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## THE OPINIONS OF THE STUDENTS OF SECONDARY EDUCATION MATHEMATICS TEACHING ON MATHEMATICAL PROOF METHODS<sup>3</sup>

### Abstract

In this study, the opinions of the students of mathematics teaching on mathematical proof methods were examined using the case study design among qualitative research methods. The data were obtained with the help of the Opinion Form on Mathematical Proof Methods. The study group consists of 10 secondary school mathematics teaching students, who are determined according to the criterion sampling method and volunteer for participating in the research. A clinical interview was held with each student, and content analysis was used in the analysis of the data. Based on the findings, it was seen that students have positive thoughts about the necessity of proving by determining mathematical proof methods.

**Keywords:** Mathematical proof methods, secondary school mathematics students, opinion on proof methods

## ORTAÖĞRETİM MATEMATİK ÖĞRETMENLİĞİ ÖĞRENCİLERİNİN MATEMATİKSEL İSPAT YÖNTEMLERİ HAKKINDAKİ GÖRÜŞLERİ

### Özet

Bu çalışmada, matematik öğretmenliği öğrencilerinin matematiksel ispat yöntemleri hakkındaki görüşleri nitel araştırma yöntemlerinden durum çalışması deseni kullanılarak incelenmiştir. Veriler Matematiksel İspat Yöntemlerine İlişkin Görüş Formu yardımıyla elde edilmiştir. Çalışma grubunu ölçüt örnekleme yöntemine göre belirlenen ve araştırmaya katılmaya gönüllü 10 ortaöğretim matematik öğretmenliği öğrencisi oluşturmaktadır. Her bir öğrenci ile klinik mülakat yapılmış ve verilerin analizinde içerik analizi kullanılmıştır. Bulgulardan, öğrencilerin matematiksel ispat yöntemlerini belirleyerek ispat yapmanın gerekliliği hakkında genellikle olumlu düşünceye sahip oldukları görülmüştür.

**Anahtar Sözcükler:** Matematiksel İspat Yöntemleri, Ortaöğretim Matematik Öğrencileri, İspat Yöntemlerine Yönelik Görüş

### INTRODUCTION

As frequently mentioned, mathematics is a field of science that requires the use of previous knowledge and obtained skills and in which the information is not only accumulated one after the other but also get intertwined (Moralı, Uğurel, Türnüklü, & Yeşildere, 2006). The proof is the basis of mathematics (Mingus & Grassl, 1999; Tall, 1998) and has a very important

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place in mathematics (Coe & Ruthven, 1994; Hanna, 2000; Martin & Harel, 1989). By using proof, why something is right can be shown, explained, and new mathematical information can be explored or created (Almeida, 2000; Knuth, 2002a). The difficulties that the students face while proving are where to start proving, how to make it, the conception information that must be used in this process and its way of usage (Weber, 2001). The difficulties that the students have result from the failure to understand the nature of proof, mathematical rules, proof techniques and strategies (Gibson, 1998; Weber, 2006).

Mathematical proof consists of universally recognized methods and is performed in two main ways such as inductive and deductive approach (Çallıalp, 1999). The deductive approach may be in the form of direct proof and indirect proof (*reductio ad absurdum*, finding a contradiction, giving contrary examples and trial) within itself.

Among the objectives of mathematics, the National Council of Teachers of Mathematics (NCTM) standards include students' gaining the skill of choosing and using different types of reasoning and proof methods at the end of the secondary education period (NCTM, 2000). In the secondary school mathematics program implemented in our country, there are two gains in "Logic Learning Area" on the concept proof in "Proof Methods Sub-Learning Area". These are as follows: "They explain the concepts of definition, axiom, theorem and proof, express the hypothesis and provision of a theorem" and "they make simple proofs by using proof methods" (Ministry of Education [ME], 2005). In addition to this, the skills of "mathematical reasoning and proving" are included in the mathematical qualifications and skills that secondary school mathematics lesson curriculum that was renewed by the Ministry of Education (ME) in 2013 aims to provide to the students. Furthermore, the renewed curriculum includes the gains of "gaining proving, proportional reasoning and probabilistic thinking skills" at 9th grade and "proving by using mathematical proof methods (giving contrary examples, contrapositive, direct proof, contradiction, and induction) (ME, 2013).

