

# Time Cost–Quality Trade-Off Optimization for Subcontractor Selection Using a Golden Ratio-Inspired Oppositional Rao-1 Algorithm

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## ABSTRACT

**Context**—Construction projects are typically divided into work packages executed by subcontractors, while general contractors remain responsible for overall quality and timely completion. Selecting appropriate subcontractors at the early project stage is a critical decision, as it directly influences project duration, cost performance, and delivered quality. This decision is inherently complex due to the conflicting nature of time, cost, and quality objectives, which must be considered simultaneously. Although multi-objective optimization techniques have been increasingly applied in construction management, many existing approaches suffer from high computational effort, slow convergence, or limited solution diversity, particularly when realistic project features such as penalties, bonuses, and deadline constraints are incorporated.

**Objective**—The objective of this study is to develop an efficient multi-objective optimization framework for subcontractor selection that explicitly addresses the time–cost–quality trade-off. The study aims to improve convergence behavior and solution quality while reducing computational effort, and to provide practical decision support for identifying balanced project plans at the early planning stage.

**Method**—This study proposes a novel algorithm, termed the golden ratio-based oppositional Rao-1 (GROL-Rao-1), which integrates golden ratio–based opposition learning into the standard parameter-less Rao-1 algorithm to enhance search efficiency. The subcontractor selection problem is formulated as a discrete multi-objective optimization model considering project duration, total cost, and overall quality, along with realistic constraints such as deadlines, penalties, and bonuses. The proposed method is validated through a real-world case study involving 20 project activities and is compared with discrete particle swarm optimization, dynamic oppositional Rao-1, NSGA-II, and NDSII-Rao-1. Performance is evaluated using objective function evaluations, hypervolume, and spread indicators, supported by statistical significance testing. The crowded distance ranking method is also applied to prioritize Pareto-optimal solutions.

**Results**—The results show that GROL-Rao-1 outperforms the benchmark algorithms by producing higher-quality and better-distributed Pareto fronts while reducing computational effort by approximately 50%. Statistical analyses confirm the significance of these improvements, and the crowded distance ranking effectively identifies compromise solutions that balance time, cost, and quality objectives.

**Conclusion**—The proposed framework provides effective support for early-stage subcontractor selection and enhances resource allocation and planning robustness in construction projects. It contributes a computationally efficient and decision-oriented approach to multi-objective optimization in construction management and offers a foundation for future research on more complex and large-scale project applications.

**Key Words**—Construction management, Golden ratio-based oppositional learning, Multi-criteria decision-making analysis, Multi-objective Rao-1 algorithm, Time-cost-quality trade-off problem.



## I. INTRODUCTION

The construction industry is inherently complex, involving multiple interdependent stakeholders, among which subcontractors play a pivotal role in project execution. Subcontractor selection is therefore a critical decision for general contractors, as it directly influences key project outcomes such as time, cost, and quality [1], [2]. Inefficient subcontractor selection may lead to project delays, cost overruns, and quality deficiencies, ultimately undermining stakeholder relationships and overall project success [2] since general contractors are accountable for subcontractor performance, selecting an optimal combination of subcontractors for different work packages at the project outset represents a strategic and high impact decision [3]. This task is further complicated by the need to balance multiple, often conflicting objectives, rendering subcontractor selection a challenging multi-criteria optimization problem [4].

Subcontractor selection inherently involves trade-offs among competing project objectives. Traditional approaches have largely emphasized cost minimization; however, empirical evidence suggests that cost driven decisions often compromise construction quality and timely project delivery [5]. To address this limitation, the time-cost-quality (TCQ) trade-off problem extends the classical time-cost trade-off (TCT), framework explicitly incorporating quality as a third objective [6]-[9]. Integrating subcontractor selection within a TCQ optimization framework therefore provides a more comprehensive and realistic decision-making structure, aligning methodological rigor with practical project management requirements.

Despite its practical relevance, TCQ optimization remains computationally challenging due to its combinatorial nature, conflicting objectives, and numerous real-world constraints. Conventional optimization techniques often struggle to capture this complexity, particularly for large-scale construction projects. Consequently, researchers have adopted multi-objective evolutionary algorithms (MOEAs) and swarm intelligence methods, including NSGA-II, particle swarm optimization (PSO), Teaching learning-based optimization (TLBO), Slime Mould Algorithm (SMA) [10], and Aquila Optimizer (AO) [11]. While these approaches have demonstrated promising results, many still suffer from slow convergence, high computational costs, and sensitivity to control parameters [12], [13].

Rao algorithms have gained attention as metaphor-free, and parameter-light optimization techniques that exhibit competitive performance across wide range of engineering applications [14]. Their inherent simplicity and reduced dependence on algorithm specific parameters make them particularly attractive for TCQ optimization problems in construction management [15]. Nevertheless, like many heuristic approaches, Rao algorithms may still encounter challenges such as premature convergence and limited exploration capability, especially in complex multi-objective search space [16], [17]. To mitigate these issues, opposition-based learning (OBL) has been incorporated into metaheuristic frameworks to enhance population diversity and accelerate convergence towards high quality solutions [18].

A recent advancement in this domain is Golden Ratio-based opposition learning (GROL), which leverages mathematical proportions to generate more effective opposite solutions, thereby improving the balance between exploration and exploitation [19]. The integration of GROL into the Rao-1 algorithm gives rise to the Golden Ratio-Inspired Oppositional Rao-1 (GROL-Rao-1), a hybrid framework with strong potential to outperform established baselines in TCQ-driven subcontractor selection problems.

