oil returns exert a positive and economically meaningful impact. VaR backtesting results support the reliability of the risk estimates. Overall, the findings demonstrate that foreign exchange and commodity markets play a decisive role in shaping stock market volatility and that GARCH-type models can be effectively employed in portfolio risk measurement and financial stability analysis.

Keywords: Stock Market Volatility, GARCH Model, Exchange Rate Effect, Crude Oil Prices, Risk Management

JEL Classification: G10, G15, C58, Q43

MAKRO-FİNANSAL FAKTÖRLERLE HİSSE SENEDİ PİYASASI OYNAKLIĞININ MODELLENMESİ: AR-X-GARCH YAKLAŞIMI

ÖZET

Bu çalışmanın amacı, 1 Ocak 2020 – 1 Kasım 2025 dönemine ait günlük veriler kullanılarak S&P 500 endeksi oynaklığının dinamiklerini incelemektir. Analiz, oynaklık kalıcılığı, kümelenme davranışı ve EUR/USD döviz kuru ile WTI ham petrol fiyatlarının hisse senedi piyasası üzerindeki etkilerine odaklanmaktadır. Bu amaçla S&P 500 endeksi, EUR/USD döviz kuru ve WTI ham petrol fiyatlarına ait günlük getiri serileri kullanılmıştır. Serilerin durağanlık özellikleri birim kök testleriyle incelenmiş

¹ Asst. Prof., Trakya University, FEAS, Edirne, Türkiye, sureyyadal@trakya.edu.tr

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ve tahmin sürecinden önce koşullu değişen varyansın varlığı doğrulanmıştır. Oynaklık dinamikleri GARCH(1,1) modeli ile, piyasalar arası etkileşimler ise AR-X-GARCH yaklaşımıyla analiz edilmiştir. Risk performansı, %99 güven düzeyinde gerçekleştirilen riske maruz değer (VaR) geriye dönük testleri ile ayrıca değerlendirilmiştir. Ampirik bulgular, tüm getiri serilerinin durağan olduğunu ve belirgin oynaklık kümelenmesi sergilediğini göstermektedir. GARCH(1,1) tahminleri güçlü oynaklık kalıcılığına işaret ederek piyasa riskinin zaman içinde evrildiğini doğrulamaktadır. AR-X-GARCH sonuçları, EUR/USD döviz kuru getirilerinin S&P 500 getirileri üzerinde negatif ve istatistiksel olarak anlamlı bir etkiye sahip olduğunu, WTI ham petrol getirilerinin ise pozitif ve ekonomik olarak anlamlı bir etkide bulunduğunu ortaya koymaktadır. VaR geriye dönük test sonuçları risk tahminlerinin güvenilirliğini desteklemektedir. Genel olarak bulgular, döviz ve emtia piyasalarının hisse senedi piyasası oynaklığının şekillenmesinde belirleyici bir rol oynadığını ve GARCH türü modellerin portföy risk ölçümü ve finansal istikrar analizlerinde etkin biçimde kullanılabileceğini göstermektedir.

Anahtar Kelimeler: Hisse Senedi Piyasası Oynaklığı, GARCH Modeli, Döviz Kuru Etkisi, Ham Petrol Fiyatları, Risk Yönetimi

JEL Sınıflandırması: G10, G15, C58, Q43

1. Introduction

Volatility is regarded as one of the foundational concepts in modern financial economics, as it reflects the degree of uncertainty and variation inherent in asset returns. Rather than focusing solely on changes in the level of prices, volatility measures the amplitude and frequency of return fluctuations over time. For this reason, it serves as a widely accepted indicator of market risk and investor uncertainty. Periods characterised by sharp price swings and unstable market behaviour are typically associated with elevated volatility, signalling greater ambiguity regarding the future path of the economy and financial markets. In contrast, periods of relatively small and infrequent price changes tend to coincide with more predictable and stable financial environments. Consequently, volatility not only plays a critical role in asset-pricing models but also forms the backbone of risk-management frameworks used by financial institutions, portfolio managers and regulators.

A key dimension of volatility relates to the way risk is transmitted across different asset classes and markets. This phenomenon, commonly referred to as volatility spillover (Diebold & Yilmaz, 2012), describes the process through which shocks originating in one market, for instance, equity, currency or commodity markets, influence the volatility dynamics of other financial segments. As capital flows have become increasingly mobile and financial markets more globally integrated, the degree of interconnectedness among markets has intensified. Events occurring in one region or asset class can rapidly affect investor sentiment worldwide, giving rise to synchronized volatility across diverse financial systems. Such spillover effects often become more pronounced during crisis periods (Bekaert et al., 2005), when uncertainty and risk aversion rise sharply and market participants react collectively to new information. As a result, understanding volatility transmission mechanisms is crucial for assessing systemic risk and monitoring financial stability at both national and global levels.

In addition to market structure and behavioural elements, macroeconomic conditions exert a powerful influence on financial-market volatility. A variety of macro-financial variables -including interest-rate developments, inflation dynamics, exchange-rate fluctuations, energy-price movements and monetary-policy decisions - shape asset valuations by altering

expectations about future cash flows and required returns. These factors may also influence investor behaviour and risk perceptions, thereby amplifying or dampening market volatility. For example, unexpected monetary-policy tightening may trigger bouts of financial turbulence, while improving macroeconomic indicators can contribute to calmer market conditions. Therefore, analysing how these macro-financial forces interact with volatility is essential not only for advancing financial-economics theory but also for guiding investment strategies and policy design.

Against this background, the present study aims to explore the dynamics of financial-market volatility and the role of key macro-financial variables in shaping these dynamics. The empirical analysis concentrates on the Standard & Poor's 500 (S&P 500) Index, representing the largest and most liquid segment of the U.S. equity market, together with two important external financial indicators: the Euro-US Dollar (EUR/USD) exchange rate and West Texas Intermediate (WTI) crude-oil prices. The interaction among these markets offers a valuable setting in which to examine cross-market volatility transmission, given the central position of the U.S. stock market in the global financial architecture and the strategic importance of both the euro and oil markets.

In particular, the study investigates whether shocks to exchange rates and oil prices are reflected in equity-market volatility, and if so, whether these effects help explain the persistence and clustering behaviour commonly observed in stock-market volatility. The issue is not only whether such external variables influence market levels or returns, but also whether they affect the intensity and duration of volatility episodes. By addressing these questions, the study contributes to a deeper understanding of the channels through which macro-financial shocks permeate equity markets.

Methodologically, the research adopts a GARCH-based modelling strategy. The standard GARCH(1,1) specification is used as a benchmark framework to model time-varying volatility and capture the well-documented features of persistence and clustering in financial returns. However, to analyse whether external macro-financial shocks modify the structure of volatility, the study extends the model into an AR-X-GARCH framework, allowing exogenous variables such as the EUR/USD exchange rate and WTI crude-oil prices to enter the conditional mean equation. This enables a direct assessment of cross-market spillovers and the extent to which these variables contribute to the observed volatility patterns in the S&P 500 Index. Such an approach maintains the analytical simplicity of a single-equation model while still providing meaningful insight into macro-financial linkages.

