ORİJİNAL MAKALE / ORIGINAL ARTICLE

Düzce Üniversitesi
Sağlık Bilimleri Enstitüsü Dergisi
2014;4(2): 9-14
ISSN: 2146-443X

sbedergi@duzce.edu.tr

A Comparative Study on Variance Components Estimation Methods*

İlkay DOĞAN[1], İbrahim KILIÇ[1]

ABSTRACT

In the study, some important methods such as; Analysis of Variance (ANOVA), Henderson III, ML (Maximum Likelihood), REML (Restricted Maximum Likelihood), and MINQUE (Minimum Norm Quadratic Unbiased Estimate) which are commonly used in literature to estimate variance components, were aimed to investigate comparatively for balanced and unbalanced data. In accordance with the experiment, this study designed with not only the data obtained from the eggs of two commercial layer herds aged 28-week and 80-week which were stored under different storage time and conditions but also with interactive and non-interactive models. Variance components related with effects of hen age, storage time and conditions on Haugh unit and egg weight were estimated with five methods (ANOVA, Henderson III, ML, REML, MINQUE). In balanced data, though the estimation of variance components in four methods were found equal to each other, error variance ratio in ML method was found higher. In unbalanced data, for the interactive model, though explanation rates of error variance to total variance are calculated approximately 14.0% for the methods ANOVA, REML and MINQUE; ML (18.32%) and Henderson III (17.39%) was found higher. Also for the non-interactive model, the rate of error variance in ANOVA, Henderson III, REML and MINQUE methods was found approximately 27.0% but for ML it was found 42.16%. According to research results, it is suggested that for the data in which balanced and normal distribution do not exist, other methods should be used except from ML, however, depending on data structure in unbalanced data it should be benefitted from REML method on condition that degree of freedom is low.

Keywords: Variance components estimation; ANOVA; Henderson methods; ML; REML; MINQUE.

Varyans Bileşenleri Tahmin Yöntemlerine Yönelik Bir Karşılaştırma

ÖZET

Araştırmada, varyans bileşenlerinin tahmin edilmesinde literatürde yaygın olarak kullanılan yöntemlerden; ANOVA, Henderson III, ML, REML, MINQUE gibi önemli yöntemlerin dengeli ve dengeli olmayan veriler için karşılaştırmalı olarak incelenmesi amaçlanmıştır. Çalışmada, yaşları 28 ve 80 haftalık olan iki ticari yumurtacı sürüden elde edilen toplam 696 adet sofralık yumurtaya ait veriler kullanılarak etkileşimli ve etkileşimsiz modeller düzenlenmiştir. Haugh birimi ve yumurta ağırlığı üzerine depolama süresi, depolama koşulu ve ana yaşına ilişkin varyans bileşenleri beş yöntem ile tahmin edilmiştir (ANOVA, Henderson III, ML, REML, MINQUE). Dengeli verilerde, varyans bileşenleri tahmini dört yöntemde birbirine eşit bulunmasına rağmen ML yönteminde hata varyans oranı yüksek bulunmuştur. Etkileşimli model için dengeli verilerde, hata varyansının toplam varyansı açıklama oranı ANOVA, REML ve MINQUE yöntemleri için yaklaşık olarak %14 bulunmuşken, ML (%18,32) ve Henderson III (%17,39) yöntemlerinde yüksek bulunmuştur. Etkileşimsiz modelde ise ANOVA, Henderson III, REML ve MINQUE yöntemlerinde hata varyansı oranı yaklaşık %27 bulunmuş fakat ML yönteminde %42,16 bulunmuştur. Çalışma sonuçlarına göre, dengeli ve normal dağılım varsayımı gerçekleşmeyen verilerde ML hariç diğer yöntemlerin kullanılabileceği, bununla birlikte verinin yapısına bağlı olarak dengeli olmayan verilerde ve serbestlik derecesinin düşük olması durumunda REML yönteminden yararlanılması önerilmiştir.

Anahtar Kelimeler: Varyans bileşenleri tahmini; ANOVA; Henderson yöntemleri; ML; REML; MINQUE.