Despite the growing number of studies addressing the TCQ trade-off in construction projects, several limitations remain. Many existing metaheuristic approaches require multiple control parameters and careful tuning, which may reduce their

robustness when applied to complex decision problems such as subcontractor selection. Although Rao algorithms provide a parameter-light optimization framework, their search capability may still be limited when dealing with multi-objective combinatorial problems. In addition, while OBL strategies have been widely used to enhance population diversity in metaheuristics, the integration of GROL with Rao-based multi-objective optimization has not yet been explored for TCQ-driven subcontractor selection problems.

Accordingly, this study contributes to existing literature in three main ways. First, subcontractor selection is formulated as a multi-objective TCQ optimization problem, allowing project managers to evaluate time, cost, and quality simultaneously within a unified decision-making framework. Second, the GROL-Rao-1 algorithm is introduced as an enhanced parameter light Rao-1 optimizer that incorporates GROL in order to strength exploration capability, improve convergence behavior, and maintain solution diversity when solving TCQ optimization problems. Third, the proposed method is evaluated against several well-known algorithms, including DPSSO, DOLRao-1, NSGA-II, and other Rao variants. In addition to generating Pareto-optimal solutions, the crowded distance ranking (CDR) is employed as a decision-making support mechanism to rank non-dominated solutions and assisted practitioners in selected balanced compromise alternatives [20], [21]. Collectively, these contributions address existing gaps in methodological development and the integration of decision-support tools within subcontractor selection research.

The remains of this paper are organized as follows. Section II reviews literature on TCQ optimization and subcontractor selection. Section III presents the proposed GROL-Rao-1 algorithm and the mathematical formulation of the TCQ model. Section IV describes experimental setup and case study. Section V discusses the results, and Section IV concludes the study.

### A. Literature Review

Time-cost trade-off problem (TCTP) constitutes a foundational research area in construction management optimization. Early contributions by Kelley and Walker [22] introduced the Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT), which enabled systematic scheduling and project duration control. Subsequent studies used deterministic optimization techniques, including linear programming [23] and dynamic programming [24] to minimize project costs while adhering to deadlines. While these approaches provided valuable theoretical insights, they relied on simplifying assumptions such as linear cost functions and deterministic activity duration, limiting their applicability to real-world construction projects. With increasing project complexity and advancements in computational intelligence, heuristic and metaheuristic approaches emerged as effective alternative for solving TCTP. Genetic Algorithm (GA) was among the earliest metaheuristic applied in construction scheduling [25], followed by simulated annealing (SA) [26], [27], ant colony optimization (ACO) [28], teaching-learning-based optimization [29], [19], PSO [30] and more recently the arithmetic optimization algorithm [31], [32]. These methods demonstrated superior capabilities in exploring large and nonlinear solution spaces and generating near-optimal schedules within reasonable computational times. Zhang and Li [6] applied differential evolution to nonlinear TCTP, while Damghani et al. [33] utilized hybrid heuristics for multi-mode project scheduling. The non-dominated sorting genetic algorithm II (NSGA-II) [12] subsequently became a widely adapted framework for multi-objective TCTP, owing to its efficiency in approximation Pareto front.

As the limitations of dual-objective time-cost frameworks became evident, researchers began to incorporate quality as an additional objective, leading to the development of time-cost-quality (TCQ) optimization models. Babu and Suresh [34]

introduced an early linear programming formulation for TCQ trade-offs, assuming linear relationships among objectives. This framework was later validated by Khang and Myint [35] through application to a cement factory project. Pollack-Johnson and Liberatore [36] extended TCQ modeling using mixed-integer programming, while Tareghian and Taheri [37] proposed electromagnetic scatter search method for discrete TCQ problems. El-Rayes and Kandil [38] employed genetic algorithms to generate TCQ Pareto fronts in highway construction projects, and Afshar et al. [8] utilized a multi-colony ant algorithm for multi-objective TCQ optimization.

The role of quality in construction optimization gained further attention with the incorporation of fuzzy logic and expert judgment. Paryzad and Eshgh [39] were among pioneers in integrating quality into construction scheduling through fuzzy modeling, enabling the representation of subjective and uncertain quality assessment. Early studies by Lam et al. [40] and Polat et al., [2] demonstrated that inadequate consideration of quality in subcontractor selection adversely affects overall project performance. With the formulization of TCQ models, multi-objective algorithms, such as NSGA-II, MAWA, SPEA2, and multi-objective particle swarm optimization (MOPSO) were increasingly adapted to balance time, cost and quality objectives. Fallah-Mehdipour et al. [41] compared NSGA-II and MOPSO, for TCQ optimization and reported superior efficiency of NSGA-II in generating well distributed pareto solutions.

More recent studies have emphasized scalability and real-world applicability of TCQ optimization models. Aminbakhsh and Abdulsattar [42] proposed a modified multi-objective genetic algorithm (MOGA) capable of handling generalized precedence relationships in multi-mode projects, validating its performance on large-scale construction problems involving more than 400 activities. This development highlights the growing interest in addressing practical constraints and computational challenges associated with TCQ optimization complex construction environment. In parallel, researchers have explored hybrid frameworks that integrate optimization algorithms with decision-making methodologies.

