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A Performance Indicator Model for the Assessment of Program Outcomes

Abstract

Outcome-Based Education (OBE) evaluates learning success by measuring the knowledge, skills, and competencies students are expected to achieve upon graduation. However, common program outcome (PO) assessment methods rely on course-level weighted averages, which limit detailed analysis and obscure the contribution of individual assessment components. To address this limitation, this study proposes a Performance Indicator Model (PIM) that measures program outcome attainment using question-level assessment data. The model incorporates curriculum-based weighting factors—such as course ECTS credits, assessment contribution ratios, and learning outcome–program outcome (LO–PO) relationships—within a hierarchical mathematical framework. This approach enables normalized aggregation of question-level performance at the program outcome level and produces scale-independent, PO-specific indicators. The effectiveness of the proposed model is demonstrated through an application scenario based on an undergraduate software engineering program. The results highlight substantial differences between PO-based indicators and traditional weighted averages, confirming the model's potential to support accreditation-aligned, data-driven quality assurance and continuous improvement processes in diverse OBE contexts.

Keywords: Evaluation methodologies, Program outcomes, Performance indicator, Micro-level assessment, Outcome-based education.

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Program Çıktılarının Değerlendirilmesi için Bir Performans Göstergesi Modeli

Öz

Çıktı Temelli Eğitim (Outcome-Based Education – OBE), öğrenme başarısını öğrencilerin mezuniyet sırasında sahip olmaları beklenen bilgi, beceri ve yetkinlikleri ölçerek değerlendirir. Ancak yaygın program çıktısı (Program Outcome – PO) değerlendirme yöntemleri, ders düzeyindeki ağırlıklı ortalamalara dayanmaktadır. Bu durum ayrıntılı analiz yapılmasını sınırlandırmakta ve bireysel değerlendirme bileşenlerinin katkısını görünmez hale getirmektedir. Bu sınırlamayı gidermek amacıyla, bu çalışma program çıktısı kazanımını soru düzeyindeki değerlendirme verilerini kullanarak ölçen bir Performans Göstergesi Modeli (Performance Indicator Model – PIM) önermektedir. Model; derslerin AKTS kredileri, değerlendirme katkı oranları ve öğrenme çıktısı–program çıktısı (LO–PO) ilişkileri gibi müfredat temelli ağırlıklandırma faktörlerini hiyerarşik bir matematiksel yapı içinde bütünleştirir. Bu yaklaşım, soru düzeyindeki performans verilerinin program çıktısı düzeyinde normalize edilerek birleştirilmesini mümkün kılmakta ve ölçekten bağımsız, program çıktısına özgü göstergeler üretmektedir. Önerilen modelin etkinliği, lisans düzeyinde bir yazılım mühendisliği programına dayalı uygulama senaryosu üzerinden gösterilmiştir. Elde edilen sonuçlar, PO tabanlı göstergeler ile geleneksel ağırlıklı ortalamalar arasında önemli farklılıklar olduğunu ortaya koyarak modelin akreditasyon odaklı, veri temelli kalite güvencesi ve sürekli iyileştirme süreçlerini destekleme potansiyelini doğrulamaktadır.

Anahtar kelimeler: Değerlendirme yöntemleri, Program çıktıları, Performans göstergesi, Mikro düzey değerlendirme, Çıktı temelli eğitim.

1. Introduction

Quality assurance and program accreditation in higher education have gained increasing global importance and now constitute one of the core components of institutional accountability. These processes are fundamentally based on clearly defined learning outcomes, the objective and reproducible measurement of these outcomes, and the continuous improvement of academic programs informed by empirical evidence. International frameworks such as the Washington Accord, the Bologna Process, and ABET have guided higher education institutions toward defining and monitoring program outcomes in a transparent, measurable, and comparable manner.

In this context, the need for traceable and evidence-based assessment models that go beyond documentation-oriented approaches has become increasingly apparent. In particular, the engineering education literature has focused on developing models and tools for the student-centered, objective, and quantitative evaluation of program outcomes. Within the framework of outcome-based education (OBE), assessment approaches based on systematic mappings between course learning outcomes (LOs) and program outcomes (POs), and implemented on a periodic basis, have been shown to support continuous quality improvement processes.

Despite the growing body of literature on outcome-based assessment, existing approaches predominantly rely on course-level aggregates, rubric-based summaries, or indirect evaluation instruments, which often obscure the granular evidence underlying program outcome attainment. Even models that employ direct assessment frequently perform early-stage aggregation, thereby limiting traceability from individual assessment items to program-level indicators. Moreover, current frameworks rarely integrate multiple weighting dimensions-such as ECTS credits, assessment contribution ratios, and LO–PO relationship strengths-within a unified and mathematically normalized structure. These limitations result in performance indicators that are insufficiently sensitive to micro-level variations and provide limited diagnostic value for continuous improvement and accreditation decision-making.

To address these gaps, this study proposes a novel Performance Indicator Model (PIM) that quantifies program outcome attainment using question-level assessment data within a hierarchical, multi-layered mathematical framework. By systematically propagating proportional question scores through LO–PO mappings and incorporating curriculum- and assessment-specific weighting mechanisms, the proposed model ensures full traceability from micro-level evidence to normalized program outcome indicators. Unlike traditional weighted averages, PIM produces scale-invariant and comparable performance measures that reveal hidden imbalances across program outcomes and support data-driven curriculum evaluation.

The remainder of this paper is organized as follows. Section 2 reviews related work on OBE, focusing on LO–PO alignment, outcome measurement models, and performance indicator–based assessment for continuous improvement. Section 3 introduces the proposed Performance Indicator Model, detailing its core components, operational steps, and computational complexity. Section 4 presents an application scenario and reports the results obtained from a representative dataset. Section 5 discusses threats to validity and model limitations, and Section 6 concludes the paper with a summary of findings and directions for future research.

2. Related Work

Outcome-Based Education (OBE) has become one of the fundamental pillars of quality assurance and accreditation processes in higher education. The OBE approach [19] emphasizes defining and assessing the knowledge, skills, and competencies that graduates are expected to possess as measurable outcomes, rather than focusing on the inputs of the educational process. In this context, the literature concentrates on how Learning Outcomes (LOs) and Program Outcomes (POs) should be aligned, which methods should be used to measure the level of outcome attainment, and how the resulting evidence should be integrated into Continuous Quality Improvement (CQI) processes [20].

OBE assessment typically follows a hierarchical structure in which the institution or degree program's vision and mission, Program Educational Objectives (PEOs), POs, and LOs are systematically aligned. One of the most critical and at the same time most problematic stages of OBE implementation is defining the relationship between LOs and POs [21]. In traditional practices, this relationship often relies on the subjective judgment of instructors and is expressed through qualitative descriptors such as "weak–moderate–strong," which reduces measurement reliability. To address this limitation, a mathematically justifiable LO–PO mapping model based on competencies and performance indicators has been proposed [18]. In this model, the level of alignment is objectively determined by calculating the degree of overlap between performance indicators defined for each program outcome and course-level learning outcomes. Similarly, it has been emphasized that aggregation from LOs to POs may lead to a loss of measurement granularity; to mitigate this issue, a more granular evaluation model based on rubric-driven, discipline-specific, and generic competencies has been developed [6].

Another mathematical approach aimed at increasing granularity is the FAPLO model, which is grounded in set theory and matrix algebra [2]. This model enables the measurement of program outcomes at the beginning, middle, and end of the program, allowing both horizontal and vertical tracking of learning progression. In a related study, a dynamic OBE model was proposed to further enhance measurement precision. This model argues that LO–PO mapping weights should not be fixed but instead derived directly from assessment data. Accordingly, examination questions and assessment instruments are explicitly linked to LOs and POs, and the weights are computed automatically from the collected evidence [3].

The question of which model should be used to measure program outcomes remains a subject of debate in the literature. Comparative studies of cumulative and culminating models have shown that both approaches are reliable; however, the culminating model yields marginally higher average PO attainment values [12]. The same study also reports that the cumulative model contributes more effectively to CQI processes by enabling early intervention and continuous monitoring. For OBE to be implemented effectively in practice, performance indicators corresponding to outcomes must be clearly defined and measurable. Particularly in project-based courses, it is recommended that ABET student outcomes be decomposed into Bloom's Taxonomy–aligned performance indicators and assessed using analytic rubrics [15]. It is emphasized that generic and ambiguous performance indicators reduce measurement reliability; therefore, subject-specific performance indicators and hybrid rubrics should be preferred [9].

The OBE literature clearly demonstrates that direct assessment methods are more reliable than indirect methods. In a study comparing different approaches to measuring course outcomes, stratified-sampling-based direct assessment was found to produce the most balanced and reliable results, whereas indirect assessments based on student perception overestimated outcome attainment by approximately 20–60% [7]. Similarly, in a model for quantitatively measuring course outcomes, directly linking exam and assignment scores to outcomes was shown to provide more meaningful data for program improvement [13].

One example of course-level OBE implementation is a hybrid OBE model developed for a control systems design course [14], in which software-supported assessment enabled quantitative tracking of LO and PO attainment. A standardized assessment instrument developed for laboratory courses allowed more objective measurement of practical skills and revealed deficiencies in problem-solving competencies [23]. At the program level [5], automated systems integrating multiple assessment instruments have been effectively used in electrical engineering programs for ABET equivalency processes [16]. Harmanani [8] proposed a bottom-up, outcome-based assessment approach for computing programs seeking ABET accreditation [17], in which course-level outcomes are mapped to ABET student outcomes [10] using an I-R-E (Introduce–Reinforce–Emphasize) matrix, and data obtained from direct and indirect assessment tools are evaluated holistically using visualization techniques and threshold-based metrics.

To extend the OBE approach beyond engineering disciplines, Bohra et al. [1] proposed a systematic and measurable framework for assessing POs in postgraduate management programs. Their four-step model enables the alignment of LOs and POs, structures assessment instruments based on Bloom's Taxonomy, and quantitatively determines PO attainment through weighted calculations.

In OBE implementations, the most critical aspect of reliably measuring the attainment of POs and LOs is the definition of Performance Indicators (PIs) that represent these outcomes. The literature emphasizes that assessment approaches based on outcomes defined through general and abstract statements yield low validity and reliability; therefore, clear, context-specific, and Bloom's Taxonomy–aligned PIs must be defined for each outcome. In particular, the use of hybrid rubrics based on specific PIs increases inter-rater consistency and more accurately reflects actual student performance [9]. In this approach, PIs decompose complex engineering tasks into measurable sub-steps, enabling both formative and summative assessment.

