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An Investigation of Pre-service Mathematics Teachers' Understanding of the Concept of the Logarithmic Function*

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Abstract

Exponential and logarithmic functions occupy an important place in both real-world modelling and mathematics curricula, as they are used to represent processes such as growth, decay, change, and scaling. Although the concept of the logarithmic function is a fundamental topic in secondary and undergraduate programmes, it is frequently reported as an area in which students and pre-service teachers experience conceptual difficulties. Given that pre-service teachers will eventually assume responsibility for teaching this topic, examining their understanding of the concept of the logarithmic function is particularly important. This qualitative study aims to investigate pre-service mathematics teachers' understanding of the concept of the logarithmic function. The participants were five pre-service secondary mathematics teachers enrolled at a public university during the spring semester of the 2024-2025 academic year. The study was conducted as a case study. Data were collected through a researcher-developed interview form consisting of four open-ended questions under three categories: (i) the definition and properties of logarithmic functions, (ii) graphs of logarithmic functions and related features, and (iii) connections between logarithmic functions and real-world contexts. Semi-structured interviews were conducted, and the data were analysed using content analysis by coding participants' statements into subcategories. The findings indicated that participants had difficulty expressing the defining conditions of logarithmic functions accurately and consistently, tended to interpret logarithmic graphs mainly as increasing, and associated real-world contexts largely with the earthquake magnitude scale. In line with these findings, learning environments should be designed to enhance pre-service teachers' understanding of the concept of the logarithmic function.

Keywords: Mathematics education, logarithmic functions, conceptual understanding, pre-service mathematics teachers.

Introduction

Function is a central concept in mathematics and is regarded as one of the fundamental components of algebra curricula. It has been stated that students' having a narrow perspective on the concept of function may adversely affect their problem-solving processes (Vinner, 1983). Understanding the function concept encompasses a range of learning outcomes that enable this concept to be used both in various contexts within mathematics and in disciplines beyond mathematics. Many mathematics educators emphasize that the concept of function is a central concept in mathematics and is essential for a deep understanding of the subject (Leinhardt et al., 1990; Trujillo et al., 2023).

When research on the concept of function is reviewed, a general picture emerges: Students often perceive the function concept as abstract and complex; therefore, they experience various difficulties and misconceptions in making sense of functions and in using their function knowledge effectively in problem solving (Aydın, 2000; Dreyfus, 1990; Kieran, 1990; Leinhardt et al., 1990; Selden & Selden, 1992; Tall & Vinner, 1981; Parhizgar et al., 2022). Focusing specifically on logarithmic functions among function types, the literature reports that students experience difficulties when solving problems related to exponential and logarithmic topics, largely because they have not sufficiently constructed the fundamental content of these topics (Gunawan & Fitra, 2021).

Exponential and logarithmic functions play a fundamental role in many university-level courses, such as analysis, differential equations, and complex analysis. The literature emphasizes that students experience various difficulties when learning these concepts; therefore, there is a need to improve the teaching of exponential and logarithmic functions (Confrey & Smith, 1995; Forster, 1998; Rahn & Berndes, 1994). It has been observed that students' work with exponential and logarithmic functions often remains at a predominantly procedural level. Studies have reported that students' conceptual meanings related to these functions are insufficient and that they frequently make errors in procedural steps as well (Confrey & Smith, 1995; Weber, 2002).

In recent years, expectations regarding mathematics education have broadened to include not only the learning of concepts and principles but also the development of problem-solving skills. Accordingly, mathematics instruction has increasingly emphasized practices grounded in learning by doing and experiencing, which activate students' cognitive and affective domains (Gafoor & Kurukkan, 2015; Laurens et al., 2018; Özreçberoğlu & Çağanağa, 2018). To enable students to move beyond merely applying procedural steps and to become effective problem solvers with strong conceptual understanding, it is necessary to provide activities and learning environments that offer opportunities for inquiry and for exploring mathematical relationships (Ministry of National Education [MoNE], 2013).

In the Republic of Türkiye, the mathematics curriculum has been restructured to adapt to changing social and technological conditions. Updated in 2024, it has been introduced as the Türkiye Century Education Model (Türkiye Yüzyılı Maarif Modeli [TYMM]) and has been implemented gradually starting from the 2024-2025 academic year. Within this model, five core skill domains are emphasized for mathematics: mathematical reasoning, mathematical problem solving, mathematical representation, data-driven thinking and decision making (working with data), and the effective use of mathematical tools and technologies (MoNE, 2024). These skills require not only the acquisition of mathematical content knowledge, but also the interpretation and questioning of this knowledge and its connection to real-life situations. Accordingly, TYMM aims for students to view mathematics not merely as a school subject, but as a fundamental component of everyday life and scientific thinking processes (Göçer & Kuzu, 2025). Within the TYMM framework, under the theme "Quantities and Changes," various function types are addressed from Grade 9 to Grade 12; within this continuity, the logarithmic function is covered at the Grade 11 level.

In Türkiye, learning outcomes related to exponential and logarithmic functions were addressed at the Grade 12 level in the 2018 Secondary School Mathematics Curriculum (MoNE, 2018). In this context, particularly during the university entrance examination preparation process, students tend to approach these topics predominantly at a procedural level without sufficiently focusing on the essence of the concepts and their underlying meanings, leaving the conceptual dimension in the background; this observation is consistent with studies reporting difficulties related to exponential and logarithmic functions (Confrey & Smith, 1995; Özer, 2023; Weber, 2002). Identifying students' learning gaps in a given mathematical topic and taking the necessary measures to remedy them is primarily the teacher's responsibility. One of the key factors influencing how effectively a teacher can fulfil this responsibility is the knowledge base they possess (Shulman, 1986; Fennema & Franke, 1992). Although subject-matter knowledge alone is not sufficient to create an effective teaching–learning environment, it is regarded as an indispensable component in designing and sustaining such an environment (Bütün, 2012; Çelik & Baki, 2007; Fennema & Franke, 1992; Lloyd & Wilson, 1998; Stein et al., 1990; Van Dooren et al., 2002). Within this framework, the present study focuses on pre-service mathematics teachers' understanding of the concept of logarithmic function as future educators.

Although logarithmic functions are among the fundamental concepts in both undergraduate mathematics teacher education programs and the secondary school mathematics curriculum, they are considered conceptually challenging for students and pre-service teachers. This makes it important to reveal how pre-service mathematics teachers think about and explain this concept, as they will be responsible for teaching logarithmic functions in classroom settings. The aim of this study is to examine secondary mathematics pre-service teachers' understanding of the concept of logarithmic function. Accordingly, the study seeks to identify how pre-service teachers (i) articulate the definition of the logarithmic function and its basic defining conditions, (ii) interpret the graphical properties of logarithmic functions, and (iii) establish connections between logarithmic functions and real-life situations. Moreover, while logarithmic functions were addressed at the Grade 12 level in the 2018 secondary school mathematics curriculum, they were moved to the Grade 11 level in the 2024 TYMM secondary school mathematics curriculum. This shift suggests that the concept has maintained its importance within the curriculum and that it is intended to be introduced at an earlier grade level. Given the central place of logarithmic functions in curricula and the fact that they are frequently reported in the literature as conceptually challenging for students and pre-service teachers, determining pre-service teachers' understanding of this concept is particularly important. In this respect, the study offers an original contribution to the literature by addressing pre-service teachers' understanding of the concept of logarithmic function not only in terms of how they define logarithmic functions, but also in terms of their understanding of the graphs of logarithmic functions and the relationships they establish with real-life situations. In the relevant literature, studies conducted in the context of pre-service teachers that address the concept of logarithmic function through these dimensions together appear to be limited.

Within this framework, the study seeks to answer the following research question: "What is the nature of pre-service mathematics teachers' understanding of the logarithmic function concept?" In line with this research question, the following sub-questions are addressed:

1. What is the nature of pre-service mathematics teachers' understanding of the definition and properties of the logarithmic function concept?

