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EXTENSIONS OF FUZZY IDEALS OF $\Gamma-$SEMIRINGS

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Abstract. In this paper, we introduce the notion of extensions of fuzzy ideals of $\Gamma-$semiring, fuzzy weakly completely prime ideals and fuzzy $3-$weakly completely prime ideal of $\Gamma-$semiring. We study the relationship between fuzzy weakly completely prime ideals, fuzzy $3-$weakly prime ideals in terms of the extension of fuzzy ideals of $\Gamma-$semiring.

1. Introduction

Semiring, the algebraic structure which is a common generalization of rings and distributive lattices, was first introduced by American mathematician Vandiver[22] in 1934 but non trivial examples of semirings had appeared in the studies on the theory of commutative ideals of rings by German Mathematician Richard Dedekind in 19th century. Semiring is an universal algebra with two binary operations called addition and multiplication where one of them distributive over the other, bounded distributive lattices are commutative semirings which are both additively idempotent and multiplicatively idempotent. A natural example of semiring which is not a ring, is the set of all natural numbers under usual addition and multiplication of numbers. In particular if I is the unit interval on the real line, then $(I, \max, \min)$ in which 0 is the additive identity and 1 is the multiplicative identity. The theory of rings and the theory of semigroups have considerable impact on the development of the theory of semirings. In structure, semirings lie between semigroups and rings. The study of rings shows that multiplicative structure of ring is independent of additive structure whereas in semiring multiplicative structure of semiring is not independent of additive stricture of semiring. Additive and multiplicative structures of a semiring play an important role in determining the structure of a semiring. Semiring, as the basic algebraic structure, was used in the areas of theoretical computer science as well as in the solutions of graph theory and optimization theory and in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches. It is well known that ideals play an important role in the study of any algebraic structures, in particular semirings.

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As a generalization of ring, the notion of a $\Gamma-$ring was introduced by Nobusawa[16] in 1964. In 1981, Sen [20] introduced the notion of a $\Gamma-$semigroup as a generalization of semigroup. The notion of a ternary algebraic system was introduced by Lehmer [7] in 1932. Lister [9] introduced the notion of a ternary ring. In 1995, Murali Krishna Rao [12] introduced the notion of a $\Gamma-$semiring as a generalization of $\Gamma-$ring, ring, ternary semiring and semiring. The set of all negative integers Z is not a semiring with respect to usual addition and multiplication but Z forms a $\Gamma-$semiring where $\Gamma = Z$. The important reason for the development of $\Gamma-$semiring is a generalization of results of rings, $\Gamma-$rings, semirings, semigroups and ternary semirings. Murali Krishna Rao and Venkateswarlu [13] introduced the notion of regular $\Gamma-$incline and field $\Gamma-$semiring.

The fuzzy set theory was developed by Zadeh [23] in 1965. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc. The fuzzification of algebraic structure was introduced by Rosenfeld [18] and he introduced the notion of fuzzy subgroups in 1971. Swamy and Swamy[21] studied fuzzy prime ideals in rings in 1988. In 1982, Liu [8] defined and studied fuzzy subrings as well as fuzzy ideals in rings. Applying the concept of fuzzy sets to the theory of $\Gamma-$ring, Jun and Lee [4] introduced the notion of fuzzy ideals in $\Gamma-$ring and studied the properties of fuzzy ideals of $\Gamma-$ring. Mandal [10] studied fuzzy ideals and fuzzy interior ideals in an ordered semiring. Das and Dutta[2] introduced the notion of extensions of fuzzy ideals in semirings.

In this paper, we introduce the notion of extensions of fuzzy ideals, fuzzy weakly completely prime ideals and fuzzy $3-$weakly completely prime ideal of $\Gamma-$semiring. We study the relationship between fuzzy weakly completely prime ideals, fuzzy $3-$weakly prime ideals in terms of the extension of fuzzy ideals of $\Gamma-$semring.

2. Preliminaries

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. [1] A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called semiring provided

(i) addition is a commutative operation.
(ii) multiplication distributes over addition both from the left and from the right.
(iii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

Definition 2.2. [12] Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a $\Gamma-$semiring, if there exists a mapping $M \times \Gamma \times M \to M$ is written (x, α, y) as $x\alpha y$ such that it satisfies the following axioms

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$, for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Every semiring R is a $\Gamma-$semiring with $\Gamma = R$ and ternary operation $x\gamma y$ as the usual semiring multiplication.

Example 2.3. [12] Let M be a a set of all rational numbers and Γ be a set of all natural numbers are commutative semigroups with respect to usual addition. Define the mapping $M \times \Gamma \times M \rightarrow M$ by $a\alpha b$ as usual multiplication for all $a, b \in M, \alpha \in \Gamma$. Then M is a $\Gamma-$semiring.

A $\Gamma-$semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0$, for all $x \in M, \alpha \in \Gamma$. Let M be a $\Gamma-$semiring. An element $1 \in M$ is said to be unity if for each $x \in M$ there exists $\alpha \in \Gamma$ such that $x\alpha 1 = 1\alpha x = x$. A $\Gamma-$semiring M is said to be commutative $\Gamma-$semiring if $x\alpha y = y\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$. An element $a \in M$ is said to be an idempotent of M if $a = a\alpha a$ for all $\alpha \in \Gamma$. Every element of M is an idempotent of M then M is said to be idempotent $\Gamma-$semiring M. A non-empty subset A of $\Gamma-$semiring M is called a $\Gamma-$subsemiring M if $(A, +)$ is a subsemigroup of $(M, +)$ and $a\alpha b \in A$, for all $a, b \in A$ and $\alpha \in \Gamma$. An additive subsemigroup I of a $\Gamma-$semiring M is said to be left (right) ideal of M if $M\Gamma I \subseteq I$ ($I\Gamma M \subseteq I$). If I is both a left and a right ideal of M then I is called an ideal of $\Gamma-$semiring M. A non-zero element a in a $\Gamma-$semiring M is said to be zero divisor if there exits non-zero element $b \in M, \alpha \in \Gamma$ such that $a\alpha b = b\alpha a = 0$.

