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Performance of multiple imputation methods for incomplete normally distributed longitudinal data

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Abstract

Missing data can often occur in longitudinal data. Multiple imputation has been very popular for longitudinal analysis in recent years. In this study, we aim to determine which multiple imputation method is superior for longitudinal normally distributed data when we use linear mixed models. In the literature, multiple imputation by chained equations (MICE) is one of the most popular methods for longitudinal data when using linear mixed models. In MICE, there are three parametric multiple imputation methods, and we compare the three methods with maximum likelihood multiple imputation. Maximum likelihood multiple imputation estimates the parameters using maximum likelihood. After the literature review, we determined that this study will be the first to compare these two approaches for longitudinal normally distributed data when we use linear mixed models. After the simulation study, maximum likelihood multiple imputation has less biased results than the three methods in MICE in terms of mean square error (MSE). The three imputation methods in MICE give closer results to each other. Therefore, we find that maximum likelihood multiple imputation is superior to MICE for longitudinal normally distributed data when we use linear mixed models. Moreover, we show that maximum likelihood multiple imputation can be used for longitudinal normally distributed data with linear mixed models.

Keywords: Longitudinal data, missing completely at random, missing at random, multiple imputation, linear mixed models

Öz

Boylamsal verilerde sıklıkla eksik veri oluşabilir. Çoklu atama, son yıllarda boylamsal veri analizi için oldukça popüler hale gelmiştir. Bu çalışmada, doğrusal karma modeller kullanıldığında boylamsal normal dağılımlı veriler için hangi çoklu atama yönteminin daha üstün olduğu belirlenmeye çalışılmaktadır. Literatürde, doğrusal karma modeller kullanıldığında boylamsal veriler için en popüler yöntemlerden biri zincirleme denklemlerle çoklu atama (MICE) yöntemidir. MICE'de üç parametrik çoklu atama yöntemi vardır ve bu üç yöntem en çok olabilirlik çoklu atama yöntemiyle karşılaştırılmaktadır. En çok olabilirlik çoklu atama, parametreleri en çok olabilirlik kullanarak tahmin eder. Literatür taramasının ardından, bu çalışmanın doğrusal karma modeller kullanıldığında boylamsal normal dağılımlı veriler için bu iki yaklaşımın karşılaştırıldığı ilk çalışma olduğu belirlenmiştir. Simülasyon çalışmasının ardından en çok olabilirlik çoklu atama yönteminin MICE'deki üç yöntemden ortalama kare hata bakımından daha az yanlı sonuçlar verdiği görülmüştür. MICE'deki üç yöntem birbirine yakın sonuçlar vermiştir. Bu nedenle, doğrusal karma modeller kullanıldığında boylamsal normal dağılımlı veriler için en çok olabilirlik çoklu atama yöntemi MICE'den daha üstün olduğu bulunmuştur. Ayrıca, en çok olabilirlik çoklu atama yönteminin, doğrusal karma modellerle birlikte boylamsal normal dağılımlı veriler için kullanılabileceği gösterilmiştir.

Anahtar sözcükler: *Boylamsal veri, tamamen rastgele eksik, rastgele eksik, çoklu atama, doğrusal karma modeller*

1. Introduction

In longitudinal data, researchers look at the influence of treatment on the disease process over time. However, there can be missing observations for several reasons in longitudinal data (e.g., people do not respond to specific questions in a survey, or the individual dies and drops out from the survey). In the literature, there are three missing data mechanisms: missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR) (Rubin, 1976). If the missing data are related to observed data, it is MAR. If the missing data are related to observed and missing observations, it is MNAR (van Buuren, 2012). If the probability of a missing value depends neither on observed data nor on unobserved data, it is MCAR (van Ginkel et al., 2020). In longitudinal data, MCAR and MAR are widely used. The listwise deletion method is the default method to solve missing data problem. However, omitting data gives rise to biased results. So, two main imputation approaches have been introduced in the literature: single imputation and multiple imputation (Jahangiri et al., 2023). Single imputation methods consist of nonparametric and parametric methods. Parametric imputation methods are mean imputation, regression imputation, and stochastic regression imputation. Nonparametric imputation methods are hot deck imputation and k-nearest neighbor imputation. In the literature, multiple imputation methods are superior to single imputation methods and listwise deletion. According to Allison (2002), multiple imputation and maximum likelihood are the main approaches to longitudinal data. In recent years, multiple imputation has been the most popular method for incomplete longitudinal data. In multiple imputation, maximum likelihood multiple imputation (MLMI) and multiple imputation by chained equations (MICE) are the two basic approaches. MLMI imputes from a multivariate normal model with an unstructured covariance matrix, and all imputed datasets are generated conditional on the maximum likelihood parameter of the model parameter (Bartlett, 2023; Schafer, 1997; von Hippel et al., 2021). Von Hippel et al. (2021) stated that MLMI is superior to the posterior draw multiple imputation, which is based on the filling of missing values with draws from the posterior predictive distribution. MICE is a type of posterior draw multiple imputation. In MICE, there are three parametric multiple imputation methods. MICE has become very popular in recent years because of its flexibility and robustness. For example, Zaninotto et al. (2017) compared full information maximum likelihood and MICE in their study. They determined that MICE is superior to full information maximum likelihood. Huque et al. (2018) determined that fully conditional specification-standard (MICE) performed well in longitudinal studies. Linear mixed model (LMM) is a statistical procedure for analyzing longitudinal data (Puspogenero et al., 2017). The superiority of LMM is proved against linear regression in longitudinal studies. Huque et al. (2020) showed that fully conditional specification-standard (MICE) is compatible with LMM in the context of longitudinal data if measurements occur at equal time intervals. However, we did not find a study related to the comparison of MLMI and MICE for longitudinal normally distributed data when we used linear mixed models. We aim to contribute to the literature with this study.