Sharma and Trivedi [43] combined NSGA-II with the Analytic Hierarchy Process (AHP) to incorporate expert preferences into TCQ optimization. Luong et al. [44] introduced an opposition-based multiple objective differential evolution (OMODE) approach for multi-mode TCQ problems, benchmarking it against MOPSO, MODE, and NSGA-II. Nguyen et al. [45] developed a parameter-free multi-objective whale optimization (MOWO) for TCQ in non-unit repetitive construction projects, while Son and Khoi [46] used the slime mold algorithm (SMA) to generate TCQ pareto fronts. Recent advances have also focused on parameter-light and metaphor-free algorithms. Yilmaz and Dede [47] proposed strength Pareto-based Rao algorithms (SP2-Rao-1 and SP2-Rao-2) for multi-objective TCQ optimization, demonstrating improving convergence behavior and reduced computational costs compared with OMODE and MOABCDE. Through these studies, quality is commonly quantified using weight indicator at the activity level and aggregated into percentage-based measures derived from expert judgment, reflecting both technical performance and sustainability considerations.

Table 1 presents a summary of representative studies in TCQ optimization and subcontractor selection, highlighting their key characteristics, limitations, and the research gaps addressed in this study.

Despite this advancement several research gaps remain. Many TCQ studies continue to rely on classical metaheuristics such as GA, PSO, and NSGA-II, which may suffer from premature convergence and scalability limitations, particularly subcontractor selection problems involving large decision spaces. Achieving an effective balance between exploration and

exploitation in multimodal objective spaces, remains unresolved. Although opposition-based learning (OBL), Rao algorithms, and golden ratio-inspired strategies have shown promise in enhancing global search efficiency [14], their application to TCQ-based subcontractor selection is still limited. Moreover, few studies integrate advanced optimization techniques with decision-support mechanisms like CDR, to assist practitioners in selecting appropriate compromise solutions from the Pareto set.

To sum up, the literature has evolved from simple time-cost optimization models to more comprehensive time-cost-quality frameworks. However, the application of modern metaheuristics that combine golden ratio-based strategies with opposition-based learning for subcontractor selection remains insufficiently explored. Addressing this gap is essential for developing robust, scalable and practically applicable optimization models capable of supporting sustainable decision-making in construction projects.

## B. Mathematical formulation for the TCQP problems

The primary aim of TCTP is to achieve an optimal trade-off between project duration and cost while adhering to defined constraints. This process generally requires an examination of the critical path, the longest chain of interdependent activities, and the identification of tasks that can be shortened or crashed to reduce the overall project timeline. Equation (1) represents the first objective of TCTP.

$$D_p = \min(\max_{j=1,\dots,m} (T_{S,j} + D_j)) = \min(\max_{j=1,\dots,m} (T_{F,j})) \quad (1)$$

Here,  $D_p$  denotes the project completion time,  $m$  refers to the total number of project activities, while  $T_{S,j}$ ,  $T_{F,j}$  and  $D_j$  correspond to the start time, finish time, and duration of the  $j$ th activity, respectively. The second objective of the TCTP is expressed in Equation (2).

$$C_p = \min(C_D + C_I) = \min(\sum_{j=1}^m c_{d,j} + D_p c_i) \quad (2)$$

In this context,  $C_p$  denotes the overall project cost, where  $C_D$  and  $C_I$  represent the total direct and indirect costs, respectively;  $c_{d,j}$  corresponds to the direct cost associated with the  $j$ th activity, and  $c_i$  indicates the indirect cost incurred per day.

Since the project quality ( $PQ$ ) is an important objective of construction project, measuring the  $PQ$  quantitatively is very challenging task. In this paper,  $PQ$  is measured in terms of project quality index ( $PQI$ ), which is estimated by taking the weighted sum of each activity quality index ( $AQI$ ). On the other hand,  $AQI$  depends upon three quality indicators i.e., labor, material, and equipment. Hence,  $AQI$  can be measured by taking the weighted sum of quality indicators in an activity and the relative weight of activities in project.

The third objective concerns project quality ( $PQ$ ). As  $PQ$  is a critical aspect of construction projects, its quantitative assessment remains a challenging task. In this study,  $PQ$  is evaluated using a Project Quality Index ( $PQI$ ), derived from the weighted sum of the Activity Quality Index ( $AQI$ ) values. Each  $AQI$ , in turn, is determined based on three quality indicators, labor, material, and equipment. Consequently,  $AQI$  is calculated by combining the weighted contributions of these indicators within an activity along with the relative weight of that activity in the overall project.

$$AQI = wt_A \sum_{k=1}^K wt_{A,k} Q_{A,k}^m \quad (3)$$

$$PQI = \sum_{A=1}^n wt_A \sum_{k=1}^K wt_{A,k} Q_{A,k}^m \quad (4)$$

where  $Q_{A,k}^m$  denotes the performance of quality indicator  $k$  in activity ( $A$ ) based on resource utilization  $m$ ;  $wt_{A,k}$  is the weight of indicator  $k$  relative to other indicators in the same activity; and  $wt_A$  is the weight of activity ( $A$ ) relative to other project activities.