Let M be a non-empty set. Then a mapping $f : M \rightarrow [0, 1]$ is called a fuzzy subset of M. Let f be a fuzzy subset of a non-empty set M. For $t \in [0, 1]$, the set $f_t = \{x \in M \mid f(x) \geq t\}$ is called a level subset of M with respect to f.

Definition 2.4. [11] A fuzzy subset μ of M is said to be fuzzy $\Gamma-$subsemiring of M if it satisfies the following conditions
(i) $\mu(x + y) \geq min\ \{\mu(x), \mu(y)\}$
(ii) $\mu(x\alpha y) \geq min\ \{\mu(x), \mu(y)\}$, for all $x, y \in M, \alpha \in \Gamma$.

Definition 2.5. [11] A fuzzy subset μ of a $\Gamma-$semiring M is called a fuzzy left (right) ideal of M if it satisfies the following conditions
(i) $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$
(ii) $\mu(x\alpha y) \geq \mu(y)$ $(\mu(x))$, for all $x, y \in M, \alpha \in \Gamma$.

Definition 2.6. [11] A fuzzy subset μ of a $\Gamma-$semiring M is called a fuzzy ideal of M if it satisfies the following conditions
(i) $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$
(ii) $\mu(x\alpha y) \geq max\ \{\mu(x), \mu(y)\}$, for all $x, y \in M, \alpha \in \Gamma$

Definition 2.7. Let μ be a non empty fuzzy subset of $\Gamma-$semiring M. Then μ is called a fuzzy prime ideal of M if

(i) $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$
(ii) $\mu(x\alpha y) = max\{\mu(x), \mu(y)\}$.

This definition can also be written as a non-constant fuzzy ideal μ of a $\Gamma-$semiring M is called fuzzy prime ideal of M if for any two fuzzy ideals μ_1 and μ_2 of M such that $\mu_1\Gamma\mu_2 \subseteq \mu$ implies that either $\mu_1 \subseteq \mu$ or $\mu_2 \subseteq \mu$.

Definition 2.8. A fuzzy ideal f of $\Gamma-$semiring M is said to be a fuzzy $k-$ideal of M if $f(x) \geq min\{f(x + y), f(y)\}$, for all $x, y \in M$.

This definition can also be written as a fuzzy ideal f of $\Gamma-$semiring M is said to be fuzzy $k-$ideal of $\Gamma-$semiring M if $f(x + a) \geq \lambda$, $f(a) \geq \lambda \Rightarrow f(x) \geq \lambda$, for all $x, a \in M, \lambda \in [0, 1]$.

Definition 2.9. A fuzzy ideal of μ of $\Gamma-$semiring M is said to be fuzzy completely prime ideal if for all $x, y \in M, t \in (o, 1], (x\alpha y)_t \in \mu$ implies $x_t \in \mu$ or $y_t \in \mu$.

A fuzzy ideal μ of $\Gamma-$semiring M is fuzzy completely prime ideal if and only if $\mu(x\alpha y) = \mu(x)$ or $\mu(x\alpha y) = \mu(y)$, for all $x, y \in M, \alpha \in \Gamma$.

Definition 2.10. [11] Let A be non-empty subset of a $\Gamma-$semiring M. The characteristic function of A is a fuzzy subset of M and it is defined by

$$\chi_{_A}(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A. \end{cases}$$

3. Extension of fuzzy ideals of $\Gamma-$semirings

In this section we introduce the notion of extensions of fuzzy ideals and fuzzy weakly completely prime ideal of $\Gamma-$semiring and study the properties related to them.

Definition 3.1. Let M be a $\Gamma-$semiring, μ be a fuzzy subset of M and $s \in M$. The fuzzy subset $< s, \mu >$: $M \to [0, 1]$ is defined by $< s, \mu > (x) = \mu(s\alpha x),\ x \in M, \alpha \in \Gamma$ is called extension of μ by s.

Example 3.2. Let $\Gamma = M = Z_0^+$. We define a fuzzy subset of μ of M as follows Z_0^+ denotes the semiring of all non-negative integers with respect to the usual addition and multiplication of integers
$\mu(n) = \begin{cases} 0, & \text{if } n = 0 \\ 0.5, & \text{if } n \text{ is even} \\ 0.3, & \text{if } n \text{ is odd} \end{cases}$. Then $< 2, \mu > (n) = \begin{cases} 1, & \text{if } n = 0 \\ 0.5, & \text{if } n \neq 0, \end{cases}$ is the extension of μ by 2.

Here extension of μ by 2 is also a fuzzy subset of M takes only the values which takes μ for the integers of multiple of 2.

Proposition 3.3. Let M be a commutative $\Gamma-$semiring. If μ is a fuzzy ideal of M and $s \in M$ then the extension of μ by s is a fuzzy ideal of M.