Mathematical induction is a proof method that is quite hard to learn in secondary education, and students have hardship in learning this method (Leung, 2005). The teachers have a great responsibility in this subject. If a teacher neglects the hardships about the induction method, the students' process of learning the mathematical induction method turns into imitating the teacher without understanding the method (Baker, 1996). While mathematical induction is in the center of university mathematics curriculum, it was observed in the studies performed that the students at bachelor's level have shortcomings in understanding this method of proof (Leung, 2005; Dubinsky & Lewin, 1986; Harel, 2002; Knuth, 2002b; Movshovitz-Hadar, 1993; Schoenfeld, 1994).

The reason for teaching the proof methods at the beginning of university education in mathematics, in which it is indispensable to encounter proof methods in almost all of its field courses, is that students can determine the method used when they encounter a proof in other lessons (Moralı, Uğurel, Türnüklü, & Yeşildere, 2006).

In our country, the studies conducted on mathematical proof have gained intensity especially in the last ten years (Güler & Dikici, 2012). The pre-condition of proving is to know the proof methods.

In the studies on mathematical proof methods, while it is generally observed that students can make proof (Altıparmak & Öziş, 2005; Baker, 1996; Güler, Özdemir, & Dikici, 2012;

İmamoğlu, 2010; Özer & Arıkan, 2001), when the studies on receiving opinions are examined, studies on receiving opinions about proof are usually encountered (Baştürk, 2010; Doruk, Özdemir, & Kaplan, 2015; Güler & Dikici, 2012; Gökkurt & Soylu, 2012; İskenderoğlu, 2010; İskenderoğlu, Baki, & Palancı, 2011; Kaplan, Doruk, Öztürk, & Duran, 2016; Kayagil, 2012; Morali, Uğurel, Türnüklü, & Yeşildere, 2006; Öçal & Güler, 2010; Varghese, 2009). The opinions of students, who will become secondary school mathematics teachers in the future, about proof methods are important. However, no direct study on the opinions of secondary school mathematics teaching students on mathematical proof methods has been encountered abroad and in our country.

The objective of this study is to determine the difficulties that the students of secondary school mathematics teaching encounter while determining the proof method and opinions about the mathematical proof methods.

## METHOD

The case study method among qualitative research methods was used in the research. In the case study, the factors in regard to a situation (environment, individuals, incidents, processes, etc.) are examined with a holistic approach, and it is focused on how these factors affect the relevant situation and are affected by it (Yıldırım & Şimşek, 2011).

### Study Group

The study group was determined in accordance with the criterion sampling method (Yıldırım & Şimşek, 2011) that is among purposive sampling methods and includes examining all of the situations fulfilling a series of criteria. The study was performed with 10 students, 2 students from each grade level, studying at the department of mathematics teaching at a state university in the Eastern Anatolia Region. The group consists of five female and five male students, whose age average is twenty-two. No real names were used in this study. The names used are nicknames.

### Data Collection Tools

#### *Opinion Form on Mathematical Proof Methods*

The opinions of the participants about mathematical proof methods in the research were taken with the “Opinion Form on Mathematical Proof Methods”. The opinion form was prepared by the researchers as 5 questions by using the relevant literature, and the consent of three faculty members was taken. The questions are in the semi-structured form, and questions were asked at the end when necessary. The opinion of a student from one category created for increasing the validity of the research was included in the findings section.

#### *Clinical Interviews*

The opinions of secondary school mathematics teaching students on mathematical proof methods were taken using clinical interviews. Before clinical interviews, the researchers expressed that the research would be conducted totally on the principle of volunteering, and also those who did not want to continue could leave it. Furthermore, the participants' permission was taken by telling them that voice recording would be performed during the interviews to be held. The interview period held with the participants lasted between twenty and forty minutes. The interviews were held in an environment where the researchers and the participant were able to talk face-to-face.