2. What is the nature of pre-service mathematics teachers' understanding of the graphs of logarithmic functions?

3. What is the nature of pre-service mathematics teachers' understanding of the relationship between logarithmic functions and real-life situations?

Method

Research Design

This study was conducted using a case study design, one of the qualitative research approaches. A case study is used to explain what occurs within a bounded system—defined in terms of time and place—and to describe and interpret the case in depth (Merriam, 1998). In this research, the case under investigation is the in-depth examination of pre-service mathematics teachers' understanding of the concept of logarithmic function.

Participants

Pre-service mathematics teachers were selected through purposive sampling, specifically criterion sampling and convenience sampling. First, in line with convenience sampling, the participants were recruited from among pre-service mathematics teachers enrolled in the undergraduate mathematics teacher education program at a public university in Ankara.

Within the framework of criterion sampling, the inclusion criteria were that the participants had successfully completed Analysis I, Analysis II, and Analysis III in the mathematics teacher education program and had volunteered to participate in the study. Analysis I, is offered in the first year of the program, and exponential and logarithmic functions are addressed within the scope of this course. In order to ensure that a certain period of time had elapsed after instruction on these topics, second-year pre-service teachers were not included in the study. Fourth-year pre-service teachers were also excluded due to such factors as the intensive workload of the Teaching Practicum course and their heightened concerns regarding post-graduation issues.

As the research required an in-depth analysis process, the number of participants was kept below ten (Yıldırım & Şimşek, 2016). Accordingly, the study was conducted with five third-year pre-service teachers enrolled in the mathematics teacher education program at a public university. Four of the participants were female and one was male. To ensure confidentiality, the participants were coded as PT1, PT2, PT3, PT4, and PT5.

Data Collection Tool

In developing the data collection tool, the relevant literature on the concept of logarithmic function was first reviewed, and a draft interview form was prepared in line with the sub-questions of the study. The questions included in the draft form were designed to reveal pre-service teachers' knowledge of the definition of logarithmic function, their understanding of its graphical properties, and the connections they established with real-life examples. The draft questions were submitted to two experts in mathematics education for evaluation in terms of content validity and clarity. In line with the expert feedback, some of the question statements were revised in terms of wording and scope, and the interview form was finalized. The finalized form consisted of four open-ended questions, which are presented below:

1. What conditions are required for a logarithmic function to be defined?
2. How would you explain a logarithmic function beyond its formal definition?

3. What can you say about the graphs of logarithmic functions? What do you pay attention to when drawing the graph of a logarithmic function?

4. Can you give an example of a logarithmic function encountered in real life?

The first two questions aim to elicit the pre-service teachers' knowledge of the definition and fundamental properties of the logarithmic function and their understanding of how they explain and/or apply this knowledge. The third question focuses on their understanding of the graphs of logarithmic functions by examining how they describe key graphical properties and what aspects they consider when sketching the graph of a logarithmic function. The fourth question aims to examine their understanding of providing examples and making connections regarding when and how logarithmic functions are used in real-life contexts.

Data Collection Process

After obtaining the necessary official permissions, the participants met with the researcher in a quiet classroom, and semi-structured interviews were conducted individually with each participant. Each interview lasted approximately fifteen minutes. The interviews were carried out with the participants' knowledge and consent and were audio-recorded using a voice recorder.

Data Analysis

The data collected in this study were analyzed using content analysis. In this process, the pre-service teachers' responses to the four questions were examined repeatedly for each question and each participant. The codes were derived from the data through repeated examination of the participants' responses. No predetermined code list was used; rather, the responses were grouped according to similarities and differences in content, and the codes were generated based on these groupings. The codes were labeled with statements that represented the pre-service teachers' solution and explanation processes. The codes were reviewed again, and the final code list was established through the researchers' joint evaluation.

During the analysis process, the researchers examined each participant's response to each question individually and matched the responses with the most appropriate codes. An inter-coder reliability coefficient of 92% was calculated, which is above the 70% threshold recommended by Miles and Huberman (1994), indicating that the coding was reliable. After the coding process, responses were classified according to the codes, and the findings were presented in tables using descriptive statistics (frequencies and percentages). In interpreting the findings, direct quotations from the participants' responses were also included where necessary.

Limitations

This study is limited to five pre-service mathematics teachers studying at a public university during the spring semester of the 2024–2025 academic year. The study is limited to the topic of logarithmic functions. The data of the study are limited to those obtained through the interview form developed by the researcher.

Ethical Permits of Research:

In this study, all the rules specified to be followed within the scope of "Higher Education Institutions Scientific Research and Publication Ethics Directive" were complied with. None of the actions specified under the heading "Actions Contrary to Scientific Research and Publication Ethics", which is the second part of the directive, have been taken.

Ethics Committee Permission Information:

Name of the committee that made the ethical evaluation = Hacettepe University Institute of Educational Sciences Research Ethics Committee

Date of ethical review decision= 5 February 2025

Ethics assessment document issue number= E-82474949-050-00004035972

Findings

The findings derived from the analysis of the data obtained through the interviews conducted to examine pre-service mathematics teachers' understanding of the logarithmic function concept in depth are presented below in relation to the research questions.

Findings Related to the Question "What Conditions Are Required for a Logarithmic Function to Be Defined?"

The prominent aspects reflected in the pre-service mathematics teachers' definitions regarding their understanding of the definition and fundamental properties of the logarithmic function were grouped into five subcategories: the base condition of the logarithmic function, the independent variable (x), establishing a connection with the exponential function, function properties, and no response/uncertainty. Table 1 presents the codes for the pre-service teachers and the subcategories associated with these codes.

$\log_a x = y$ therefore, the following table was prepared;

Table 1.

Pre-service Mathematics Teachers' Statements Regarding the Definition and Properties of the Logarithmic Function

Subcategory	Code	Pre-service teachers	*f*	%
Base condition (a)	Positive base (a > 0)	PT2, PT4, PT5	3	38
	a > 1	PT1, PT4	2	
	a ≠ 1	PT2	1	
	Positive integer base (a is a positive integer)	PT1, PT4	2	
Independent variable of the logarithmic function (x)	Magnitude condition of x (x > 0 or x > 1)	PT1, PT2, PT4, PT5	4	28
	Non-equality condition of x (x ≠ 0 or x ≠ 1)	PT4, PT5	2	
Relating to the exponential function	Symbolic representation ($x = a^y$)	PT1	1	10
	Emphasis on the inverse function	PT2	1	
Function properties	Emphasis on injectivity and surjectivity	PT2	1	10
	Emphasis on continuity	PT3	1	
No response / Uncertainty	Awareness of lack of knowledge	PT1, PT3, PT5	3	14

Under the subcategory "base condition (a)," the pre-service teachers used different statements. Within the code "emphasis on a positive base (a>0)," they stated that the base must be positive. In this regard, PT2, PT4, and PT5 indicated that the base should be a positive number or a positive real number. Under the code "a>1," there were explanations suggesting that the base must be greater than 1. Accordingly, PT1 and PT4 stated that the base of the logarithmic function should be a number greater than 1. Under the code "a ≠ 1," participants expressed that the base of a logarithmic function cannot be equal to 1. Finally, under the code "integer," there were explanations in which the base was considered as an integer. The pre-service teachers' statements are presented below:

"Let $log_a x = y$; among these given numbers, a needs to be greater than… I couldn't really remember…" (PT1).

"The exponent goes from one to infinity. And I think the base is a positive integer…" (PT4).

The codes under the subcategory "independent variable (x) of the logarithmic function" are explained as follows.

Under the code "magnitude condition of x," there were statements indicating that x must satisfy a certain magnitude condition. In this context, PT1, PT2, PT4, and PT5 stated that x>0 or x>1 is required.