Despite the growing body of research on volatility spillovers, the existing literature remains fragmented in both theoretical focus and methodological design. First, many studies examine the effects of exchange rates and oil prices on stock markets separately, rather than assessing their joint role within a unified volatility framework. Second, a large part of the literature relies on multivariate GARCH models or spillover index approaches that are highly useful for measuring interdependence across markets, yet these frameworks often prioritise system-wide transmission patterns over the parsimonious identification of how specific macro-financial variables contribute to the volatility dynamics of a benchmark equity index. Third, relatively fewer studies provide a single-equation assessment of how major macro-financial variables shape S&P 500 volatility in the post-2020 period, which has been character-

ised by exceptional global uncertainty. Finally, although volatility persistence and spillover effects are widely documented, the extent to which exchange-rate and oil-price returns improve the explanatory and risk-forecasting performance of a benchmark GARCH specification remains insufficiently addressed. Unlike much of the existing literature, which primarily relies on multivariate spillover frameworks, this study adopts a parsimonious single-equation AR-X-GARCH approach that allows for direct interpretation of macro-financial effects on volatility. In addition, the analysis focuses on the post-2020 period characterised by unprecedented global shocks, thereby providing updated empirical evidence on volatility dynamics under extreme uncertainty. In this context, the present study contributes to the literature in three ways. It jointly evaluates the role of EUR/USD and WTI returns in explaining S&P 500 volatility, adopts a parsimonious AR-X-GARCH framework that allows direct interpretation of macro-financial effects, and complements in-sample volatility estimation with VaR backtesting in order to assess the practical adequacy of the model for risk measurement. Accordingly, the study is guided by the following research question: to what extent do exchange-rate and oil-price dynamics improve our understanding and prediction of equity-market volatility beyond the standard GARCH structure?

The study is organised as follows. First, a review of the existing theoretical and empirical literature is presented to situate the research within its broader academic context and to highlight key findings and debates in the field. The next section introduces the data set, outlines the theoretical foundations of the models employed and explains the econometric methodology applied in the analysis. This is followed by the presentation and interpretation of the empirical results, where particular emphasis is placed on the effect of exchange-rate and oil-price fluctuations on equity-market volatility. Finally, the paper concludes with a discussion of the main results, their implications for market participants and policymakers and potential avenues for future research.

Overall, by jointly examining volatility behaviour in financial, foreign-exchange and commodity markets within a unified framework, this study seeks to contribute to the growing body of evidence on macro-financial spillovers and their implications for risk assessment in an increasingly integrated global financial system.

2. Volatility and Cross-Market Dependence

Volatility has long been recognised in the financial economics literature as a fundamental concept that reflects the degree of uncertainty and fluctuation in asset prices. Early contributions such as Mandelbrot (1997) demonstrated that financial returns deviate from the normal distribution, display “fat-tailed” characteristics and exhibit volatility clustering. These findings contradict the assumption of constant variance embedded in traditional asset-pricing models and paved the way for the development of modelling frameworks capable of capturing time-varying volatility.

A major step in this direction was taken by Engle (1982), who introduced the ARCH model, while Bollerslev (1986) extended this framework by proposing the GARCH model, enabling a more flexible specification of conditional variance dynamics. Subsequent extensions such as EGARCH (Nelson, 1991), TGARCH (Zakoian, 1994) and GJR-GARCH (Glosten et al., 1993) allowed researchers to account for asymmetry and leverage effects, thereby providing a more realistic representation of volatility behaviour in financial markets.

Research on volatility spillovers has broadened this field by examining how shocks originating in one market are transmitted to others. Diebold & Yilmaz (2009, 2012) proposed an econometric framework to quantify volatility spillovers across global financial markets. Similarly, Engle et al. (1990) documented significant volatility spillovers across major foreign-exchange market segments, showing that shocks originating in one market can affect subsequent volatility in other markets. Further evidence of significant volatility interactions between stock markets and exchange rates is reported by Kanas (2000).

From a macroeconomic perspective, Schwert (1989) and Hamilton (1983) showed that macroeconomic uncertainty - such as inflation, interest-rate and oil-price shocks - plays a crucial role in shaping financial-market volatility. Building on this line of research, Kilian (2009) highlighted the importance of distinguishing between demand- and supply-driven oil-price shocks and demonstrated that these shocks influence commodity and equity markets in different ways.

More recent contributions combine GARCH-type models with exogenous variables and causality-in-variance approaches to better capture market dependence. For example, Arouri et al. (2012) employed a multivariate GARCH framework to analyse the oil-stock market nexus in Europe and documented significant volatility spillovers. Bhargava & Konku (2023) investigated the impact of exchange-rate fluctuations on US equity-market returns using the S&P 500 Index as a benchmark for market performance over the period 2000–2019. Drawing on VAR/VECM and GARCH family models, they found that exchange-rate volatility in currencies such as the Australian dollar, Canadian dollar and euro significantly affects the volatility of the S&P 500. Their findings also indicated asymmetric spillover effects, whereby depreciation shocks in foreign currencies exert stronger influences on stock-market volatility than appreciation shocks. This result is broadly consistent with the flow-oriented model of Dornbusch & Fischer (1980), which suggests that currency depreciation supports export competitiveness and firm profitability, thereby raising equity prices. Conversely, the portfolio-balance model (Branson et al., 1977) argues that increases in equity prices attract foreign capital inflows and lead to domestic-currency appreciation. Despite these theoretical perspectives, the empirical literature has yet to reach a clear consensus, with studies reporting positive, negative or insignificant relationships between exchange rates and equity returns (Bhargava & Konku, 2023).

In much the same way as exchange-rate movements, fluctuations in oil-price volatility also shape financial-market behaviour by influencing factors such as firms' production costs, inflation expectations and the overall mood of investors. Using monthly observations spanning the period 1986–2018, Balcilar et al. (2021) applied the Diebold–Yilmaz (2012) spillover index to explore how returns and volatility are transmitted between the S&P 500 Index, crude oil and gold. Their analysis shows that oil acts as a key hub in the information–transmission network, with volatility spillovers flowing in both directions across markets. These cross-market linkages strengthened noticeably during episodes of financial stress - most clearly during the 2008 global financial crisis - providing evidence that rising economic uncertainty intensifies market interdependence. The authors also argue that the growing financialisation of commodity markets after 2002 weakened the safe-haven characteristics traditionally associated with oil and gold, thereby limiting the diversification benefits of including commodities in investment portfolios.

Building on this strand of work, Bouazizi et al. (2022) analysed how shocks to global oil-price volatility spread to exchange rates and equity returns in major oil-importing economies such as the United States, Germany and Japan. Using ARMA–GJR–GARCH models, they report that volatility stemming from the global crude-oil market has a stronger influence on financial variables than volatility originating from domestic oil prices. This finding highlights the fact that oil operates primarily as a global risk factor rather than a country-specific one. Their Granger-causality evidence further suggests that volatility shocks tend to spill over into both currency and equity markets, although the magnitude and direction of these effects differ across economies.