INTRODUCTION

In recent years evaluations of methods related to estimating variance components are very important for some of the researchers who are engaged in scientific work about statistics. Researchers in the field of applied statistics, science and

[1] Afyon Kocatepe Üniversitesi Veteriner Fakültesi, Biyoistatistik Anabilim Dalı

* This study is presented in International Congress in Honour of Professor Hari M. Srivastava, August 23-26 2012, Bursa.

Correspondence: İbrahim KILIÇ, kilicibrahim@hotmail.com

especially health sciences tend to emphasize on the importance of this issue because of the need for the estimation of variance components. Thus in models which are used in animal breeding in the field of animal husbandry, determination of the genetic variance and error variance in the total variation is of great importance. Because in determining the heritability which is the key parameter at genetic improving, in addition to the error, the components of other fixed and random effects should also be estimated.

After Fisher (1) had defined variance analysis method in 1925, it has been using widely in order to test the significance of the effects of treatments. If the effects of several factors are examined on one dependent variable, the rate of each factor in total variance can be calculated. In other words, variance components can be calculated. According to Crump (2), another use of analysis of variance enables to estimate variance components.

In a study which is made for estimate of variance components, let i=1,2,…,a are number of groups and j=1,2,…,n are the number of observations;

$$y_{ij}=\mu+\alpha_i+e_{ij} \qquad [1]$$

linear model has been using. In this model; y_{ij} states j. observation in i. group, μ states for general mean, α_i states for effect of i. group and ε_{ij} states for random error.

The basic principle to obtain estimation of variance components using ANOVA method, equalization of mean squares to their expected values and then solving the obtained linear equalities (3,4). For balanced data, the expected value of mean squares of between groups was calculated by [2]. For unbalanced data (i=1,2,...,a and j=1,2,...,n_i), the expected value of mean squares of between groups was calculated by [3].

$$\mathrm{E(MSBG)} = \mathrm{E}\left(\frac{n\sum_{i=1}^{a}(\bar{y}_{i.}-\bar{y}_{..})^2}{a-1}\right) \qquad [2]$$

$$\mathrm{E(MSBG)} = \mathrm{E}\left(\frac{\sum_{i=1}^{a}n_i(\bar{y}_{i.}-\bar{y}_{..})^2}{a-1}\right) \qquad [3]$$

Henderson I, II, III methods have widely used in order to estimate variance components. Henderson I is the easiest method to calculate among these methods. Henderson II method can be used for the random models. In this method, if there are fixed effects in the model, data should be corrected regarding to fixed effects beforehand, and later, Henderson I method is applied. Thus, the biases on the estimation of variance components can be eliminated. Henderson III method is the most suitable of the three methods used to calculate the variance components. This method has been moving around the difficulties of fixed elements in the model. Even though elements of the model are correlated, they are able to provide unbiased estimates (3,5). If linear model is assumed that;

$$y_a=\sum_{(i=1)}^{p} b_i x_{ia}+e_a \qquad (a=1,2,\ldots,N) \qquad [4]$$

Where x's are known, e's have mean zero, are uncorrelated, and have common variance $(\sigma_e)^2$.

p' indicates the number of independent variables in least squares equations. Error variance $(\sigma_e)^2$ is found by this formula.

$$\mathrm{E}(\textstyle\sum_a (y_a)^2 - R(b_1,\ldots,b_p)) = (N-p')(\sigma_e)^2 \qquad [5]$$

Then, σ^2 is found by substituting the found error variance in the below formula. Where q is the number of independent variables in least squares equations (3).

$$\sum_{(i=q+1)}^{p}\sum_{(j=q+1)}^{p}(C_{ij}-\lambda_{ij})E(b_i b_j)+(p'-q')(\sigma_e)^2$$
$$=\sum\sum_{(i=q+1)}^{p}(C_{ii}-\lambda_{ii})\sigma^2+(p'-q')(\sigma_e)^2 \qquad [6]$$