2. Material and Methods

2.1. Linear Mixed Models in Longitudinal Data

For dealing with longitudinal data, linear mixed effects models are usually used. These models explain the correlation of observations via random effects and estimate the effects of predictor variables on repeated outcomes (Blood et al., 2010).

Let $y_i = (y_{i1}, y_{i2}, \dots, y_{in})^T$ be the n_i-repeated measures of a continuous outcome for subject i ∈ (1,2,…,n), and $x_i = (x_{i1}, x_{i2}, \dots, x_{in_i})^T$ represents continuous covariate and $t_i = (t_{i1}, t_{i2}, \dots, t_{in_i})^T$ represents measurement times. Linear mixed models explain the association between measured outcome and covariates (Huque et al., 2020).

$$y_i \mid x_i , t_i = \beta_0 + \beta_1 x_i + \beta_2 t_i + b_{0i} + b_{1i} t_i + \varepsilon_i, \qquad i=1,2,\dots,n$$

$\beta = (\beta_0, \beta_1, \beta_2)$ is the fixed-effect vector, $b_i = (b_{0i}, b_{1i}) \sim N(0,G)$ shows the random vector (G is the variance components matrix) , and $\varepsilon_i = (\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{in_i}) \sim N(0, \Phi_i = \sigma_{\varepsilon_i}^2 I)$, where I is the identity matrix with n_i x n_i. Also, ε_i and b_i are independent of each other (Huque et al., 2020).

2.2. Multiple Imputation Methods for Longitudinal Normally Distributed Data

In recent years, for analyzing missing data, multiple imputation and full information maximum likelihood are popular approaches (Allison, 2002; Little & Rubin, 2002; Von Hippel, 2016). Maximum likelihood integrates the missing data out of the likelihood. Multiple imputation fills in each missing value via randomization of plausible imputations. The two methods are available in software. The estimation of maximum likelihood needs iterative maximization of the likelihood (Von Hippel, 2016).

In MLMI, values can be imputed conditionally on the maximum likelihood estimates. It is simple but not common and serves as a bridge to the new multiple imputation methods. The most popular multiple imputation methods are posterior draw multiple imputation, which fills in missing values with draws from the posterior predictive distribution (Anderson, 1957; Wang et al., 1998; Rubin, 1987; Von Hippel, 2016). Von Hippel et al. (2021) stated that MLMI is superior to the posterior draw multiple imputation approach. However, MLMI has not been compared with MICE by the researchers in longitudinal normally distributed data before.

MICE is also known as fully conditional specification (FCS) and has become popular in recent years because of its flexibility and unbiased estimates. MICE creates imputations per variable by specifying the univariate distribution of each incomplete variable conditional on all other variables. The MICE algorithm yields random draws from the posterior distribution, which is a Bayesian simulation approach. In the MICE algorithm, the joint distribution does not actually exist, and so MICE is different from joint modeling (Javadi et al., 2021).

In MICE, there are three parametric multiple imputation methods for longitudinal normally distributed data. MICE-Predictive Mean Matching (MICE-PMM) is an advantageous method because it keeps nonlinear relationships in the data. In PMM, firstly, with estimating the conditional distribution of the missing variable, the imputation process works. PMM uses a donor pool from the completely observed data. PMM imputes value by randomly drawing value from the donor pool. Small donor pools, such as 5, yield better results, and so we determined the number of donor pools as 5 (Schwerter et al., 2024; van Buuren, 2018; Kleinke, 2017). MICE using Bayesian linear regression (MICE-BAYES) is another parametric approach in MICE. MICE-BAYES imputes univariate missing data by Bayesian linear regression, also known as the normal model. It is equivalent to the Schafer's NORM method, which is proposed by Schafer in 1997. MICE-BAYES uses Cholesky decomposition, which is useful for efficient solutions in simulation studies (van Buuren, 2012; van Buuren et al., 2021). MICE using linear regression with bootstrap (MICE-BOOT) is the last parametric approach in MICE. Imputation under the normal linear model with the bootstrap algorithm computes univariate imputations by randomly drawing a sample from the part of the data (van Buuren, 2012; Heitjan et al., 1991). In MICE-BOOT, firstly, individual conditional distributions are specified for each variable. Then, all regression models are fitted iteratively, and values are imputed as random draws from the distribution of the regression predictions. This process is repeated k times and can be viewed as a Gibbs sampler (Schomaker et al., 2018).

3. Simulation Study

The simulation study consists of four steps. Firstly, different scenarios are constructed from a multivariate normal distribution with a zero mean vector and variance-covariance matrix. We specified 3 correlated variables (3 time points). We assume an exchangeable correlation structure or compound symmetry (equal time points).