Another critical dimension of PI-based assessment [22] is the classification of performance indicators according to cognitive, affective, and psychomotor domains. Automated OBE systems developed in this direction can indicate not only whether program outcomes have been achieved, but also in which learning domains and at what depth they have been realized [11]. Similarly, PI-based analytic rubrics [4] used in project- and design-oriented courses enable the evidence-based assessment required by accreditation frameworks such as ABET and ensure that the results can be directly fed into CQI processes [15]. Taken together, the literature demonstrates that performance indicators constitute a fundamental building block in OBE systems, serving as a bridge between abstract outcomes and measurable learning behaviors. While technological advancements in online assessment platforms facilitate the collection of granular performance data [24], the primary focus of such tools often remains on administrative efficiency and user interface design, rather than on the methodological frameworks needed to transform this data into valid, traceable, and program-level outcome indicators.

3. Material and Method

In this study, a novel Performance Indicator Model (PIM) is proposed to address the loss of granularity and the limited discriminative capability frequently observed in existing approaches for assessing POs. Rather than evaluating student achievement solely at the course or learning outcome level, the proposed model quantifies PO attainment based on individual assessment questions, enabling a more precise and traceable measurement of outcome achievement. This approach provides an evaluation framework aligned with the principles of objectivity, transparency, and repeatability required in accreditation processes.

The proposed model is formulated through a hierarchical mathematical structure that explicitly accounts for the multi-layered nature of the assessment process. Within this structure, course ECTS credits, contribution ratios of assessment components (e.g., midterm exams, final exams, homework, and projects), and the strength of relationships between learning outcomes and program outcomes are integrated in a unified manner. The proportional performance obtained from each assessment question is combined with these weighting factors to produce normalized performance indicators at the program outcome level. This formulation ensures scale-invariant and comparable results across different courses and assessment types.

This section first presents the core components that define the conceptual framework and data structure of the proposed model. Subsequently, the ECTS-based weighting scheme, the mapping mechanism between learning outcomes and program outcomes, and the thresholding process used to identify meaningful outcome relationships are described in detail. Finally, the mathematical formulation employing proportional scores and weighting vectors is introduced, explaining how performance indicators for program outcomes are computed.

The conceptual structure and data model of the proposed Performance Indicator Model (PIM) are illustrated through the UML class diagram presented in Fig. (1). The diagram formalizes a data organization specifically designed to support micro-level measurement of student performance with respect to POs. The core entities of the model include Student, ProgramOutcome, Course, LearningOutcome, Exam, ExamQuestion, and ExamQuestionScore, which collectively represent the hierarchical structure of outcome-based assessment.

At the center of the model lies the $LOPORelationship$ class, which defines and manages the mapping between LOs and POs. This class enables the systematic association of each exam question with its corresponding PO through the δ parameter defined in the mathematical formulation of the model. As a result, question-level performance evidence is consistently aligned with program-level evaluation targets.

The multilayered relationships depicted in the figure ensure full traceability of student achievement, spanning from ECTS-weighted courses to individual exam questions. Moreover, the diagram clarifies how the fundamental mathematical components of the PIM—namely the proportional score (γ), the outcome association indicator (δ), and the weighting vector (ω)—are directly derived from the underlying assessment data, thereby establishing a coherent link between the conceptual model and the proposed analytical framework.

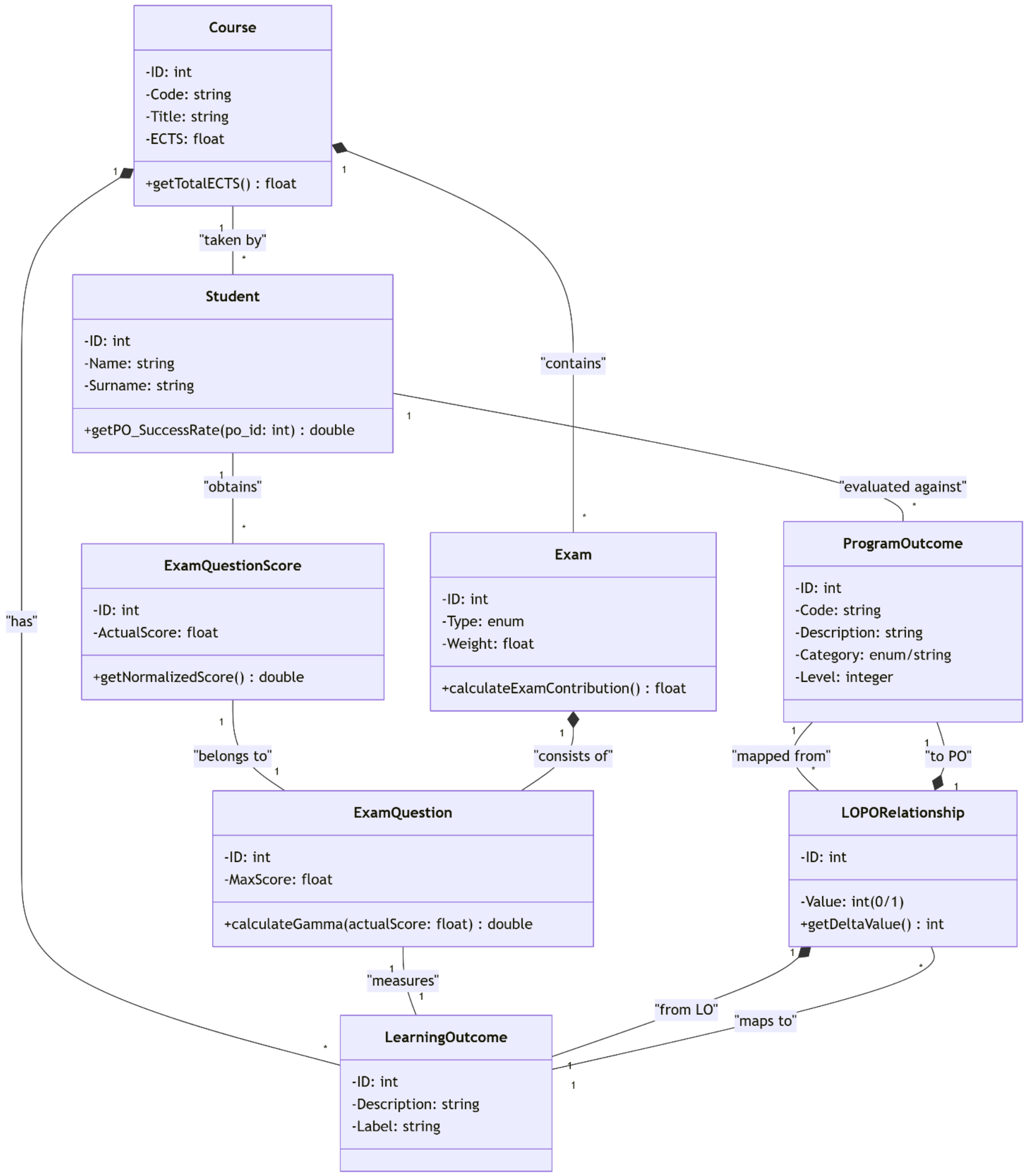

Figure 1. Conceptual data model of the proposed Performance Indicator Model (PIM)

3.1. Core Components of the PIM

This subsection introduces the fundamental components that constitute the PIM and underpin its analytical framework. The model is structured around a set of interrelated entities and parameters designed to capture student performance at the micro level while maintaining alignment with program-level learning objectives. These components collectively enable the integration of assessment data, curricular structure, and outcome relationships within a unified evaluation framework.

Specifically, the core components define how student performance evidence is collected at the question level, how learning outcomes and program outcomes are formally linked, and how multiple weighting mechanisms—such as course ECTS credits, assessment contribution ratios, and outcome relationship strengths—are incorporated into the model. The roles and mathematical definitions of these components are detailed in the following

3.1.1. ECTS Credit and Workload Calculation

The European Credit Transfer and Accumulation System (ECTS) is a standardized credit framework developed within the scope of the European Higher Education Area (EHEA) and the Bologna Process. It is based on the student workload required to achieve the intended learning outcomes of a course. The ECTS credit of a course represents the estimated total amount of time a student is expected to spend in order to attain these learning outcomes. This workload includes all academic activities such as lectures, laboratory sessions, project development, individual study, preparation for examinations, and similar tasks. According to widely accepted practice, 1 ECTS credit corresponds to approximately 25-30 hours of student workload.

The ECTS credit value of a course is determined by calculating the total workload associated with all learning activities required to achieve the course's intended learning outcomes. This workload covers lecture hours, laboratory work, assignments and project preparation, individual study periods, exam preparation, and other relevant learning activities. To ensure that the estimation of this workload is systematic and measurable, institutions utilize an ECTS Workload Table. As illustrated in Fig. (2), this table categorizes each learning activity separately and specifies the estimated number of hours associated with each activity.

ECTS Workload Calculation Table			
Learning-Teaching Activity	**Weeks**	**Hours**	**Workload**
Class Hours (Theoretical)	14	3	42
Class Hours (Practical)	14	2	28
Other Activities	14	4	56
Midterm Exam	1	1	1
Homework/Assignment	3	6	18
Final Exam	1	2	2
		Total Workload	**147**
		Total Workload / 30	**4.90**
		ECTS Credits	**5**

Figure 2. ECTS workload calculation table

The ECTS credit assigned to each course constitutes one of the key components of the proposed PIM. As a core instrument of the Bologna Process, ECTS serves as a standardized system for quantifying student workload and ensuring academic transparency and comparability across higher education institutions. Beyond facilitating international student mobility, ECTS provides a quantitative representation of the academic weight of each course and the workload it imposes on students. Accordingly, transcripts and graduation documents typically report student achievement using weighted grade-point averages (GPA) in which ECTS credits play a central role. Thus, the ECTS value of a course is a strong indicator of its relative importance within the curriculum and its potential contribution to learning outcomes.

In the proposed model, the ECTS credit of a course is treated as a fundamental parameter for weighting its contribution to POs. Courses with higher ECTS credits are expected to exert a greater influence on students' overall PO achievement. For this reason, ECTS serves as an objective and central component within the model's weighting mechanism.

3.1.2. Mapping learning outcomes to program outcomes

Courses, which constitute the fundamental building blocks of an academic program, are designed to enable students to acquire specific knowledge, skills, and competencies. These intended targets are defined as course Learning Outcomes (LOs). LOs describe, in clear, measurable, and observable terms, what a student who successfully completes a course will know, be able to do, and be competent in. For example, a learning outcome for a mathematics course may be stated as: "the ability to solve basic engineering problems using the concepts of derivatives and integrals."