Maximum likelihood (ML) estimation of variance components from data on a continuous variable is often confined to situations based on the normality assumptions. The likelihood function is defined as (4);

$$L=\frac{\exp\{-\frac{1}{2\sigma_e^2}[\sum_i\sum_j(y_{ij}-\mu)^2-\sum_i\frac{\sigma_\alpha^2}{\sigma_e^2+n_i\sigma_\alpha^2}(y_{i.}-n_i\mu)^2]\}}{(2\pi)^{\frac{1}{2}N}\sigma_e^{2\left[\frac{1}{2}(N-a)\right]}\prod_{i=1}^{a}(\sigma_e^2+n_i\sigma_\alpha^2)^{\frac{1}{2}}} \qquad [7]$$

Since maximizing L parameter values is equal to maximizing its natural logarithm, and logL, which we denote by l= logL, is often a more tractable function than L, we deal with;

$$l=\log L=\log L(\mu,V|y)$$
$$=-\frac{1}{2}N\log 2\pi-\frac{1}{2}(N-a)\log\sigma_e^2-\frac{1}{2}\sum_i\log(\sigma_e^2+n_i\sigma_\alpha^2)$$
$$-\frac{\sum_i\sum_j(y_{ij}-\mu)^2}{2\sigma_e^2}+\frac{1}{2\sigma_e^2}\sum_i\frac{\sigma_\alpha^2}{\sigma_e^2+n_i\sigma_\alpha^2}(y_{i.}-n_i\mu)^2 \qquad [8]$$

Restricted maximum likelihood (REML), which is an adaptation of maximum likelihood method, maximizes just one part of likelihood. It means to maximize the part of likelihood which does not include μ for one way classification. Frequently mentioned characteristic feature of REML estimation is that though it is not related with fixed effect estimation, it take into account degree of freedom about the fixed effects of the model as a maximum likelihood method. In order to obtain the restricted likelihood were benefited from equation [9].

$$L(\sigma_e^2,\sigma_\alpha^2|SSB,SSW)=\frac{\exp\left[-\frac{1}{2}\left(\frac{SSW}{\sigma_e^2}+\frac{SSB}{\lambda}\right)\right]}{(2\pi)^{\frac{1}{2}(an-1)}\sigma_e^{2\left[\frac{1}{2}a(n-1)\right]}\lambda^{\frac{1}{2}(a-1)}(an)^{\frac{1}{2}}} \qquad [9]$$

Equation [9] means that likelihood function of $(\sigma_e)^2$ and $(\sigma_\alpha)^2$ depending on Sum of Squares Between Groups (SSBG) and Sum of Squares Within Groups (SSWG). This is named as restricted likelihood for balanced data with one-way classification (4,6).

General stages about obtaining quadric unbiased estimators including minimum norm and maximum variance features were presented by (7-9). Minimum Norm Quadratic Unbiased Estimation (MINQUE) procedure seeks to minimize the differences with natural estimator $\sum_{(i=1)}^{p} l_i\,\beta_i'\beta_i/n_i$ and proposed estimators Y'AY=β' U'AUβ subject to invariance and unbiasedness conditions, by using Euclidean norm. Where Euclidean norm is defined for any symmetric matrix M;

$\|M\|=\{tr[M^2]\}^{(1/2)}$ [10]

Namely MINQUE proposed to minimize matrix;

$\|U'AU-\Delta\|$ [11]

For this purpose MINQUE problem will change as minimizing the $tr[(AV)^2]$ subject to invariance and unbiasedness conditions. Where A is a symmetric matrix chosen subject to the conditions which guarantee the estimator's unbiasedness and invariance to changes in α (5).
Rao (10) proposes to minimize the variance of Y'AY subject to the conditions for unbiasedness and invariance. The variance of Y'AY is given by;

$Var(Y'AY)=2tr[(AW)^2]+\sum\sum_{(i=1)}^{p} K_i\sigma_i^4 tr(AV_i)^2$ [12]

where W is defined as $W=\sigma_1^2V_1+\cdots+\sigma_p^2V_p$, K_i is the common kurtosis of the variables in β_i. Under normality; when β_is are normally distributed, the kurtosis terms are zero; so that