Under the code "non-equality condition of x," PT4 and PT5 stated that x≠0 or x≠1 is required.

The participants' statements from which these codes were derived are given below:

"If I say $log_a x = y$, then x has to be greater than zero…" (PT2).

"For a logarithmic function to be defined, the inside part shouldn't be zero… In $log_a x = y$ here, what I mean, professor, is that x shouldn't be zero. That's it; I can't remember anything else at the moment." (PT5).

The codes under the subcategory "relating to the exponential function" are as follows.

Under the code "symbolic representation," participants provided explanations indicating that a logarithmic expression can be written symbolically in exponential form.

Under the code "emphasis on the inverse function," participants made statements in which the logarithmic function was considered in an inverse relationship with the exponential function. In this regard, PT2 described the logarithmic function as the inverse of the exponential function. The participants' statements are presented below:

"We should be able to write this expression as $x=a^y$; maybe that could be, characteristically, the definition of the logarithm…" (PT1).

"Because the logarithmic function is the inverse of the exponential function…" (PT2).

The codes under the subcategory "function properties" were expressed as follows.

Under the code "emphasis on injectivity and surjectivity," participants made statements associating the logarithmic function with being one-to-one and onto. In this context, PT2 stated that a logarithmic function should be one-to-one and onto.

Under the code "emphasis on continuity," there were statements referring to the continuity of the logarithmic function. In this regard, PT3 characterized the logarithmic function as a continuous function.

Finally, under the subtheme "no response/uncertainty," the code "awareness of lack of knowledge" includes statements indicating that the pre-service teachers did not consider their knowledge about logarithmic functions sufficient or could not recall certain points. In this context, PT1, PT3, and PT5 stated that they could not remember some aspects related to the definition or defining conditions of the logarithmic function, or that they were aware of their lack of knowledge in this regard.

Findings Related to the Question "How Would You Explain a Logarithmic Function Beyond Its Formal Definition?"

The prominent aspects reflected in the pre-service mathematics teachers' explanations of the logarithmic function beyond its formal definition were grouped into three subcategories: the exponential–logarithmic relationship, logarithmic scaling and representation, and symbolic representation. Table 2 presents the codes for the pre-service teachers and the subcategories associated with these codes.

Table 2.

Pre-service Mathematics Teachers' Statements Regarding the Non-formal Definitions of the Logarithmic Function

Subcategory	Code	Pre-service teachers	*f*	%
Exponential–logarithmic relationship	Logarithm as an exponent	PT1	1	17
Logarithmic scaling and representation	Intensity of increase	PT2	1	33
	Representing large numbers	PT5	1	
Symbolic representation	Association with the "log" symbol	PT3	1	50
	Association with the "ln" symbol	PT3, PT4	2	

When the pre-service teachers' statements regarding how they made sense of the logarithmic function concept beyond its formal definition were examined, it was found that, under the code "logarithm as an exponent," there were explanations linking the logarithmic function concept to an exponential expression. In this context, PT1 stated:

"If the given number is a certain power of the number aaa, then the value of x here will correspond to that. In other words, we can reach a conclusion such as which number is the nth power of which number." (PT1).

Within the subcategory "logarithmic scaling and representation," under the code "intensity of increase," there were statements in which the logarithmic function was discussed in relation to the magnitude of increases. In this context, PT2 stated:

"I know that the logarithmic function is used to explain not certain increases—for example, not increases by tens—but larger increases. For instance, in the earthquake example, the difference between 6.5 and 7.5 is not just 1; there is an increase of thousands." (PT2).

Under the code "representing large numbers," PT5 stated:

"It feels like a representation that helps us express large numbers. With a logarithmic base, we can write large numbers more easily." (PT5).

Finally, within the subtheme "symbolic representation," under the code "association with the 'log' symbol," there were explanations indicating that the concept of logarithm was directly associated with the expression "log." Under the code "association with the 'ln' symbol," there were explanations in which the logarithmic function was specifically associated with the symbol "ln." In this context, PT4 stated:

"When I think of the logarithmic function, it mostly comes to mind in the form with 'ln.' Questions are always in the 'ln' form." (PT4).

Findings Related to the Question "What Can You Say About the Graphs of Logarithmic Functions? What Do You Pay Attention to When Drawing the Graph of a Logarithmic Function?"

The pre-service mathematics teachers' explanations regarding their understanding of the graphs of logarithmic functions were coded under the following subcategories: graph construction strategies, increasing/decreasing function, asymptote, and no response. In this section, the codes under each subcategory and the corresponding participant statements are presented descriptively. Table 3 presents the codes for the pre-service teachers and the subcategories associated with these codes.

Table 3.

Pre-service Mathematics Teachers' Statements Regarding the Graphs of Logarithmic Functions

Subcategory	Code	Pre-service teachers	*f*	%
Graph construction strategies	Focusing on the domain/range	PT3, PT5	2	14
Increasing/decreasing function	General perception of increasing behavior Continuous but non-linear Domain/range-focused approach Dependent on the base Sign of the logarithmic expression	PT1, PT5 PT2 PT3, PT5 PT2, PT4 PT5	2 1 2 2 1	58
Asymptote	Approaching but not reaching Horizontal asymptote Approaching the line (y=1)	PT4 PT1 PT1	1 1 1	21
No response	Awareness of lack of knowledge	PT3	1	7

Within the subcategory "graph construction strategies," under the code "focusing on the domain/range," there were statements indicating that, when constructing the graph of a logarithmic function, participants relied on the domain and range. In this context, PT3 and PT5 stated that they proceeded by considering the domain and the range while drawing the graph.

Under the subcategory "increasing/decreasing function," different types of statements regarding whether the function increases or decreases were coded. Under the code "general perception of increasing behavior," there were explanations suggesting that a logarithmic function is generally an increasing function. Under the code "continuous but non-linear," PT2 stated that the increase occurs continuously but in a non-linear manner. Under the code "domain/range-focused approach," there were statements indicating that the domain and range were consulted to determine whether the function is increasing or decreasing. Under the code "dependent on the base," participants stated that whether the function is increasing or decreasing is determined by the base. In this regard, PT2 and PT4 noted that the function is decreasing when the base is between 0 and 1, and increasing when the base is greater than 1. Under the code "sign of the logarithmic expression," PT5 stated that whether the function is increasing or decreasing is determined by the sign in front of the logarithmic expression. Examples of the participants' statements are provided below:

"Whether it is increasing or decreasing is determined by the base. If the base is between zero and one, it is decreasing; if it is greater than one, it is increasing…" (PT2).

"When drawing the graph, we look at the domain; it cannot be negative. Whether it is increasing or decreasing is determined by the sign in front of the logarithm. Generally, it is increasing…" (PT5).

Under the subcategory "asymptote," statements regarding the asymptotic behavior of logarithmic functions were coded. Under the code "approaching but not reaching," there were explanations indicating that the graph approaches a certain value but does not reach it. Under the code "horizontal asymptote," there were statements suggesting that the logarithmic function has a horizontal asymptote. Under the code "approaching the line y=1," there were explanations indicating that the graph approaches the line y=1. The participants' statements are presented below:

"I remember that it generally approaches some value but cannot reach it…" (PT4).

"I think it has a horizontal asymptote. It should be an increasing function that approaches 1 as you move to the right." (PT1).

Finally, under the subcategory "no response," statements reflecting uncertainty regarding participants' own knowledge level were coded. Under the code "awareness of lack of knowledge," there were statements indicating that some properties of the graph of the logarithmic function could not be recalled. In this context, PT3 stated that they *"could not remember very well"* some details related to the graph.

Findings Related to the Question "Can You Give an Example of a Logarithmic Function Encountered in Real Life?"

Pre-service mathematics teachers' understanding of the real-life uses of logarithmic functions was coded under the following subcategories: scale-based examples, emphasis on the logarithmic base, and the exponential-function context. Table 4 presents the codes for the pre-service teachers and the subcategories associated with these codes.