Learning outcomes serve as the focal point of curriculum design, the selection of instructional methods, and ultimately the assessment and evaluation processes. A key instrument of outcomes-based education and accreditation practices is the LO–PO matrix, which demonstrates the relationship between course-level learning outcomes and the broader program-level graduation targets defined as Program Outcomes (POs). An example LO–PO relationship matrix is presented in Fig. (3).

	PO #01		PO #02			PO #03		PO #04		PO #05		PO #06		PO #07		PO #08		PO #09			PO #10		PO#11	
	1.1	1.2	2.1	2.2	2.3	3.1	3.2	4.1	4.2	5.1	5.2	6.1	6.2	7.1	7.2	8.1	8.2	9.1	9.2	9.3	10.1	10.2	11.1	11.2
LO #1	4								2															
LO #2		4		5																				
LO #3					3													3						
LO #4								1				2		4										
LO #5						5																		
LO #6													3				4							
LO #7									4															

Figure 3. An Example LO–PO Relationship Matrix

This matrix systematically illustrates how each course in the program's annual curriculum aligns with the POs. Exam questions used in the assessment of a course are expected to correspond to the course's designated LOs. Consequently, the PO to which a given exam question contributes is not determined directly at the PO-level; rather, it is inferred through the LO that the question assesses and the corresponding relationship value specified in the LO–PO matrix.

The proposed model systematically maps the LOs defined for each course to the corresponding POs through a predefined relationship matrix, as illustrated in the example in Fig. (3). Let C denote a course offered within the annual curriculum of an academic program. The relationships between the LOs of this course and the POs are represented by the matrix $\mathrm{R}^{(C)} \in \mathbb{R}^{\mathrm{L}\times\mathrm{P}}$, as shown in Eq. (1), where L is the number of LOs and P is the number of POs. In this matrix, the columns correspond to the POs, the rows correspond to the course's LOs, and each entry $r_{i,j}^{(C)}$ indicates the degree of relationship between LO_i and PO_j on a scale of $[0, 5]$ A value of 0 denotes no relationship, values of $1-2$ indicate a low level of contribution, 3 indicates a moderate contribution, and values of $4-5$ represent a strong contribution.

$$R^{(C)} = \begin{bmatrix} r_{1,1}^{(C)} & r_{1,2}^{(C)} & \cdots & r_{1,P}^{(C)} \\ r_{2,1}^{(C)} & r_{2,2}^{(C)} & \cdots & r_{2,P}^{(C)} \\ \vdots & \vdots & \ddots & \vdots \\ r_{L,1}^{(C)} & r_{L,2}^{(C)} & \cdots & r_{L,P}^{(C)} \end{bmatrix} \quad i \in \{1,\ldots,L\}, \quad j \in \{1,\ldots,P\} \tag{1}$$

This scale is an analytical rating approach commonly used in mapping MÜDEK/ABET program outcomes to course-level learning outcomes. By ensuring that the assessment focuses solely on questions aligned with the relevant outcomes, this approach enhances the accuracy and meaningfulness of the model. The relationship matrix $R^{(C)}$thus enables the establishment of a consistent and traceable linkage between macro-level (program) objectives and micro-level (question) performance.

3.1.3. Threshold-based filtering of significant relationships

To isolate only the meaningful LO–PO relationships for subsequent computations, a threshold value τ is introduced. In this study, the threshold parameter is set to τ = 0 so that all LO–PO relationships defined in the mapping matrix are retained, ensuring that no potential outcome contribution is excluded from the analysis. For a given LO_i, the row vector $p^{(i)} = \left(r_{i,1}^{(C)}, r_{i,2}^{(C)}, \dots, r_{i,P}^{(C)}\right)$is extracted from the matrix $R^{(C)}$. The index set of the POs that exhibit a significant relationship with LO_i is then defined as in Eq. (2):

$$S_\tau(i) = \left\{ j \in \{1, \dots, P\} \;\middle|\; r_{i,j}^{(C)} \geq \tau \right\} \tag{2}$$

Next, the matched set that contains both the PO indices and their corresponding relationship strengths is constructed as shown in Eq. (3):

$$\Psi_S(i) = \left\{ \left(j, r_{i,j}^{(C)}\right) \;\middle|\; j \in S_\tau(i) \right\} \tag{3}$$

In the proposed model, only the relationship strengths within the set $\Psi_S(i)$are used for weighting. Accordingly, for a given LO_i_i, the collection of relationship scores that contribute to the model is defined as in Eq. (4):

$$A^{(i)} = \left\{ p_j \;\middle|\; \left(j, p_j\right) \in \Psi_S(i),\; p_j = r_{i,j}^{(C)} \right\} \tag{4}$$

3.1.4. Weight vector of performance indicator components

Let course C have an ECTS credit value denoted by $ECTS^{(C)}$. Assume that the course includes several examinations indexed by e (e.g., midterm, final). Each examination e is assigned a predefined weight $E_w^{(C,e)} \in [0,1]$ within the overall grading scheme of the course (for instance, 0.4 for a midterm weighted at 40%). Every question q in examination e is associated with exactly one learning outcome LO_i Given these components, the weighted contribution of a question to the Performance Indicator is directly proportional to the total relationship strength between the LO being assessed and the corresponding POs. This contribution is represented by the weight vector $\vec{\omega}^{(C,e,q)}$and is formulated as shown in Eq. (5).

$$\begin{aligned} \vec{\omega}^{(C,e,q)} &= ECTS^{(C)} \cdot E_w^{(C,e)} \cdot \sum_{p \in A^{(LO(q))}} \frac{p}{5} \\ \vec{\omega}^{(C,e,q)} &= \left(\omega_1^{(C,e,q)}, \omega_2^{(C,e,q)}, \dots, \omega_P^{(C,e,q)}\right) \end{aligned} \tag{5}$$

Here, P is the number of POs and $A^{(LO(q))}$denotes the set of relationship scores associated with all Program Outcomes that are significantly related (i.e., $p \geq \tau$) to the Learning Outcome $LO(q)$ targeted by question q. The values $p \in \{1,2,\dots,5\}$ are integers taken from the LO–PO relationship matrix $R^{(C)}$ and indicate the contribution level between the corresponding LO–PO pair. The summation term quantitatively represents the total contribution potential of the respective LO to the POs. The constant 5 in the denominator is the maximum attainable relationship score; dividing by this value normalizes the total relationship strength, ensuring that the weights derived from different LOs remain on a single, consistent scale.

3.1.5. Normalization vector

In the proposed model, a normalization procedure is applied to ensure that the achievement level of each PO can be expressed within the range [0, 1] or as a percentage. The normalization vector $\vec{\Omega}$ represents the total weighted contributions of all questions across all examinations in all curriculum courses that contribute to a given PO. It is computed as shown in Eq. (6).

$$\vec{\Omega} = \sum_{C}\sum_{e}\sum_{q}\vec{\omega}^{(C,e,q)} = (\Omega_1, \Omega_2, \ldots, \Omega_P) \tag{6}$$

Here, P is the number of POs.

3.1.6. Proportional score

The primary data input of the model is the normalized representation of students' performance on each exam question. The proportional question score γ is defined as the ratio of the raw score obtained by a student on a given question to the maximum attainable score for that question. For a particular student, let $\gamma_{C,e,q} \in [0, 1]$ denote the proportional question score achieved on question q of examination e in course C. Accordingly, the proportional question score for a student's response to a specific question is computed as shown in Eq. (7).

$$\gamma_{C,e,q} = g_{C,e,q} / G_{C,e,q} = g_{C,e,q} \cdot G_{C,e,q}^{-1} \tag{7}$$

Here, $\gamma_{C,e,q}$ represents the proportional score, taking a value between 0 and 1 for question q in examination e of course C $g_{C,e,q}$denotes the student's raw score for that question, and $G_{C,e,q}$ denotes the maximum possible score. The proportional question score calculation standardizes performance comparisons across questions with different point values, thereby converting all questions into a common, comparable scale within the range [0, 1].

3.1.7. Formulation of the performance indicator

In the final stage of the computation, the Performance Indicator (PI) associated with each program outcome is derived by aggregating the student's weighted proportional scores at the question level and normalizing this aggregate by the maximum attainable contribution for the corresponding outcome. This formulation ensures that performance is evaluated relative to the full assessment potential defined by the curriculum structure. Formally, the vector of performance indicators is defined in Eq. (8).

$$\overrightarrow{\mathrm{PI}} = \frac{\sum_{C}\sum_{e}\sum_{q}\gamma_{C,e,q} \cdot \vec{\omega}^{(C,e,q)}}{\vec{\Omega}} = (\mathrm{PI}_1, \mathrm{PI}_2, \ldots, \mathrm{PI}_P), \tag{8}$$

Here, $\gamma_{C,e,q}$ represents the proportional score, and $\vec{\omega}^{(C,e,q)}$ represents the corresponding question-level weight vector. The normalization vector $\vec{\Omega}$ contains the maximum attainable weighted contributions for each program outcome. Accordingly, the performance indicator for a specific program outcome PO_j is given by

$$\mathrm{PI}_j = (\Omega_j)^{-1} \cdot \sum_{C}\sum_{e}\sum_{q}\left(\gamma_{C,e,q} \cdot \omega_j^{(C,e,q)}\right) \tag{9}$$

In this expression, the numerator represents the cumulative weighted performance obtained from all questions contributing to PO_jwhile the denominator Ω_j corresponds to the maximum possible weighted contribution that would be achieved if the student attained a perfect score ($\gamma = 1$) on all relevant questions. Consequently, PI_j is interpreted as a normalized achievement ratio in the range [0, 1], providing a scale-invariant and comparable measure of the student's attainment level with respect to the targeted program outcome.

3.1.8. Methodology for Determining LO–PO Relationship Strengths

Although LO–PO relationship matrices are widely used in outcome-based education systems, their construction often relies on expert judgment. To reduce subjectivity and increase methodological transparency, a structured procedure was adopted in this study. The determination of LO–PO relationship strengths followed a three-stage process:

- **Stage 1. Outcome Decomposition:** Each PO was decomposed into measurable performance indicators (PIs) aligned with the cognitive levels of Bloom's Taxonomy. This decomposition ensured that each outcome corresponds to clearly observable and assessable competencies.

- **Stage 2. LO–PI Alignment:** Each course LO was analyzed with respect to the defined performance indicators associated with the program outcomes. The degree of alignment between an LO and the corresponding PO was evaluated through expert consensus based on the relevance of the targeted competencies.