$Var(Y'AY)=2tr[(AW)^2]$ [13]

Thus MIVQUE (Minimum Variance Quadratic Unbiased Estimation) under normality is identical to the MINQUE (5).
In variance components estimation methods, the effects of the factors on the dependent variables are explained with different models. These effects can be explained with fixed effect models, which are consisted of finite level of a factor; random effect models, which are chosen by random sampling between from infinite levels of a factor and mixed effect models, which are consisted from fixed effects and random effects (4,11,12).
Features of random effect model and fixed effect model for one-way classification are illustrated in Table 1 (4).
It is possible to see a lot of study in both national and international literature about variance components in different fields such as stock farming, biology and medicine (13-16).
In scientific studies related to estimating the variance components, although both simulation and real data are used in estimating variance components, in different studies with the same objective different methods are used and it is becoming a discussion topic as which method in which situations produce better results. In this context, in the study, some important methods such as; Analysis of Variance (ANOVA), Henderson, Maximum Likelihood (ML), Restricted Maximum Likelihood (REML), Minimum Norm Quadratic Unbiased Estimation (MINQUE) which are commonly used in literature to estimate variance components, were aimed to investigate comparatively for balanced-unbalanced data and interactive/non-interactive models.

MATERIAL AND METHODS

In this study, egg weight and Haugh unit which are obtained from 696 table eggs obtaining from two commercial layer herds aged 28-week (young) and 80-week (old) were used (17). Eggs in both age groups were stored in periods of 0, 15, 30 and 45 days. A part of eggs in each storage time were packaged with stretch film and a part was left in open area. On the other hand, the significance of interactions between variables was determined with multiple variable variance analysis (MANOVA) and the interactions which were considered significant were used in variance components estimation. The variance components pertinent to impact of hen age, storage time and condition of Haugh unit and egg weight were estimated with ANOVA Henderson, ML, REML and MINQUE methods. After the correction of the data for all the fixed effects in model, since Henderson II method includes Henderson I on the corrected data and in

Table 1. Features of random effect model and fixed effect model

Characteristic	Fixed Effects Model	Random Effects Model
Model equation	$y_{ij}=\mu+\alpha_i+e_{ij}$	$y_{ij}=\mu+\alpha_i+e_{ij}$
Mean of y_{ij}	$E(y_{ij})=\mu+\alpha_i$	$E(y_{ij}\|\alpha_i)=\mu+\alpha_i$ $E(y_{ij})=\mu$
α_i	Fixed	$\alpha_i \sim i.i.d.(0,\sigma_\alpha^2)$
e_{ij}	$y_{ij}-E(y_{ij})=y_{ij}-(\mu+\alpha_i)$ $e_{ij\sim i.i.d.(0,\sigma_e^2)}$	$y_{ij}-E(y_{ij}\|\alpha_i)=y_{ij}-(\mu+\alpha_i)$ $e_{ij\sim i.i.d.(0,\sigma_e^2)}$
$E(e_{ij}\alpha_i)$	$\alpha_i E(e_{ij})=0$	$E(e_{ij}\alpha_i)=0$
$Var(y_{ij})$	σ_e^2	$\sigma_\alpha^2+\sigma_e^2$
$cov(y_{ij},y_{i'j'})$	$\begin{cases}\sigma_e^2 & \text{for } i=i' \text{ and } j=j'\\ 0 & \text{otherwise}\end{cases}$	$\begin{cases}\sigma_\alpha^2+\sigma_e^2 & \text{for } i=i' \text{ and } j=j'\\ \sigma_\alpha^2 & \text{for } i=i' \text{ and } j\neq j'\\ 0 & \text{otherwise}\end{cases}$

i.i.d: Independent Identically Distribution

this research it is not used variables having fixed effects, there is not any comparison with this method and Henderson III is used instead.
The interactive mathematical model used in the estimation of variance components was expressed as follows;

$$Y_{ijkl}=\mu+a_i+b_j+c_k+[(ab)_{ij}+(ac)_{ik}+(bc)_{jk}+(abc)_{ijk}]+e_{ijkl} \quad [14]$$