Table 4.

Pre-service Mathematics Teachers' Statements Regarding the Real-life Uses of the Logarithmic Function

Subcategory	Code	Pre-service teachers	f	%
Scale-based examples	Earthquake magnitude scale	PT2, PT4, PT5	3	50
Emphasis on the logarithmic base	Base-10 logarithm	PT1	1	17
Exponential-function context	Biology / bacterial growth	PT3, PT4	2	33

Under the code "earthquake magnitude scale," there were statements referring to the use of logarithms in scales related to earthquakes. In this context, PT2, PT4, and PT5 stated that logarithms are used in earthquake applications and in the earthquake magnitude scale, and that a logarithmic structure is employed to measure earthquakes or to express earthquake magnitudes. The participants' statements are presented below:

"...For example, in the earthquake case, the difference between 6.5 and 7.5 is not just 1; there is an increase of thousands." (PT2).

"We used it in earthquake applications to measure it." (PT5).

Under the code "base-10 logarithm," PT1 made statements specifically emphasizing base 10. PT1 noted that base-10 logarithms are widely used and stated that, when giving an example, one could proceed by considering logarithms with base 10.

Within the subcategory "exponential-function context," under the code "biology/bacterial growth," there were statements indicating that logarithmic and exponential structures were discussed in a biological context. PT4 stated:

"It is used in the earthquake scale and in the rate of bacterial growth..." (PT4).

Discussion and Conclusion

In this study, pre-service mathematics teachers' understanding of the concept of logarithmic function was examined in depth. To this end, three sub-questions were addressed. The first sub-question focused on the participants' understanding of the definition of the logarithmic function. The prominent aspects in the definitions provided by the pre-service teachers were grouped into five subcategories: the base condition of the logarithmic function, the independent variable (x), relating the logarithmic function to the exponential function, function properties, and no response/uncertainty. An examination of the distribution of codes indicates that the highest proportion clustered under the base condition (aaa) subcategory (38%), followed by the independent variable (x) subcategory (28%). In contrast, the subcategories of relating to the exponential function and function properties were represented at lower rates (10% and 10%, respectively), and codes reflecting no response/uncertainty also appeared at a non-negligible level (14%).

The findings regarding the base condition of the logarithmic function indicate that the pre-service teachers had difficulty expressing the conditions that determine the base of a logarithmic function in a complete and general form. They tended to reduce the base conditions to specific cases and articulated them in a limited manner. One group of participants emphasized that the base must be positive ($a>0$), demonstrating a basic awareness that the base cannot be negative (PT2, PT4, PT5). Some participants restricted the base with respect to 1, stating that the base must be greater than 1 ($a>1$) or cannot be equal to 1 ($a\neq1$) (PT1, PT2, PT4). In addition to these findings, the fact that some participants described the base as an "integer" or a "positive integer" (PT1, PT4) suggests that situations in which the base can be a real number were overlooked, and that the base condition was generally represented in their thinking as "positive," "greater than one," and "mostly an integer."

The findings regarding the independent variable of the logarithmic function indicate that a considerable proportion of the pre-service teachers referred to the condition x>0x>0x>0 and emphasized that “the expression inside must be positive” (PT1, PT2, PT4). In addition, some participants expressed the condition for the independent variable in terms of “non-equality” (x≠0, x≠1) and, particularly through statements such as “the inside part should not be zero,” treated the condition x>0 in a narrowed form as x≠0 (PT4, PT5). This suggests that the participants could not fully differentiate, at a cognitive level, between the conditions a≠1 and x>0 in the definition of the logarithmic function; although they recalled that there are “certain values that cannot be equal,” they had difficulty clarifying which variable each condition corresponds to. In the subcategory “no response/uncertainty,” the coding of three participants (PT1, PT3, PT5) as “awareness of lack of knowledge” indicates that some pre-service teachers recognized their insufficiencies regarding the definition and properties of logarithmic functions and did not feel confident about this topic. The conclusions drawn from the findings of the present study are consistent with studies reporting that students experience difficulties in defining logarithmic functions accurately and consistently (Akkuş, 2004; Campo-Meneses et al., 2021).

The prominent aspects in the pre-service mathematics teachers’ explanations reflecting their understanding of the logarithmic function concept beyond its formal definition were identified under three subcategories: the exponential–logarithmic relationship, logarithmic scaling/representation, and symbolic representation. The fact that the codes “intensity of increase” (PT2) and “representing large numbers” (PT5), both under the subcategory “logarithmic scaling and representation,” together accounted for 33% indicates that some pre-service teachers made sense of the logarithmic function particularly in terms of explaining large increases and representing large numbers in a more compact form. For these participants, the logarithmic function is positioned not only as an algebraic expression but also as a tool that rescale magnitudes. The highest proportion in the table, however, was observed in the subcategory “symbolic representation” (50%). The statements grouped under the codes “association with the ‘log’ symbol” and “association with the ‘ln’ symbol” suggest that, for a substantial portion of the participants, the symbols log and ln were the first associations that came to mind when thinking about logarithmic functions (PT3, PT4). This indicates that, within the group, the logarithmic function was predominantly evoked through symbolic notation. This result is consistent with Kastberg’s (2002) study, which reported that students often view the concept of logarithmic function primarily as a symbolic representation or as a procedure to be carried out.

The second sub-question of the study focused on how the pre-service teachers interpreted the graphs of logarithmic functions and their understanding of these graphs. The pre-service mathematics teachers’ explanations regarding the graph of the logarithmic function were coded under the subcategories of graph construction strategies, increasing/decreasing function, asymptote, and no response. Overall, the participants tended to focus primarily on increasing/decreasing behavior (58%) and asymptotic behavior (21%) when making sense of the graph of a logarithmic function. However, in both areas, accurate criteria (e.g., base-dependent behavior and the idea of approaching but not reaching) co-occurred with confused or incomplete representations (e.g., determining increasing/decreasing behavior based on the sign, referring to a horizontal asymptote, or claiming that the graph approaches the line y=1). While some participants addressed the graph more systematically through the domain/range and the base condition, others confused the behavior of logarithmic functions with the graph of the exponential function or interpreted it based on superficial cues such as coefficients and the sign of the logarithmic expression. These results are consistent with the findings of Quinio and Cuarto (2023), who reported that students experience difficulties in making sense of the behavior of logarithmic function graphs and in constructing accurate graphical representations.

The third sub-question of the study focused on the pre-service teachers' understanding of providing examples and making connections regarding when and how logarithmic functions are used in real-life contexts. Three of the five participants (PT2, PT4, PT5) explained the relationship between logarithmic functions and real life through the earthquake scale; this indicates that more than half of the participants primarily associated logarithmic functions with Richter-type scales. This preference may be related to the fact that the earthquake scale is frequently used as an example in applications of logarithmic functions at the secondary school level. This finding is consistent with the 2018 secondary school mathematics curriculum, which presents contexts such as earthquake magnitude (Richter scale), pH value, and sound intensity (decibel) as examples of real-life situations modeled by exponential and logarithmic functions (MoNE, 2018). Gol Tabaghi (2007) also refers to logarithmic scales such as Richter, pH, and decibel as examples of applications of the logarithmic function in different fields. In addition, Budinski and Takači (2013) discuss the Richter scale as one of the real-life contexts used in the teaching of logarithms. In addition, two participants (PT3, PT4; 33%) referred to biological, exponential processes such as bacterial growth when providing an example of a logarithmic function. Taken together, these findings suggest that pre-service teachers tended to make sense of logarithmic functions in real life mainly through familiar examples such as the earthquake scale, while some participants also invoked exponential growth processes in their explanations, at times addressing logarithmic and exponential relationships in an intertwined manner. The fact that participants referred to exponential processes such as bacterial growth while explaining logarithmic functions is consistent with studies showing that exponential and logarithmic models are often taught around the same real-life contexts and that students have difficulty distinguishing between these two concepts (Borji et al., 2023; Engbersen, 2009).