Research that considers oil prices and exchange rates together further supports the idea that these variables jointly shape equity-market volatility. Basher et al. (2012) develop a structural VAR framework to analyse the dynamic interdependence among oil prices, exchange rates and equity indices in emerging markets. They report that positive oil-price shocks lead to declines in both emerging-market equity prices and the US dollar in the short term, illustrating the rapid transmission of commodity-market shocks into financial markets. In a related study, Kondo et al. (2019) employ GARCH models and identify a one-way volatility spillover running from the S&P 500 Index to crude-oil returns, suggesting that equity-market sentiment can also influence energy markets.

Mollick & Sakaki (2019) use VAR and GARCH models, with a DCC-GARCH model using data for 14 major currencies between 1999 and 2017 to explore the interaction between exchange rates, oil prices and global equity returns. Their findings show that commodity-linked currencies – such as the Australian dollar, Canadian dollar and Russian rouble – tend to appreciate when oil prices rise, whereas traditional safe-haven currencies including the Japanese yen and Swiss franc typically weaken when global equity markets strengthen. These results emphasise the dual nature of oil as both a key driver of real economic activity and a global financial risk factor that simultaneously affects equity and currency markets. Recent empirical research also highlights the role of financial stress and market conditions in shaping the relationship between oil prices and stock markets. For example, Reboredo & Rivera-Castro (2014) provide wavelet-based evidence that the interdependence between oil prices and stock markets becomes significantly stronger during periods of financial crisis, revealing clear patterns of contagion and time-varying co-movement across markets. Similarly, Shahzad et al. (2020) show that asset relationships (in the context of gold and bitcoin) in financial markets are highly state-dependent, with hedging and safe-haven properties becoming more pronounced during turbulent periods. These findings suggest that the transmission of shocks across oil, currency and equity markets intensifies under conditions of heightened uncertainty, reinforcing the importance of modelling frameworks that account for time-varying volatility and cross-market linkages. Complementing this evidence, Zhang (2024) analyses daily data from 30 July 2016 to 1 August 2023 and reports a negative relationship between the US dollar index and the S&P 500 Index based on VAR model estimates. Complementing the international literature, numerous studies have investigated volatility spillovers in Türkiye and other emerging economies. Polat & Kılıç (2022) analysed stock–exchange-rate interactions in BRICS countries using weekly data for 2004–2019 and a VAR-EGARCH framework, identifying significant bidirectional linkages between returns and volatility. Doğan (2025) examined volatility spillovers from USD/TRY and EUR/TRY exchange-rate volatility to the Borsa Istanbul 100 (BIST 100) Index and major

sector indices using Fourier variance-causality tests and found unidirectional causality from exchange-rate volatility to stock-market volatility. Şak & Öcal Özkaya (2022) applied the Diebold–Yilmaz spillover framework to daily data for 2000–2022 and showed that fluctuations in the US dollar exchange rate significantly affect BIST 100 volatility.

Yaşar Akçalı et al. (2019) employed a DCC-GARCH model to examine dynamic relations between the BIST 100 Index and several global indicators - including the JP Morgan Emerging Markets Bond Index (Global), Dow Jones Global Indices, the US Dollar Index, the CBOE SPX Volatility Index (VIX) and Brent crude oil prices - and documented persistent volatility clustering across these markets. Şenol (2020) investigated volatility transmission among Borsa Istanbul, exchange-rate and oil markets using GARCH and DCC-GARCH approaches and reported bidirectional volatility spillovers between exchange rates and stock returns, alongside unidirectional spillovers from oil to both markets. Pazarıcı et al. (2022), using an ARDL bounds-testing approach, concluded that exchange rates, risk premia and fear indices significantly influence the Turkish equity market. Similarly, Bostancı & Doğru (2023) identified significant volatility spillovers among the USD/TRY exchange rate, WTI oil prices and selected Borsa Istanbul sector indices using DCC-GARCH models. Çıtak & Kendirli (2019) reported that oil-price shocks do not generate long-run asymmetric effects on Turkish equity or exchange-rate markets, implying relative resilience of Turkish financial assets to oil shocks. Altın (2024) found a negative relationship between oil prices and the Borsa Istanbul, while the relationship between gold prices and the Borsa Istanbul was positive. Dinc Cavlak (2024) confirmed the presence of dynamic conditional correlations between US and Turkish sustainability indices and showed that oil prices have a significant causal effect on these correlations. Using frequency-domain causality tests, Karhan & Aydın (2018) detected short-run causality from Brent crude prices to the BIST 100 Index.

Taken together, the existing literature provides substantial evidence that oil prices and exchange rates play a crucial role in shaping stock-market volatility through multiple transmission channels. However, important gaps remain. Much of the prior research either focuses on multivariate spillover frameworks or examines these macro-financial variables in isolation. While multivariate GARCH and spillover index approaches are highly informative for tracing cross-market dependence, they are not always designed to provide a parsimonious assessment of how specific external variables affect the volatility dynamics of a single benchmark market within an interpretable conditional variance framework. In addition, although the time-varying and state-dependent nature of these relationships has been widely acknowledged, the extent to which exchange-rate and oil-price dynamics enhance the explanatory and predictive performance of standard volatility models remains insufficiently explored, particularly for the S&P 500 in the post-2020 period.

Building on these insights, the present study contributes to the literature in several important ways. First, it jointly examines the effects of EUR/USD exchange-rate movements and WTI crude-oil price fluctuations on S&P 500 volatility within a unified AR-X-GARCH framework. Second, it provides updated empirical evidence for the 2020–2025 period, characterised by unprecedented global uncertainty, including pandemic-related disruptions and geopolitical shocks. Third, by incorporating macro-financial variables into the conditional mean equation within a GARCH-based volatility framework, the study evaluates their contribution to improving volatility modelling and risk measurement. In doing so, the paper offers new insights

into the interconnected nature of global financial markets and provides a more comprehensive understanding of volatility dynamics in a highly integrated financial environment.

3. Methodology

The selection of an appropriate methodological framework is of central importance when analysing volatility dynamics in financial markets. Volatility is not only a statistical property of return series, but also a key measure of financial risk, market fragility and investor sentiment. Therefore, the econometric model used to estimate and interpret volatility has a direct impact on the conclusions drawn regarding financial stability, portfolio risk, and the transmission of macro-financial shocks. Indeed, model specification plays a decisive role in determining the statistical and economic significance of volatility spillovers across markets, since different specifications can lead to markedly different conclusions about the strength and direction of cross-market transmission. In this sense, the methodological design of the present study is not merely a technical detail, but a core determinant of analytical validity.