- **Stage 3. Numerical Scaling:** The strength of the relationship between learning outcomes and program outcomes was quantified using an ordinal contribution scale ranging from 0 to 5. In this scale, a value of 0 indicates that the learning outcome has no observable relationship with the corresponding program outcome, whereas higher values represent progressively stronger levels of contribution. Specifically, a score of 1 denotes a very weak contribution, indicating minimal alignment between the learning outcome and the targeted program outcome. A score of 2 represents a weak contribution, suggesting a limited but identifiable relationship. A value of 3 indicates a moderate contribution, where the learning outcome meaningfully supports the achievement of the program outcome. A score of 4 reflects a strong contribution, demonstrating substantial alignment between the learning outcome and the program outcome. Finally, the highest value of 5 represents a very strong contribution, indicating that the learning outcome plays a primary and dominant role in achieving the corresponding program outcome.

To enhance evaluation reliability and ensure academic consistency, the LO–PO mappings were determined through the joint evaluation of the course coordinator and the instructors responsible for delivering the course. During this process, the scope of the course learning outcomes, course contents, assessment instruments, and their cognitive alignment with the program outcomes were analyzed collectively. Subsequently, the contribution level of each learning outcome to the relevant program outcome was evaluated using the predefined 0–5 scale, and the final relationship value was determined through consensus among the instructors. This collaborative evaluation approach leverages the expertise of the academic staff directly involved in the course design and assessment processes, thereby supporting the development of LO–PO relationship matrices that are systematic, consistent, and contextually grounded.

3.2. Operational steps and pseudocode of the PIM

Within the proposed Performance Indicator Model (PIM), the Performance Indicator PI_j for a student with respect to a specific Program Outcome PO_j is computed by following the systematic steps outlined below:

- **Step 1. Definition of Data and Parameters:** The set of courses C to be included in the assessment and the ECTS credit value of each course $ECTS^{(\boldsymbol{C})}$are identified. For each course, the examinations e and their corresponding weight contributions to the course grade $E_w^{(C,e)}$are specified. Each question q in each examination is mapped to one LO of the course, and the LO–PO relationship matrix $R^{(\boldsymbol{C})}$ is used to determine the POs significantly related to that LO. The raw score obtained by the student for each question $g_{C,\boldsymbol{e},\boldsymbol{q}}$ and the maximum attainable score $G_{C,e,q}$ are recorded.

- **Step 2. Calculation of the Proportional Question Score:** For every exam question, the proportional score $\gamma_{C,e,q}$, which normalizes the student's performance to the interval $[0,1]$, is computed according to Eq. (7).

▪ **Step 3. Determination of the Component Weight Vector:** Considering the total contribution potential of the corresponding LO to the POs, the component weight vector $\vec{\omega}^{(C,e,q)}$ for each question is computed using Eq. (5).

▪ **Step 4. Calculation of the Normalization Vector:** The normalization vector is obtained by summing the component weights of all questions across all examinations of all assessed courses, as defined in Eq. (6).

▪ **Step 5. Computation of the Performance Indicator:** In the final step, the normalized performance indicator for PO_j is calculated by dividing the total weighted proportional scores achieved by the student by the normalization vector, as shown in Eq. (8). This yields a comprehensive weighted average that incorporates all assessment components.

The computational workflow of the proposed Performance Indicator Model (PIM) is formalized in Algorithm (1). The pseudocode operationalizes the theoretical formulation of the model into an executable procedure by traversing the relevant curriculum components and aggregating student performance evidence. The algorithm reflects the hierarchical structure of the assessment process—progressing from individual exam questions to program outcomes—while integrating the multidimensional weighting mechanisms defined in Eq. (3–6).

3.3. Time complexity analysis of the PIM

The computational complexity of the PIM stems from its systematic processing of all assessment elements within a curriculum through a four-tiered hierarchical structure. The algorithm traverses the following dimensions: the total number of courses (N), the average number of examinations per course (M), the average number of questions per examination (K), and the total number of POs (P). Within the innermost loop, for each individual question q, the algorithm must: (1) identify the associated LO index i; (2) scan the i-th row of the course-specific LO-PO relationship matrix $R^{(C)}$ to extract all relationship scores $r_{i,j}$ that meet or exceed the predefined threshold τ; (3) compute the normalized sum of these significant scores; and (4) update the accumulators for the weighted score and normalization vector. The operation that dominates the per-question computational cost is the scanning of the LO-PO matrix row, which requires examining all P Program Outcomes. It is important to note that while the algorithm references LOs, the LO dimension (L) does not appear as an additional multiplicative factor in the complexity analysis because each question is mapped to exactly one LO. Therefore, the worst-case time complexity of Algorithm 1 is $O(N \cdot M \cdot K \cdot P)$, representing a polynomial-time complexity that is linear with respect to each of the four defining parameters.

Algorithm 1. Pseudocode of the Performance Indicator Model (PIM)

Require:
- StudentID: Student identifier (e.g. student number)
- $\mathrm{PO_j}$: target (j-th) program outcome
- τ: Relationship threshold value (default is 0)
- Curriculum $= \{\mathrm{C_1, C_2, \dots, C_n}\}$

Initialize:
- $\Sigma \leftarrow 0$ // Weighted score sum
- $\Omega_j \leftarrow 0$ // Normalization vector

Output:
- $PI_j \in [0,1]$ // Performance indicator

```
FOR EACH course C ∈ Curriculum DO                    ▷ Course loop: ∀C ∈ Curriculum
    FOR EACH exam e ∈ Exams(C) DO                    ▷ Exam loop: ∀e ∈ Exams(C)
        FOR EACH question q ∈ Questions(e) DO        ▷ Question loop: ∀q ∈ Questions(e)
```

$\gamma \leftarrow g_{C,e,q}/G_{C,e,q}$ ▷ **Eq. (7):** *Calculate PQ score*
$i \leftarrow LO_{index(q)}$ ▷ *Identify the LO measured by this question*
$A^{(i)} \leftarrow \{R^{(C)}[i,j] \mid R^{(C)}[i,j] \geq \tau\}$ ▷ **Eq. (4):** *Set of significant relationship scores*
$\vec{\omega}^{(C,e,q)} \leftarrow ECTS^{(C)} \times E_w^{(C,e)} \times \sum_{p \in A^{(i)}} p/5$ ▷ **Eq. (5):** *Compute component weight*
IF $R^{(C)}[i, \mathbf{PO_j}] \geq \tau$ **THEN** ▷ *Check if question contributes to target PO_j*
  $\vec{\Sigma} \leftarrow \vec{\Sigma} + \gamma \cdot \omega_j^{(C,e,q)}$ ▷ *Add weighted score if contributes*
END IF
$\Omega_j \leftarrow \Omega_j + \omega_j^{(C,e,q)}$ ▷ *Update normalization vector (all questions)*
END FOR
END FOR
END FOR
$PI_j \leftarrow \vec{\Sigma} / \Omega_j$ ▷ **Eq. (9):** *Final Performance Indicator*
Return PI_j

In practical academic settings, these parameters assume bounded, realistic values. A representative undergraduate engineering curriculum might consist of $N \approx 40 - 60$ courses, each with $M \approx 2 - 4$ graded examinations (e.g., midterm and final), $K \approx 5 - 15$ open-ended questions per exam, and $P \approx 8 - 12$ defined Program Outcomes (e.g., as per ABET criteria). Even under a pessimistic upper-bound scenario, this yields an upper bound on the order of 10^4 to 10^5 basic operations— a negligible computational load relative to contemporary hardware capabilities. Moreover, the average-case complexity is typically lower due to the inherent sparsity in LO-PO mappings; each Learning Outcome is meaningfully correlated with only a subset αP (where $0 < \alpha \leq 1$) of the total Program Outcomes. A significant performance optimization can be realized by precomputing the set of significant relationship scores $A^{(i)}$ for each Learning Outcome i during an initialization phase. This preprocessing step, performed once per course, reduces the cost of the critical per-question operation from $O(P)$ to $O(1)$ for a simple set retrieval, thereby lowering the overall time complexity to $O(N \cdot M \cdot K)$. The space complexity of the model is modest, primarily dictated by the storage of the LO-PO relationship matrices for all courses, amounting to $O(N \cdot L \cdot P)$, where L is the maximum number of Learning Outcomes per course—an easily manageable requirement for institutional databases.

4. Application Scenario and Results

In this section, the theoretical framework of the proposed Performance Indicator Model (PIM) is applied step by step to a simplified yet realistic undergraduate software engineering program scenario, demonstrating the practical operation of the model using concrete data. The scenario is constructed based on a representative four-course curriculum, associated program outcomes, learning outcomes, outcome relationship matrices, and performance data of a sample student. The primary objective is to illustrate how micro-level assessment data collected at the question level are transformed into meaningful macro-level performance indicators at the program outcome level, while ensuring full traceability and reproducibility of the computation process.

The section first describes the dataset and underlying assumptions employed in the application scenario. Subsequently, the algorithmic steps of the proposed model are demonstrated through representative calculations. Finally, the resulting performance indicators are analyzed and discussed to assess the validity and interpretability of the proposed approach.

4.1. Dataset and Application Scenario

The application is based on the structured dataset presented in Tables 1–6. This dataset has been constructed under the following realistic assumptions:

(1) Curriculum Structure: The scenario consists of four compulsory courses representing different semesters of a software engineering undergraduate program—MAT101, PHY202, SWE205, and SWE308 (Table 1). The ECTS credit assigned to each course is employed as the academic weighting factor in the proposed model.

Table 1. Sample curriculum used in the application scenario

Course Code	Course Title	T	P	Local Credit	ECTS
MAT101	Mathematics	5	0	5	6
PHY202	Physics	3	0	3	3
SWE205	Object Oriented Programming	3	2	4	5
SWE308	Database Management Systems	2	2	3	4

(2) Program Outcomes: The graduation objectives of the program are defined through five fundamental program outcomes (POs) in alignment with MÜDEK/ABET criteria (Table 2). These outcomes encompass the categories of knowledge (PO_1), skills (PO_2 and PO_3), and competence (PO_4 and PO_5) and constitute the final evaluation units of the proposed model.

Table 2. Program outcomes

POID	Category	Description
PO1	Knowledge	Apply fundamental mathematics, science, and engineering knowledge within the context of software engineering.
PO2	Skills	Use contemporary methods and tools effectively for analyzing, designing, developing, and evaluating software systems.
PO3	Skills	Demonstrate creative, systematic, and algorithmic thinking to solve complex software problems.
PO4	Competence	Exhibit competence in designing and operating data management and information system infrastructures.
PO5	Competence	Apply professional ethics, teamwork, communication, and lifelong learning principles in software engineering practice.