This study presents pre-service mathematics teachers' understanding of the concept of logarithmic function in terms of definition, graph, and giving real-life examples. The findings show that pre-service teachers were able to express some fundamental elements related to logarithmic functions; however, in some cases, they articulated the defining conditions incompletely or incorrectly, experienced uncertainties in their knowledge of graphical properties, and explained real-life examples mostly through limited contexts. This suggests that not only procedural knowledge but also conceptual understanding is important in the teaching of the concept of logarithmic function. Indeed, the literature emphasizes that exponential and logarithmic concepts involve difficulties for students, that understanding the idea of logarithm requires more than simply presenting the definition and having students carry out procedural applications, and that real-life contexts can support the understanding of these concepts (Borji et al., 2023; Kuper & Carlson, 2020; Weber, 2002). In this respect, the study contributes to the literature by addressing pre-service mathematics teachers' understanding of the concept of logarithmic function through multiple dimensions.

Recommendations

This study is based on in-depth interviews conducted with a limited number of pre-service teachers. For future research, it is recommended to work with larger samples; to include different participant groups (e.g., high school students and teachers); and to support and examine understandings of the logarithmic function concept through quantitative data in a comparative manner.

Based on the findings of the study, it is important that the teaching of logarithmic functions should not be limited to the procedural aspect of the concept, but should also address its definition, graphical properties, and real-life examples together. In order to support pre-service mathematics teachers' understanding of the concept of logarithmic function, it may be useful to include instructional processes that center conceptual learning. In this respect, it may be recommended to increase graph-based activities, to address different representations of logarithmic functions together, and to integrate technology-supported applications into the teaching process.

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Conflict Statement

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Notice of Use of Artificial Intelligence

The authors did not use any artificial intelligence tools in the research, authorship, or publication of this article

Matematik Öğretmeni Adaylarının Logaritmik Fonksiyon Kavramına Yönelik Anlayışlarının İncelenmesi

Özet

Günümüzde üstel ve logaritmik fonksiyonlar; büyüme, azalma, değişim ve ölçeklendirme süreçlerinin modellenmesi açısından hem gerçek yaşamda hem de matematik öğretim programlarında önemli bir yere sahiptir. Logaritmik fonksiyon, ortaöğretim ve lisans düzeyindeki programlarda temel kavramlar arasında bulunmasına rağmen öğrenciler ve öğretmen adayları tarafından kavramsal açıdan güçlük yaşanan konular arasında gösterilmektedir. Öğretmen adaylarının ileride bu konuyu sınıf ortamında öğretme sorumluluğunu üstlenecek olmaları, logaritmik fonksiyona ilişkin anlayışlarının incelenmesini önemli kılmaktadır. Çalışmanın amacı, matematik öğretmeni adaylarının logaritmik fonksiyon kavramına yönelik anlayışlarını incelemektir. Araştırmanın katılımcılarını, 2024-2025 eğitim öğretim yılı bahar döneminde bir devlet üniversitesinde öğrenim gören beş ortaöğretim matematik öğretmeni adayı oluşturmaktadır. Çalışma, nitel araştırma yöntemlerinden durum çalışması deseniyle yürütülmüştür. Veri toplama aracı olarak araştırmacı tarafından geliştirilen ve dört açık uçlu sorudan oluşan bir görüşme formu kullanılmıştır. Sorular, "logaritmik fonksiyonun tanımı ve özellikleri", "logaritmik fonksiyonun grafikleri ve özellikleri" ve "logaritmik fonksiyon ile gerçek yaşam ilişkisi" olmak üzere üç kategori altında toplanmıştır. Veriler yarı yapılandırılmış görüşmeler yoluyla toplanmış, içerik analiziyle çözümlenmiş ve öğretmen adaylarının ifadeleri alt kategoriler ve kodlar altında sınıflandırılmıştır. Bulgular, öğretmen adaylarının logaritmik fonksiyonun tanım koşullarını doğru ve tutarlı biçimde ifade etmekte zorlandıklarını, logaritmik fonksiyonların grafiklerini genellikle artan bir yapı olarak yorumladıklarını ve gerçek yaşam bağlamlarını ağırlıklı olarak deprem ölçeği örneğiyle sınırlı tuttuklarını göstermektedir. Elde edilen bulgular doğrultusunda, öğretmen adaylarının logaritmik fonksiyon kavramına ilişkin anlayışlarını geliştirmeye yönelik öğrenme ortamlarının tasarlanması önerilmektedir.

Anahtar Kelimeler: Matematik eğitimi, logaritmik fonksiyonlar, kavramsal anlayış, matematik öğretmeni adayları.

Giriş

Fonksiyon kavramına ilişkin araştırmalar incelendiğinde genel olarak şu tablo ortaya çıkmaktadır: Öğrenciler fonksiyon kavramını çoğu zaman soyut ve karmaşık olarak algılamakta; bu nedenle, fonksiyonları anlamlandırmada ve fonksiyonlara ilişkin bilgilerini problem çözmede etkili bir şekilde kullanmada çeşitli güçlükler ve kavram yanılgıları yaşamaktadırlar (Aydın, 2000; Dreyfus, 1990; Kieran, 1990; Leinhardt vd., 1990; Selden & Selden, 1992; Tall & Vinner, 1981; Parhizgar vd., 2022). Fonksiyon türleri içerisinde logaritmik fonksiyona odaklanıldığında ise alan yazınında öğrencilerin üstel ve logaritma konularının temel içeriklerini yeterince yapılandıramadıkları için bu konularla ilgili soruları çözerken güçlükler yaşadıkları gözlemlenmiştir (Gunawan & Fitra, 2021). Literatür incelendiğinde üstel ve logaritmik fonksiyonların daha çok işlemsel düzeyde kaldığı gözlemlenmektedir. Öğrencilerin bu kavramlara ilişkin anlamlarının kavramsal açıdan yetersiz kaldığı ve işlem basamaklarında da sıkça hatalar yaptıkları rapor edilmiştir (Confrey & Smith, 1995; Weber, 2002).

Son yıllarda matematik eğitimine ilişkin beklentiler; kavram ve ilke öğrenimi ile problem çözme becerisinin geliştirilmesini de kapsayacak biçimde genişlemiştir. Bu nedenle matematik öğretiminde, öğrencilerin bilişsel ve duyuşsal yönlerini etkinleştiren, yaparak-yaşayarak öğrenmeye dayalı uygulamalara ağırlık verildiği görülmektedir (Gafoor & Kurukkan 2015; Laurens vd., 2018; Özreçberoğlu & Çağanağa, 2018).

Türkiye Cumhuriyeti'nde matematik dersi öğretim programı, değişen toplumsal ve teknolojik koşullara uyum sağlamak amacıyla yeniden yapılandırılmış ve 2024 yılında güncellenerek Türkiye Yüzyılı Maarif Modeli [TYMM] adıyla 2024-2025 eğitim-öğretim yılından itibaren kademeli olarak uygulamaya konulmuştur. TYMM, öğrencilerin matematiği yalnızca ders kapsamında değil, gündelik yaşamın ve bilimsel düşünme süreçlerinin temel unsurlarından biri olarak görmelerini hedeflemektedir (Göçer & Kuzu, 2025). TYMM çerçevesinde "Nicelikler ve Değişimler" teması altında 9. sınıftan 12. sınıfa kadar farklı fonksiyon türlerine yer verilmekte; logaritmik fonksiyon ise bu bütünlük içerisinde 11. sınıf düzeyinde ele alınmaktadır. Logaritmik fonksiyonlar, 2018 ortaöğretim matematik dersi öğretim programında 12. sınıf düzeyinde ele alınırken, 2024 TYMM ortaöğretim matematik dersi öğretim programında 11. sınıf düzeyine taşınmış ve böylece kavramın öğretim programlarındaki önemini korumaya devam ettiği gibi daha erken sınıf düzeyinde ele alınmasının hedeflendiği görülmüştür. Logaritmik fonksiyonun öğretim programlarında temel bir yer tutması ve alan yazınında öğrenciler ile öğretmen adayları için kavramsal açıdan zorlayıcı bir kavram olarak gösterilmesi, geleceğin öğreticileri olan öğretmen adaylarının bu kavrama ilişkin anlayışlarının belirlenmesini özellikle önemli kılmaktadır.