The AR-X-GARCH framework adopted in this study allows for a richer representation of financial reality compared to standard GARCH models, since it explicitly incorporates macro-financial factors such as exchange-rate and oil-price movements. This structure captures how shocks arising in the currency and commodity markets propagate into equity-market volatility, reflecting the growing interconnectedness of global financial systems. Moreover, the assumption of a Student-t distribution for the error term improves the model's ability to represent heavy-tailed behaviour, which is a well-documented feature of financial returns. These methodological choices collectively enhance the robustness and explanatory depth of the empirical findings.

Although asymmetric GARCH specifications such as EGARCH and GJR-GARCH are capable of capturing leverage effects, the present study employs a symmetric GARCH framework in order to maintain model parsimony and facilitate direct interpretation of the effects of exogenous macro-financial variables. Preliminary analyses indicated no substantial improvement in model performance when asymmetric specifications were considered.

However, despite these strengths, certain limitations should be acknowledged. First, the analysis is based on daily data, and therefore does not capture intraday volatility dynamics or high-frequency market microstructure effects, which may provide additional insights into short-term risk behaviour. Second, the model assumes linear relationships in the conditional mean equation, whereas the underlying economic mechanisms governing cross-market linkages may exhibit non-linear and regime-dependent dynamics, particularly during periods of financial stress. Third, although the EUR/USD exchange rate and WTI crude-oil prices are widely recognised as key global indicators, they do not fully represent the broader set of macro-financial factors that may influence equity-market volatility, such as monetary-policy expectations, geopolitical developments and credit-market conditions.

Another limitation concerns model risk. Even when statistical diagnostics suggest a good fit, there remains the possibility that alternative specifications -such as asymmetric GARCH models or multivariate volatility frameworks-might yield different parameter estimates or risk-forecasting results. Finally, Value-at-Risk is itself a controversial risk metric, as it summarises downside risk using a single quantile and does not provide direct information

about the magnitude of losses beyond the threshold. In this context, the issue of model risk and the limitations of Value-at-Risk (VaR) as a risk metric have been widely discussed in the literature. While VaR is a standard tool for measuring downside risk, it has been criticised for its inability to capture the magnitude of extreme losses beyond the specified quantile and for lacking subadditivity under certain conditions (Artzner et al., 1999; Jorion, 2007). These limitations imply that VaR should be interpreted as a partial measure of risk rather than a comprehensive indicator of tail risk. Accordingly, the results of the VaR analysis in this study are evaluated with these conceptual limitations in mind.

Recognising these limitations ensures that the results of the study are interpreted with appropriate caution. At the same time, the chosen methodology offers a powerful and policy-relevant lens through which to examine the evolving interaction between equity, currency and commodity markets in a period of heightened financial uncertainty.

3.1. Research Objective and Hypotheses

The primary objective of this study is to conduct a comprehensive analysis of equity-market volatility dynamics during the period 1 January 2020–1 November 2025, a time-frame characterised by heightened uncertainty in global financial markets. In this context, the study aims to examine the persistence and clustering behaviour of volatility in the S&P 500 Index, and to assess how macro-financial shocks originating from foreign-exchange and commodity markets are transmitted to equity-market volatility. In particular, the analysis evaluates the role of fluctuations in the EUR/USD exchange rate and WTI crude-oil prices in shaping both the level and persistence of stock-market volatility, thereby providing insights into the mechanisms of cross-market volatility transmission.

This research aims to deepen the understanding of the interconnections within financial markets and to offer evidence-based insights for policymakers and market participants regarding risk management and financial stability assessment. Drawing from the theoretical framework and existing empirical studies, the following hypotheses are proposed:

H1: Financial market returns exhibit statistically significant patterns of volatility clustering and persistence. This hypothesis suggests that fluctuations in equity returns do not occur randomly over time. Instead, periods of high volatility tend to be followed by further turbulence, while calm periods are typically succeeded by similarly stable market conditions. In other words, volatility displays a tendency to “cluster,” and shocks to market uncertainty may have long-lasting effects rather than fading immediately.

H2: Fluctuations in the EUR/USD exchange rate have a significant impact on S&P 500 returns. This hypothesis proposes that developments in the foreign-exchange market spill over into equity markets. Exchange-rate movements may alter investor expectations, capital flows, and global risk perceptions, thereby amplifying or dampening fluctuations in U.S. stock-market returns.

H3: Changes in WTI crude oil prices positively and significantly affect S&P 500 market returns. This hypothesis argues that oil-price shocks act as an important external source of uncertainty for equity markets. Since oil prices influence production costs, inflation dynamics, and macroeconomic performance, changes in crude-oil prices may affect S&P 500 returns and contribute to broader market volatility dynamics.

H4: Including macro-financial variables in the model improves its explanatory power compared to standard GARCH models. This hypothesis assumes that models based solely on past returns and volatility provide an incomplete picture of market dynamics. Incorporating external financial variables—such as exchange rates and oil prices—should enhance the model's ability to explain and predict volatility, reflecting the interconnected nature of modern financial markets.

In order to test the above hypotheses within a rigorous empirical setting, the following section describes the data sources and modelling strategy adopted in the study.

3.2. Data and Model

In this study, the dynamics of financial-market volatility are examined using the Standard & Poor's 500 Index (S&P 500), the Euro–US Dollar (EUR/USD) exchange rate and West Texas Intermediate (WTI) crude-oil prices. The data cover the period from 1 January 2020 to 1 November 2025 and were obtained from the Yahoo Finance platform. All variables used in the empirical analysis were converted into percentage logarithmic returns according to Equation (1):

$$r_t(\%) = 100 \times \ln \left(\frac{P_t}{P_{t-1}} \right) \quad (1)$$

where P_t denotes the price at time t , P_{t-1} the price in the previous period, and $r_t(\%)$ the percentage logarithmic return on the S&P 500. All series were aligned based on common trading days to ensure consistency across markets. Observations corresponding to non-overlapping dates were removed, and missing values were handled through listwise deletion. Since logarithmic returns are computed using both the current and the previous price observation, the transformation necessarily results in the loss of one initial observation. After computing the log-return series for all variables, their stationarity properties were examined using the Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) unit-root tests. In both tests, the null hypothesis states that the series contain a unit root.