Proof. Obviously $< s, \mu >$ is a fuzzy subset of M. Let $x, y \in M, \alpha, \beta \in \Gamma$.

$$\begin{aligned} < s, \mu > (x + y) &= \mu(s\alpha(x + y)) \\ &= \mu(s\alpha x + s\alpha y) \\ &\geq min\{\mu(s\alpha x), \mu(s\alpha y)\} \\ &= min\{< s, \mu > (x), < s, \mu > (y)\}. \\ \text{Hence } < s, \mu > (x + y) &\geq min\{< s, \mu > (x), < s, \mu > (y)\}. \\ \text{And also } < s, \mu > (x\beta y) &= \mu(s\alpha x\beta y) \\ &\geq \mu(s\alpha x) \\ &=< s, \mu > (x) \\ \text{Similarly } < s, \mu > (x\beta y) &\geq< s, \mu > (y). \end{aligned}$$

Thus $< s, \mu >$ is a fuzzy ideal of M. ☐

The converse of the above proposition may not be true. This follows from the following example

Example 3.4. Let $M = \Gamma = Z_0^+$. We define a fuzzy subset $\mu : M \to [0,1]$ as follows

$$\mu(n) \; = \; \begin{cases} 1, & \text{if } n = 0 \\ 0.5, & \text{if } 1 \leq n \leq 4 \\ 0.2, & \text{if } n > 4 \end{cases}.$$

Then μ is not a fuzzy ideal of M, since $\mu(2+4) \ngeq min\{\mu(2), \mu(4)\}$. Now

$$< 5, \mu > (n) \; = \; \begin{cases} 1, & \text{if } n = 0 \\ 0.2, & \text{if } n \neq 0 \end{cases}.$$

Then clearly $< 5, \mu >$ is a fuzzy ideal of M.

Proposition 3.5. Let M be a commutative $\Gamma-$semiring. If μ is a fuzzy k-ideal of M and $s \in M$ then the extension of μ by s is a fuzzy k-ideal of M.

Proof. By Proposition 3.3, $< s, \mu >$ is a fuzzy ideal of M. Since μ is a fuzzy k-ideal of M, $\mu(s\alpha x) \geq \min\{\mu(s\alpha x + s\alpha y), \mu(s\alpha y)\}$, for all $x, y \in M, \alpha \in \Gamma$. Then $< s, \mu > (x) \geq \min\{< s, \mu > (x+y), < s, \mu > (y)\}$, for all $x, y \in M, \alpha \in \Gamma$. There fore $< s, \mu >$ is a fuzzy $k-$ideal of M. □

Here also the converse of the above proposition may not be true. This follows from the following example.

Example 3.6. Let $\Gamma = M = Z_0^+$. We define a fuzzy subset $\mu : M \to [0,1]$ as
$\mu(n) \; = \; \begin{cases} 1, & \text{if } n = 0 \\ 0.2, & \text{if } 1 \leq n \leq 7 \\ 0.5, & \text{if } n > 7 \end{cases}$. Then clearly μ is a fuzzy ideal of M.

But μ is not a fuzzy $k-$ideal of M, since $\mu(6) \ngeq \; min\{\mu(6+9), \mu(9)\}$. Now

$$< 9, \mu > (n) \; = \; \begin{cases} 1, & \text{if } n = 0 \\ 0.5, & \text{if } n \neq 0 \end{cases}.$$

By Proposition 3.3, $< 9, \mu >$ is a fuzzy ideal of M. Let $x, y \in M$.

(1). If $x = 0$ then $< 9, \mu > (x) = 1 \geq \min\{< 9, \mu > (x), \; < 9, y > (y)\}$.

(2). If $x \neq 0$ then $< 9, n > (x+y) = 0.5$.
Now $< 9, \mu > (x) = 0.5 = \min\{< 9, \mu > (x+y), < 9, \mu > (y)\}$.
Hence the extension of the fuzzy ideal μ by 9 is a fuzzy $k-$ideal of M.

Definition 3.7. If μ is a fuzzy subset of Γ-semiring M then we define
supp $\mu = \{s \in M \mid \mu(s) > 0\}$.

Proposition 3.8. Let μ be a fuzzy ideal if Γ-semiring of M and $s \in M$. Then

(i) $\mu \; \subseteq < s, \mu >$

(ii) $< (s\alpha)^{n-1}s, \mu > \subseteq < (s\alpha)^n s, \mu >, \alpha \in \Gamma$, for every natural number n

(iii) If $\mu(s) > 0$ then supp $< s, \mu >= M$.

Proof. Let μ be a fuzzy ideal of Γ-semiring M and $s \in M, \alpha \in \Gamma$.

(i) Since μ is a fuzzy ideal of Γ-semiring M. By Proposition 3.3, $< s, \mu >$ is a ideal of M.
Now $< s, \mu > (x) = \mu(s\alpha x) \geq \mu(x)$, for all $x \in M, \alpha \in \Gamma$.
Therefore $\mu \subseteq < s, \mu >$.

(ii) For all natural number n and for all $x \in M,\ \alpha, \beta \in \Gamma$,

$$\begin{aligned} < (s\alpha)^n s, \mu > (x) &= \mu((s\alpha)^n s\beta x) \\ &= \mu(s\alpha(s\alpha)^{n-1} s\beta x) \\ &\geq \mu((s\alpha)^{n-1} s\beta x) \\ &=< (s\alpha)^{n-1} s, \mu > (x). \end{aligned}$$

Therefore $< (s\alpha)^n s, \mu > \subseteq < (s\alpha)^{n-1} s, \mu >$.

(iii) Let $\mu(s) > 0, x \in M$. Now

$$\begin{aligned} &< s, \mu > (x) = \mu(s\alpha x) \geq \mu(s) > 0. \\ &\Rightarrow\ < s, \mu > (x) \geq 0 \\ &\Rightarrow x \in supp < s, \mu >, \quad \text{for all } x \in M \\ &\Rightarrow M \subseteq supp < s, \mu > . \end{aligned}$$

By Definition 3.7, clearly supp$< s, \mu > \subseteq M$.
Hence $M = supp < s, \mu >$.

□

Proposition 3.9. [14] If μ is a fuzzy subset of a $\Gamma-$semiring M then μ is a fuzzy prime ideal of M if and only if Im $\mu = \{1, \alpha\}$, where $\alpha \in [0, 1)$ and $\mu_0 = \{x \in M \mid \mu(x) = \mu(0)\}$ is a fuzzy prime ideal of M.