Table 1. Comparative analysis of TCQ optimization studies.

| Ref.         | Algorithm                       | Objectives                              | Key features                                | Limitation / Gap  | Contribution of this study  |
|--------------|---------------------------------|---|---|---|---|
| [22]         | CPM                             | Time                                    | Deterministic scheduling                    | Single objective.   | Extends to multi-objective TCQ optimization   |
| [23]         | Linear Programming              | Time, Cost                              | Efficient for linear problems               | Simplified assumptions, no quality  | Incorporates quality and nonlinear decision space   |
| [24]         | Dynamic Programming             | Time, Cost                              | Optimal for small problems                  | Computationally expensive for large problems  | Uses metaheuristics for large-scale combinatorial problems  |
| [25]         | GA                              | Time, Cost                              | Early metaheuristic for TCTP                | Parameter tuning required   | Parameter-light optimization framework  |
| [26]         | SA                              | Time, Cost                              | Good exploration capability                 | Slow convergence, local optima risk   | Improved convergence via opposition learning  |
| [28]         | ACO                             | Time, Cost                              | Suitable for discrete problems              | Parameter sensitivity   | Reduced parameter dependency  |
| [30]         | PSO                             | Time, Cost                              | Effective for large-scale problems          | Premature convergence, parameter tuning   | Improved exploration-exploitation balance   |
| [33]         | Hybrid Heuristics               | Time, Cost, Quality                     | Handles multi-mode problems                 | Complex structure   | Simpler and efficient optimization structure  |
| [34]         | Linear Programming              | Time, Cost, Quality                     | Early TCQ formulation                       | Assumes linear relationships  | Handles nonlinear and discrete decision space   |
| [38]         | GA                              | Time, Cost, Quality                     | Generates Pareto solutions                  | Parameter sensitivity   | Parameter-less optimization approach  |
| [40]         | Artificial Neural Network (ANN) | Subcontractor Prequalification          | Decision support model                      | Does not generate Pareto solutions  | Provides optimization-based decision-making with Pareto-optimal subcontractor combinations                            |
| [41]         | NSGA-II                         | Time, Cost, Quality                     | Pareto-based MOEA                           | Requires parameter tuning, diversity issues   | Improved Pareto diversity and convergence   |
| [43]         | NSGA-II + AHP                   | Time, Cost, Quality                     | Integrates decision-making                  | Depends on expert judgment  | Data-driven optimization without subjective bias  |
| [44]         | OMODE                           | Time, Cost, Quality                     | Opposition-based DE                         | Still parameter-dependent   | Parameter-light opposition-based Rao framework  |
| [2]          | GA                              | Subcontractor Selection (Cost, Quality) | Optimization-based selection                | Limited to selection focus, does not integrate full TCQ trade-off with scheduling         | Extends subcontractor selection to integrated TCQ optimization with scheduling constraints                            |
| [46]         | SMA                             | Time, Cost, Quality                     | Good global search Ability                  | May require tuning and slow convergence   | Faster convergence and reduced evaluations  |
| [47]         | SP2-Rao-1/Rao-2                 | Time, Cost, Quality                     | Parameter-less optimization                 | Limited exploration in complex spaces   | Improved exploration via golden ratio-based opposition  |
| (This Study) | GROL-Rao-1                      | Time, Cost, Quality                     | Hybrid Rao golden ratio opposition learning | Performance not yet validated on very large-scale projects or multiple benchmark datasets | Improved exploration-exploitation balance, enhanced Pareto diversity, and 50% reduction in computational effort (NFE) |

Typically, activities include multiple quality indicators such as labor, material, and equipment. The weights associated with labor, material, and equipment in the  $PQI$  were determined based on expert judgment and project-specific considerations.

## II. RAO OPTIMIZATION ALGORITHM

Rao optimization algorithms [14] form a class of metaheuristic techniques inspired by natural mechanisms. Variants such as Rao-1, Rao-2, and Rao-3 have been widely utilized for diverse optimization tasks because of their simplicity, adaptability, and effectiveness.

$$X'_{j,k,i} = X_{j,k,i} + r_{1,j,i}(X_{j,best,i} - X_{j,worst,i}) \quad (5)$$

where  $X_{j,best,i}$  and  $X_{j,worst,i}$  denote the best and worst candidates for the  $j$ th variable in the  $i$ th iteration, respectively, while  $X_{j,k,i}$  represents the updated value. The term  $r_{1,j,i}$  is a random number generated within the range  $[0, 1]$  for the  $j$ th variable in that iteration.

## III. OPPOSITION-BASED LEARNING (OBL)

OBL [18] is based on evaluating the opposite of a given assumption. By comparing a candidate solution with its opposite, the search process can be accelerated. This concept is especially useful when integrated into metaheuristic optimization

algorithms. An opposite point in  $D$ -dimensional space is shown in Fig. 1.

$$X_j^o = a_j + b_j - X_j, \quad j = 1, \dots, D \quad (6)$$

### A. GROL population initialization

The opposite population denotes the randomly generated initial population, whereas the GROL-based population is produced through the GROL mechanism, potentially offering better search guidance. The GROL initialization is defined as follows:

$$oP_{i,j}^{grol} = P_{i,j} + \phi r_1(r_2(a_j + b_j - P_{i,j}) - P_{i,j}) \quad (7)$$

To improve GROL's efficiency, boundary constraints are checked. In this process  $r_1$  and  $r_2$  are random values, while  $\phi$  is set to 1 during initialization and 0.618 in the generation-jumping phase.

Let  $X \in [a, b]$  be a real number in the search space. Its opposite number  $X^o$ , and the random opposite number is defined as  $X^{Ro} = rand \times X^o$ , with  $rand \in [0, 1]$  introducing asymmetry. The golden ratio-based opposite number  $X^{gro}$  is calculated using a golden ratio factor  $\phi > 0$ , enhancing the GROL approach.