In the model, Y_{ijkl} states for the observation pertinent to l. egg in group wherein i. states for hen age, j. states for storage time and k. states for storage condition, μ states for general mean, a_i i. states for impact of hen age, b_j j. states for impact of storage time, c_k k. states for impact of storage condition, e_{ijkl} states for random error with $N(0,\sigma^2)$ parameter. Besides, $(ab)_{ij}$ $(ac)_{ik}$, $(bc)_{jk}$ and $(abc)_{ijk}$ indicate the impact pertinent to interactions between hen age, storage time and storage condition and it is used in the model pertinent to dependent variable (Haugh unit) wherein interactions are found to be significant. As for the egg weight, the interactions between hen age, storage time and storage condition which were found to be insignificant are not included in the model ($Y_{ijkl}=\mu+a_i+b_j+c_k+e_{ijkl}$). In the model, it was assumed that all factors were random and different experimental design was constructed to compare the variance components methods in terms of balanced and unbalanced data.
In the study, SPSS (17.0) for Windows program is used in the analysis of the estimation of variance components related to egg weight and Haugh unit.

RESULTS

In the study, for the balanced data the effect of hen age on egg weight and Haugh unit are calculated equal by using four methods and the lowest estimation is given by ML method. According to other methods the highest rate is obtained by ML method for storage time. Though the effect of the storage condition on Haugh unit is found 0%, the effect of the storage condition on egg weight is found 0.36% by using ML method. In error variance, rates obtained from ML method are much higher than the equaled to each other methods (Table 2).
The results of the analysis for variance components of unbalanced data are presented in Table 3. In the estimation of variance components related to Haugh unit, Henderson III gives the highest rate (36.59%) for hen age, however, the lowest rate (23.05%) is obtained from ML method. To determine the effect of hen age on egg weight, the highest estimation is given by MINQUE (72.31%) and the lowest estimation is obtained by ML method (56.56%).
Findings about the comparison of error variance ratios according to variance components estimation methods are given in Table 4. Accordingly, in unbalanced data, though the explanation ratios of total variance of error variance for interactive method calculated as about 14% for ANOVA, REML and MINQUE methods, it is calculated higher with ML (18.32%) and Henderson III (17.39%) methods. In the non-interactive model, error variance ratio for ANOVA, Henderson III, REML and MINQUE methods is found about 27%, but it is found 42.16% by ML. As in balanced data the error ratio was calculated higher than the others.

DISCUSSION

According to the results, the reason of finding higher for both balanced and unbalanced data of error variance ratios obtained by ML method can be explained with the inhomogeneity of variances and not having normal

Table 2. Variance components analysis results for balanced data

Variance Components	Interactive Model (Haugh Unit)		Non-interactive Model (Egg Weight)	
	Other Methods	ML	Other Methods	ML
Hen Age	30.51%	16.36%	71.18%	55.45%
Storage Time	20.29%	22.01%	0.89%	1.35%
Storage Condition	4.80%	0.00%	0.23%	0.36%

ML: Maximum Likelihood

Table 3. Variance components analysis results for unbalanced data

Variance Components	Interactive Model (Haugh Unit)		Non-interactive Model (Egg Weight)	
	Highest	Lowest	Highest	Lowest
Hen Age	Henderson III (36.59%)	ML (23.05%)	MINQUE (72.31%)	ML (56.56%)
Storage Time	ML (44.79%)	Henderson III (37.02%)	ML (0.54%)	MINQUE (0.28%)
Storage Condition	ANOVA (1.3%)	ML (0.0%)	ML (0.74%)	REML (0.48%)

ANOVA: Analysis of Variance, ML: Maximum Likelihood, REML: Restricted Maximum Likelihood, MINQUE: Minimum Norm Quadratic Unbiased Estimate