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}, \Sigma = \begin{pmatrix} {\sigma_1}^2 & \sigma_1\sigma_2\rho_{12} & \sigma_1\sigma_3\rho_{13} \\ \sigma_1\sigma_2\rho_{12} & {\sigma_2}^2 & \sigma_2\sigma_3\rho_{23} \\ \sigma_1\sigma_3\rho_{13} & \sigma_2\sigma_3\rho_{23} & {\sigma_3}^2 \end{pmatrix}$$

The population variances were set to 1. Three different correlations (0.9, 0.6, and 0.3) were included. We determined n = 100 for a small sample size and n = 500 for a large sample size. Before generating missing values, we performed linear mixed models analysis on the complete dataset. For comparing mean responses at different time points, one modeling strategy is to use indicator variables.

$$\mathrm{E}(Y_{ij}) = \beta_0 + \beta_1 X_{ij1} + \beta_2 X_{ij2}$$

In this study, the indicator variables are X_{ij1} and X_{ij2} (the second and the third variables), and the first variable is referent. Three fixed effect parameters are calculated (β_0, β_1, and β_2). β_0 is the intercept parameter.

The second step is deleting data under MCAR and MAR. Under MCAR, we deleted 10%, 20%, and 30% of the three variables. Under MAR, we deleted 10%, 20%, and 30% of the X_1 and X_2 variables. According to the MAR assumption, missingness in X_1 and X_2 is related to X_3 and is not related to X_1 and X_2.

R is the matrix that stores the location of the missing data in the variables of X. ψ includes the parameters of the missing data model. The data X = (X_1, X_2, X_3) and the three variables have the correlation ρ between them. X_1 and X_2 have missing values and X_3 is fully observed.

$$\Pr(R_{X_1,X_2}=0) = \psi_0 + \frac{e^{X_1,X_2}}{1+e^{X_1,X_2}}\psi_{X_1,X_2} + \frac{e^{X_3)}}{1+e^{X_3}}\psi_{X_3)}$$

Where $\psi = (\psi_0, \psi_{X_1,X_2}, \psi_{X_3})$. ψ_{MCAR}=(ρ, 0, 0) and ψ_{MAR}=(0, 0, 1)

$\Pr(R_{X_1,X_2}=0) = \rho$ for MCAR

$\mathrm{logit}\,(\Pr(R_{X_1,X_2}=0)) = X_3$, for MAR (van Buuren,2012).

In the third step, we applied the four imputation methods. In the fourth step, we performed linear mixed models analysis on the imputed dataset. For each scenario, we simulated 1000 data sets to evaluate the performance of the four methods. We used seven R (R Core Team, 2025) packages (mlmi, missMethods, tidyr, stats, lme4, mice, and MASS). Our evaluation criteria are the average bias and mean square error (MSE). With repetition size R, the average bias and MSE are defined as

$$\text{Average bias} = \frac{\sum_{i=1}^{R}\left(\beta_{comp(i)} - \beta_{imp(i)}\right)}{R}$$

$$\text{MSE} = \frac{\sum_{i=1}^{R}\left(\beta_{comp(i)} - \beta_{imp(i)}\right)^2}{R}$$

β_{comp} represents true parameter values of the complete data and β_{imp} represents parameter values of the imputed data. The average bias and MSE with smaller values show a smaller discrepancy between missing data and complete data analysis scenarios. Three fixed effect parameters are evaluated (β_0, β_1, and β_2).

Tables 1, 2, 3, 4, 5, and 6 show the average bias of the three parameters for the four methods under MCAR and MAR. The average bias may not give correct results for comparison of the four methods because some of the values are positive and some of the values are negative. Therefore, we do not compare the four methods in terms of average bias.

Tables 7, 8, 9, 10, 11, and 12 and Figures 1, 2, and 3 show the MSE values of the three parameters for the four methods under MCAR and MAR. Under MCAR, For β_0, MLMI yields less biased results than the three methods in MICE for small sample size. For β_1, MLMI yields less biased results than the three methods in MICE for small sample size. For β_2, MLMI yields less biased results than the three methods in MICE for small sample size. However, MLMI is not superior to the other methods in all the scenarios for large sample size. This result shows that MLMI is affected by the sample size. Under MAR, For β_0, MLMI yields less biased results than the three methods in MICE for small and large sample sizes. For β_1, MLMI yields less biased results than the three methods in MICE for small and large sample sizes. For β_2, MLMI yields less biased results than the three methods in MICE for small and large sample sizes. We can say that MLMI is more effective under MAR than MCAR. Moreover, the methods yielded less biased results for large sample size than small sample size. The methods have less biased results for strong correlation than weak correlation. As the percentage of missingness increases, MSE and bias increase. The results of the four methods are similar for the three parameters (β_0, β_1, β_2). MSE values of the three methods in MICE are close to each other. All in all, MLMI gives the closest results to the complete data for MCAR and MAR. This can be seen in Figures 1, 2, and 3.