(3) Learning Outcomes and Relationships: For each course, three measurable learning outcomes (LOs) are defined (Table 3). The contribution relationships between LOs and program outcomes (POs) are quantified using course-specific LO–PO relationship matrices, with values ranging from 0 *(no relationship)* to 5 *(very strong contribution)* (Table 4). These matrices constitute the foundation of traceability and weighting within the proposed model.

Table 3. Learning outcomes of the curriculum courses

Course Code	LOID	Learning Outcome
MAT101	$\mathbf{LO_1}$	Apply differentiation and integration techniques.
	$\mathbf{LO_2}$	Use linear algebra concepts in engineering applications.
	$\mathbf{LO_3}$	Interpret basic probability and statistical concepts.
PHY202	$\mathbf{LO_1}$	Apply fundamental mechanics principles in engineering analysis.
	$\mathbf{LO_2}$	Explain basic electricity and magnetism concepts.
	$\mathbf{LO_3}$	Evaluate physical models using quantitative methods.
SWE205	$\mathbf{LO_1}$	Use fundamental OOP concepts correctly.
	$\mathbf{LO_2}$	Apply design patterns in appropriate contexts.
	$\mathbf{LO_3}$	Design and develop an OOP-based software component.

Course Code	LOID	Learning Outcome
SWE308	LO_1	Explain the relational data model and data independence.
	LO_2	Apply normalization techniques in database design.
	LO_3	Perform basic database operations using SQL.

Table 4 presents the LO–PO relationship matrices, which constitute the core of the proposed model. These matrices are constructed separately for each course, where rows represent the course learning outcomes (LOs) and columns correspond to the program outcomes (POs). The cell values, ranging from 0 to 5, indicate the strength of contribution of a given LO to a specific PO. For instance, in matrix (a) corresponding to the MAT101 course, LO_1 contributes to PO_1 at a level of “5” *(very strong),* while its contribution to PO_2 is “0” *(no contribution)*. These matrices provide a structured mapping that governs how question-level performance evidence is propagated and aggregated at the program outcome level.

Table 4. LO-PO Matrices for (a) MAT101, (b) PHY202, (c) SWE205, (d) SWE308 courses, showing the mapped contribution values (0–5) between learning outcomes (LOs) and program outcomes (POs).

$$\begin{array}{c|ccccc} & \mathbf{PO_1} & \mathbf{PO_2} & \mathbf{PO_3} & \mathbf{PO_4} & \mathbf{PO_5} \\ \hline \mathbf{LO_1} & 5 & 0 & 2 & 0 & 0 \\ \mathbf{LO_2} & 4 & 2 & 2 & 1 & 0 \\ \mathbf{LO_3} & 3 & 0 & 0 & 0 & 0 \end{array}$$

(a)

$$\begin{array}{c|ccccc} & \mathbf{PO_1} & \mathbf{PO_2} & \mathbf{PO_3} & \mathbf{PO_4} & \mathbf{PO_5} \\ \hline \mathbf{LO_1} & 5 & 0 & 0 & 0 & 0 \\ \mathbf{LO_2} & 3 & 2 & 1 & 0 & 0 \\ \mathbf{LO_3} & 4 & 2 & 2 & 0 & 0 \end{array}$$

(b)

$$\begin{array}{c|ccccc} & \mathbf{PO_1} & \mathbf{PO_2} & \mathbf{PO_3} & \mathbf{PO_4} & \mathbf{PO_5} \\ \hline \mathbf{LO_1} & 0 & 4 & 5 & 0 & 0 \\ \mathbf{LO_2} & 0 & 5 & 3 & 1 & 0 \\ \mathbf{LO_3} & 2 & 4 & 3 & 0 & 2 \end{array}$$

(c)

$$\begin{array}{c|ccccc} & \mathbf{PO_1} & \mathbf{PO_2} & \mathbf{PO_3} & \mathbf{PO_4} & \mathbf{PO_5} \\ \hline \mathbf{LO_1} & 0 & 3 & 0 & 4 & 1 \\ \mathbf{LO_2} & 0 & 2 & 0 & 4 & 0 \\ \mathbf{LO_3} & 3 & 4 & 0 & 4 & 2 \end{array}$$

(d)

(4) Assessment Plan: The grading scheme of each course is specified in detail, including the weight percentages of assessment components such as midterm exams, final exams, homework, and projects, as well as the number of questions in each examination (Table 5). In addition, each examination question is explicitly mapped to the learning outcome (LO) it assesses. This information constitutes a necessary input for the computation of question-level contribution weights (ω).

Table 5. Assessment components of the courses: assessment types, weight distribution, and question-to-LO mapping.

Course Code	Assessment Type	Weight (%)	No. of Questions	Question-to-LO Mapping
MAT101	Midterm exam	40	3	**Q1**→ LO_1, **Q2**→ LO_2, **Q3**→ LO_1
	Final exam	60	3	**Q1**→ LO_1, **Q2**→ LO_3, **Q3**→ LO_2
PHY202	Midterm exam	30	2	**Q1**→ LO_1, **Q2**→ LO_2
	Final exam	70	2	**Q1**→ LO_1, **Q2**→ LO_3
SWE205	Midterm exam	30	3	**Q1**→ LO_1, **Q2**→ LO_1, **Q3**→ LO_2
	Homework	20	1	**Q1**→ LO_2
	Final exam	50	2	**Q1**→ LO_2, **Q2**→ LO_3
SWE308	Midterm exam	25	2	**Q1**→ LO_1, **Q2**→ LO_2
	Project	25	1	**Q1**→ LO_3
	Final exam	50	3	**Q1**→ LO_1, **Q2**→ LO_2, **Q3**→ LO_3

(5) Student Performance Data: The raw scores obtained by a representative student for each question across all assessment components of the four courses, along with the corresponding maximum scores, are provided in Table 6. Based on these data, the normalized proportional score or P-Score (γ) for each question is computed within the interval [0,1]. The proportional score explicitly captures variations in the student's performance across different question types and examinations, thereby enabling a detailed analysis of learning processes and the identification of areas of strength and weakness.

Table 6. Exam results and proportional score (P-Score) of a student

Course Code	Examination	Question No	Actual Score	Max Score	P-Score (γ)
MAT101	Midterm	Q1	27	30	0.9000
		Q2	24	30	0.8000
		Q3	25	40	0.6250
	Final	Q1	5	25	0.2000
		Q2	35	35	1.0000
		Q3	25	40	0.6250
PHY202	Midterm	Q1	40	50	0.8000
		Q2	45	50	0.9000
	Final	Q1	35	40	0.8750
		Q2	50	60	0.8333
SWE205	Midterm	Q1	30	30	1.0000
		Q2	20	35	0.5714
		Q3	25	35	0.7143
	Homework	Q1	95	100	0.9500
	Final	Q1	38	50	0.7600
		Q2	20	50	0.4000
SWE308	Midterm	Q1	25	45	0.5555
		Q2	40	55	0.7273
	Project	Q1	85	100	0.8500
	Final	Q1	20	25	0.8000
		Q2	18	25	0.7200
		Q3	30	50	0.6000

(6) Model Parameter: In the computations, the significance threshold for relationships is set to $\tau = 0$. This implies that all LO–PO relationships with values greater than or equal to zero (i.e., 0, 1, 2, 3, 4, or 5) in the LO–PO matrices are taken into account.

This comprehensive and structured dataset provides all input parameters required by the model, thereby enabling the initiation of the computational procedure detailed in the following subsection.

4.2. Execution of the PIM computational procedure

In this section, the mathematical formulation of the Performance Indicator Model (PIM) introduced in Section 3 is applied step by step using the curriculum structure, assessment plan, and student performance data presented in the preceding subsections. The primary objective is to clearly demonstrate how micro-level assessment data collected at the question level are transformed into meaningful and normalized performance indicators at the PO level. All computations are presented through tables to ensure the reproducibility of the proposed model.

The computation process begins with the identification of the assessment components of each course and the corresponding question-to–learning outcome (LO) mappings. The assessment components include various evaluation instruments such as midterm exams (ME: Midterm Exam), final exams (FE: Final Exam), homework (H: Homework), and projects (P: Project). Each component contributes to the course grade with a predefined weight, as specified in the assessment plan presented in Table 5. The mapping of each assessment question to the learning outcome it measures constitutes the primary input for the subsequent question-level contribution calculations.

In the next stage, the raw scores obtained by the student for each assessment question are divided by the maximum possible score of the corresponding question to compute the proportional question score (γ). This operation follows the proportional scoring approach defined in Eq. (7) and eliminates scale differences arising from variations across exams and questions. Consequently, all questions are converted into a normalized and comparable performance measure within the range [0,1]

Subsequently, a weight vector (ω) is computed for each question. According to Eq. (5), this computation is based on the product of three fundamental components: (i) the ECTS credit of the course, (ii) the contribution ratio of the corresponding assessment component (ME, FE, H, or P) to the course grade, and (iii) the strength of the relationship between the learning outcome measured by the question and the program outcomes. At this stage, the relationship values obtained from the LO–PO mapping matrices are filtered using the predefined threshold value τ and normalized by dividing them by the maximum scale value of 5. In this manner, the relative contribution of each question to different program outcomes is quantitatively expressed.

The question-level weight vectors and proportional scores obtained across all courses and assessment components are presented in detail on a course-by-course basis in Tables 8–11. These tables explicitly show the contributions of individual questions to the relevant program outcomes for each assessment component (ME, FE, H, and P), thereby making the micro-level operation of the model transparent. Subsequently, the results obtained for all courses are consolidated in Table 12, where the total weight components for each program outcome and the normalization vector required for the computation of performance indicators are derived.

In the final stage, the Performance Indicator (PI) for a specific program outcome is computed using Eq. (8). This calculation is based on dividing the sum of the weighted proportional scores obtained from all questions contributing to the given program outcome by the maximum attainable weighted contribution for that outcome. The resulting PI value is interpreted as a normalized achievement ratio within the interval [0,1], providing a quantitative and comparable measure of the extent to which the student has attained the corresponding program outcome. Unlike traditional course-level weighted averages, this approach enables the disaggregated evaluation of learning attainment at the program outcome level and produces highly meaningful outputs for accreditation processes.

All parameters, variables, and notations used in the computation of the performance indicators are summarized in Table 7 to ensure conceptual clarity and facilitate the interpretation of the mathematical expressions. This table provides the definitions of the core quantitative elements of the model, including ECTS credits, assessment component weights, question-level weight vectors, and proportional scores.