Table 4. Comparison of error variance ratios according to methods

		ANOVA	Henderson III	ML	REML	MINQUE
Balanced Data	Interactive Model (Haugh Unit)	20.84%	20.84%	26.31%	20.84%	20.84%
	Non-Interactive Model (Egg Weight)	27.70%	27.70%	42.84%	27.70%	27.70%
Unbalanced Data	Interactive Model (Haugh Unit)	14.25%	17.39%	18.32%	14.86%	14.82%
	Non-Interactive Model (Egg Weight)	27.52%	26.84%	42.16%	27.08%	26.88%

ANOVA: Analysis of Variance, ML: Maximum Likelihood, REML: Restricted Maximum Likelihood, MINQUE: Minimum Norm Quadratic Unbiased Estimate

distribution at sub levels of each factor related to Haugh unit and egg weight data. Because, for the data which is consist of continuous variables, maximum likelihood estimation of variance components is based on generally normality distribution (4-6).

It is an expected result that the other methods, except from ML, give the same estimation related to variance components. Because ML method doesn't take into account the degree of freedom related to effects in model (4). Patterson and Thompson (18) report that one of the important features of REML method gives the equal estimation to ANOVA methods (Henderson I, II, III) for the balanced data. In another study (19), age of onset to lay, egg production, egg yield and average egg weight of each individual are collected at the end of the period of egg production and Henderson III, ML, REML and MINQUE methods which are commonly used to estimate variance components, were compared. As a result of the research, statistical differences were not generally found among the methods used for the estimation of variance components. The obtained result can be explained with the data collected from population that had a balanced design.

Khatree and Gill (20) made some comparisons for different experimental design and ANOVA was emphasized as the most favourite method to estimate . In contrast with REML was emphasized as the most favourite method to estimate. On the other hand, Patterson and Thompson (18) notified that due to Henderson III method is not practical in terms of calculation and estimators of ANOVA have negative estimate for unbalanced data, REML method gives the best result. Though Henderson (21) notified that if the degrees of freedom belonging to effects of factors are no more; REML and ML for balanced data and for unbalanced data REML give the best results. On the other hand, MINQUE estimation is the same as the first iteration of REML and it is also necessary to use the prior value for both MINQUE and REML. These connections of REML and MINQUE preoccupy that MINQUE is not a practical method for the estimation of variance components (4,22,23).

As a result of the research performed by Yolcu et al. (24) it have been informed that from variance components estimations made with different methods REML and ML methods which are based on probability theory gave more consistent results than ANOVA, and negative estimation problem of ANOVA were eliminated. Also, it have been informed that REML method eliminates the bias caused by ML and in the studies with unbalanced data, using the REML method will be effective in terms of more accurate estimation.

CONCLUSION

In accordance with this information and findings, it can be said that for balanced and normal distribution assumptions unrealized data in animal science except for the ML method, the other methods can be used. On the other hand, it can be suggested that subject to data structure REML method can be used when the degree of freedom is low and data is unbalanced animal science. In addition to this it is expected that evaluation of the obtained results in papers which will be done with different experimental designs and data which have different distribution and comparison of the evaluation with the research results will contribute to the statistical literature as well as the researchers in different areas in need of these methods.

Acknowledgements

The authors are grateful to Alper YILMAZ and Zehra BOZKURT for their support to obtain the data of this study.

REFERENCES

1. Fisher RA. Statistical Methods for Research Workers. Edinburgh and London: Oliver & Boyd; 1925.
2. Crump SL. The Estimation of Variance Components in Analysis of Variance. Biometrics. 1946; 2(1):7-11.
3. Henderson CR. Estimation of Variance and Covariance Components. Biometrics. 1953; 9(2): 223-52.
4. Searle SR, Casella G, McCulloch CE. Variance components. New Jersey: A John Wiley & Sons Inc.; 2006.
5. Sahai H, Ojeda MM. Analysis of Variance for Random Models, Volume II: Unbalanced Data. Boston, Basel, Berlin: Birkhäuser; 2005.
6. Searle SR. Linear Models. New York: A John Wiley & Sons Inc.; 1971.
7. Hartley H O, Searle SR. A Discontinuity in Mixed Model Analysis. Biometrics. 1969; 25(3): 573-6.