### Data Analysis

Content analysis was used in the analysis of the data. The “categorical analysis” among content analysis methods was used. In this process, the stages of coding the data, creating the categories, regulating the categories, defining and interpreting the data were monitored (Corbin & Strauss, 2007).

The voice records were transcribed, summarized and interpreted in the study. Then, codes and categories were created in 4 questions. Codes and categories were not created only in the analysis of the question “How do you decide on which category to use while making a mathematical proof?” The categories in Table-1 are the categories in the literature. The categories in Table-2, Table-3 and Table-4 were established by researchers. When the categories were examined, it was seen that there is only one participant in certain categories, there is more than one participant in certain categories, and the opinions of certain participants are included in more than one categories.

### FINDINGS AND INTERPRETATION

In this section, the findings obtained were primarily presented by making tables together with the answers that secondary school mathematics teaching students gave to clinical interview questions and the answers that led to the creation of these categories, the clinical interview held with each student was summarized, and the opinions of the researchers were included at the second stage.

#### Findings Concerning the Mathematical Proof Methods of the Students

The question “Could you give information about mathematical proof methods?” was directed in order to get the opinion of secondary school mathematics teaching students on mathematical proof methods. The answers given by the students to these questions were gathered under the categories of induction, deduction, direct proof, indirect proof, method of reductio ad absurdum, contradiction method, trial method, proof by giving contrary examples. The categories and the students in these categories are given together in Table-1.

**Table 1.** Knowledge of the Students about Mathematical Proof Methods

Categories	Induction	Deduction	Direct Proof	Indirect Proof	Method of reductio ad absurdum	Contradiction Method	Trial Method	Proof by Giving Contrary Examples Method
Zeynep	x	x	x	x				
Selim	x	x			x			
Erkan	x	x			x	x		
Pınar	x	x			x			
Eda			x	x	x			x
Berke	x	x	x	x	x	x	x	x
Taner	x	x			x			
Hümeyra	x	x			x	x		x
Gökhan	x	x	x		x			
Canan	x	x	x	x	x			

In general, the students told the names of the proof methods. Especially the induction, deduction, and reductio ad absurdum methods were remembered by almost all of the students. Only one student was able to remember the trial method. The content of proof methods could not be expressed clearly by most of the students. Some students expressed that they could use the proof methods although they could not tell the definition of proof methods.

### Findings Concerning the Meanings Attributed to Mathematical Proof Methods by the Students

The students were asked the question “What do mathematical proof methods mean for you?” in order to learn what mathematical proof methods mean for the students. The answers given by the students to these questions were gathered under the categories of getting rid of memorization (Zeynep, Selim, Berke, Taner, Hümeýra), accuracy (Zeynep, Erkan, Berke, Gökhan), permanence (Selim, Hümeýra, Gökhan), generalization (Pınar, Taner), building logic (Pınar, Berke), convenient start (Zeynep), objectivity (Erkan), easiness (Eda), trust (Gökhan), unnecessary (Canan). In Table-2, the categories and the answer of a student from each category were given, respectively, based on the frequency of repeating the response categories.