Subsequently, the conditional mean equation of the S&P 500 return series was estimated to identify the presence of ARCH effects, which indicate time-varying volatility:

$$r_t = \mu + \varepsilon_t \quad (2)$$

Here, μ represents the mean and ε_t the error term. Following Engle's (1982) procedure, an auxiliary regression was estimated using the squared residuals from Equation (2) to test for ARCH effects:

$$\hat{\varepsilon}_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \hat{\varepsilon}_{t-i}^2 + u_t \quad (3)$$

disturbances are homoskedastic, whereas the alternative hypothesis implies conditional heteroskedasticity. The test statistic is computed as nR^2 , the product of the sample size and the coefficient of determination from the auxiliary regression, which asymptotically follows a chi-square distribution with q degrees of freedom. Rejection of the null indicates serial correlation in the squared residuals, confirming the presence of ARCH effects in the return series. Given this result, volatility dynamics are modelled using the GARCH(1,1) specification introduced

by Bollerslev (1986). In this model, conditional variance depends on both lagged squared residuals and lagged conditional variances (Terzioğlu, 2018). The parameters are estimated under the standard regularity conditions $\omega > 0$, $\alpha_j \geq 0$, $\beta_j \geq 0$, and $\alpha_j + \beta_j < 1$, which ensure positivity and covariance-stationarity of the variance process. Accordingly, the conditional variance equation is expressed as:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (4)$$

To better capture the heavy-tailed behaviour frequently observed in financial returns, the error term is assumed to follow a Student-t distribution:

$$\varepsilon_t = z_t \sqrt{h_t}, \quad z_t \sim t_v(0, 1) \quad (5)$$

where z_t denotes the standardized residuals and represents the degrees of freedom, reflecting the fat-tailed nature of the distribution. To further explain the volatility dynamics of the S&P 500, an AR-X-GARCH(1,1) model is employed, in which the return depends on its own lagged value as well as exogenous variables:

$$r_t = c + \phi_1 r_{t-1} + \gamma_1 x_{1,t} + \gamma_2 x_{2,t} + \varepsilon_t \quad (6)$$

Here, r_t denotes the S&P 500 return at time t , r_{t-1} its one-period lag, while $x_{1,t}$ and $x_{2,t}$ represent the returns on the EUR/USD exchange rate and WTI crude-oil price, respectively. Consistent with stylised facts in the literature, the error term is again assumed to follow a Student-t distribution. Conditional volatility is modelled through a GARCH process in which variance evolves according to past shocks and past conditional variances. The lag structure in the AR-X specification was determined based on information criteria and diagnostic tests, with the AR(1) structure providing the most parsimonious and statistically adequate representation.

Model adequacy and the behaviour of the residuals are assessed using a set of standard diagnostic tests. The Ljung–Box test is employed to detect autocorrelation, the ARCH–LM test is used to determine whether the residuals still exhibit conditional heteroskedasticity, and the Jarque–Bera test evaluates deviations from the assumed distributional form. To evaluate the risk-forecasting performance of the selected model, Value at Risk (VaR) is computed at the 99% confidence level using Equation (7):

$$VaR_T = \mu + \sigma T^{-1}(\alpha, v) \sqrt{\frac{v-2}{v}} \quad (7)$$

where μ denotes the mean return on the S&P 500, σ its volatility, v the degrees of freedom of the Student-t distribution, and $T(\cdot)$ the cumulative distribution function of the standardized Student-t distribution (Xu, 2017). The VaR value represents the loss threshold that is not expected to be exceeded with 99% probability in the following period. VaR violations are then defined as:

$$violation_t = \begin{cases} 1, & Actual_t < VaR_t \\ 0, & other \end{cases} \quad (8)$$

Let x denote the total number of violations and T the total number of observations. The observed violation rate is therefore calculated as $\hat{p}=x/T$. To determine whether the VaR model generates the correct coverage rate, the Kupiec Unconditional Coverage (UC) test is applied (Kupiec, 1995). The null hypothesis states that the expected and observed violation probabilities are equal. The test statistic is given by:

$$LR_{UC} = -2[\ln L(p) - \ln L(\hat{p})] \sim \chi^2(1) \quad (9)$$

Having introduced the Value-at-Risk framework and the statistical procedure used to assess the accuracy of the estimated risk measures, the next step is to present the empirical findings. In this context, the following section reports the estimation results obtained from the GARCH-based volatility models and the VaR back-testing exercises. The analysis first evaluates the behaviour of return volatility and the significance of the explanatory variables, and subsequently examines whether the proposed risk-measurement framework provides adequate coverage at the selected confidence levels. These results form the empirical basis for assessing the interaction between financial markets and for evaluating the effectiveness of the modelling strategy adopted in the study.

4. Results

The volatility patterns observed in financial markets provide valuable insights into risk perception and the nature of interactions across markets. The empirical findings reveal how volatility in equity, foreign-exchange and commodity markets evolves over time, and indicate that these dynamics reflect not only market-specific factors but also broader financial inter-linkages. The results offer an overall perspective on how uncertainty is distributed across different asset classes and help to clarify the mechanisms through which market conditions and cross-market connections shape financial volatility.

In this context, the descriptive statistics presented in Table 1 highlight notable differences in risk and return characteristics among the markets under consideration. The S&P 500 Index displays the highest standard deviation and the widest minimum–maximum range, indicating greater volatility and uncertainty relative to the other asset classes. By contrast, the EUR/USD exchange rate exhibits a lower level of dispersion and fewer extreme values, suggesting a comparatively stable structure. WTI crude-oil returns record the highest mean return, while at the same time presenting moderate volatility accompanied by noticeable price fluctuations.

Taken together, these findings suggest that equity markets are associated with higher levels of risk, foreign-exchange markets tend to be more stable, and commodity markets occupy an intermediate position in terms of the risk–return trade-off.

As reported in Table 2, the ADF and PP unit-root test results consistently show that all return series are stationary, regardless of whether the specification excludes a constant, includes a constant, or incorporates both a constant and a trend. For the S&P 500, the EUR/USD exchange rate and WTI crude-oil prices, the test statistics are significant at the 1% level, leading to a clear rejection of the unit-root hypothesis. In practical terms, this implies that the series fluctuate around a stable mean and do not display a stochastic trend. Hence, the data are already in a form suitable for modelling volatility, and no additional differencing or transformation is required.

Table 1: Risk and Return Characteristics among the Financial Markets

Statistic	S&P 500	EUR/USD	WTI
Number of observations	1463	1463	1463
Mean	0.040902	0.002069	0.054063
Standard Deviation	3.220128	0.483426	1.335240
Minimum	-28.220613	-2.814408	-12.765220
Maximum	31.963363	2.753679	9.089488

Table 2: Stationarity Evidence in Financial Markets

Variable	Test	None	Constant	Constant&Trend
S&P 500	ADF	-6.6556***	-6.6745***	-6.7620***
	PP	-36.9409***	-36.9279***	-36.9094***
EUR/USD	ADF	-37.1362***	-37.1244***	-37.1169***
	PP	-37.1589***	-37.1469***	-37.1419***
WTI	ADF	-11.3889***	-11.4987***	-11.4996***
	PP	-44.6956***	-44.8403***	-44.8313***

Note: *** p < 0.01

The ARCH–LM statistics reported in Table 3 provide clear evidence that the variance of S&P 500 returns is not constant over time, confirming the presence of conditional heteroskedasticity. This finding is consistent with the well-established stylised facts of financial return series. Furthermore, the estimated ARCH and GARCH coefficients are both positive and statistically significant, indicating strong volatility clustering and a high degree of persistence. These results suggest that shocks to market volatility are not short-lived, but instead tend to have prolonged effects over time.