Bu doğrultuda araştırmanın amacı, ortaöğretim matematik öğretmeni adaylarının logaritmik fonksiyon kavramına yönelik anlayışlarını incelemektir. Bu amaç doğrultusunda "Matematik öğretmeni adaylarının logaritmik fonksiyon kavramına yönelik anlayışları nasıldır?" sorusuna yanıt aranmıştır. Bu problem doğrultusunda ise aşağıdaki alt problemlere yanıt aranmıştır.

1. Matematik öğretmeni adaylarının logaritmik fonksiyon kavramının tanımı ve özelliklerine yönelik anlayışları nasıldır?

2. Matematik öğretmeni adaylarının logaritmik fonksiyonların grafiklerine yönelik anlayışları nasıldır?

3. Matematik öğretmeni adaylarının logaritmik fonksiyon kavramı ile gerçek yaşam durumları arasındaki ilişkiye yönelik anlayışları nasıldır?

Yöntem

Araştırma Deseni

Bu araştırma, nitel araştırma yaklaşımlarından durum çalışması deseniyle yürütülmüştür. Durum çalışması, zaman ve mekân bakımından sınırları belirlenmiş bir sistemde ele alınan olgunun ayrıntılı biçimde betimlenmesi ve yorumlanmasını amaçlamaktadır (Merriam, 1998). Bu çalışmada incelenen durum, matematik öğretmeni adaylarının logaritmik fonksiyon kavramına yönelik anlayışlarının derinlemesine ortaya konulmasıdır.

Katılımcılar

Çalışma grubundaki öğretmen adayları amaçlı örnekleme kapsamında ölçüt örnekleme ve kolay ulaşılabilir örnekleme ile belirlenmiştir. Katılımcılar, Ankara'da bir devlet üniversitesinin matematik öğretmenliği lisans programında öğrenim gören öğretmen adaylarından seçilmiştir. Ölçüt olarak Analiz I, Analiz II ve Analiz III derslerini başarıyla tamamlamış olma ve araştırmaya gönüllü katılım esas alınmıştır. Derinlemesine analiz gerektiren yapı nedeniyle çalışma grubu 10'u geçmeyecek şekilde sınırlandırılmış (Yıldırım & Şimşek, 2016) ve araştırma üçüncü sınıfta öğrenimine devam eden beş öğretmen adayıyla (dördü kadın, biri erkek) yürütülmüştür. Gizlilik ilkesi doğrultusunda katılımcılar Ö1-Ö5 şeklinde kodlanmıştır.

Veri Toplama Aracı

Araştırmanın verileri, araştırmacı tarafından geliştirilen ve dört problem durumundan oluşan görüşme formu aracılığıyla toplanmıştır. Formda yer alan problem durumları aşağıda verilmiştir:

1. Logaritmik fonksiyonun tanımlı olması için gereken koşullar nelerdir?

2. Logaritmik fonksiyonu formal tanımı dışında nasıl açıklarsınız?

3. Logaritmik fonksiyonların grafiklerine ilişkin neler söyleyebilirsiniz? Logaritmik fonksiyonun grafiğini çizerken nelere dikkat edersiniz?

4. Gerçek yaşamda karşımıza çıkan bir logaritmik fonksiyon örneği verir misiniz?

İlk iki problem, öğretmen adaylarının logaritmik fonksiyonun tanımı ve temel özelliklerine ilişkin bilgilerini ve bu bilgileri açıklama/uygulama biçimlerine yönelik anlayışlarını ortaya çıkarmayı, üçüncü problem adayların logaritmik fonksiyonların grafiklerine ilişkin anlayışlarını; grafiklerin temel özelliklerini nasıl ifade ettiklerini ve bir logaritmik fonksiyonun grafiğini çizerken hangi noktalara dikkat ettiklerini ve dördüncü problem logaritmik fonksiyonun gerçek yaşamda hangi durumlarda ve nasıl kullanıldığına dair örnek verme ve ilişkilendirme becerilere yönelik anlayışlarını incelemeyi hedeflemektedir.

Veri Toplama Süreci

Gerekli yasal izinlerin alınmasının ardından, katılımcılarla sessiz bir sınıfta bir araya gelinmiş ve her bir katılımcı ile bireysel olarak yaklaşık on beş dakika süren yarı yapılandırılmış görüşmeler gerçekleştirilmiştir. Yapılan görüşmeler, öğretmen adaylarının bilgisi ve onayı dâhilinde yürütülmüş ve ses kayıt cihazı aracılığıyla kaydedilmiştir.

Veri Analizi

Bu çalışmada toplanan veriler içerik analizi kullanılarak çözümlenmiştir. Bu süreçte, öğretmen adaylarının dört soruya verdikleri yanıtlar, her bir soru ve her bir katılımcı için tekrar tekrar incelenmiştir. Kodlar, öğretmen adaylarının yanıtlarının yinelemeli biçimde incelenmesi sonucunda veriden türetilmiştir. Önceden belirlenmiş bir kod listesi kullanılmamış; bunun yerine yanıtlar, içerik bakımından benzerlik ve farklılıklarına göre gruplandırılmış ve kodlar bu gruplamalar temel alınarak oluşturulmuştur. Kodlar, öğretmen adaylarının çözüm ve açıklama süreçlerini yansıtan ifadelerle etiketlenmiştir. Kodlar yeniden gözden geçirilmiş ve araştırmacıların ortak değerlendirmesi sonucunda nihai kod listesi oluşturulmuştur. Veri analiz sürecinde, araştırmacılar her bir öğretmen adayının her bir probleme verdiği yanıtları tek tek incelemiş ve bu yanıtları en uygun kodlarla eşleştirmiştir. Kodlayıcılar arası güvenirlik oranının %92 olarak hesaplanması ve bu oranın Miles ve Huberman'ın (1994) önerdiği %70 sınırının üzerinde olması, yapılan kodlamaların güvenilir olduğunu göstermektedir. Kodlama sürecinin ardından yanıtlar kodlara göre sınıflandırılmış ve elde edilen bulgular betimsel istatistikler (frekans ve yüzde) aracılığıyla tablo hâlinde sunulmuştur. Bulguların yorumlanmasında, gerektiği yerlerde öğretmen adaylarının yanıtlarından doğrudan alıntılara da yer verilmiştir.

Sınırlılıklar

Araştırma, 2024–2025 akademik yılının bahar döneminde bir devlet üniversitesinde öğrenim görmekte olan beş matematik öğretmeni adayı ile sınırlandırılmıştır. Çalışma kapsam bakımından logaritmik fonksiyon konusu ile sınırlıdır. Ayrıca araştırmanın verileri, araştırmacılar tarafından geliştirilen görüşme formundan elde edilen verilerle sınırlıdır.

Araştırmanın Etik İzinleri:

Bu çalışmada "Yükseköğretim Kurumları Bilimsel Araştırma ve Yayın Etiği Yönergesi" kapsamında uyulması gerektiği belirtilen tüm kurallara uyulmuştur. Yönergenin ikinci bölümü olan "Bilimsel Araştırma ve Yayın Etiğine Aykırı Eylemler" başlığı altında belirtilen eylemlerin hiçbiri gerçekleştirilmemiştir.