**Table 2.** Opinions of the Students on the Meaning of Mathematical Proof Methods

Categories	Opinions of the Students
Getting Rid of Memorization	<b>Taner:</b> Frankly speaking, proof methods reduce my memorization; I mean you can kind of find the formula yourself when you learn the conceptual basis of something.
Accuracy	<b>Berke:</b> Proof methods show that a particular subject, something that I cannot achieve is right.
Permanence	<b>Hümeýra:</b> I believe that the students perceive easier than memorizing by doing methods such as reductio ad absurdum and similar methods rather than writing the theorem and proving it; it will be permanent this way.
Generalization	<b>Taner:</b> It actually means generalization for me; we reveal something of which we cannot find a contrary example.
Building Logic	<b>Berke:</b> It explains us where the formulae come from; we use them in the solution of the problems, but they help us better explain where and how they come, this shows us what the subjects we examine are based on.
Convenient Start	<b>Zeynep:</b> ...Upon thinking mathematically in order to be able to start proving, we should know these, so that we can start from a point; I mean, after all, we will make a proof, if we know what we will do and which means we will use, and if we choose which way we can use in accordance with the question, what it requires will happen.
Objectivity	<b>Erkan:</b> Actually, the best part of mathematics is that we can prove what we say, when we say theorem, it is a theorem everywhere, I mean we cannot falsify here, it cannot be falsified elsewhere too, it is used the same way; thus, proof methods keep people objective in daily life.
Easiness	<b>Eda:</b> I can think of easiness, frankly speaking, I cannot think of anything else.
Trust	<b>Gökhan:</b> Being mathematically proved gives one a sense of trust, we say that something is true.
Unnecessary	<b>Canan:</b> ...I am a bit curious, but I still consider it unnecessary, I do not think that it will be useful for me in the future; I find it interesting, but I am still kind of doubtful whether it is necessary.

Except for Canan, the students have positive opinions. Canan expressed that it is unnecessary to know mathematical proof methods, as they will not use these methods when they become teachers. It is seen that the answers given by the students with positive opinions to this question are mostly gathered under the category of getting rid of memorization. In other words, the students think that knowing and using proof methods will relieve them of memorization.

**Findings Concerning the Opinions of the Students on the Importance of Mathematical Proof Methods**

The students were asked the question “What is the importance of the proof method to be used while making mathematical proving?” as for the importance of the method to be used while making mathematical proofs. The answers given to this question were gathered under the categories of easiness (Zeynep, Selim, Erkan, Pınar, Eda, Gökhan), accuracy (Selim, Berke, Taner), convenient start (Erkan, Berke), expressibility (Eda, Hümeýra), preventing time loss (Hümeýra, Gökhan), precision (Pınar), comprehensibility (Eda), destroying creativity (Canan) and unimportant (Zeynep). In Table-3, the categories and the answer of a student from each category were given, respectively, based on the frequency of repeating the response categories.

**Table 3.** Opinions of the Students on the Importance of Mathematical Proof Methods

<b>Categories</b>	<b>Opinions of the Students</b>
Easiness	<b>Zeynep:</b> Why is it important? Maybe, it can make easier if we know the logic at a particular moment, the direct proof is this, and the indirect proof is that, if we determine this at the beginning, maybe, it can be easier.
Accuracy	<b>Selim:</b> I think it is like if you use another proof while you are supposed to use induction, you may not find it.
Convenient Start	<b>Berke:</b> When we cannot determine the proof method, we cannot even determine how to start or how to make a beginning; determining the proof method is so important.
Expressibility	<b>Eda:</b> I think that it is possible in terms of being explanatory in terms of explaining in the sense of expression what we mean to the person we explain to.
Preventing Time Loss	<b>Gökhan:</b> Proving a question that will be proved with induction using deduction is a time loss, or proving a question that will be proved using the direct proof method is likewise.
Precision	<b>Pınar:</b> For example, if we have determined that we will solve that question by giving a contrary example, we know that there is a problem with this question, and we can solve this by giving a contrary example, then we find the course of the question; so we somehow seem to solve it all.
Comprehensibility	<b>Eda:</b> In terms of being more comprehensible.
Destroying Creativity	<b>Canan:</b> ...knowing these seems to limit creativity. I say, I limit it, there are induction and deduction, I cannot make any other proof, I used to try harder before knowing these two methods, I think it limits the thoughts a bit.
Unimportant	<b>Zeynep:</b> First, I take the question, and I deal with it, I start saying that it happens there, but I do not think whether the way I prove is direct or indirect, I cannot decide that, I do it without determining the proof method.