4. Discussion and Conclusion

In this study, we aim at comparing the four multiple imputation methods for longitudinal normally distributed data when we use linear mixed models. In the literature, there is no study related to this issue. Therefore, we aim to contribute to the literature with this study. Firstly, we gave information about linear mixed models and multiple imputation methods. In the simulation study, we generated three correlated variables from multivariate normal distribution under MCAR and MAR assumptions. Multivariate normal distribution is an important assumption in longitudinal normally distributed data, and three correlated variables can be extended. Our evaluation criteria are the average bias and MSE. When calculating bias and MSE, we used a linear mixed models approach, which is a common method in longitudinal studies. According to the simulation results, MLMI has less biased results than MICE-PMM, MICE-BAYES, and MICE-BOOT under MAR. MLMI has less biased results than MICE-PMM, MICE-BAYES, and MICE-BOOT for small sample size under MCAR. For large sample size, we can say that no method is superior to the others under MCAR. MICE-BAYES and MICE-BOOT were not affected by missing data mechanisms. MICE-PMM yielded less biased results under MCAR than under MAR. This is compatible with the literature. For example, Morris et al. (2014) determined that MICE-PMM yielded larger degrees of bias under MAR than under MCAR.

Nooraee (2018) compared maximum likelihood with MICE-PMM in longitudinal questionnaire data. PMM was applied at both item and scale levels, and maximum likelihood is inferior to the PMM. Shaw et al. (2022) showed that PMM is superior to the regression imputation and complete case analysis for incomplete longitudinal data. However, we found that MLMI is superior to the parametric MICE methods for longitudinal normally distributed data when we use linear mixed models. There are limitations to the study. This study is limited to longitudinal normally distributed continuous data. Non-normal and categorical data are out of the scope. Two different sample sizes (100 and 500) did not change the simulation results for MAR. For longitudinal data, especially a sample size of 500, it is enough for obtaining true simulation results. The repetition size of 1000 was a reasonable iteration size to compare the four methods in the simulation study for longitudinal data. In this study, the evaluation criterion is MSE. According to Chiu et al. (2022), MSE is one of the most widely used metrics in the literature.

Conflicts of Interest: The authors declare no conflict of interest.

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Appendix : Tables and Figures

Table 1. Average bias of β_0 for the five methods under MCAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	0.000539	0.000417	0.000417	0.000462
0.9	100	20	0.002676	-0.000125	-0.000377	0.000224
0.9	100	30	-0.013566	0.000146	-0.000473	-0.001994
0.6	100	10	0.000683	0.001163	-0.000141	0.000011
0.6	100	20	0.008287	0.004186	0.000348	0.002089
0.6	100	30	-0.018785	0.000661	-0.000203	-0.002706
0.3	100	10	0.002469	0.001449	0.001340	0.001516
0.3	100	20	0.011061	0.001795	0.000212	0.002096
0.3	100	30	-0.018518	0.003533	0.004034	0.000455
Large Sample Size						
0.9	500	10	-0.005800	0.000405	0.000377	0.000078
0.9	500	20	0.011136	0.000146	0.000002	0.000215
0.9	500	30	0.004832	0.000151	-0.000242	-0.000507
0.6	500	10	-0.010788	0.001080	0.000737	0.000256
0.6	500	20	0.023001	0.000045	-0.000117	-0.000040
0.6	500	30	0.007459	0.000464	0.000369	-0.000179
0.3	500	10	-0.013739	0.000940	0.000668	-0.000339
0.3	500	20	0.028777	-0.001078	-0.000391	-0.000454
0.3	500	30	0.008844	0.001035	0.008487	0.000213

Note=PM: Percentage of missingness

Table 2. Average bias of β_1 for the five methods under MCAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	0.004640	-0.000266	-0.000653	-0.000129
0.9	100	20	-0.000014	0.000238	0.001549	0.000782
0.9	100	30	0.016972	0.000048	0.000550	0.004772
0.6	100	10	0.010462	0.000341	0.000528	0.001062
0.6	100	20	0.000225	-0.005747	0.002073	-0.000442
0.6	100	30	0.036836	0.000535	0.000505	0.009329
0.3	100	10	0.011429	-0.000125	-0.001509	-0.000411
0.3	100	20	0.000997	-0.001862	0.001981	-0.000511
0.3	100	30	0.042746	0.000426	-0.004524	0.003755
Large Sample Size						
0.9	500	10	0.002391	-0.000386	-0.000478	-0.000189
0.9	500	20	-0.014064	0.000420	-0.000147	0.000084
0.9	500	30	0.006367	0.000106	-0.000073	0.000188
0.6	500	10	0.004709	-0.000778	-0.000847	-0.000405
0.6	500	20	-0.026264	-0.000165	0.000213	0.000732
0.6	500	30	0.009863	-0.001427	-0.000789	-0.000501
0.3	500	10	0.006463	-0.000513	-0.000610	0.000436
0.3	500	20	-0.032244	0.000997	0.000182	0.001249
0.3	500	30	0.009225	-0.001314	-0.001305	-0.000936

Note=PM: Percentage of missingness

Table 3. Average bias of β_2 for the five methods under MCAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	-0.009466	0.000044	-0.001056	-0.000652
0.9	100	20	-0.001931	-0.001544	0.000932	0.000235
0.9	100	30	0.015846	-0.000481	-0.000409	0.002513
0.6	100	10	-0.016986	-0.002140	-0.001009	0.000127
0.6	100	20	-0.000466	-0.004523	0.001383	0.000241
0.6	100	30	0.033349	-0.000655	-0.000408	0.002756
0.3	100	10	-0.022735	-0.001555	-0.003473	-0.002028
0.3	100	20	0.001576	-0.001595	0.001577	-0.000395
0.3	100	30	0.047772	-0.002848	-0.004297	0.002343
Large Sample Size						
0.9	500	10	0.008587	-0.000462	-0.000560	-0.000382
0.9	500	20	-0.016221	-0.000541	-0.000049	-0.000288
0.9	500	30	-0.008150	0.000026	0.000072	0.000858
0.6	500	10	0.017463	-0.001107	-0.000352	-0.000254
0.6	500	20	-0.031181	-0.000004	-0.000018	-0.000109
0.6	500	30	-0.016464	-0.000959	-0.000975	0.000285
0.3	500	10	0.023029	0.000204	0.000429	0.000870
0.3	500	20	-0.038783	0.000522	0.000326	0.001948
0.3	500	30	-0.019967	-0.001877	-0.000898	-0.001367