Etik Kurul İzin Bilgileri:

Etik değerlendirmeyi yapan kurulun adı = Hacettepe Üniversitesi Eğitim Bilimleri Enstitüsü Araştırma Etik Kurulu

Etik Kurul Etik inceleme karar tarihi = 5 Şubat 2025

Etik değerlendirme belgesi konu numarası = E-82474949-050-00004035972

Bulgular

Görüşmelerden elde edilen veriler, araştırma soruları doğrultusunda betimsel olarak çözümlenmiş ve bulgular dört başlık altında sunulmuştur.

Logaritmik Fonksiyonun Tanımlı Olması İçin Gereken Koşullar Nelerdir? Sorusuna İlişkin Bulgular

Öğretmen adaylarının logaritmik fonksiyonun tanımı/temel koşullarına ilişkin açıklamaları beş alt kategoride toplanmıştır: taban koşulu (a), bağımsız değişken koşulu (x), üstel fonksiyonla ilişki kurma, fonksiyon özellikleri ve cevap yok/belirsizlik.

$\log_a x = y$ olacak şekilde aşağıdaki tablo düzenlenmiştir.

Tablo 1.

Matematik Öğretmeni Adaylarının Logaritmik Fonksiyonun Tanımına ve Özelliklerine Yönelik İfadeleri

Alt kategori	Kod	Öğretmen adayları	*f*	%
Taban koşulu(a)	Pozitif taban	Ö2, Ö4, Ö5	3	38
	a>1	Ö1, Ö4	2	
	a≠ 1	Ö2	1	
	Tam sayı	Ö1, Ö4	2	
Logaritmik fonksiyonun bağımsız değişkeni (x)	x'in büyüklük koşulu	Ö1, Ö2, Ö4, Ö5	4	28
	x'in eşit olmama kosulu	Ö4, Ö5	2	
Üstel fonksiyon ile ilişki kurma	Sembolik gösterim	Ö1	1	10
	Ters fonksiyon vurgusu	Ö2	1	
Fonksiyon özellikleri	Birebir ve örtenlik	Ö2	1	10
	Süreklilik vurgusu	Ö3	1	
Cevap yok / Belirsizlik	Bilgi eksikliğini farklında olma	Ö1, Ö3, Ö5	3	14

Tablo 1'de yer alan dağılım, ifadelerin en çok taban koşulu (%38) ve bağımsız değişken koşulu (%28) üzerinde yoğunlaştığını göstermektedir. Taban koşulu kapsamında adayların bir kısmı "tabanın pozitif olması (a>0)" vurgusu yaparken (Ö2, Ö4, Ö5), bazıları tabanı "a>1" ile sınırlamış (Ö1, Ö4) ya da "a≠1" koşulunu belirtmiştir (Ö2). Bazı adayların tabanı "tamsayı/pozitif tam sayı" olarak ele alması, taban koşulunun genellenmesinde güçlük yaşandığına işaret etmiştir. Bağımsız değişken koşulunda ise x>0 ifadesi yaygın olmakla birlikte, bazı adayların bu koşulu "x≠0/x≠1" biçiminde daralttığı ve a≠1 ile x>0 koşullarını net bir biçimde ayrıştıramadığı görülmüştür.

Logaritmik Fonksiyonun Formal Tanımı Dışında Nasıl Açıklarsınız? Sorusuna İlişkin Bulgular

Formal tanım dışındaki anlamlandırmalar üç alt kategoride toplanmıştır: üstel–logaritmik ilişki, logaritmik ölçekleme/temsil ve sembolik temsil.

Tablo 2.

Matematik Öğretmeni Adaylarının Logaritmik Fonksiyonun Formal Olmayan Tanımlarına Yönelik İfadeleri

Alt kategori	Kod	Öğretmen adayları	*f*	%
Üstel-logaritmik ilişkisi	Üs olarak logaritma	Ö1	1	17
Logaritmik ölçekleme ve temsil	Artış şiddeti	Ö2	1	33
	Büyük sayıları ifade etmek	Ö5	1	
Sembolik temsil	Log sembolü imgesi	Ö3	1	50
	Ln sembolik imgesi	Ö3, Ö4	2	

Formal tanım dışındaki anlayışlarda üç eğilim öne çıkmıştır: üstel–logaritmik ilişki, logaritmik ölçekleme/temsil ve sembolik temsil. "Artış şiddeti" ve "büyük sayıları ifade etmek" kodlarının birlikte %33 oranında görülmesi, bazı adayların logaritmik fonksiyonu büyüklükleri ölçeklendiren bir araç olarak anlamlandırabildiğini göstermektedir. Ancak en yüksek yoğunluk "sembolik temsil"dedir (%50): Adayların önemli bir kısmı logaritmik fonksiyon denildiğinde öncelikle log ve özellikle ln sembollerini çağrıştıran bir anlayış oluşturmuştur (Ö3, Ö4).

Logaritmik Fonksiyonların Grafiklerine İlişkin Neler Söyleyebilirsiniz? Logaritmik Fonksiyonun Grafiğini Çizerken Nelere Dikkat Edersiniz? Sorusuna İlişkin Bulgular

Grafiklere ilişkin açıklamalar; grafik oluşturma stratejileri, artan/azalanlık, asimptot ve cevap yok kategorilerinde toplanmıştır.

Tablo 3.

Matematik Öğretmeni Adaylarının Logaritmik Fonksiyonun Grafiklerine Yönelik İfadeleri

Alt kategori	Kod	Öğretmen adayları	*f*	%
Grafik oluşturma stratejileri	Tanım/ değer kümesine odaklanmak	Ö3, Ö5	2	14
Artan/azalan fonksiyon	Genel artanlık algısı	Ö1, Ö5	2	58
	Sürekli ama doğrusal olmayan	Ö2	1	
	Tanım/değer kümesine odaklı	Ö3, Ö5	2	
	Tabana bağlı	Ö2, Ö4	2	
	Logaritmik fonksiyon işareti	Ö5	1	
Asimptot	Yaklaşma-ulaşamama	Ö4	1	21
	Yatay asimptot	Ö1	1	
	y=1 doğrusuna yaklaşma	Ö1	1	
Cevap yok	Bilgi eksikliğini farklında olma	Ö3	1	7

Bu dağılıma göre en yüksek yoğunluk artan/azalanlık kategorisindedir (%58). Bazı adaylar artanlık/azalanlığı tabanla ilişkilendirmiş ($0<a<1$ ise azalan; $a>1$ ise artan) (Ö2, Ö4); bazıları ise “genel artanlık” gibi genelleyici söylemler kullanmıştır (Ö1, Ö5). Grafiklere ilişkin bulgularda adaylar çoğunlukla artanlık-azalanlık ve asimptotik davranış üzerinde durmuş; bazı adayların artanlık-azalanlığı tabana bağlı doğru ölçütle gerekçelendirdiği, bazılarının ise işaret gibi yüzeysel ipuçlarına dayalı yorum yaptığı görülmüştür.

Gerçek Yaşamda Karşımıza Çıkan Bir Logaritmik Fonksiyon Örneği Verir Misiniz? Sorusuna İlişkin Bulgular

Gerçek yaşam örnekleri üç alt kategoride toplanmıştır: ölçek temelli örnekleme, logaritmanın tabanına ilişkin vurgu ve üstel fonksiyon bağlamı.

Tablo 4.

Matematik Öğretmeni Adaylarının Logaritmik Fonksiyonun Gerçek Yaşamda Kullanım Alanlarına İlişkin İfadeleri

Alt kategori	Kod	Öğretmen adayları	*f*	%
Ölçek temeli örnekleme	Deprem ölçeği	Ö2, Ö4, Ö5	3	50
Logaritma tabanına ilişkin vurgu	Logaritma on tabanı	Ö1	1	17
Üstel fonksiyon bağlamı	Biyoloji/ bakteri çoğalması	Ö3, Ö4	2	33

Gerçek yaşamla ilişkilendirmede en yaygın örnek deprem ölçeği olmuş; daha sınırlı düzeyde pH/ses şiddeti türü ölçek mantığına atıf ve biyoloji/bakteri çoğalması gibi üstel süreçlere yönelen açıklamalar da gözlenmiştir.