Table 7. Definitions of parameters and notation employed in performance indicator calculations

	Parameter	MAT101	PHY202	SWE205	SWE308
	ECTS credit	$\mathrm{ECTS}^{(\mathrm{M})}$	$\mathrm{ECTS}^{(\mathrm{P})}$	$\mathrm{ECTS}^{(\mathrm{S})}$	$\mathrm{ECTS}^{(\mathrm{D})}$
ME	Assessment weight	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{M,m})}$	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{P,m})}$	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{S,m})}$	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{D,m})}$
	Weight vector	$\vec{\omega}^{(\mathrm{M,m,q})}$	$\vec{\omega}^{(\mathrm{P,m,q})}$	$\vec{\omega}^{(\mathrm{S,m,q})}$	$\vec{\omega}^{(\mathrm{D,m,q})}$
	Proportional score	$\gamma_{\mathrm{M,m,q}}$	$\gamma_{\mathrm{P,m,q}}$	$\gamma_{\mathrm{S,m,q}}$	$\gamma_{\mathrm{D,m,q}}$
H	Assessment weight	–	–	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{S,h})}$	–
	Weight vector	–	–	$\omega_{\mathrm{S,h,1}}$	–
	Proportional score	–	–	$\gamma_{\mathrm{S,h,1}}$	–
P	Assessment weight	–	–	–	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{D,p})}$
	Weight vector	–	–	–	$\omega_{\mathrm{D,p,1}}$
	Proportional score	–	–	–	$\gamma_{\mathrm{D,p,1}}$
FE	Assessment weight	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{M,f})}$	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{P,f})}$	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{S,f})}$	$\mathrm{E}_{\mathrm{w}}^{(\mathrm{D,f})}$
	Weight vector	$\vec{\omega}^{(\mathrm{M,f,q})}$	$\vec{\omega}^{(\mathrm{P,f,q})}$	$\vec{\omega}^{(\mathrm{S,f,q})}$	$\vec{\omega}^{(\mathrm{D,f,q})}$
	Proportional score	$\gamma_{\mathrm{M,f,q}}$	$\gamma_{\mathrm{P,f,q}}$	$\gamma_{\mathrm{S,f,q}}$	$\gamma_{\mathrm{D,f,q}}$

Tables 8–11 present the question-level weight vectors computed for the assessment components of each course in the curriculum based on Eq. (5). These tables include, in sequence, the type of assessment, the mapping information indicating which LO each question measures, the calculated question-level weight vector, and the corresponding weight components associated with each program outcome (PO_j). In this manner, the tables clearly demonstrate how micro-level assessment data are transformed into quantitative inputs that can be utilized at the program outcome level.

To improve clarity, the computation of the weight vector is illustrated through an example from the MAT101 course. The first term of Eq. (5) represents the ECTS credit, which reflects the academic weight of the course. According to Table 1, the ECTS value of MAT101 is $\mathrm{ECTS}^{(\mathrm{M})} = 6$. The second term in the equation corresponds to the contribution ratio of the relevant assessment component to the course grade. For instance, the weight of the midterm examination for MAT101 is defined as $\mathrm{E}_{\mathrm{w}}^{(\mathrm{M,m})} = 40\%$ (i.e., 0.4) in Table 5.

The third term of Eq. (5) incorporates the relationship between assessment questions and program outcomes through learning outcomes. The learning outcome measured by each exam question is identified using the "Question-to-LO Mapping" column in Table 5. Based on this information, the corresponding row of the LO–PO relationship matrix $R_{i,j}^{(M)}$for the MAT101 course (Table 4(a)) is selected. Using the predefined threshold value $\tau \geq 0$ all program outcome relationship values in this row that satisfy the threshold condition are retained; formally, this set is expressed as $\left\{R_{i,j}^{(M)} \middle| \; j = 1, \dots, P, \; R_{i,j}^{(M)} \geq \tau\right\}$. Each selected relationship value is then normalized by dividing it by the maximum scale value of 5, yielding the weight components used in Eq. (5).

As an illustrative example, the weight coefficient for the first question (Q1) of the midterm examination (ME) in the MAT101 course, reported in Table 8, is computed as $\omega_1^{(M,e,q)} = 2.40$ by following the steps described above. Since Q1 is associated with LO_1, the relationship value between LO_1 and PO_1 in Table 4(a) is 5. When combined with the course ECTS credit (6) and the midterm examination weight (0.4), this example clearly illustrates how question-level contributions are quantitatively propagated to the PO level.

Table 8. Calculation of assessment weight vector based on question–LO mapping and POs for MAT101

	Mapping	$\vec{\omega}^{(M,e,q)}$	PO$_1$: $\omega_1^{(M,e,q)}$	PO$_2$: $\omega_2^{(M,e,q)}$	PO$_3$: $\omega_3^{(M,e,q)}$	PO$_4$: $\omega_4^{(M,e,q)}$	PO$_5$: $\omega_5^{(M,e,q)}$
ME	Q1→ LO$_1$	$\vec{\omega}^{(M,m,1)}$	$6 \times 0.4 \times \frac{5}{5} = 2.40$	$6 \times 0.4 \times \frac{0}{5} = 0.00$	$6 \times 0.4 \times \frac{2}{5} = 0.96$	$6 \times 0.4 \times \frac{0}{5} = 0.00$	$6 \times 0.4 \times \frac{0}{5} = 0.00$
	Q2→ LO$_2$	$\vec{\omega}^{(M,m,2)}$	$6 \times 0.4 \times \frac{4}{5} = 1.92$	$6 \times 0.4 \times \frac{2}{5} = 0.96$	$6 \times 0.4 \times \frac{2}{5} = 0.96$	$6 \times 0.4 \times \frac{1}{5} = 0.48$	$6 \times 0.4 \times \frac{0}{5} = 0.00$
	Q3→ LO$_1$	$\vec{\omega}^{(M,m,3)}$	$6 \times 0.4 \times \frac{5}{5} = 2.40$	$6 \times 0.4 \times \frac{0}{5} = 0.00$	$6 \times 0.4 \times \frac{2}{5} = 0.96$	$6 \times 0.4 \times \frac{0}{5} = 0.00$	$6 \times 0.4 \times \frac{0}{5} = 0.00$
FE	Q1→ LO$_1$	$\vec{\omega}^{(M,f,1)}$	$6 \times 0.6 \times \frac{5}{5} = 3.60$	$6 \times 0.6 \times \frac{0}{5} = 0.00$	$6 \times 0.6 \times \frac{2}{5} = 1.44$	$6 \times 0.6 \times \frac{0}{5} = 0.00$	$6 \times 0.6 \times \frac{0}{5} = 0.00$
	Q2→ LO$_3$	$\vec{\omega}^{(M,f,2)}$	$6 \times 0.6 \times \frac{3}{5} = 2.16$	$6 \times 0.6 \times \frac{0}{5} = 0.00$	$6 \times 0.6 \times \frac{0}{5} = 0.00$	$6 \times 0.6 \times \frac{0}{5} = 0.00$	$6 \times 0.6 \times \frac{0}{5} = 0.00$
	Q3→ LO$_2$	$\vec{\omega}^{(M,f,3)}$	$6 \times 0.6 \times \frac{4}{5} = 2.88$	$6 \times 0.6 \times \frac{2}{5} = 1.44$	$6 \times 0.6 \times \frac{2}{5} = 1.44$	$6 \times 0.6 \times \frac{1}{5} = 0.72$	$6 \times 0.6 \times \frac{0}{5} = 0.00$

Table 9. Calculation of assessment weight vectors based on question–LO mapping and POs for PHY202

	Mapping	$\vec{\omega}^{(P,e,q)}$	PO$_1$: $\omega_1^{(P,e,q)}$	PO$_2$: $\omega_2^{(P,e,q)}$	PO$_3$: $\omega_3^{(P,e,q)}$	PO$_4$: $\omega_4^{(P,e,q)}$	PO$_5$: $\omega_5^{(P,e,q)}$
ME	Q1→ LO$_1$	$\vec{\omega}^{(P,m,1)}$	$3 \times 0.3 \times \frac{5}{5} = 0.90$	$3 \times 0.3 \times \frac{0}{5} = 0.00$	$3 \times 0.3 \times \frac{0}{5} = 0.00$	$3 \times 0.3 \times \frac{0}{5} = 0.00$	$3 \times 0.3 \times \frac{0}{5} = 0.00$
	Q2→ LO$_2$	$\vec{\omega}^{(P,m,2)}$	$3 \times 0.3 \times \frac{3}{5} = 0.54$	$3 \times 0.3 \times \frac{2}{5} = 0.36$	$3 \times 0.3 \times \frac{1}{5} = 0.18$	$3 \times 0.3 \times \frac{0}{5} = 0.00$	$3 \times 0.3 \times \frac{0}{5} = 0.00$
FE	Q1→ LO$_1$	$\vec{\omega}^{(P,f,1)}$	$3 \times 0.7 \times \frac{5}{5} = 2.10$	$3 \times 0.7 \times \frac{0}{5} = 0.00$	$3 \times 0.7 \times \frac{0}{5} = 0.00$	$3 \times 0.7 \times \frac{0}{5} = 0.00$	$3 \times 0.7 \times \frac{0}{5} = 0.00$
	Q2→ LO$_3$	$\vec{\omega}^{(P,f,2)}$	$3 \times 0.7 \times \frac{4}{5} = 1.68$	$3 \times 0.7 \times \frac{2}{5} = 0.84$	$3 \times 0.7 \times \frac{2}{5} = 0.84$	$3 \times 0.7 \times \frac{0}{5} = 0.00$	$3 \times 0.7 \times \frac{0}{5} = 0.00$

Table 10. Calculation of assessment weight vectors based on question–LO mapping and POs for SWE205