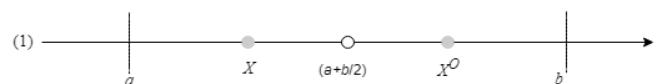


Figure 1. The original OBL pattern (own research).

$$X^{gro} = X + \phi^o \times rand \times (rand \times X^o - X) \quad (8)$$

Similarly, a multi-dimension dynamic opposite point can be defined as

$$X_j^{gro} = X_j + \phi \times rand \times (rand \times X_j^o - X_j), j = 1, \dots, D \quad (9)$$

**1. GROL-Rao-1 algorithm for TCTP optimization**

This section provides an overview of the oppositional Jaya algorithm. Rao-1 serves as the primary optimization framework, with GROL integrated to enhance the optimization process within the TCTP model. Figure 2 illustrates the flowchart of the GROL-Rao-1 algorithm.

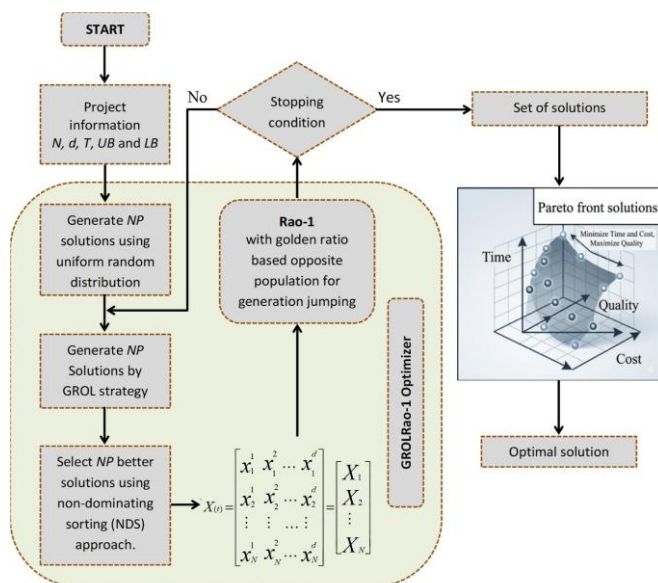
The parameter  $\phi$  used in the generation-jumping phase is set to the classical Golden Ratio ( $\approx 0.618$ ). This choice leverages the inherent balance properties of the Golden Ratio to enhance exploration and exploitation during the search process.

**IV. EMPIRICAL EXAMPLE of 20-ACTIVITY PROJECT**

This study evaluates the proposed optimization approach using a trade center project in Samandıra, Istanbul, completed in 2014. The project cost \$7.31 million, covered 2000 m<sup>2</sup>, and had a 500-day completion schedule under a turnkey contract. The project comprises 20 construction work packages, each executable by three alternative subcontractors offering different activity duration, cost, and quality performance levels. The subcontractor selection problem is formulated as a multi-objective time–cost–quality (TCQ) optimization to determine the optimal combination of subcontractors that minimizes duration and cost while maximizing quality. The decision space is extremely large due to the problem's combinatorial structure. With three subcontractor alternatives per activity, the total number of potential solutions reaches 3.4 billion alternatives, making exhaustive enumeration impractical. This complexity yields a challenging benchmark for evaluating metaheuristic optimization algorithms in construction planning. The model incorporates indirect project costs of \$230.42 per day, early completion incentives of \$345.62 per day, and delay penalties of \$460.83 per day. The 500-day contractual deadline affects overall cost through these mechanisms, and project quality is evaluated with equal importance weights for all work packages. The optimization framework was implemented in MATLAB, and the algorithm's performance was compared with the Discrete Particle Swarm Optimization (DPSO) method, using parameters recommended by Reyes-Sierra and Coello [48]. The subcontractor alternatives and their attributes are presented in Table 2. The case study provides a realistic environment for