Note=PM: Percentage of missingness

Table 4. Average bias of β_0 for the five methods under MAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	-0.002367	0.009533	0.000509	0.000665
0.9	100	20	0.001191	0.019695	0.000624	-0.000116
0.9	100	30	0.006595	0.030943	0.000841	-0.001392
0.6	100	10	-0.005270	0.008227	0.001236	-0.000631
0.6	100	20	0.003963	0.015093	0.001247	0.001063
0.6	100	30	0.014271	0.021608	0.001702	0.002925
0.3	100	10	-0.006756	0.004583	-0.000798	-0.001501
0.3	100	20	0.004262	0.008163	0.000679	-0.001007
0.3	100	30	0.015769	0.014114	-0.001017	-0.003606
Large Sample Size						
0.9	500	10	0.001079	0.002940	0.000132	0.000173
0.9	500	20	-0.003980	0.005543	0.000401	-0.000128
0.9	500	30	0.003053	0.008657	0.000896	0.000316
0.6	500	10	0.001634	0.001325	0.000321	-0.000476
0.6	500	20	-0.006821	0.003938	0.000511	0.001305
0.6	500	30	0.006367	0.005626	0.000825	0.000652
0.3	500	10	0.001874	-0.000629	-0.000211	-0.000294
0.3	500	20	-0.009255	0.001774	-0.001702	0.000096
0.3	500	30	0.003942	0.001081	-0.002773	-0.000611

Note=PM: Percentage of missingness

Table 5. Average bias of β_1 for the five methods under MAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	-0.000662	-0.000666	-0.001313	-0.000066
0.9	100	20	0.002510	-0.000694	0.000854	0.000415
0.9	100	30	0.004093	0.001053	-0.000240	0.000158
0.6	100	10	0.000375	-0.002183	-0.001061	0.000938
0.6	100	20	0.000464	-0.001545	-0.003069	-0.002590
0.6	100	30	0.009073	0.002494	-0.000508	0.003040
0.3	100	10	0.001287	-0.001769	-0.000465	-0.001028
0.3	100	20	0.005140	0.001463	-0.000387	-0.000997
0.3	100	30	0.005258	-0.005436	0.001362	0.003270
Large Sample Size						
0.9	500	10	0.000841	0.000305	0.000286	0.000123
0.9	500	20	-0.001091	-0.000417	0.000026	0.000550
0.9	500	30	0.000623	-0.000001	-0.001211	-0.001453
0.6	500	10	0.000928	-0.000481	0.000206	-0.000056
0.6	500	20	-0.001916	-0.001102	0.000907	-0.000313
0.6	500	30	0.000979	0.000229	-0.001968	-0.000950
0.3	500	10	0.001463	0.000972	-0.000051	-0.000224
0.3	500	20	-0.001134	-0.000928	0.001087	-0.000482
0.3	500	30	0.007415	0.004356	0.007664	0.002562

Note=PM: Percentage of missingness

Table 6. Average bias of β_2 for the five methods under MAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	0.002367	-0.009533	-0.000509	-0.000665
0.9	100	20	-0.001191	-0.019695	-0.000624	0.000116
0.9	100	30	-0.006595	-0.030943	-0.000841	0.001392
0.6	100	10	0.005270	-0.008227	-0.001236	0.000631
0.6	100	20	-0.003963	-0.015093	-0.001247	-0.001063
0.6	100	30	-0.014271	-0.021608	-0.001702	-0.002925
0.3	100	10	0.006756	-0.004583	0.000798	0.001501
0.3	100	20	-0.004262	-0.008163	-0.000679	0.001007
0.3	100	30	-0.015769	-0.014114	0.001017	0.003606
Large Sample Size						
0.9	500	10	-0.001079	-0.002940	-0.000132	-0.000173
0.9	500	20	0.003980	-0.005543	-0.000401	0.000128
0.9	500	30	-0.003053	-0.008657	-0.000896	-0.000316
0.6	500	10	-0.001634	-0.001325	-0.000321	0.000476
0.6	500	20	0.006821	-0.003938	-0.000511	-0.001305
0.6	500	30	-0.006367	-0.005626	-0.000825	-0.000652
0.3	500	10	-0.001874	0.000629	0.000211	0.000294
0.3	500	20	0.009255	-0.001774	0.001702	-0.000096
0.3	500	30	-0.003942	-0.001081	0.002773	0.000611