	Mapping	$\vec{\omega}^{(S,e,q)}$	PO$_1$: $\omega_1^{(S,e,q)}$	PO$_2$: $\omega_2^{(S,e,q)}$	PO$_3$: $\omega_3^{(S,e,q)}$	PO$_4$: $\omega_4^{(S,e,q)}$	PO$_5$: $\omega_5^{(S,e,q)}$
ME	Q1→ LO$_1$	$\vec{\omega}^{(S,m,1)}$	$5 \times 0.3 \times \frac{0}{5} = 0.00$	$5 \times 0.3 \times \frac{4}{5} = 1.20$	$5 \times 0.3 \times \frac{5}{5} = 1.50$	$5 \times 0.3 \times \frac{0}{5} = 0.00$	$5 \times 0.3 \times \frac{0}{5} = 0.00$
	Q2→ LO$_1$	$\vec{\omega}^{(S,m,2)}$	$5 \times 0.3 \times \frac{0}{5} = 0.00$	$5 \times 0.3 \times \frac{4}{5} = 1.20$	$5 \times 0.3 \times \frac{5}{5} = 1.50$	$5 \times 0.3 \times \frac{0}{5} = 0.00$	$5 \times 0.3 \times \frac{0}{5} = 0.00$
	Q3→ LO$_2$	$\vec{\omega}^{(S,m,3)}$	$5 \times 0.3 \times \frac{0}{5} = 0.00$	$5 \times 0.3 \times \frac{5}{5} = 1.50$	$5 \times 0.3 \times \frac{3}{5} = 0.90$	$5 \times 0.3 \times \frac{1}{5} = 0.30$	$5 \times 0.3 \times \frac{0}{5} = 0.00$
H	Q1→ LO$_2$	$\vec{\omega}^{(S,h,1)}$	$5 \times 0.2 \times \frac{0}{5} = 0.00$	$5 \times 0.2 \times \frac{5}{5} = 1.00$	$5 \times 0.2 \times \frac{3}{5} = 0.60$	$5 \times 0.2 \times \frac{1}{5} = 0.20$	$5 \times 0.2 \times \frac{0}{5} = 0.00$
FE	Q1→ LO$_2$	$\vec{\omega}^{(S,f,1)}$	$5 \times 0.5 \times \frac{0}{5} = 0.00$	$5 \times 0.5 \times \frac{5}{5} = 2.50$	$5 \times 0.5 \times \frac{3}{5} = 1.50$	$5 \times 0.5 \times \frac{1}{5} = 0.50$	$5 \times 0.5 \times \frac{0}{5} = 0.00$
	Q2→ LO$_3$	$\vec{\omega}^{(S,f,2)}$	$5 \times 0.5 \times \frac{2}{5} = 1.00$	$5 \times 0.5 \times \frac{4}{5} = 2.00$	$5 \times 0.5 \times \frac{3}{5} = 1.50$	$5 \times 0.5 \times \frac{0}{5} = 0.00$	$5 \times 0.5 \times \frac{2}{5} = 1.00$

Table 11. Calculation of assessment weight vectors based on question–LO mapping and POs for SWE308

	Mapping	$\vec{\omega}^{(D,e,q)}$	$PO_1: \omega_1^{(D,e,q)}$	$PO_2: \omega_2^{(D,e,q)}$	$PO_3: \omega_3^{(D,e,q)}$	$PO_4: \omega_4^{(D,e,q)}$	$PO_5: \omega_5^{(D,e,q)}$
ME	Q1→ LO_1	$\vec{\omega}^{(D,m,1)}$	$4 \times 0.25 \times \frac{0}{5} = 0.00$	$4 \times 0.25 \times \frac{3}{5} = 0.60$	$4 \times 0.25 \times \frac{0}{5} = 0.00$	$4 \times 0.25 \times \frac{4}{5} = 0.80$	$4 \times 0.25 \times \frac{1}{5} = 0.20$
	Q2→ LO_2	$\vec{\omega}^{(D,m,2)}$	$4 \times 0.25 \times \frac{0}{5} = 0.00$	$4 \times 0.25 \times \frac{2}{5} = 0.40$	$4 \times 0.25 \times \frac{0}{5} = 0.00$	$4 \times 0.25 \times \frac{4}{5} = 0.80$	$4 \times 0.25 \times \frac{0}{5} = 0.00$
P	Q1→ LO_3	$\vec{\omega}^{(D,p,1)}$	$4 \times 0.25 \times \frac{0}{5} = 0.00$	$4 \times 0.25 \times \frac{3}{5} = 0.60$	$4 \times 0.25 \times \frac{0}{5} = 0.00$	$4 \times 0.25 \times \frac{4}{5} = 0.80$	$4 \times 0.25 \times \frac{1}{5} = 0.20$
FE	Q1→ LO_1	$\vec{\omega}^{(D,f,1)}$	$4 \times 0.5 \times \frac{0}{5} = 0.00$	$4 \times 0.5 \times \frac{3}{5} = 1.20$	$4 \times 0.5 \times \frac{0}{5} = 0.00$	$4 \times 0.5 \times \frac{4}{5} = 1.60$	$4 \times 0.5 \times \frac{1}{5} = 0.40$
	Q2→ LO_2	$\vec{\omega}^{(D,f,2)}$	$4 \times 0.5 \times \frac{0}{5} = 0.00$	$4 \times 0.5 \times \frac{2}{5} = 0.80$	$4 \times 0.5 \times \frac{0}{5} = 0.00$	$4 \times 0.5 \times \frac{4}{5} = 1.60$	$4 \times 0.5 \times \frac{0}{5} = 0.00$
	Q3→ LO_3	$\vec{\omega}^{(D,f,3)}$	$4 \times 0.5 \times \frac{3}{5} = 1.20$	$4 \times 0.5 \times \frac{4}{5} = 1.60$	$4 \times 0.5 \times \frac{0}{5} = 0.00$	$4 \times 0.5 \times \frac{4}{5} = 1.60$	$4 \times 0.5 \times \frac{2}{5} = 0.80$

All question-level weight vectors and proportional scores obtained from Tables 8–11 are consolidated in Table 12 to enable a comprehensive evaluation at the program outcome level. This table simultaneously presents the quantitative contribution of each assessment question to the corresponding program outcome across all courses and assessment components in the curriculum, along with the student's proportional performance achieved from these questions. The values reported in the final row of Table 12 represent the total weight components for each program outcome, which serve as the normalization vector used in the computation of the Performance Indicator (PI).

Table 12. Question-level assessment weight vectors and proportional scores for performance indicators

Course	Assessment	Question	$\gamma_{C,e,q}$	$PO_1 \leftarrow \omega_1^{(C,e,q)}$	$PO_2 \leftarrow \omega_2^{(C,e,q)}$	$PO_3 \leftarrow \omega_3^{(C,e,q)}$	$PO_4 \leftarrow \omega_4^{(C,e,q)}$	$PO_5 \leftarrow \omega_5^{(C,e,q)}$
MAT101	ME	Q1	0.9000	2.4000	0.0000	0.9600	0.0000	0.0000
		Q2	0.8000	1.9200	0.9600	0.9600	0.4800	0.0000
		Q3	0.6250	2.4000	0.0000	0.9600	0.0000	0.0000
	FE	Q1	0.2000	3.6000	0.0000	1.4400	0.0000	0.0000
		Q2	1.0000	2.1600	0.0000	0.0000	0.0000	0.0000
		Q3	0.6250	2.8800	1.4400	1.4400	0.7200	0.0000
PHY202	ME	Q1	0.8000	0.9000	0.0000	0.0000	0.0000	0.0000
		Q2	0.9000	0.5400	0.3600	0.1800	0.0000	0.0000
	FE	Q1	0.8750	2.1000	0.0000	0.0000	0.0000	0.0000
		Q2	0.8333	1.6800	0.8400	0.8400	0.0000	0.0000
SWE205	ME	Q1	1.0000	0.0000	1.2000	1.5000	0.0000	0.0000
		Q2	0.5714	0.0000	1.2000	1.5000	0.0000	0.0000
		Q3	0.7143	0.0000	1.5000	0.9000	0.3000	0.0000
	H	Q1	0.9500	0.0000	1.0000	0.6000	0.2000	0.0000
	FE	Q1	0.7600	0.0000	2.5000	1.5000	0.5000	0.0000
		Q2	0.4000	1.0000	2.0000	1.5000	0.0000	1.0000
SWE308	ME	Q1	0.5555	0.0000	0.6000	0.0000	0.8000	0.2000
		Q2	0.7273	0.0000	0.4000	0.0000	0.8000	0.0000
	P	Q1	0.8500	0.0000	0.6000	0.0000	0.8000	0.2000
	FE	Q1	0.8000	0.0000	1.2000	0.0000	1.6000	0.4000
		Q2	0.7200	0.0000	0.8000	0.0000	1.6000	0.0000
		Q3	0.6000	1.2000	1.6000	0.0000	1.6000	0.8000
$\Omega = \sum \omega$:				**22.7800**	**18.2000**	**14.2800**	**9.4000**	**2.6000**

According to Table 12, the computation of the performance indicator (PI_j) is carried out as follows. All variables, indices, and notations employed at this stage strictly conform to the definitions provided in Table 7. The indices appearing in the question-level weight component $\omega_j^{(C,e,q)}$ and the proportional score term $\gamma_{C,e,q}$ explicitly and transparently indicate the course, the assessment component, and the specific question from which each contribution originates.

Based on the notation in Eq. (9), the term $\omega_j^{(M,m,q)}$ denotes the contribution weight of the q-th question in the midterm examination (m) of the MAT101 course (M) to the j-th program outcome. Correspondingly, $\gamma_{M,m,q}$ represents the proportional score obtained by a student for the same question. In a similar manner $\omega_j^{(M,f,q)}$ expresses the contribution weight of the q-th question in the final examination (f) of the MAT101 course to the j-th program outcome. As another example, in the term $\omega_j^{(S,h,1)}$, the index S refers to the SWE205 course, h denotes the homework assessment component, and the index 1 indicates that the homework consists of a single question or topic. The corresponding $\gamma_{S,h,1}$ term represents the student's score for that homework, converted into a proportional score.

$$\mathrm{PI}_j = \frac{\begin{gathered}\left(\sum_{q=1}^{3} \omega_j^{(M,m,q)} \cdot \gamma_{M,m,q} + \omega_j^{(M,f,q)} \cdot \gamma_{M,f,q}\right) + \\ \left(\sum_{q=1}^{2} \omega_j^{(P,m,q)} \cdot \gamma_{P,m,q} + \omega_j^{(P,f,q)} \cdot \gamma_{P,f,q}\right) + \\ \left(\sum_{q=1}^{3} \omega_j^{(S,m,q)} \cdot \gamma_{S,m,q}\right) + \left(\omega_j^{(S,h,1)} \cdot \gamma_{S,h,1}\right) + \left(\sum_{q=1}^{2} \omega_j^{(S,f,q)} \cdot \gamma_{S,f,q}\right) + \\ \left(\sum_{q=1}^{2} \omega_j^{(D,m,q)} \cdot \gamma_{D,m,q}\right) + \left(\omega_j^{(D,p,1)} \cdot \gamma_{D,p,1}\right) + \left(\sum_{q=1}^{3} \omega_j^{(D,f,q)} \cdot \gamma_{D,f,q}\right)\end{gathered}}{\begin{gathered}\left(\sum_{q=1}^{3} \omega_j^{(M,m,q)} + \omega_j^{(M,f,q)}\right) + \left(\sum_{q=1}^{2} \omega_j^{(P,m,q)} + \omega_j^{(P,f,q)}\right) + \\ \left(\sum_{q=1}^{3} \omega_j^{(S,m,q)}\right) + \left(\omega_j^{(S,h,1)}\right) + \left(\sum_{q=1}^{2} \omega_j^{(S,f,q)}\right) + \\ \left(\sum_{q=1}^{2} \omega_j^{(D,m,q)}\right) + \left(\omega_j^{(D,p,1)}\right) + \left(\sum_{q=1}^{3} \omega_j^{(D,f,q)}\right)\end{gathered}} \tag{9}$$

The numerator obtained by aggregating these contributions over all courses (C), assessment components (e), and questions (q) contributing to PI_j represents the student's actual total weighted performance with respect to the j -th program outcome. The denominator of Eq. (9), given by the term $\sum \omega_j^{(C,e,q)}$ and reported in the last row of Table 12, corresponds to the maximum attainable weighted contribution for the same program outcome as defined by the curriculum and the assessment structure; it therefore serves as the normalization vector.