Proposition 3.10. If μ is a fuzzy semiprime ideal of a commutative Γ-semiring M then $\mu(x\alpha x) = \mu(x)$, for all $x \in M$.

Proof. Let μ be a fuzzy semiprime ideal of a commutative Γ-semiring and $x \in M, \alpha \in \Gamma$. Then we have $\mu(x\alpha x) \geq \mu(x)$. There fore $\mu(x\alpha x) = \mu(x)$, for all $x \in M$. □

Corollary 3.11. If μ is a fuzzy semiprime ideal of a commutative $\Gamma-$semiring M then $< x, \mu >=< x\alpha x, \mu >$, for $x \in M$ and $\alpha \in \Gamma$.

Proof. Suppose μ is a fuzzy semiprime ideal of a commutative $\Gamma-$semiring $M,\ x, y \in M$ and $\alpha, \beta \in \Gamma$.

$$\begin{aligned} < x\alpha x, \mu > (y) &= \mu(x\alpha x\beta y) > \mu(x\alpha y) \\ &=< x, \mu > (y), \text{ for all } y \in M. \\ \text{Then } < x, \mu > &\subseteq < x\alpha x, \mu > . \\ \text{Now } \mu(x\alpha y) &= \mu\Big[(x\alpha y)\beta(x\alpha y)\Big], \quad \text{by Proposition 3.10} \\ &= \mu\Big[(x\alpha x)\beta(y\alpha y)\Big] \\ &\geq \mu\Big[(x\alpha x)\beta y\Big] \\ \Rightarrow < x, \mu > (y) &\geq < x\alpha x, \mu > (y) \\ \Rightarrow < x\alpha x, \mu > &\subseteq < x, \mu > . \\ \text{Hence } < x, \mu > &=< x\alpha x, \mu > . \end{aligned}$$

□

Definition 3.12. Let A be a subset of Γ-semiring M and $x \in M$. Define $< x, A >= \{s \in M \mid x\alpha s \in A\}$, for all $\alpha \in \Gamma$.

Proposition 3.13. Let A be a subset of $\Gamma-$semiring M and $x \in M$. Then $< s, \lambda_A >= \lambda_{<s,A>}$, for all $s \in M$, where λ_A denotes the characteristic function of A.

Proof. Let $x \in M, \alpha \in \Gamma$. Now $< s, \lambda_A > (x) = \lambda_A(s\alpha x) = 1$ or 0.

(i). If $< s, \lambda_A > (x) = 1$ then $\lambda_A(s\alpha x) = 1$, for all $s, x \in M, \alpha \in \Gamma$.

$$\Rightarrow s\alpha x \in A, \text{ for all } s, x \in M, \alpha \in \Gamma.$$
$$\Rightarrow x \in < s, A > .$$
$$\Rightarrow \lambda_{<s,A>}(x) = 1.$$

(ii). If $< s, \lambda_A > (x) = 0$ then $\lambda_A(s\alpha x) = 0$, for all $s \in M, \alpha \in \Gamma$.
$\Rightarrow s\alpha x \notin A$
$\Rightarrow x \notin < s, A >$
$\Rightarrow \lambda_{<s,A>}(x) = 0.$
Hence $< s, \lambda_A >= \lambda_{<s,A>}$.

□

Definition 3.14. Let μ be a fuzzy ideal of Γ-semiring M. Then μ is called a fuzzy weakly completely prime ideal if $\mu(x_1\alpha x_2) = max\ \{\mu(x_1), \mu(x_2)\}$, for all $x_1, x_2 \in M, \alpha \in \Gamma$.

Proposition 3.15. [19] Let μ be a fuzzy ideal of Γ-semiring. Then μ is a fuzzy completely prime ideal of M if and only if the following conditions hold

(i). $\mu(0) = 1$
(ii). $Im\mu = \{1, \alpha\},\ \alpha \in [0, 1)$
(iii). $\mu_0 = \{x \in M \mid \mu(x) = \mu(0)\}$ is a completely prime ideal of M.

Proposition 3.16. Every fuzzy completely prime ideal of a Γ-semiring M is fuzzy weakly completely prime.

Proof. Let μ be a fuzzy completely prime ideal of Γ-semiring M. By Proposition 3.15, μ_0 is a fuzzy completely prime ideal of M and $Im\mu = \{1, \alpha\}$, where $\alpha \in [0, 1)$. Let $x_1, x_2 \in M, \beta \in \Gamma$.

(i). Let $\mu(x_1\beta x_2) = 1$. Then $x_1\beta x_2 \in \mu_0$.
Since μ_0 is a completely prime ideal of M, $x_1 \in \mu_0$ or $x_2 \in \mu_0$.
Thus $\mu(x_1) = 1$ or $\mu(x_2) = 1$.
Therefore $\max\{\mu(x_1), \mu(x_2)\} = 1$. Hence $\mu(x_1\beta x_2) = max\{\mu(x_1), \mu(x_2)\}$.

(ii). Let $\mu(x_1\beta x_2) \neq 1$. Then $\mu(x_1\beta x_2) = \alpha$.
Thus $\mu(x_1) = \alpha$ and $\mu(x_2) = \alpha$, otherwise $\mu(x_1) = 1$ or $\mu(x_2) = 1$.
This implies that $x_1\beta x_2 \in \mu_0$. That is $\mu(x_1\beta x_2) = 1$, which is a contradiction. So $\max\{\mu(x_1), \mu(x_2)\} = \alpha$.
Hence $\mu(x_1\beta x_2) = max\{\mu(x_1), \mu(x_2)\}$.