Note=PM: Percentage of missingness

Table 7. Mean square error values of β_0 for the five methods under MCAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	**0.000204**	0.000395	0.000391	0.000366
0.9	100	20	**0.000590**	0.000916	0.000989	0.000933
0.9	100	30	**0.001354**	0.002062	0.001874	0.001953
0.6	100	10	**0.000744**	0.001353	0.001352	0.001404
0.6	100	20	**0.002119**	0.002990	0.003107	0.003022
0.6	100	30	**0.003407**	0.005323	0.005191	0.005631
0.3	100	10	**0.001182**	0.001824	0.002069	0.001980
0.3	100	20	**0.002904**	0.004252	0.004365	0.004205
0.3	100	30	**0.004656**	0.007779	0.007680	0.007826
Large Sample Size						
0.9	500	10	0.000074	0.000066	**0.000071**	**0.000071**
0.9	500	20	0.000227	0.000187	0.000191	**0.000186**
0.9	500	30	**0.000252**	0.000369	0.000382	0.000413
0.6	500	10	0.000254	0.000246	0.000248	**0.000241**
0.6	500	20	0.000847	0.000597	0.000611	**0.000540**
0.6	500	30	**0.000683**	0.001068	0.001105	0.001098
0.3	500	10	0.000408	0.000390	0.000388	**0.000384**
0.3	500	20	0.001337	0.000925	0.000944	**0.000889**
0.3	500	30	**0.000902**	0.001472	0.001556	0.001520

Note=PM: Percentage of missingness

Table 8. Mean square error values of β_1 for the five methods under MCAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	**0.000449**	0.000770	0.000720	0.000714
0.9	100	20	**0.001095**	0.001783	0.001637	0.001739
0.9	100	30	**0.002116**	0.003257	0.003086	0.002920
0.6	100	10	**0.001770**	0.002664	0.002605	0.002728
0.6	100	20	**0.004088**	0.005807	0.006020	0.005930
0.6	100	30	**0.007546**	0.001054	0.001084	0.001067
0.3	100	10	**0.002471**	0.003732	0.003997	0.003711
0.3	100	20	**0.006154**	0.008890	0.009013	0.008656
0.3	100	30	**0.010882**	0.015297	0.015961	0.015432
Large Sample Size						
0.9	500	10	**0.000083**	0.000129	0.000135	0.000136
0.9	500	20	0.000381	**0.000317**	0.000333	0.000328
0.9	500	30	**0.000375**	0.000605	0.000604	0.000604
0.6	500	10	**0.000308**	0.000486	0.000501	0.000487
0.6	500	20	0.001374	0.001168	0.001252	**0.001109**
0.6	500	30	**0.001239**	0.001935	0.002069	0.002028
0.3	500	10	**0.000498**	0.000789	0.000781	0.000765
0.3	500	20	0.002117	0.001843	0.001881	**0.001822**
0.3	500	30	**0.001810**	0.002958	0.003040	0.003020

Note=PM: Percentage of missingnes

Table 9. Mean square error values of β_2 for the five methods under MCAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	**0.000452**	0.000743	0.000712	0.000698
0.9	100	20	**0.000983**	0.001718	0.001635	0.001679
0.9	100	30	**0.001998**	0.003328	0.003267	0.003022
0.6	100	10	**0.001616**	0.002485	0.002598	0.002653
0.6	100	20	**0.003815**	0.005859	0.006198	0.005850
0.6	100	30	**0.007183**	0.010588	0.010976	0.011108
0.3	100	10	**0.002933**	0.003845	0.004216	0.004013
0.3	100	20	**0.006149**	0.008874	0.009668	0.008982
0.3	100	30	**0.010699**	0.015398	0.014877	0.015899
Large Sample Size						
0.9	500	10	0.000143	**0.000125**	0.000129	0.000142
0.9	500	20	0.000438	0.000320	0.000337	**0.000312**
0.9	500	30	**0.000404**	0.000624	0.000620	0.000635
0.6	500	10	0.000571	0.000516	**0.000480**	0.000510
0.6	500	20	0.001578	0.001156	0.001201	**0.001035**
0.6	500	30	**0.001419**	0.002036	0.002070	0.002189
0.3	500	10	0.000987	**0.000731**	0.000768	0.000753
0.3	500	20	0.002505	0.001898	0.001876	**0.001836**
0.3	500	30	**0.002139**	0.002868	0.003038	0.002969

Note=PM: Percentage of missingness

Table 10. Mean square error values of β_0 for the five methods under MAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	**0.000310**	0.000615	0.000479	0.000491
0.9	100	20	**0.000536**	0.001597	0.001199	0.001089
0.9	100	30	**0.001200**	0.003034	0.002189	0.002331
0.6	100	10	**0.001039**	0.001594	0.001628	0.001483
0.6	100	20	**0.002007**	0.004241	0.003820	0.003680
0.6	100	30	**0.004147**	0.007901	0.007764	0.007576
0.3	100	10	**0.001538**	0.002368	0.002321	0.002211
0.3	100	20	**0.002827**	0.005748	0.005676	0.005391
0.3	100	30	**0.005806**	0.009764	0.010976	0.010809
Large Sample Size						
0.9	500	10	**0.000064**	0.000105	0.000098	0.000102
0.9	500	20	**0.000153**	0.000264	0.000228	0.000202
0.9	500	30	**0.000252**	0.000500	0.000410	0.000415
0.6	500	10	**0.000220**	0.000316	0.000310	0.000311
0.6	500	20	**0.000478**	0.000745	0.000750	0.000773
0.6	500	30	**0.000810**	0.001386	0.001380	0.001382
0.3	500	10	**0.000318**	0.000444	0.000452	0.000446
0.3	500	20	**0.000717**	0.001106	0.001018	0.001016
0.3	500	30	**0.001124**	0.001894	0.001941	0.001946