The majority of the students are of the opinion that the proof methods are important while making mathematical proof; however, Canan and Zeynep have negative opinions on this. Canan

thinks that knowing proof methods kills creativity while Zeynep sais that the proof method is not important, and proving can be performed without having regard to the method.

#### **Findings Concerning the Opinions of the Students on Determining the Method They Will Use While Making Mathematical Proof**

The students were asked the question “How do you decide on which method to use while making a mathematical proof?” when determining the method they will use while making a mathematical proof. The students tried to express their opinions on this subject. However, Canan said that she tried to do it the same way upon seeing the stereotypes in previous proofs; while Selim and Zeynep said that they did not think much of the proof methods while making the proofs. It was observed that most of the students do not have an adequate opinion on this subject.

#### **Findings Concerning the Opinions of the Students on Developing the Ability to Correctly Choose the Method They Use While Making a Mathematical Proof**

The students were asked the question “What are the ways of developing the ability to choose correctly the method that must be used while making a mathematical proof?” in order to develop the ability to choose correctly the method to be used while making a mathematical proof. The answers given by the students were gathered under the categories of making a lot of proofs (Zeynep, Pınar, Berke, Taner, Hümeyra, Gökhan), having a good command of proof methods (Zeynep, Selim, Berke, Hümeyra, Gökhan, Canan), making proof in different ways (Erkan, Pınar), looking at the proofs already made (Erkan), not memorizing (Selim), working consciously (Eda), analysing the proofs (Eda), going for generalizations (Berke), having a good command of the subject (Canan), and having good reasoning skills (Canan). In Table-4, the categories and the answer that belongs to a student from each category are shown together.

**Table 4.** Opinions of the Students on Developing the Ability to Choose the Method They Use While Making a Mathematical Proof Correctly

<b>Categories</b>	<b>Opinions of the Students</b>
Making A Lot of Proofs	<b>Berke:</b> I think making a lot of proofs means how a proof will start by making different proofs; i.e. making a lot of proofs means a lot of example solutions.
Having A Good Command of Proof Methods	<b>Canan:</b> It is necessary to know the proof method well, it is necessary to know where to use it, whether the deduction is used instead of induction and the differences between them so that I can use this method when the time comes, then I should examine the processes of the sub-steps.
Making Proof in Different Ways	<b>Erkan:</b> When it comes to the ways of developing the ability to choose accurately, I generally look at previously made proofs and think whether I can do it otherwise, we start to see upon looking for an alternative way and constantly using these; there was not a single proof of this theorem, there were different types of proof, I think it can be developed when trying to prove it in different ways.
Looking at Previously Made Proofs	<b>Erkan:</b> When it comes to the ways of developing the ability to choose accurately, I generally look at previously made proofs
Not Memorizing	<b>Selim:</b> I’m sure that this is possible by moving mathematics away from memorization and understanding the logic behind the methods, for example, how to use the method of reductio ad absurdum or how to make things in the contradiction method, etc. We can also estimate it at

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	the beginning of the question, in such a way that we use this here, etc.
Working Consciously	<b>Eda:</b> If we actually work consciously, that's it.
Analyzing the Proofs	<b>Eda:</b> When we look at the theorem, we don't say what's proof method of this; we just understand it, memorize it, and pass generally, but when we say that's solved like this, I mean if we analyze it, we will understand what it is.
Going for Generalization	<b>Berke:</b> By making a lot of proofs and seeing how the proof will start, we could make a generalization on which proof method to use starting from the words in the propositions.
Having a Good Command of the Subject	<b>Canan:</b> I guess it is necessary to know a lot, I mean it is necessary to have a good command of the subject, you should have good field knowledge, I mean, I should have a good command of my field, I need to know mathematics well, the method used in mathematics, for example, I cannot do an experiment so I must definitely use an abstract method in mathematics. Thus, I believe I need to know the methods, processes, and their steps well.
Having Good Reasoning Skills	<b>Canan:</b> I guess it is necessary to have good reasoning skills while making reasoning, I think I can find it from there not by memorizing...