Thus μ is a fuzzy weakly completely prime ideal of Γ-semiring M. □

Theorem 3.17. *Let μ be a fuzzy weakly completely prime ideal of* Γ*-semiring M. If $x \in M$ such that $\mu(x) = \inf\limits_{y \in M} \mu(y)$ then $< x, \mu >= \mu$.*

Proof. Let $y \in M$. Clearly $\inf_{y \in M} \mu(y)$ exists in $[0,1]$.
Then $< x, \mu > (y) = \mu(x\alpha y) \ \cdots \ (1)$.
Since μ is a fuzzy weakly completely prime ideal of M, $\mu(x\alpha y) = max\{\mu(x), \mu(y)\}$.
Then either $\mu(x\alpha y) = \mu(x)$ or $\mu(x\alpha y) = \mu(y) \ \cdots \ (2)$.
Let $\mu(x\alpha y) \neq \mu(y)$. Then by (2) , $\mu(x\alpha y) = \mu(x) \ \cdots \ (3)$.
Since μ is a fuzzy ideal of M, By Proposition 3.8, $\mu \subseteq < x, \mu >$

$$\begin{aligned}
&\Rightarrow \mu(y) \leq < x, \mu > (y) \\
&\Rightarrow \mu(y) \leq \mu(x\alpha y) \\
&\Rightarrow \mu(y) \leq \mu(x).
\end{aligned}$$

$$\text{Also since } \mu(x) = \inf_{y \in M} \mu(y) \Rightarrow \mu(x) \leq \mu(y).$$

From $\mu(x) = \mu(y)$ and $\mu(x\alpha y) = \mu(y)$. Which is a contradiction.

$$\begin{aligned}
&\text{Hence } \mu(x\alpha y) = \mu(y). \\
&\Rightarrow < x, \mu > (y) = \mu(y), \text{ for all } y \in M. \\
&\text{Hence } < x, \mu >= \mu.
\end{aligned}$$

□

Proposition 3.18. Let M be a Γ-semiring and μ be a fuzzy completely prime ideal of M. If $x \in M$ such that $x \notin \mu_0$ then $< x, \mu >= \mu$.

Proof. Let μ be a fuzzy completely prime ideal of Γ-semiring M. Then by Proposition 3.15, μ_0 is a completely prime ideal of M.
Suppose $Im\mu = \{1, \alpha\}$. Let $s \in M, \beta \in \Gamma$.

(i) . Let $s \in \mu_0$. Then $x\beta s \in \mu_0$.
Therefore $< x, \mu > (s) = 1 \Rightarrow 1 = \mu(x\beta s) = \mu(s)$.

(ii). Let $s \notin \mu_0$. Then $x\beta s \notin \mu_0$, as μ_0 is a completely prime ideal of M.
Thus $< x, \mu > (s) = \mu(x\beta s) = \alpha$.
So $< x, \mu > (s) = \mu(s)$, for all $s \in M$. Hence $< x, \mu >= \mu$.

□

Theorem 3.19. [14] *Let I be a prime ideal of a Γ semiring M. Then characteristic function λ_I is a fuzzy prime ideal of M.*

Proposition 3.20. Let M be a commutative Γ-semiring and I be an ideal of M. If I is prime ideal of M then for $x \in M$ such that $x \notin I$, $< x, \lambda_I >= \lambda_I$.

Proof. Let M be a commutative Γ-semiring and $x \in M$. Since I is a prime ideal of Γ-semiring M. By Proposition 3.19, λ_I is a fuzzy prime ideal of M. Now $x \notin I$, $\lambda_I(x) = 0$. Then $x \notin \lambda_I$. Thus by Proposition 3.18, $< x, \lambda_I >= \lambda_I$. □

Proposition 3.21. Let M be a commutative Γ-semiring and μ be a fuzzy prime ideal of M. Then $< x, \mu >$ is a fuzzy prime ideal of M for $x \in M$ such that $x \notin \mu_0$.

Proof. Since M is a commutative Γ-semiring, μ is a completely prime ideal of M. Then by Proposition 3.18, $x \notin \mu_0$ and $< x, \mu >= \mu$.
Thus $< x, \mu >$ is a fuzzy prime ideal of M. □

Proposition 3.22. Let M be a Γ-semiring and μ be a fuzzy prime ideal of M. If $x \notin \mu_0$ then $< x, \mu >= \lambda_M$.

Proof. Since $x \in \mu_0,\ x\alpha s \in \mu_0$, for all $s \in M, \alpha \in \Gamma$.
Then $< x, \mu > (s) = \mu(x\alpha s) = 1 = \lambda_s(s)$, for all $s \in M$.
Hence $< x, \mu >= \lambda_M$. □

Theorem 3.23. *Let M be a commutative Γ-semiring and μ be a fuzzy subset of M such that $< s, \mu >= \mu$, for every $s \in M$. Then μ is a constant.*

Proof. Let M be a commutative a Γ-semiring, μ be a fuzzy subset of M such that $< s, \mu >= \mu$, for every $s \in M$ and $x, y \in M$.
Then $< x, \mu >= \mu$ and $< y, \mu >= \mu$. Thus

$$\mu(y) =< x, \mu > (y) = \mu(x\alpha y) = \mu(y\alpha x) =< y, \mu > (x) = \mu(x).$$

Hence μ is a constant. □

Theorem 3.24. *Let M be a Γ-semiring, μ be a fuzzy ideal of M and $Im\mu = \{1, \alpha\}$. Suppose $< y, \mu >= \mu$, for all those $y \in M$ for which $\mu(y) = \alpha$. Then μ is a fuzzy weakly completely prime ideal of M.*

Proof. Let $x_1, x_2 \in M$. Since μ is a fuzzy ideal of M. Then we have $\mu(x_1\beta x_2) \geq \mu(x_1)$ and $\mu(x_1\beta x_2) \geq \mu(x_2)\ \cdots\ (1)$.