Note=PM: Percentage of missingness

Table 11. Mean square error values of β_1 for the five methods under MAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	**0.000405**	0.000543	0.000529	0.000529
0.9	100	20	**0.000591**	0.001155	0.001243	0.001186
0.9	100	30	**0.001218**	0.002026	0.002421	0.002221
0.6	100	10	**0.001577**	0.002140	0.002158	0.002009
0.6	100	20	**0.002271**	0.004531	0.004620	0.004541
0.6	100	30	**0.004532**	0.008329	0.008920	0.008760
0.3	100	10	**0.002707**	0.003239	0.003450	0.003416
0.3	100	20	**0.004150**	0.008534	0.009211	0.007811
0.3	100	30	**0.007874**	0.014740	0.015646	0.015750
Large Sample Size						
0.9	500	10	**0.000083**	0.000102	0.000101	0.000109
0.9	500	20	**0.000166**	0.000246	0.000244	0.000234
0.9	500	30	**0.000292**	0.000465	0.000460	0.000459
0.6	500	10	**0.000351**	0.000381	0.000406	0.000405
0.6	500	20	**0.000673**	0.000936	0.000886	0.000889
0.6	500	30	**0.001153**	0.001661	0.001773	0.001660
0.3	500	10	**0.000587**	0.000668	0.000683	0.000687
0.3	500	20	**0.001197**	0.001575	0.001643	0.001661
0.3	500	30	**0.002177**	0.003174	0.003137	0.002954

Note=PM: Percentage of missingness

Table 12. Mean square error values of β_2 for the five methods under MAR.

Parameter			Methods			
Rho	n	PM				
Small Sample Size			MLMI	MICE-PMM	MICE-BAYES	MICE-BOOT
0.9	100	10	**0.000310**	0.000615	0.000479	0.000491
0.9	100	20	**0.000536**	0.001597	0.001199	0.001089
0.9	100	30	**0.001200**	0.003034	0.002189	0.002331
0.6	100	10	**0.001039**	0.001594	0.001628	0.001483
0.6	100	20	**0.002007**	0.004241	0.003820	0.003680
0.6	100	30	**0.004147**	0.007901	0.007764	0.007576
0.3	100	10	**0.001538**	0.002368	0.002321	0.002211
0.3	100	20	**0.002827**	0.005748	0.005676	0.005391
0.3	100	30	**0.005806**	0.009764	0.010976	0.010809
Large Sample Size						
0.9	500	10	**0.000064**	0.000105	0.000098	0.000102
0.9	500	20	**0.000153**	0.000264	0.000228	0.000202
0.9	500	30	**0.000252**	0.000500	0.000410	0.000415
0.6	500	10	**0.000220**	0.000316	0.000310	0.000311
0.6	500	20	**0.000478**	0.000745	0.000750	0.000773
0.6	500	30	**0.000810**	0.001386	0.001380	0.001382
0.3	500	10	**0.000318**	0.000444	0.000452	0.000446
0.3	500	20	**0.000717**	0.001106	0.001018	0.001016
0.3	500	30	**0.001124**	0.001894	0.001941	0.001946

Note=PM: Percentage of missingness

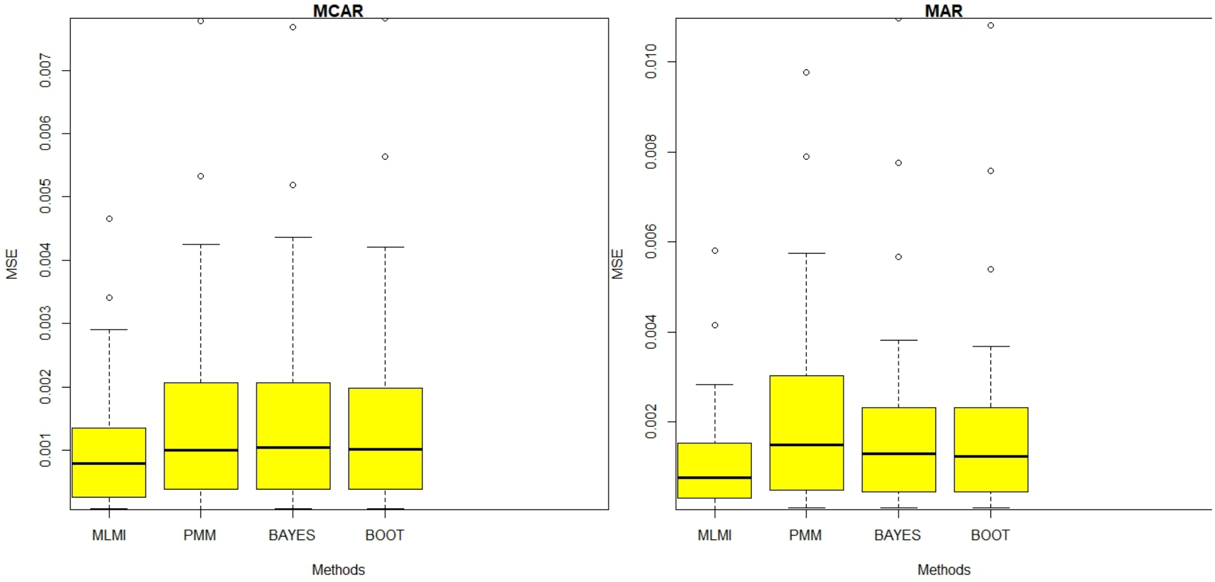

Figure 1. Boxplot of MSE for β_0 for the four methods under MCAR and MAR.