**Table 2.** Subcontractor alternatives based on work packages.

| Act. | Pred.      | Option | T (days) | C (\$)   | Q (%) |
|------|------------|--------|----------|----------|-------|
| 1    |            | 1      | 90       | 1244608  | 74    |
|      |            | 2      | 95       | 1235346  | 70    |
|      |            | 3      | 100      | 1193548  | 68    |
| 2    | 1          | 1      | 60       | 170368.7 | 79    |
|      |            | 2      | 68       | 167050.7 | 77    |
|      |            | 3      | 72       | 163502.3 | 75    |
| 3    | 1          | 1      | 120      | 1506682  | 71    |
|      |            | 2      | 130      | 1470415  | 69    |
|      |            | 3      | 140      | 1405945  | 65    |
| 4    | 2, 3       | 1      | 30       | 85437.79 | 87    |
|      |            | 2      | 33       | 83041.47 | 86    |
|      |            | 3      | 35       | 80000    | 83    |
| 5    | 3          | 1      | 120      | 420368.7 | 90    |
|      |            | 2      | 125      | 410092.2 | 85    |
|      |            | 3      | 130      | 404331.8 | 80    |
| 6    | 3          | 1      | 90       | 255483.9 | 85    |
|      |            | 2      | 95       | 253824.9 | 83    |
|      |            | 3      | 100      | 253271.9 | 80    |
| 7    | 4          | 1      | 30       | 108940.1 | 72    |
|      |            | 2      | 33       | 107695.9 | 70    |
|      |            | 3      | 36       | 106866.4 | 69    |
| 8    | 4          | 1      | 120      | 584562.2 | 77    |
|      |            | 2      | 130      | 585345.6 | 85    |
|      |            | 3      | 135      | 603686.6 | 95    |
| 9    | 4          | 1      | 120      | 479032.3 | 82    |
|      |            | 2      | 125      | 502350.2 | 89    |
|      |            | 3      | 130      | 507235   | 93    |
| 10   | 7          | 1      | 20       | 123133.6 | 70    |
|      |            | 2      | 27       | 121152.1 | 75    |
|      |            | 3      | 32       | 119631.3 | 78    |
| 11   | 10         | 1      | 50       | 182258.1 | 88    |
|      |            | 2      | 55       | 179907.8 | 92    |
|      |            | 3      | 60       | 178894   | 94    |
| 12   | 10         | 1      | 20       | 80967.74 | 74    |
|      |            | 2      | 18       | 79354.84 | 68    |
|      |            | 3      | 15       | 77603.69 | 65    |
| 13   | 10         | 1      | 15       | 56682.03 | 71    |
|      |            | 2      | 14       | 55668.2  | 72    |
|      |            | 3      | 13       | 54654.38 | 69    |
| 14   | 11         | 1      | 15       | 40921.66 | 86    |
|      |            | 2      | 13       | 42488.48 | 91    |
|      |            | 3      | 12       | 43778.8  | 94    |
| 15   | 14         | 1      | 20       | 68341.01 | 88    |
|      |            | 2      | 18       | 70322.58 | 91    |
|      |            | 3      | 16       | 71428.57 | 95    |
| 16   | 13, 12, 15 | 1      | 15       | 65852.53 | 79    |
|      |            | 2      | 16       | 63456.22 | 75    |
|      |            | 3      | 19       | 62811.06 | 68    |
| 17   | 16,6       | 1      | 45       | 103917.1 | 81    |
|      |            | 2      | 42       | 105990.8 | 77    |
|      |            | 3      | 35       | 106958.5 | 73    |
| 18   | 17         | 1      | 20       | 129585.3 | 95    |
|      |            | 2      | 25       | 128387.1 | 90    |
|      |            | 3      | 30       | 122488.5 | 87    |
| 19   | 18         | 1      | 20       | 638847.9 | 87    |
|      |            | 2      | 26       | 617557.6 | 82    |
|      |            | 3      | 32       | 593732.7 | 79    |
| 20   | 5          | 1      | 60       | 80875.58 | 91    |
|      |            | 2      | 55       | 83041.47 | 85    |
|      |            | 3      | 50       | 83870.97 | 82    |



**Figure 2.** Flowchart of the GROL-Rao-1 algorithm (own research).

assessing optimization algorithms in solving multi-objective construction management problems.

The results in Table 3 show that the GROL-Rao-1 algorithm consistently achieved competitive or superior outcomes compared to DPSO. For most combinations, GROL-Rao-1 yielded shorter project completion times and comparable or lower costs, while maintaining quality at the same or slightly better levels. Notably, it achieved these results with half the iterations and only 25,000 function evaluations, compared to 50,000 for DPSO. This highlights the efficiency and effectiveness of GROL-Rao-1 in balancing time, cost, and quality in large-scale subcontractor selection problems.

**Table 3.** Comparison of nine Pareto optimal subcontractor sequences for each work package.

| S. No. | Optimal subcontractor combinations      | This study, (GROL-Rao-1) |         |       | DPSO, Polat et al. [2] |         |       |
|--------|---|--------------------------|---------|-------|------------------------|---------|-------|
|        |   | PCT*                     | PCC     | PQu   | PCT                    | PCC     | PQu   |
| 1      | 1 3 1 1 1 3 1 1 1 1 1 2 1 3 3 1 3 1 1 1 | 458                      | 6545209 | 81.25 | 465                    | 6524954 | 81,25 |
| 2      | 1 1 1 1 1 1 3 1 1 1 1 3 1 3 3 1 3 1 1 2 | 458                      | 6573827 | 82.15 | 475                    | 6498758 | 81,85 |
| 3      | 1 2 3 1 2 2 1 1 1 1 1 2 3 3 3 1 3 1 1 1 | 478                      | 6433965 | 80.55 | 475                    | 6479654 | 80,95 |
| 4      | 1 3 1 1 3 3 1 1 1 1 1 3 3 3 3 2 3 1 1 1 | 459                      | 6522882 | 80.00 | 488                    | 6409447 | 81,90 |
| 5      | 1 3 1 1 1 3 1 1 1 1 1 2 1 3 3 1 3 1 1 1 | 458                      | 6545209 | 81.25 | 489                    | 6408733 | 81,75 |
| 6      | 3 1 3 3 3 1 3 2 1 1 2 3 2 3 3 2 2 2 3 2 | 529                      | 6342307 | 79.50 | 500                    | 6346037 | 80,80 |
| 7      | 1 3 1 1 3 3 3 1 1 3 3 3 3 1 1 1 3 1 1 1 | 463                      | 6509933 | 80.00 | 495                    | 6349655 | 80,30 |
| 8      | 1 3 1 1 2 3 1 3 2 1 1 2 2 3 3 1 3 1 1 2 | 473                      | 6557214 | 81.40 | 493                    | 6392834 | 81,65 |
| 9      | 3 1 2 1 2 2 1 2 1 1 1 1 1 3 3 1 3 1 1 1 | 478                      | 6455117 | 81.65 | 500                    | 6371244 | 81,50 |
|        | Pop. size                               |                          | 100     |       |                        | 100     |       |
|        | No. of iterations                       |                          | 250     |       |                        | 500     |       |
|        | NFE                                     |                          | 25000   |       |                        | 50000   |       |

PCT\* = Project completion time, PCC = Project completion cost, PQu = Project quality

Table 4 shows that GROL-Rao-1 consistently produced higher-quality solutions (up to 82.15) compared to NDSII-Rao-1 (max 80.10) and DOLRao-1 (max 81.55). Although its project completion cost was slightly higher in some cases, GROL-Rao-1 achieved the shortest completion times (458 days) with fewer iterations and only 25,000 function evaluations.