Tartışma ve Sonuç

Bu çalışmada matematik öğretmeni adaylarının logaritmik fonksiyon kavramına yönelik anlayışları üç alt problem doğrultusunda incelenmiştir. Birinci alt problem kapsamında adayların logaritmik fonksiyonun tanımına yönelik açıklamaları; taban koşulu, bağımsız değişken (x), üstel fonksiyon ile ilişki kurma, fonksiyon özellikleri ve cevap yok/belirsizlik olmak üzere beş alt kategoride toplanmıştır. Taban koşuluna ilişkin bulgular, adayların tabanı belirleyen koşulları genel ve tam biçimiyle ifade etmekte zorlandıklarını göstermektedir. Adayların bir bölümü tabanın pozitif olması gerektiğini (a>0) vurgulamış (Ö2, Ö4, Ö5); bazıları tabanı a>1 ile sınırlandırmış veya a≠1 koşulunu belirtmiştir (Ö1, Ö2, Ö4). Bunun yanı sıra tabanı "tamsayı/pozitif tam sayı" olarak ifade eden açıklamalar da görülmüştür (Ö1, Ö4). Bağımsız değişken koşulunda adayların önemli bir kısmı x>0 koşuluna atıf yapmış (Ö1, Ö2, Ö4); bazı adaylar ise "eşit olmama" biçiminde (x≠0, x≠1) ifade ederek x>0 koşulunu daraltmıştır (Ö4, Ö5). Bu bulgular, adayların a≠1 ve x>0 koşullarını net olarak ayrıştıramadıklarını göstermektedir. Elde edilen sonuçlar, öğrencilerin logaritmik fonksiyonu doğru ve tutarlı biçimde tanımlamakta zorlandıklarını raporlayan çalışmalarla paralellik göstermektedir (Akkuş, 2004; Campo-Meneses vd., 2021).

Formal tanım dışındaki anlayışlar üç alt kategoride toplanmıştır: üstel–logaritmik ilişkisi, logaritmik ölçekleme/temsil ve sembolik temsil. Logaritmik ölçekleme/temsil kapsamında "artış şiddeti" (Ö2) ve "büyük sayıları ifade etmek" (Ö5) kodları birlikte %33 düzeyinde yer alırken, en yüksek oran "sembolik temsil" alt kategorisinde görülmüştür (%50). "Log sembolü imgesi" ve "ln sembolik imgesi" ifadeleri, logaritmik fonksiyon denildiğinde adayların önemli bir kısmının öncelikle log ve ln sembollerini çağrıştırdığını göstermektedir (Ö3, Ö4). Bu bulgu, öğrencilerin logaritmik fonksiyon kavramını çoğunlukla sembolik bir gösterim ya da yapılacak bir işlem olarak gördüklerini belirten sonuçlarla örtüşmektedir (Kastberg, 2002).

İkinci alt problemde logaritmik fonksiyonların grafiklerine ilişkin anlayışlar incelenmiştir. Adayların açıklamaları; grafik oluşturma stratejileri, artan/azalan fonksiyon, asimptot ve cevap yok alt kategorileri altında kodlanmıştır. Genel olarak adaylar grafiği yorumlarken artanlık (%58) ve asimptotik davranış (%21) üzerinde yoğunlaşmıştır; ancak bu iki boyutta hem doğru ölçütlerin (tabana bağlı artış/azalış; yaklaşma–ulaşamama fikri) hem de karışmış/eksik temsillerin (işarete dayalı artanlık) birlikte bulunduğu görülmüştür. Bu sonuç, öğrencilerin logaritmik fonksiyon grafiğini anlamlandırmada ve doğru grafik temsili oluşturmada zorlandıklarını belirten çalışmalarla benzerlik göstermektedir (Quinio & Cuarto, 2023).

Üçüncü alt problem kapsamında adayların logaritmik fonksiyonu gerçek yaşamla ilişkilendirme biçimleri incelenmiştir. Beş katılımcıdan üçü (Ö2, Ö4, Ö5), logaritmik fonksiyonlarla gerçek yaşam arasındaki ilişkiyi deprem ölçeği üzerinden açıklamış; bu durum, katılımcıların yarısından fazlasının logaritmik fonksiyonları öncelikle Richter türü ölçeklerle ilişkilendirdiğini göstermiştir. Bu tercih, deprem ölçeğinin ortaöğretim düzeyindeki logaritmik fonksiyon uygulamalarında sıklıkla kullanılan bir örnek olmasıyla açıklanabilir. Bu bulgu, deprem büyüklüğü (Richter ölçeği), pH değeri ve ses şiddeti (desibel) gibi bağlamları üstel ve logaritmik fonksiyonlarla modellenen gerçek yaşam durumlarına örnek olarak sunan 2018 Ortaöğretim Matematik Dersi Öğretim Programı ile de tutarlıdır (Millî Eğitim Bakanlığı [MEB], 2018). Gol Tabaghi (2007) de Richter, pH ve desibel gibi logaritmik ölçekleri, logaritmik fonksiyonun farklı alanlardaki uygulamalarına örnek olarak göstermektedir. Ayrıca Budinski ve Takači (2013), Richter ölçeğini logaritmaların öğretiminde kullanılan gerçek yaşam bağlamlarından biri olarak ele almaktadır. Bunun yanında iki katılımcı (Ö3, Ö4; %33), logaritmik fonksiyona örnek verirken bakteri çoğalması gibi üstel örneklere de değinmiştir. Bu bulgu, bazı öğretmen adaylarının logaritmik ve üstel fonksiyonlar arasındaki ayrımı net biçimde kuramadıklarını göstermekte ve öğrencilerin bu kavramları ayırt etmekte zorlanabildiğini ortaya koyan çalışmalarla örtüşmektedir (Borji vd., 2023; Engbersen, 2009).

Bu çalışma, matematik öğretmeni adaylarının logaritmik fonksiyon kavramına ilişkin anlayışlarını tanım, grafik ve gerçek yaşam örnekleri boyutlarında incelemiştir. Elde edilen bulgular, öğretmen adaylarının logaritmik fonksiyonlara ilişkin bazı temel unsurları ifade edebildiklerini; ancak tanımlayıcı koşulları eksik ya da yanlış ifade etme, grafik özelliklerine ilişkin belirsizlik yaşama ve gerçek yaşam örneklerini sınırlı bağlamlar üzerinden açıklama gibi güçlüklerle karşılaştıklarını göstermiştir. Bu durum, logaritmik fonksiyon kavramının öğretiminde yalnızca işlemsel bilgiye değil, kavramsal anlamaya da odaklanılması gerektiğine işaret etmektedir.

Öneriler

Bu çalışma, sınırlı sayıda öğretmen adayı ile yürütülen derinlemesine görüşmelere dayanmaktadır. Gelecek araştırmalarda daha geniş örneklemlerle çalışılması; farklı katılımcı gruplarının (lise öğrencileri ve öğretmenler) sürece dâhil edilerek logaritmik fonksiyon kavramına ilişkin anlayışların nicel verilerle desteklenmesi ve karşılaştırmalı biçimde incelenmesi önerilmektedir.

Matematik öğretmeni adaylarının logaritmik fonksiyon kavramına ilişkin anlayışlarını desteklemek amacıyla, kavramsal öğrenmeyi merkeze alan öğretim süreçlerine yer verilmesi önerilmektedir. Bu doğrultuda, grafik temelli etkinliklerin artırılması, logaritmik fonksiyonların farklı temsillerinin birlikte ele alınması ve teknoloji destekli uygulamaların öğretim sürecine entegre edilmesi önemli görülmektedir.