8. Rao CR. Estimation of Heteroscedastic Variances in Linear Models. Journal of the American Statistical Association. 1970; 65(329): 161-72.
9. Rao CR. Estimation of Variance and Covariance Components in Linear Models. Journal of the American Statistical Association. 1972; 67(337): 112-5.
10. Rao CR. Minimum Variance Quadratic Unbiased Estimation of Variance Components. Journal of Multivariate Analysis. 1971; 1(4): 445-56.
11. Smith DW, Hocking RR. Maximum Likelihood Analyses of the Mixed Model: The Balanced Case. Communications in Statistics - Theory and Methods. 1978; 7(13): 1253-66.
12. Andrade DF, Helms RW. Maximum Likelihood Estimates in the Multivariate Normal with Patterned Mean and Covariance via the EM Algorithm. Communications in Statistics - Theory and Methods. 1984; 13(18): 2239-51.
13. Doğan İ, Özçelik M, Doğan N. Tekrarlı Ölçüm Düzenlerinde Varyans Analizi Yöntemi ile Holştaynlarda Süt Veriminin Kalıtım Derecesinin Tahmini. Turkish Journal of Veterinary and Animal Sciences. 1999; 23(2): 269-76.
14. Saatçi M, Dewi I, Ulutaş Z. Effects of Embryo-transfer, Recipient-donor Mother and Environment on Lamb Weaning Weight and Variance Components. Turkish Journal of Veterinary and Animal Sciences. 2002; 26(2): 215-20.
15. Koyuncu M, Duru S. Estimates of (Co)Variance Components for Direct and Maternal Effects on Birth Weight of Karacabey Merino Lambs. Turkish Journal of Veterinary and Animal Sciences. 2009; 33(3): 235-40.
16. Gevrekçi Y, Oğuz İ, Akşit M, Önenç A, Özdemir D, Altan Ö. Heritability and Variance Component Estimates of Meat Quality in Japanese Quail (Coturnix coturnix japonica). Turkish Journal of Veterinary and Animal Sciences. 2009; 33(2): 89-94.
17. Yılmaz AA, Bozkurt Z. Ana Yaşı, Depolama Süresi ve Streç Film ile Paketlemenin Sofralık Yumurtaların İç ve Dış Kalite Özelliklerine Etkisi. Lalahan Hayvancılık Araştırma Enstitüsü Dergisi. 2008; 48(2): 81-91.
18. Patterson HD, Thompson R. Recovery of Inter-block Information when Block Sizes are Unequal. Biometrika. 1971; 58(3): 545-54.
19. Akbaş Y, Settar P, Türkmut L. Kanatlılarda Yumurta Verimi Özellikleri için dört Farklı Varyans Komponent Tahminleme Yönteminin Karşılaştırılması. International Poultry Congress; 1993 May 13-14; İstanbul; 1993. p. 459-66.
20. Khattree R, Gill DS. Comparison of Some Estimates of Variance Components Using Pitman’s Nearness Criterion. Proceedings, American Statistical Association, Statistical Computing Section. Alexandria, VA: ASA; 1988. p. 133-6.
21. Henderson CR. Applications of linear models in animal breeding. Ontario: Guelph; 1984.
22. Hocking RR, Kutner MH. Some Analytical and Numerical Comparisons of Estimators for the Mixed A.O.V. Model. Biometrics. 1975; 31(1): 19-28.
23. Brown KG. Asymptotic Behavior of MINQUE-type Estimators of Variance Components. The Annals of Statistics. 1976; 4(4): 746-54.
24. Yolcu Hİ, Balcıoğlu MS, Fırat MZ, Karabağ K. Beyaz Yeni Zelanda Tavşanlarında Canlı Ağırlıklara ait Varyans Unsurlarının Farklı Yöntemlerle Tahmini. Akdeniz Üniversitesi Ziraat Fakültesi Dergisi. 2004; 17(1): 81-5.