This demonstrates its efficiency and robustness in solving the TCQT optimization problem, offering a better balance of time, cost, and quality.

Figure 3 presents the value path plot for the TCQT problem. The blue line shows dominated solutions, while the orange line represents archived non-dominated solutions from the GROL-Rao-1 algorithm. The distribution highlights the trade-offs where higher quality or shorter time generally increase costs, providing decision-makers with valuable alternatives. Hence, this visualization provides alternatives to the decision-makers to facilitate the decision-making process.

**A. Empirical example of 20-activity project**

Due to the complexity of multi-objective problems, no single performance metric exists. In this study, the HV metric is used to assess convergence and diversity, as illustrated in Fig. 4 and Table 5, consistent with prior literature [13].

**Table 4.** Result comparison between Rao-1 variants for 20-activity project.

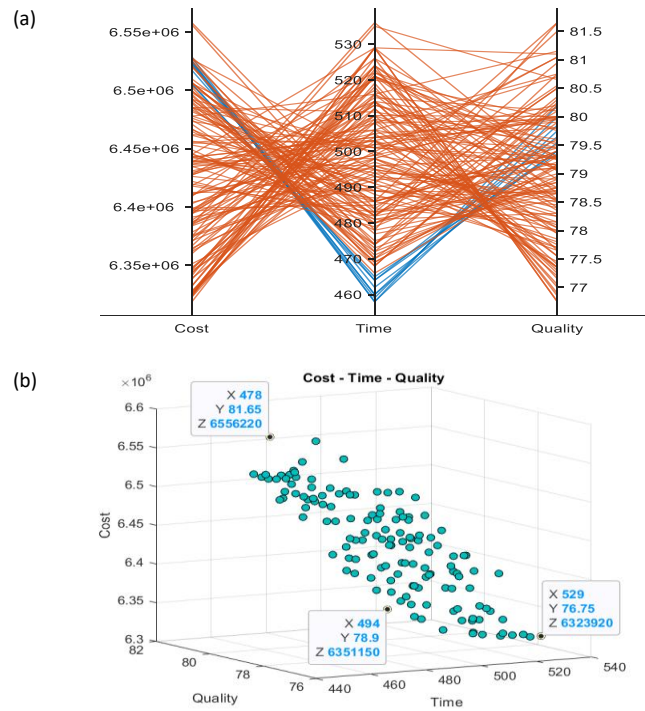
| Rao-1 variants | This study |         |         |       |     |       |
|----------------|------------|---------|---------|-------|-----|-------|
|                | PCT        | PCC     | PQu     | NOP   | NOI | NFE   |
| NDSII-Rao-1    | 464        | 6514103 | 79.00   | 100   | 300 | 30000 |
|                | 472        | 6485993 | 80.10   |       |     |       |
|                | 463        | 6519795 | 78.85   |       |     |       |
|                | 477        | 6455002 | 79.20   |       |     |       |
|                | 489        | 6389427 | 78.95   |       |     |       |
|                | 463        | 6509933 | 80.00   |       |     |       |
|                | 501        | 6357905 | 77.75   |       |     |       |
|                | 507        | 6412376 | 77.60   |       |     |       |
|                | 500        | 6332537 | 78.85   |       |     |       |
|                | DOLRao-1   | 465     | 6527491 | 79.65 | 100 | 300   |
| 469            |            | 6518781 | 80.60   |       |     |       |
| 468            |            | 6514887 | 80.80   |       |     |       |
| 470            |            | 6488850 | 80.25   |       |     |       |
| 493            |            | 6363712 | 78.40   |       |     |       |
| 500            |            | 6336869 | 78.15   |       |     |       |
| 486            |            | 6406730 | 79.85   |       |     |       |
| 460            |            | 6553550 | 81.55   |       |     |       |
| 470            |            | 6563182 | 80.40   |       |     |       |
| GROL-Rao-1     |            | 458     | 6545209 | 81.25 | 100 | 250   |
|                | 458        | 6573827 | 82.15   |       |     |       |
|                | 478        | 6433965 | 80.55   |       |     |       |
|                | 459        | 6522882 | 80.00   |       |     |       |
|                | 458        | 6545209 | 81.25   |       |     |       |
|                | 529        | 6342307 | 79.50   |       |     |       |
|                | 463        | 6509933 | 80.00   |       |     |       |
|                | 473        | 6557214 | 81.40   |       |     |       |
|                | 478        | 6455117 | 81.65   |       |     |       |

**B. Statistical validation of algorithm performance via Wilcoxon signed rank test**

To statistically validate the performance of the proposed GROL-Rao-1 algorithm, pairwise Wilcoxon Signed-Rank tests were conducted against comparative algorithms, including DPSO [2], Plain Rao-1, DOLRao-1, and NSGA-II. The analysis was based on 10 independent runs for each algorithm. The tests were performed separately for HV and Sp metrics with a significance level of  $\alpha = 0.05$ . The results obtained, as presented in Table 6, indicate that the proposed GROL-Rao-1 statistically outperforms competing methods for both HV and Sp metrics. These findings demonstrate that the integration of golden ratio-based opposition learning into the Rao-1 framework significantly enhances solution quality and Pareto front distribution.