In addition, the sum of the ARCH and GARCH coefficients is close to unity, further confirming the high persistence of volatility in the S&P 500 market. The persistence of volatility can also be expressed in terms of its half-life, defined as the time required for the impact of a shock to decay by half. The half-life is calculated as $\ln(0.5)/\ln(\alpha + \beta)$. Based on the estimated parameters, the half-life of volatility shocks is approximately 15 trading days, indicating that the effects of market disturbances remain significant for several weeks.

This behaviour is consistent with the well-known stylised facts of equity markets, where financial shocks tend to cluster rather than dissipate instantaneously (Cont, 2001). Both the standard GARCH specification and the extended AR–X–GARCH model confirm that increases in market uncertainty are typically followed by additional waves of volatility. In other words, shocks to the system leave a persistent effect on investor expectations and risk perception rather than being absorbed immediately by the market.

Within this framework, the ARCH coefficient (α_i) captures the short-run impact of past shocks on current volatility, the GARCH coefficient (β_i) reflects volatility persistence, and the

constant term (ω_l) represents the long-run average variance level. The statistical significance and magnitude of these parameters jointly confirm the strong clustering and persistence of volatility in the S&P 500 market.

Once the EUR/USD exchange rate and WTI crude-oil prices are incorporated into the empirical framework, the underlying financial dynamics become more clearly observable. The estimation results show that EUR/USD returns have a negative and statistically significant effect on S&P 500 returns, whereas WTI crude-oil returns exert a positive and highly significant impact. These findings underscore the important role of exchange-rate and commodity-market dynamics in shaping equity-market behaviour.

The results are broadly consistent with the existing literature. In particular, the negative relationship between exchange-rate movements and stock-market returns aligns with the findings of Ehrmann et al. (2011), who document significant interactions between currency markets and equity markets. Similarly, the positive and statistically significant impact of oil-price movements is in line with the findings of Hamilton (1983) and Kilian (2009), who emphasise the macroeconomic importance of oil-price shocks in shaping financial-market dynamics.

More recent studies also support these results. For example, Joo & Park (2021) report a significant relationship between oil-price dynamics and stock-market behaviour, while Bilal et al. (2024) find that oil-price shocks positively affect stock-market performance in emerging markets. In addition, Reboredo & Rivera-Castro (2014) show that the dependence between oil prices and stock markets intensifies during periods of financial stress.

However, unlike much of the existing literature, which typically relies on multivariate or spillover-based approaches, the present study provides evidence within a parsimonious single-equation AR-X-GARCH framework. This allows for a more direct and interpretable assessment of how macro-financial variables influence equity-market dynamics, thereby offering a complementary perspective to the existing body of research.

In particular, the negative coefficient on the exchange rate indicates that EUR/USD and S&P 500 returns move in opposite directions, with U.S. dollar appreciation being generally associated with stronger equity-market performance. This relationship may reflect stronger investor confidence in U.S. assets and capital flows toward U.S. financial markets. In this context, U.S. dollar appreciation may coincide with stronger equity-market performance, consistent with the portfolio-balance perspective.

The positive impact of oil-price returns may reflect the pro-cyclical nature of commodity markets, where rising oil prices are often associated with stronger global demand conditions and improved economic expectations. In this sense, oil-price increases may signal favourable macroeconomic conditions, which in turn support equity-market performance and contribute to higher return levels. At the same time, oil-price movements remain an important source of macroeconomic uncertainty, as higher energy costs may increase production expenses, compress corporate profit margins, reinforce inflationary pressures, and affect expectations regarding monetary-policy responses. These channels highlight the broader relevance of oil-price dynamics for equity-market conditions, although the estimated WTI coefficient in the present model directly captures its relationship with S&P 500 returns rather than a direct effect on conditional volatility.

The improvement in model fit following the inclusion of exchange-rate and oil-price variables — as reflected in the higher log-likelihood and lower AIC values — highlights the strong integration between global currency, commodity and equity markets. From a practical perspective, this finding implies that shocks originating outside the stock market can be transmitted rapidly into equity-price dynamics and risk behaviour. Consequently, investors operate in a risk environment shaped not only by domestic developments but also by broader macro-financial conditions.

To formally assess whether the extended specification provides a statistically superior fit, a likelihood ratio (LR) test is employed to compare the standard GARCH(1,1) model with the AR-X-GARCH(1,1) model. The GARCH model is treated as the restricted model, while the AR-X-GARCH model is considered the unrestricted model. The LR statistic is computed as:

$$LR = 2(\ln L_{AR-X-GARCH} - \ln L_{GARCH}) = 67.34 \quad (10)$$

The likelihood ratio (LR) test strongly favours the AR-X-GARCH specification over the standard GARCH model. The LR statistic (67.34) is highly significant, indicating that the inclusion of macro-financial variables significantly improves the model's explanatory power. With three degrees of freedom, this value exceeds the critical chi-square threshold even at the 1% significance level. Accordingly, the null hypothesis that the additional parameters are jointly equal to zero is rejected. This result confirms that the inclusion of the autoregressive component together with the macro-financial variables significantly enhances the explanatory power of the volatility model.

Although the residuals still exhibit heavy-tailed behaviour, implying that extreme price movements remain an inherent feature of the data, the results collectively deliver a consistent message: equity-market dynamics are significantly associated with developments in the foreign-exchange and oil markets. As such, stock-market risk cannot be accurately assessed without considering broader global financial conditions and cross-market spillover mechanisms.

Post-estimation diagnostic tests indicate that the GARCH specifications adequately capture the conditional heteroskedasticity in S&P 500 returns. The ARCH-LM and Ljung-Box statistics show no remaining ARCH effects or serial correlation in the standardized residuals. While normality (Jarque-Bera test, J-B) is rejected, the use of a Student-t distribution effectively accommodates excess kurtosis, supporting the overall adequacy of the model.

Figure 1 displays the standardized residuals obtained from the GARCH model together with the estimated conditional volatility series. The fact that the standardized residuals fluctuate around zero without exhibiting any clear systematic pattern suggests that the model successfully captures both the mean and variance dynamics of the return series. The conditional volatility, on the other hand, shows clear evidence of volatility clustering: following a sharp rise during the early part of the sample—coinciding with heightened market uncertainty—volatility gradually declines and stabilizes over time. This pattern confirms the time-varying nature of volatility and supports the suitability of the GARCH framework for modelling volatility in S&P 500 returns.

Table 3: Volatility Dynamics and Macro-Financial Spillovers in the S&P 500 Index

S&P 500 ARCH-LM (10) 227.8268***		
Parameter	Constant Mean – GARCH(1,1)	AR-X – GARCH(1,1)
c (μ)	0.1212 (0.0540)**	0.0792 (0.0545)
$\phi_1(r_{t-1})$	—	0.0298 (0.2630)
γ_1 (EUR/USD)	—	-0.2784(0.1270)*
γ_2 (WTI)	—	0.4356 (0.1270)***
ω	0.2954 (0.0940)***	0.2832 (0.0746)***
α_1	0.1251(0.0229)***	0.1248 (0.0215)***
β_1	0.8307 (0.0268)***	0.8301 (0.0229)***
v (shape)	6.8912(1.204)***	7.4248 (1.39)***
Log-L	-3313.23	-3279.56
AIC	6636.46	6575.12
ARCH-LM (10)	13.9547	17.8101
J-B	630.5084***	438.6297***
Q(20)	18.7472	18.1932

Note: Values in parentheses represent standard errors. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively.