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A great majority of the students think that making a lot of proofs and having a good command of proof methods will develop the ability to choose correctly the method to be used while making a mathematical proof.

### **RESULT, DISCUSSION, AND SUGGESTIONS**

The students, in general, remembered the names of mathematical proof methods, but they failed to exhibit the same success as of the content. This supports the view that high school and university students have significant difficulties (Baker, 1996) conceptually and operationally concerning the proof techniques. That some of the students said that they could use the proof methods although they could not express them is consistent with the result that most of the students focused on the operational side of mathematical induction rather than its conceptual aspect (Baker, 1996). A great majority of the students think that knowing and using the proof methods will save them from memorizing the proof while determining the method will provide the ease while making a proof. These opinions of the students overlap with the finding that pre-service teachers generally have positive opinions on mathematical proof (Güler & Dikici, 2012; Güler, 2013). That the students generally do not have sufficient views about deciding on which method they will use while making a proof overlaps with the findings that the opinions of pre-service teachers about proof are not yet formed fully (Moralı, Uğurel, Türnüklü, & Yeşildere, 2006; Güler, Özdemir, & Dikici, 2012), the skills of pre-service teachers on proving using the induction method are low (Güler, Özdemir, & Dikici, 2012), the students fail to determine the appropriate proof method (Sarı Uzun & Bülbül, 2013), the students fail to use the proving methods and techniques sufficiently (Özer & Arıkan, 2002) they have hardship in choosing the correct method while proving and fail to have a full command of the process (Peşken Sağır, 2013). That students determine the proof method by lessons and stereotypes is in accordance with the findings that students focus on a particular proof method (Sarı Uzun & Bülbül, 2013),



pre-service teachers try to use the proof methods by memorization and have incomplete and inaccurate information about the methods (Peşken Sağır, 2013). The majority of the students think that developing the ability to choose correctly the method to be used while making a proof will only be ensured by making a lot of proofs and having a good command of proof methods.

All of the students participating in the research think that proof methods have an important place in proving. Thus, students' opinions regarding the use of proof methods are generally positive; however, they have problems in the effective using of proof methods. They think that the time allocated for proof methods in lessons is insufficient. That students who will become secondary school mathematics teachers have sufficient information, they can use these proof methods and have positive opinions about these methods are important. For proof methods are included in the high school and university program. The shortness of time leads students to memorization without understanding the logic behind the proofs. Thus, the time allocated for proof methods should be increased, or a lesson called proof methods should be added to the first grade. It is also necessary to prefer induction when determining the method of proof. Furthermore, university students should be made to understand the role and importance of knowing the proof methods by performing studies on proof methods.

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### **GENİŞLETİLMİŞ ÖZET**

Matematiksel ispat evrensel olarak kabul gören yöntemlerden oluşmakta, tümevarım ve tümdengelim olmak üzere iki temel yoldan yapılmaktadır (Çallıalp, 1999). Tümdengelim de kendi içerisinde doğrudan ispat ve dolaylı ispat (olmayana ergi, çelişki bulma, aksine örnek verme ve deneme yöntemiyle ispat) şeklinde olabilmektedir.

Alan derslerinin hemen hepsinde ispatla karşılaşmanın kaçınılmaz olduğu matematikte, ispat yöntemlerine lisans eğitiminin başlangıcında yer verilmesinin nedeni, öğrencilerin diğer derslerde bir ispatla karşılaştıklarında kullanılan yöntemi belirleyebilmeleridir (Moralı, Uğurel, Türnüklü, & Yeşildere, 2006).

Ülkemizde matematiksel ispat ile ilgili yapılan araştırmalar özellikle son on yıl içerisinde yoğunluk kazanmıştır (Güler & Dikici, 2012). İspat yapabilmenin ön koşulu ispat yöntemlerini bilmektir. Bu çalışmanın amacı, ortaöğretim matematik öğretmenliği öğrencilerinin matematiksel ispat yöntemleri hakkındaki görüşlerinin ve ispat yöntemini belirlerken karşılaştıkları güçlüklerin belirlenmesidir.