**C. Multi-criteria decision making (MCDM)**

Table 7 presents three high-performing solutions for the 20-activity project obtained using the Crowding Distance Rank (CDR) mechanism. The solutions are evaluated based on PCT, PCC, and PQu values, with the CDR reflecting their diversity on the Pareto front. The first solution, with an infinite crowding distance, ranks first, indicating it is the most isolated and contributes most to diversity. The second and third solutions have crowding distances of 0.771 and 0.693, ranking second and



**Figure 3.** (a) Value path of time–cost–quality, (b) TCQT Pareto-optimal solutions for 20 activities.

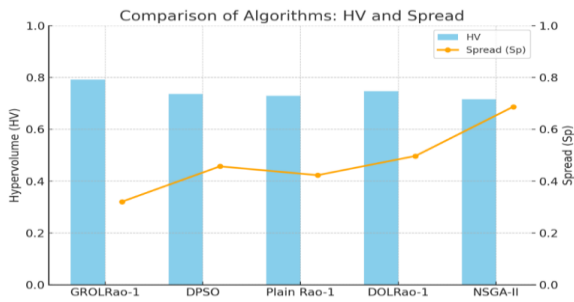


Figure 4. Graphical representation of the HV and Sp metrics.

third, respectively, and represent slightly less diverse yet high-quality trade-off solutions.

### V. CONCLUSION

This study presented GROL-Rao-1 algorithm for optimizing the time–cost–quality trade-off in subcontractor selection. By incorporating golden ratio-based opposition learning, the method improved convergence speed, enhanced solution quality, and reduced computational effort by nearly 50% compared to state-of-the-art algorithms. Applied to a real-world 20-activity construction project, GROL-Rao-1 achieved shorter completion times, higher quality, and competitive costs while requiring fewer iterations and function evaluations.

A real-world case study involving 20 construction activities demonstrated the practical effectiveness of GROL-Rao-1. Compared to DPSO and other advanced algorithms such as SMA, GROLAO, NSGA-II, and Rao-1 variants, GROL-Rao-1 consistently achieved superior or competitive results. Specifically, it delivered shorter project completion times (as low as 458 days), higher quality levels (up to 82.15), and comparable costs, while requiring only half the iterations and function evaluations of competing methods. This efficiency highlights its potential for large-scale, complex construction optimization problems.

Future work can extend GROL-Rao-1 to involve sustainability metrics, multi-mode scheduling, and uncertainty modeling to better reflect real-world complexities in construction projects. Also, hybridization with learning-based or adaptive optimization strategies may further improve solution quality and robustness for large-scale, multi-objective construction planning problems. Overall, GROL-Rao-1 provides a robust and practical framework for methodological advancement and applied construction management decision support.

Compared to previous studies, GROL-Rao-1 combines methodological innovation with real-world performance gains. Its superior HV and Sp metrics indicate better solution diversity and distribution, overcoming common issues like premature

Table 5. Performance comparison of the algorithms.

| Algorithms  | NPFs | HV    | Sp    |
|-------------|------|-------|-------|
| GROL-Rao-1  | 10   | 0.792 | 0.321 |
| DPSO [2]    | 10   | 0.736 | 0.457 |
| Plain Rao-1 | 10   | 0.729 | 0.423 |
| DOLRao-1    | 10   | 0.747 | 0.497 |
| NSGA-II     | 10   | 0.716 | 0.687 |

convergence in traditional Rao-1 or NSGA-II approaches. Overall, GROL-Rao-1 provides a robust and computationally efficient framework, with future extensions possible for sustainability aims, multi-mode scheduling, and hybrid optimization strategies.

### AUTHOR STATEMENT

**Plagiarism Check**—The article has been scanned with iThenticate and found to be compliant with the journal's plagiarism policy.

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Table 6. Wilcoxon signed-rank test results for performance metrics (HV and Sp).

| Algorithm  | Metrics | Comparison Algorithm | p-value | Significant ( $\alpha=0.05$ ) |
|------------|---------|----------------------|---------|-------------------------------|
| GROL-Rao-1 | HV      | DPSO [2]             | 0.014   | Yes                           |
| GROL-Rao-1 | HV      | Plain Rao-1          | 0.008   | Yes                           |
| GROL-Rao-1 | HV      | DOLRao-1             | 0.021   | Yes                           |
| GROL-Rao-1 | HV      | NSGA-II              | 0.002   | Yes                           |
| GROL-Rao-1 | Sp      | DPSO [2]             | 0.033   | Yes                           |
| GROL-Rao-1 | Sp      | Plain Rao-1          | 0.027   | Yes                           |
| GROL-Rao-1 | Sp      | DOLRao-1             | 0.049   | Yes                           |
| GROL-Rao-1 | Sp      | NSGA-II              | 0.001   | Yes                           |

Table 7. Three high-performing solutions for the 20-activity project using CDR mechanism

| GROL-Rao-1 |         | Crowding distance rank (CDR) |                  | CDR order |
|------------|---------|------------------------------|------------------|-----------|
| PCT        | PCC     | PQu                          |                  |           |
| 458        | 6545209 | 81.25                        | Inf ( $\infty$ ) | 1         |
| 478        | 6455117 | 81.65                        | 0,771            | 2         |
| 529        | 6342307 | 79.50                        | 0.693            | 3         |

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