Figure 1: Standardized Residuals and Conditional Volatility of the S&P 500 Index

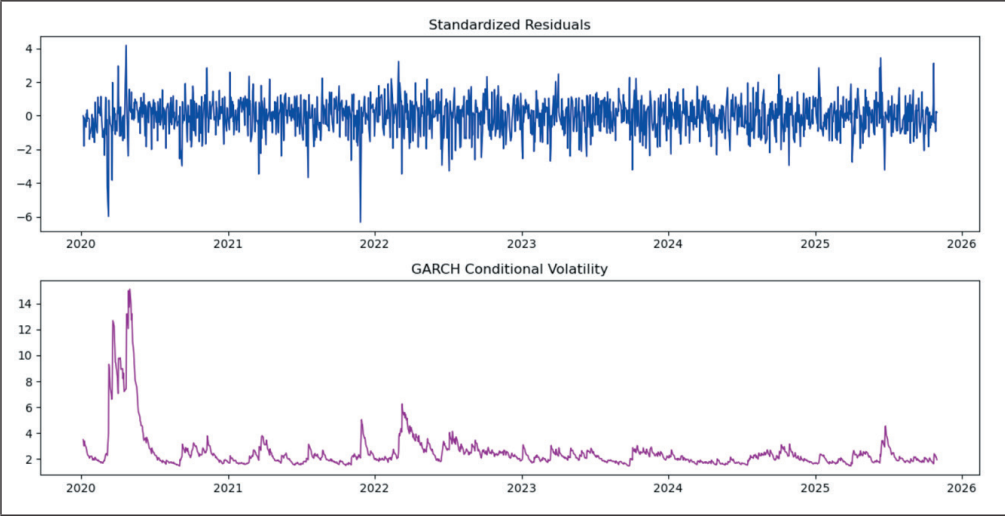
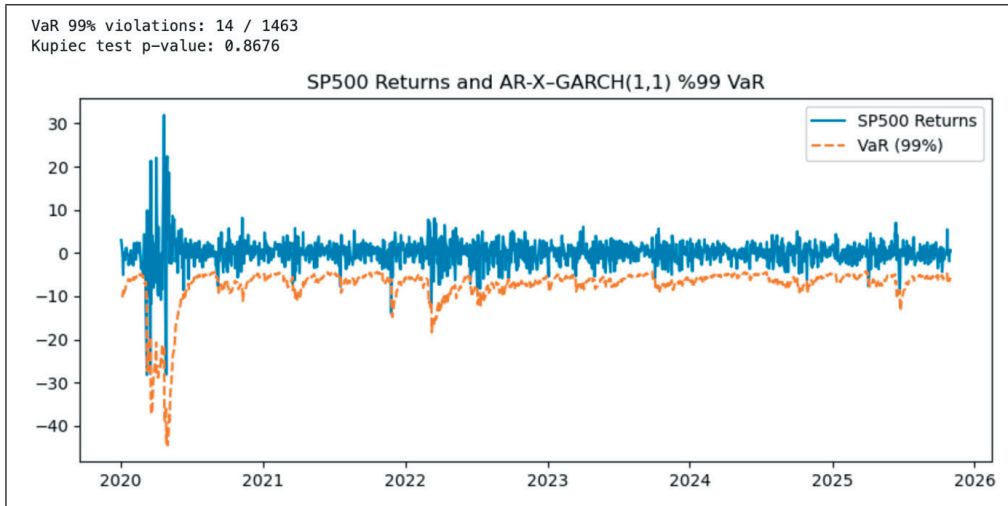


Figure 2 presents the evolution of S&P 500 daily returns together with the 99% Value-at-Risk (VaR) estimates obtained from the AR-X-GARCH(1,1) model. The chart illustrates that the VaR series responds sensitively to changing market conditions. Periods marked by elevated uncertainty and sharp price swings are accompanied by visibly higher VaR estimates, in-

dicating a greater probability of experiencing large negative returns. Conversely, during calmer phases the VaR boundary narrows, reflecting a reduction in downside risk. This confirms that the model is able to track fluctuations in risk appetite and market sentiment through time rather than assuming a constant risk structure.

Figure 2: S&P 500 Returns and 99% Value at Risk (VaR) Estimated by the AR-X-GARCH(1,1) Model



The early part of the sample -characterised by pandemic-induced financial stress - displays particularly large spikes in the VaR series, highlighting the extent to which tail-risk increased during that period. As conditions normalise, both realised volatility and the VaR threshold gradually converge toward more moderate levels, though occasional bursts of turbulence continue to generate temporary increases in estimated risk. This time-varying pattern is consistent with the presence of volatility clustering and reinforces the argument that risk management frameworks must adapt dynamically to changing financial environments.

Out of a total of 1,463 trading days, only 14 observations breach the 99% VaR boundary. This frequency is broadly consistent with theoretical expectations at the 1% tail probability, implying that the model neither systematically overstates risk nor understates it. The Kupiec unconditional coverage test confirms this conclusion statistically, as the null hypothesis that the predicted and realised violation rates are equal cannot be rejected. In other words, the VaR model appears to produce a risk measure with appropriate coverage accuracy.

From a financial-risk-management perspective, this outcome is highly relevant. The ability of the AR-X-GARCH model to adjust the VaR threshold in response to heightened uncertainty means that it can identify periods during which the likelihood of extreme portfolio losses rises materially. This information can assist traders and portfolio managers in decisions regarding leverage, hedging strategies, margin requirements and capital-allocation rules. Risk controllers may also use these signals to trigger early-warning mechanisms or tighten exposure limits when downside risk becomes unusually elevated.

At the same time, the fact that a small number of extreme losses still occur — and that the return distribution remains heavy-tailed — serves as a reminder that market risk can never be eliminated entirely, even with sophisticated modelling tools. Nevertheless, the evidence suggests that the AR-X-GARCH approach provides a credible and informative risk-measurement framework, capturing both the cyclical nature of volatility and the influence of macro-financial drivers included in the model. By successfully aligning predicted risk with realised outcomes, the model enhances the reliability of VaR-based risk assessment in the S&P 500 market.

Overall, the results point to a clear and persistent pattern in equity-market volatility, suggesting that periods of turbulence tend to reinforce themselves rather than fading quickly. This persistence means that shocks continue to shape investor expectations and pricing behaviour over time. The analysis also shows that developments in the foreign-exchange and oil markets play a meaningful role in this process. Movements in exchange rates and energy prices appear to feed into equity-market risk, indicating that the stock market is closely connected to broader global financial and commodity dynamics.