Çalışmada nitel araştırma yöntemlerinden durum çalışması modeli kullanılmıştır. Çalışma grubu amaçlı örnekleme yöntemlerinden biri olan ve önceden belirlenmiş bir dizi ölçütü karşılayan bütün durumların çalışılmasını içeren ölçüt örnekleme yöntemine (Yıldırım & Şimşek, 2011), ve gönüllülük esasına göre belirlenmiştir. Araştırma, Doğu Anadolu Bölgesi'nde bir devlet üniversitenin ortaöğretim matematik öğretmenliği bölümünde öğrenim gören her bir sınıf düzeyinden 2 öğrenci olmak üzere 10 öğrenci ile gerçekleştirilmiştir.

Araştırmada, katılımcıların matematiksel ispat yöntemleri hakkındaki görüşleri "Matematiksel İspat Yöntemlerine İlişkin Görüş Formu" ile alınmıştır. Görüş formu, araştırmacılar tarafından ilgili alan yazından yararlanılarak 5 soru olarak hazırlanmıştır. Sorular yarı yapılandırılmış şekildedir ve gerektiğinde sonda sorular yöneltilmiştir.

Ortaöğretim matematik öğretmenliği öğrencilerinin matematiksel ispat yöntemleri hakkındaki görüşleri klinik mülakatlar ile alınmıştır. Katılımcılarla yapılan mülakat süresi yirmi ile kırk dakika aralığında olmuştur.

Verilerin çözümlenmesinde içerik analizi kullanılmıştır. İçerik analizi yöntemlerinden "kategorisel analiz" kullanılmıştır. Bu süreçte; verilerin kodlanması, kategorilerin oluşturulması, kategorilerin düzenlenmesi, bulguların tanımlanması ve yorumlanması aşamaları izlenmiştir (Corbin & Strauss, 2007).

Çalışmada ses kayıtları yazıya dökülmüş, özetlenmiş ve yorumlanmıştır. Daha sonra 4 soruda kod ve kategoriler oluşturulmuştur. Yalnız "Matematiksel ispat yaparken hangi yöntemi

kullanacağınıza nasıl karar verirsiniz?" sorusunun çözümlenmesinde kod ve kategoriler oluşturulmamıştır. Araştırmanın geçerliliğini artırmak için oluşturulan her bir kategoriden bir öğrenci görüşüne bulgular kısmında yer verilmiştir.

Araştırmaya katılan öğrencilerin tamamı ispat yöntemlerinin ispat yapmada önemli bir yere sahip olduğunu düşünmektedir. Bu yüzden, öğrencilerin ispat yöntemlerini kullanmanın gerekliliği ile ilgili görüşleri genelde olumlu yöndedir; fakat ispat yöntemlerini etkin bir şekilde kullanmakta sorun yaşamaktadırlar. Derslerde ispat yöntemlerine ayrılan sürenin yetersiz olduğunu düşünmektedirler. Ortaöğretim matematik öğretmeni olacak öğrencilerin matematiksel ispat yöntemleri hakkında yeterli bilgiye sahip olması, bu ispat yöntemlerini kullanabilmeleri ve bu yöntemler hakkında olumlu görüşe sahip olmaları önemlidir. Çünkü lise ve üniversite programında ispat yöntemleri yer almaktadır. Sürenin azlığı öğrencileri ispatların mantığını anlamadan ezbere yöneltmektedir. O yüzden ispat yöntemlerine ayrılan süre arttırılmalı ya da birinci sınıfa ispat yöntemleri adlı bir ders konulmalıdır. Aynı zamanda ispat yaparken yöntemin belirlenmesinde buluş yolu tercih edilmeli ve ispat yöntemleri birbirleriyle karşılaştırmalı olarak öğretilmelidir. Ayrıca ispat yöntemleri ile ilgili çalışmalar yapılarak ispat yöntemlerini bilmenin ispat yapmadaki rolü ve önemi üniversite öğrencilerine kavratılmalıdır.