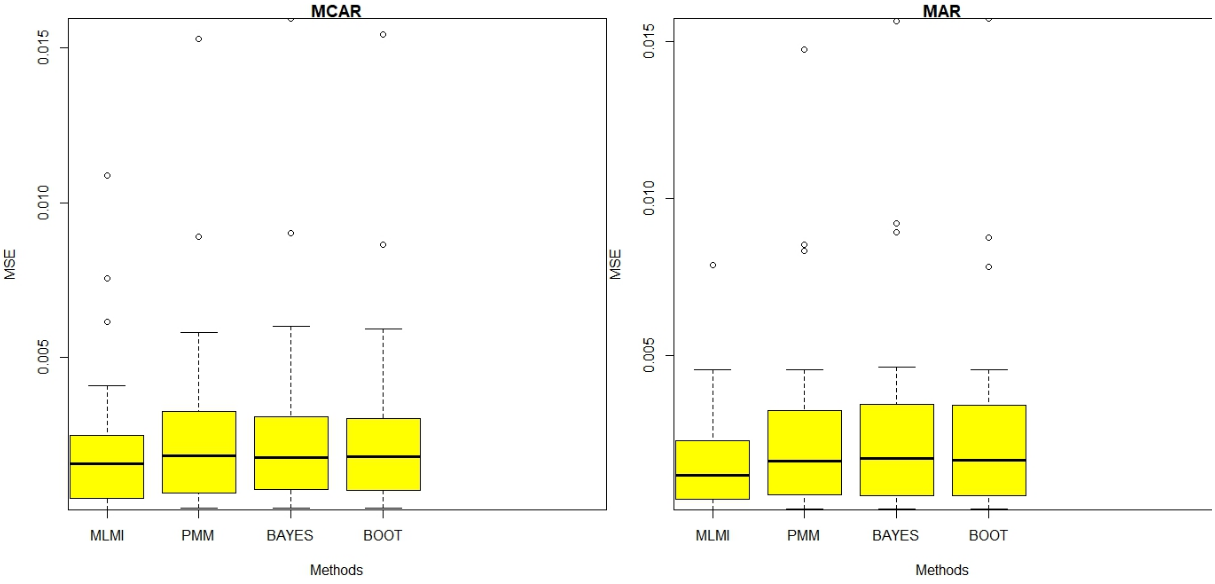

Figure 2. Boxplot of MSE for β_1 for the four methods under MCAR and MAR.

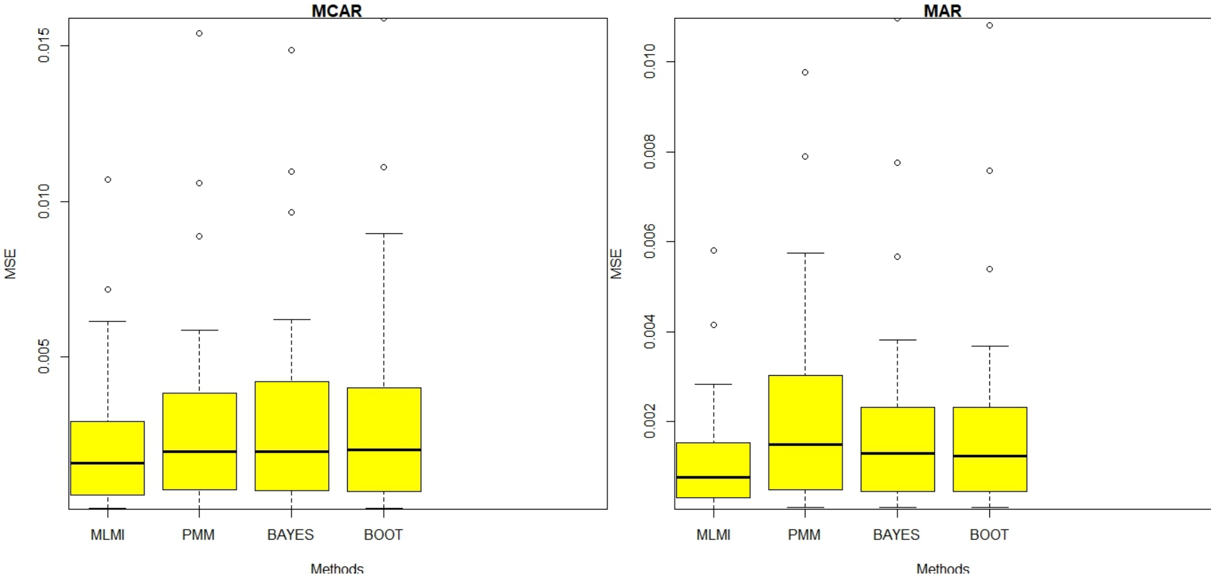

Figure 3. Boxplot of MSE for β_2 for the four methods under MCAR and MAR.