The fact that the model performs better once these macro-financial variables are included implies that volatility cannot be fully understood by looking at equity returns alone. Instead, investors and policymakers need to account for external influences that alter funding conditions, cost structures, inflation expectations and overall market sentiment. At the same time, the strong performance in risk forecasting shows that models which incorporate such variables are better equipped to detect periods when downside risk is building up.

From a practical perspective, these findings have several important implications. For portfolio managers, they underline the need to monitor exchange-rate and oil-price developments as part of the risk-management process, since shocks in these markets can quickly spill over into equities. For regulators and policymakers, the results highlight the potential for macro-financial shocks to threaten financial stability by amplifying market volatility. In short, the evidence reinforces the idea that equity-market risk is increasingly global in nature, shaped not only by domestic factors but also by shifts in key international financial and commodity markets.

5. Conclusion and Discussion

Volatility is widely regarded as one of the most informative indicators of uncertainty and risk in financial markets. Understanding how volatility behaves through time becomes particularly critical during phases of heightened macro-financial stress, when global linkages intensify and shocks originating in one market are transmitted rapidly across borders. Against this backdrop, the present study investigates volatility dynamics in the S&P 500 Index over the exceptionally turbulent period from 2020 to 2025, during which global markets were repeatedly affected by economic disruptions, geopolitical shocks and policy-driven uncertainty. The analysis is conducted within a unified AR-X-GARCH framework that integrates both EUR/USD exchange-rate movements and WTI crude-oil price fluctuations into the modelling structure. By incorporating these macro-financial drivers into the equity-return equation within a GARCH-based volatility framework, the study broadens the typical approach in the literature, which has often examined the influence of exchange rates and oil prices separately. Instead, the findings reported here highlight the combined and interactive nature of cross-market risk transmission.

The empirical results provide strong evidence in support of H1, confirming that the S&P 500 return series exhibits pronounced volatility clustering and persistent risk dynamics. In practical terms, periods of elevated market turbulence tend to be followed by further episodes of instability, while tranquil conditions also display persistence. Shocks to market uncertainty do not dissipate instantly; rather, their effects remain embedded in the system for an extended period. The sum of the ARCH and GARCH coefficients ($\alpha + \beta \approx 0.96$) indicates a high degree of volatility persistence. This persistence can also be expressed in terms of its half-life, defined as the time required for the impact of a shock to decay by half. Based on the estimated parameters, the half-life of volatility shocks is approximately 15 trading days, suggesting that the effects of market disturbances remain significant for several weeks. These findings are consistent with the stylised facts of financial time series and reinforce the view that equity-market risk evolves in a path-dependent and time-varying manner rather than following a memory-free process (Ratliff-Crain et al., 2023).

Turning to H2, the results indicate that fluctuations in the EUR/USD exchange rate exert a statistically meaningful influence on U.S. equity-market returns. EUR/USD returns are negatively and significantly associated with S&P 500 returns, indicating that developments in the foreign-exchange market are closely linked to equity-market performance. This relationship may reflect changes in investor expectations, capital flows, and global risk perceptions. These findings suggest that the foreign-exchange market plays a significant role in shaping U.S. equity-market dynamics, supporting the hypothesis that exchange-rate movements are transmitted to stock-market returns.

Consistent with H3, the analysis shows that WTI crude-oil returns are positively and significantly associated with S&P 500 returns. This finding supports the view that oil-price movements constitute an important external factor for equity markets. Changes in energy prices can influence production costs, corporate profit margins, inflation expectations, and monetary-policy responses, thereby affecting expectations regarding future economic performance and corporate earnings. The significant relationship between oil-price and equity returns highlights the central role of commodity markets in broader financial-market dynamics (Hamilton, 1983).

Finally, the results provide support for H4, indicating that the extended AR-X-GARCH specification provides additional explanatory information relative to the standard GARCH model. The inclusion of exchange-rate and oil-price variables enables the model to account for macro-financial influences on S&P 500 return dynamics while conditional volatility is captured through the GARCH process. This finding underscores the interconnected nature of modern financial markets, where market dynamics cannot be fully understood solely through past returns and volatility. Instead, macro-financial factors play an important role in shaping equity-market behaviour (Bekaert et al., 2013).

The empirical findings of this study are broadly consistent with the existing literature while also providing important extensions. In particular, the results align with Joo and Park (2021), who document a statistically significant relationship between oil-price dynamics and stock-market behaviour, as well as with Bilal et al. (2024), who find that oil-price shocks exert a positive effect on stock-market performance in emerging markets. However, unlike these studies, the present analysis focuses on a major developed market and integrates both ex-

change-rate and oil-price dynamics within a unified AR-X-GARCH framework. As such, the findings provide evidence of broader cross-market linkages among equity, foreign-exchange, and commodity markets.

At the same time, it is important to note that the estimated relationships reflect conditional dependence rather than strict causality. The coefficients capture information transmission and co-movement between markets rather than unidirectional causal effects. Exchange-rate movements may influence corporate earnings, capital flows and investor sentiment, while oil-price fluctuations affect production costs, inflation expectations and monetary-policy outlooks. Moreover, feedback effects from equity markets to currency and commodity markets may also exist, suggesting the presence of bidirectional interactions. Capturing such dynamics would require multivariate or causality-based modelling approaches, which lie beyond the scope of the present study.

Despite the strong empirical performance of the AR-X-GARCH framework, several limitations should be acknowledged. In particular, the model does not explicitly account for asymmetries in volatility responses to positive and negative shocks. Financial markets are known to exhibit leverage effects, whereby negative shocks tend to increase volatility more strongly than positive shocks of similar magnitude. Although such effects are commonly modelled using EGARCH, TGARCH or GJR-GARCH specifications, the present study adopts a symmetric GARCH framework for the sake of parsimony and interpretability. As a result, the estimated volatility dynamics should be interpreted as average conditional responses rather than as fully capturing asymmetric risk behaviour.

Future research may extend the present framework by incorporating asymmetric models, regime-switching specifications or multivariate volatility structures in order to better capture nonlinearities and feedback mechanisms in financial markets. Such extensions would provide a richer understanding of volatility transmission and further enhance the robustness of empirical findings.

Overall, the study provides strong empirical evidence that macro-financial forces play a decisive role in shaping equity-market risk. By integrating exchange-rate and oil-price dynamics into a GARCH-based framework, the analysis offers a more comprehensive and realistic depiction of volatility behaviour in an increasingly interconnected global financial system. The results highlight the importance of monitoring external financial conditions alongside domestic market indicators and contribute to the broader literature on volatility transmission and financial stability.

Statements and Declarations

The author declares no conflicts of interest and received no financial or material support from any organization for the submitted work.

Declaration of AI Use

During the preparation of this manuscript, the author used Claude (Anthropic) for language editing and grammatical refinement. Following the use of this tool, the author reviewed, edited, and verified all content and takes full responsibility for the final published article.

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