case (i) . Suppose $\mu(x_1\beta x_2) = \mu(x_1)$. Then by (1), $\mu(x_1) \geq \mu(x_2)$.
$\Rightarrow\ \max\{\mu(x_1), \mu(x_2)\} = \mu(x_1) = \mu(x_1\beta x_2)$.

case (ii) . Suppose $\mu(x_1\beta x_2) \neq \mu(x_2)$.
Then $\mu(x_1)$ can not be a maximal element of $\mu(M)$, otherwise $\mu(x_1) = 1 = \mu(x_1\beta x_2)$, which is a contradiction.
Thus $\mu(x_1) = \alpha$ and by hypothesis,

$$\begin{aligned} &< x_1, \mu >= \mu \\ \Rightarrow\ &< x_1\mu > (x_2) = \mu(x_2) \\ \Rightarrow\ &\mu(x_1\beta x_2) = \mu(x_2). \end{aligned}$$

Hence $\mu(x_2) = \mu(x_1\beta x_2) \geq \mu(x_2)$ and therefore $\mu(x_1\beta x_2) = max\{\mu(x_1), \mu(x_2)\}$.
Hence μ is a fuzzy weakly completely prime ideal of M.

□

Proposition 3.25. Let M be a commutative Γ-semiring and μ be fuzzy weekly completely prime ideal of M. Then $< x, \mu >$ is a fuzzy weekly completely prime ideal of M, for every $x \in M$.

Proof. Let μ be a commutative $\Gamma-$semiring M and μ be fuzzy weekly completely prime ideal of M. Since μ is a fuzzy ideal of a commutative Γ-semiring M, By Proposition 3.3, $< x, \mu >$ is fuzzy ideal of M for every $x \in M$. Let $y, z \in M$ and $\beta, \gamma, \delta_1, \delta_2$. Then

$$\begin{aligned} < x, y > (y\beta z) &= \mu(x\gamma y\beta z) \\ &= max\{\mu(x\gamma y), \mu(z)\} \\ &= max\{max\{\mu(x), \mu(y)\}, \mu(z)\} \\ &= max\{max\{\mu(x), \mu(y)\}, max\{\mu(x), \mu(z)\}\} \\ &= max\{\mu(x\delta_1 y), \mu(x\delta_2 z)\} \\ &= max\{< x, \mu > (y), < x, \mu > (z)\}. \end{aligned}$$

Thus $< x, \mu >$ is a fuzzy weekly completely prime ideal of M. □

Proposition 3.26. [14] If I is a semiprime ideal of a $\Gamma-$semiring M. Then the characteristic function λ_I of I is a fuzzy semi prime ideal of M.

Proposition 3.27. Let M be a commutative $\Gamma-$semiring and μ be a fuzzy semiprime ideals of M. Then $< x, \mu >$ is a fuzzy semiprime ideals of M, for every $x \in M$.

Proof. Let $x \in M$. Since μ is a fuzzy semiprime ideals of M. By Proposition 3.3, $< x, \mu >$ is a fuzzy ideal of M. Let $y \in M, \alpha, \beta \in \Gamma$. Then

$$< x, \mu > (y\beta y) = \mu(x\alpha y\beta y) \geq \mu(x\beta y) =< x, \mu > (y).$$

$$\begin{aligned} \text{Now } < x, \mu > (y) &= \mu(x\beta y) = \mu[(x\beta y)\gamma(x\beta y)] \\ &= \mu[(x\beta x)\gamma(y\beta y)] \\ &\geq \mu[x\gamma(y\beta y)] \\ &=< x, \mu > (y\beta y). \end{aligned}$$

Hence $< x, \mu > (y\beta y) =< x, \mu > (y)$, for all $y \in M, \beta \in \Gamma$. Thus $< x, \mu >$ is a fuzzy semiprime ideal of M. □

Corollary 3.28. Let M be a commutative Γ-semiring, $\{\mu_i\}_{i\in\Delta}$ be a non empty family of fuzzy semiprime ideals of M and $\mu = \inf\limits_{i\in\Delta} \mu_i$. Then for any $x \in M$, $< x, \mu >$ is a fuzzy semiprime ideals of M.

Proof. Obviously μ is a fuzzy subset of M. Let $x, y \in M,\ \alpha \in \Gamma$. Then

$$\begin{aligned} \mu(x + y) &= \inf_{i\in\Delta} \mu_i(x + y) \\ &\geq \inf_{i\in\Delta} \min\{\mu_i(x), \mu_i(y)\} \\ &= \min\{\inf_{i\in\Delta} \mu_i(x), \inf_{i\in\Delta} \mu_i(y)\} \\ &= \min\{\mu(x), \mu(y)\}. \\ \text{And } \mu(x\alpha y) &= \inf_{i\in\Delta} \mu_i(x\alpha y) \geq \inf_{i\in\Delta} \mu_i(x) = \mu(x). \end{aligned}$$

Similarly, $\mu(x\alpha y) \geq \mu(y)$. Thus μ is a fuzzy ideal of M. Now

$$\begin{aligned} \mu(y\alpha y) &= \inf_{i\in\Delta} \mu_i(y\alpha y) = \inf_{i\in\Delta} \mu_i(y), \text{ since each } \mu_i \text{ is semiprime.} \\ &= \mu(y). \end{aligned}$$

Therefore μ is a fuzzy semiprime ideal of M.
Hence $< x, \mu >$ is a fuzzy semiprime ideal of M, for all $x \in M$. □

Corollary 3.29. Let M be a commutative semiprime $\Gamma-$semiring and $\{P_i\}_{i\in\Delta}$ be a non empty family of semiprime ideals of M and $P = \bigcap\limits_{i\in\Delta} P_i \notin \phi$. Then $< x, \lambda_P >$ is a fuzzy semiprime ideals of M, for every $x \in M$.

Proof. Obviously $P = \bigcap\limits_{i\in\Delta} P_i \notin \phi$ is a semiprime ideals M.
Then by Proposition 3.26, λ_P is a fuzzy semiprime ideals of M.
Thus by Proposition3.27, $< x, \lambda_P >$ is a fuzzy semiprime ideal of M, for every $x \in M$. □

Corollary 3.30. Let M be a commutative $\Gamma-$semiring and μ be a fuzzy weakly complete prime ideal of M. If μ is not a constant then μ is not a maximal fuzzy weakly completely ideal of M.

Proof. Since μ is a fuzzy weakly complete prime ideal of $\Gamma-$semiring M,
by Proposition 3.25, $< x, \mu >$ is a fuzzy weakly complete prime ideal of M, for all $x \in M$.
Also there exists $x \in M$ such that $\mu \subset< x, \mu >$.
Otherwise $\mu =< x, \mu >$, for all $x \in M$.
Then by Theorem 3.23, μ is a constant, which is a contradiction.
Therefore μ is not a maximal fuzzy weakly complete ideal of M. □

Theorem 3.31. *Let M be a commutative $\Gamma-$semiring and μ be a fuzzy semiprime ideal of M. Then $\mu = \inf\{< x, \mu >| \ x \in M\}$.*

Proof. By Proposition 3.8, $\mu \subseteq< x, \mu >$, for all $x \in M$.
Let $\mu_1 \subseteq< x, \mu >$, for all $x \in M$ and $y \in M$. Then

$$\mu_1(y) \leq< y, \mu > (y) = \mu(y\alpha y) = \mu(y), \quad \text{since } \mu \text{ is a semi prime.}$$

Thus $\mu_1 \subseteq \mu$. So $\mu = \inf\{< x, \mu >| \ x \in M\}$. □

4. Fuzzy 3-weakly completely prime ideal

In this section, we introduce the notion of fuzzy $3-$weakly completely prime ideal of $\Gamma-$semiring. We study the relationship between fuzzy weakly completely prime ideals, fuzzy $3-$weakly prime ideals in terms of the extension of fuzzy ideals of $\Gamma-$semring.

Definition 4.1. Let M be a $\Gamma-$semiring. A fuzzy ideal of μ of M is called fuzzy $3-$weakly completely prime ideal if for every $x_1, x_2, x_3 \in M, \ \alpha \in \Gamma$,

$$\begin{aligned} \mu(x_1\alpha x_2\alpha x_3) &= max\{\mu(x_1\alpha x_2), \mu(x_1\alpha x_3)\} \\ &= max\{\mu(x_2\alpha x_3), \mu(x_2\alpha x_1)\} \\ &= max\{\mu(x_3\alpha x_1), \mu(x_3\alpha x_2)\} \end{aligned}$$

Example 4.2. Let $M = 2\mathbb{Z}_0^+$ and $\Gamma = \mathbb{Z}^+, \mathbb{Z}^+$ be the set of positive integers.
We defines a fuzzy subset μ of M as follows
$\mu(0) = 1, \mu(2) = 0.3$ and $\mu(2n) = 0.5$ for $n \geq 2$.
Obviously μ is a fuzzy ideal of M. Let $x_1, x_2, x_3 \in M$.
(i). At least one of x_1, x_2, x_3 is 0 then

$$\begin{aligned} \mu(x_1\alpha x_2\alpha x_3) = 1 &= max\{\mu(x_1\alpha x_2), \mu(x_1\alpha x_3)\} \\ &= max\{\mu(x_2\alpha x_3), \mu(x_2\alpha x_1)\} \\ &= max\{\mu(x_3\alpha x_1), \mu(x_3\alpha x_2)\}. \end{aligned}$$

(ii). If each of x_1, x_2 and x_3 are non zero then each of
$x_1\alpha x_2\alpha x_3, \ x_1\alpha x_2, \ x_2\alpha x_3, \ x_3\alpha x_1, \ x_3\alpha x_2$ are greater than or equal to 0.5

$$\begin{aligned} \mu(x_1\alpha x_2\alpha x_3) = 1 &= max\{\mu(x_1\alpha x_2), \mu(x_1\alpha x_3)\} \\ &= max\{\mu(x_2\alpha x_3), \mu(x_2\alpha x_1)\} \\ &= max\{\mu(x_3\alpha x_1), \mu(x_3\alpha x_2)\}. \end{aligned}$$

Then μ is a fuzzy 3-weakly completely prime ideal of M.

Proposition 4.3. Let M be a commutative Γ-semiring and μ be a fuzzy ideal of M. If μ is a weakly completely prime ideal of M then μ is a fuzzy $3-$weakly completely prime ideal of M.

Proof. Let $x_1, x_2, x_3 \in M, \alpha \in \Gamma$. Since μ is a fuzzy weakly completely prime ideal of M then $\mu(x_i \alpha x_j) = \mu(x_j \alpha x_i)$, for all $i, j = 1, 2, 3, \cdots, \alpha \in \Gamma$. Now

$$\begin{aligned}
\mu(x_1\alpha x_2\alpha x_3) &= \max\{\mu(x_1), \mu(x_2)\} \\
&\leq \max\{\mu(x_1\alpha x_2), \mu(x_2\alpha x_3)\} \\
&\leq \max\{\mu(x_1\alpha x_2\alpha x_3), \mu(x_1\alpha x_2\alpha x_3)\} \\
&= \mu(x_1\alpha x_2\alpha x_3). \\
\text{Then } \mu(x_1\alpha x_2\alpha x_3) &= max\{\mu(x_1\alpha x_2), \mu(x_2\alpha x_3)\} \\
&= \max\{\mu(x_2\alpha x_1), \mu(x_2\alpha x_3)\} \\
\text{Also } \mu(x_1\alpha x_2\alpha x_3) &= \max\{\mu(x_1\alpha x_2), \mu(x_3)\} \\
&= \max\{\mu(x_2\alpha x_1), \mu(x_3)\} \\
&= \mu(x_2\alpha x_1\alpha x_3). \\
\text{and } \mu(x_1\alpha x_2\alpha x_3) &= \max\{\mu(x_1\alpha x_2), \mu(x_3)\} \\
&\leq \max\{\mu(x_1\alpha x_2), \mu(x_1\alpha x_3)\} \\
&= \max\{\mu(x_2\alpha x_1), \mu(x_1\alpha x_3)\} \\
&\leq \max\{\mu(x_2\alpha x_1\alpha x_3), \mu(x_2\alpha x_1\alpha x_3)\} \\
&= \mu(x_2\alpha x_1\alpha x_3) = \mu(x_1\alpha x_2\alpha x_3). \\
\text{Thus } \mu(x_1\alpha x_2\alpha x_3) &= \max\{\mu(x_1\alpha x_2), \mu(x_1\alpha x_3)\} \\
\text{Now } \mu(x_1\alpha x_2\alpha x_3) &= \max\{\mu(x_1), \mu(x_2\alpha x_3)\} \\
&= max\{\mu(x_3\alpha x_1), \mu(x_2\alpha x_3)\} \\
&= max\{\mu(x_1\alpha x_3), \mu(x_2\alpha x_3)\} \\
&\leq \max\{\mu(x_2\alpha x_1\alpha x_3), \mu(x_1\alpha x_2\alpha x_3)\} \\
&= \max\{\mu(x_1\alpha x_2\alpha x_3), \mu(x_1\alpha x_2\alpha x_3)\} \\
&= \max\{\mu(x_1\alpha x_2\alpha x_3)\}. \\
\text{Therefore } \mu(x_1\alpha x_2\alpha x_3) &= \max\{\mu(x_3\alpha x_1), \mu(x_2\alpha x_3)\} \\
&= \max\{\mu(x_3\alpha x_1), \mu(x_3\alpha x_2)\}.
\end{aligned}$$

Hence μ is a fuzzy $3-$weakly completely prime ideal of M. □

Remark 4.4. If a $\Gamma-$semiring M contains the multiplicative identity then the notions of fuzzy weakly completely prime ideal and fuzzy $3-$weakly completely prime ideal coincide. In general, the converse of the proposition 4.3 is not true . This follows from the following example.

Example 4.5. Let $M = 2\mathbb{Z}_0^+, \Gamma = \mathbb{Z}^+$. We define a fuzzy subset μ of M as follows. $\mu(0) = 1, \mu(2) = 0.3$ and $\mu(2n) = 0.5$ for $n \geq 2$. By Example 4.2, μ is a fuzzy $3-$weakly completely prime ideal of M. But μ is not a fuzzy weakly completely prime ideal of M, since $\mu(2\alpha 2) \neq max\{\mu(2), \mu(2)\}$.

Theorem 4.6. *Let M be a commutative $\Gamma-$ semiring and μ be a fuzzy ideal of M. Then μ is a fuzzy $3-$weakly completely prime ideal of M if and only if any extension of μ by x, where $x \in M$, is a fuzzy weakly completely prime ideal of M.*

Proof. Suppose μ is a fuzzy $3-$weakly completely prime ideal of $\Gamma-$semiring M and $x \in M, \alpha \in \Gamma$. Then

$$\begin{aligned} < x, \mu > (x_1 \alpha x_2) &= \mu(x \alpha x_1 \alpha x_2). \\ &= max\{\mu(x\alpha x_1), \mu(x \alpha x_2)\} \\ &= max\{< x, \mu > (x_1), < x, \mu > (x_2)\}, \text{ for all } x_1, x_2 \in M, \alpha \in M. \end{aligned}$$

Thus $< x, \mu >$ is a fuzzy weakly completely prime fuzzy ideal of M, for every $x \in M$.

Conversely suppose $< x, \mu >$ is a fuzzy weakly completely prime ideal of M, for every $x \in M$. Then

$$\begin{aligned} \mu(x_1 \alpha x_2 \alpha x_3) &=< x, \mu > (x_2 \alpha x_3) \\ &= \max\{< x_1, \mu > (x_2), < x_1, \mu > (x_3)\} \\ &= \max\{\mu(x_1 \alpha x_2), \mu(x_1 \alpha x_3)\}. \end{aligned}$$

and since μ is commutative and by Definition 4.1, μ is a fuzzy $3-$weakly completely prime ideal of M. □

Corollary 4.7. Let M be a commutative $\Gamma-$semiring with unity 1 and μ be a fuzzy $3-$weakly completely prime ideal of M. Then μ is a fuzzy weakly completely prime ideal of M.

Proof. Now $< 1, \mu > (x) = \mu(1\alpha x) = \mu(x)$, for all $x \in M, \alpha \in \Gamma$.
Then $< 1, \mu >= \mu$.
Since μ is a fuzzy $3-$weakly completely prime ideal of M and by Theorem 4.6, $< 1, \mu >$ is a fuzzy weakly completely prime ideal of M. Then

$$\begin{aligned} < 1, \mu > (x \alpha y) &= \max\{< 1, \mu > (x), < 1, \mu > (y)\} \\ \mu(x \alpha y) &= \max\{\mu(x), \mu(y)\}, \text{ for all } x, y \in M, \alpha \in \Gamma. \end{aligned}$$

Hence μ is a fuzzy weakly completely prime ideal of M.

□

The proof of the following corollary follows from Theorems 3.31 and 4.6.

Corollary 4.8. Let M be a commutative $\Gamma-$semiring and μ be a fuzzy ideal of M. If μ is a fuzzy $3-$weakly completely prime ideal and fuzzy semi prime ideal of M then μ is the infimum of all fuzzy weakly completely prime ideal of M containing μ.

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