The resulting PI_j values obtained through this normalization process are reported in Table 13 and can be interpreted as scale-independent and directly interpretable performance indicators defined within the interval [0,1]. In this way, all course-level assessments, including examinations, homework, and question-level performance data defined in the application scenario, are transformed into integrated and quantitative indicators at the program outcome level.

Table 13. Performance indicators by program outcomes

Program Outcome	**Performance Indicator**	
PO_1	$\mathrm{PI}_1 = \sum \omega_1^{(C,e,q)} \cdot \gamma_{C,e,q} / \sum \omega_1^{(C,e,q)}$	$\mathrm{PI}_1 = 15.4389/22.7800$ $\mathrm{PI}_1 = \mathbf{0.6777}$
PO_2	$\mathrm{PI}_2 = \sum \omega_2^{(C,e,q)} \cdot \gamma_{C,e,q} / \sum \omega_2^{(C,e,q)}$	$\mathrm{PI}_2 = 12.9291/18.2000$ $\mathrm{PI}_2 = \mathbf{0.7104}$
PO_3	$\mathrm{PI}_3 = \sum \omega_3^{(C,e,q)} \cdot \gamma_{C,e,q} / \sum \omega_3^{(C,e,q)}$	$\mathrm{PI}_3 = 9.5917/14.2800$ $\mathrm{PI}_3 = \mathbf{0.6717}$
PO_4	$\mathrm{PI}_4 = \sum \omega_4^{(C,e,q)} \cdot \gamma_{C,e,q} / \sum \omega_4^{(C,e,q)}$	$\mathrm{PI}_4 = 6.7165/9.4000$ $\mathrm{PI}_4 = \mathbf{0.7145}$
PO_5	$\mathrm{PI}_5 = \sum \omega_5^{(C,e,q)} \cdot \gamma_{C,e,q} / \sum \omega_5^{(C,e,q)}$	$\mathrm{PI}_5 = 1.4811/2.6000$ $\mathrm{PI}_5 = \mathbf{0.5697}$

Fig. (4) illustrates the program outcome–based performance profile of an individual student computed using the proposed PIM. The figure clearly demonstrates that, despite relatively balanced course grades, student achievement is not uniformly distributed across program outcomes. While certain outcomes exhibit high attainment levels, others remain noticeably lower, revealing latent performance disparities that cannot be observed through conventional weighted averages. This outcome-based profiling confirms that macro-level indicators may obscure program outcome–specific deficiencies and underscores the necessity of micro-level assessment for accurate and diagnostic evaluation.

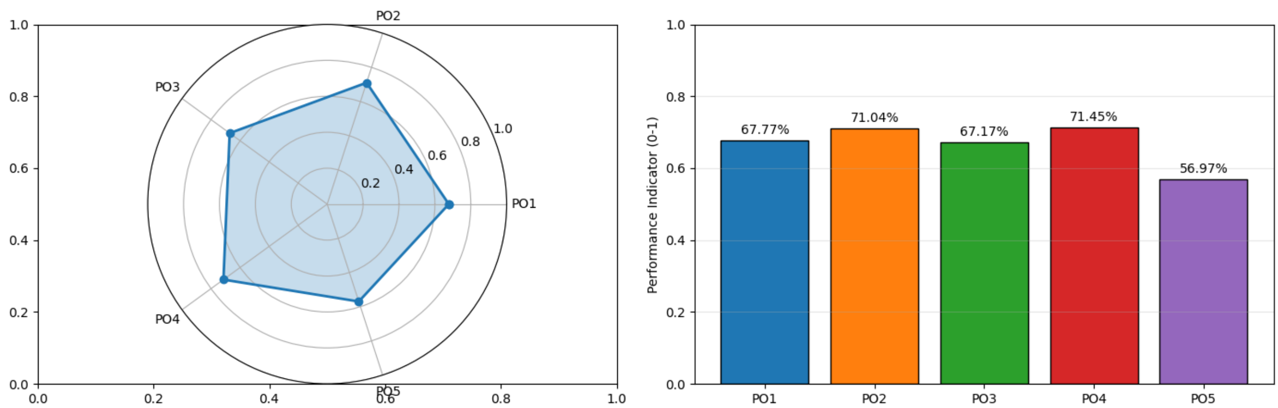

Figure 4. Individual student PO performance profile

Fig. (5) illustrates the hierarchical aggregation of program outcome (PO) contribution weights (ω) across three evaluation levels: individual questions, course–assessment combinations, and complete courses. At the question level, the figure reveals a highly heterogeneous distribution of contribution weights, indicating that individual assessment items contribute unevenly to different program outcomes.

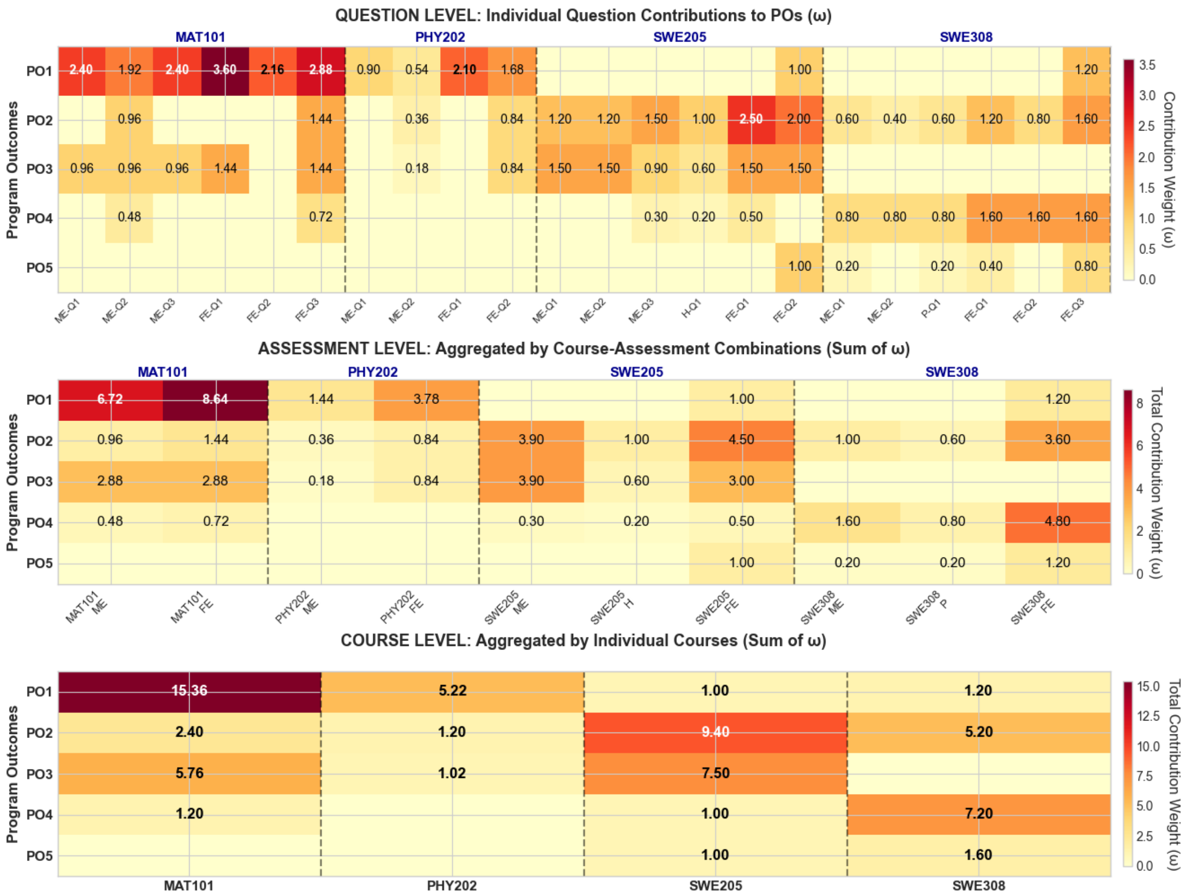

Figure 1. Hierarchical analysis of PO contribution weights (ω) at different aggregation levels

This micro-level variability reflects the combined effects of ECTS credits, assessment weights, and LO–PO relationship strengths embedded in the model. When aggregated at the assessment level, these dispersed contributions consolidate into more structured patterns, making explicit how specific exams or assignments dominate the attainment of particular program outcomes. For example, midterm and final examinations in MAT101 and SWE205 emerge as primary contributors to PO_1, PO_2, and PO_3, whereas SWE308 exhibits stronger associations with competence-oriented outcomes such as PO_4 and PO_5. At the course level, the aggregation further clarifies the relative influence of each course on the overall PO structure, revealing imbalances that are not apparent at lower levels. Notably, MAT101 exerts a dominant influence on knowledge-based outcomes, while SWE205 disproportionately contributes to skill-oriented outcomes. This hierarchical visualization demonstrates the traceability of the proposed PIM from question-level evidence to program-level indicators and highlights how micro-level assessment design decisions propagate upward to shape the final program outcome performance profile.

Fig. (6) depicts the Course–Program Outcome Contribution Network derived from the proposed PIM, visualizing the strength and distribution of contributions between curriculum courses and program outcomes. In this network representation, nodes correspond to courses and program outcomes, while edge thickness encodes the magnitude of the aggregated contribution weights (ω). The figure highlights a non-uniform contribution structure, where certain courses exhibit strong, targeted influences on specific program outcomes rather than uniformly supporting all outcomes. For instance, MAT101 demonstrates a pronounced contribution to knowledge-oriented outcomes, particularly PO_1, whereas SWE205 emerges as a central contributor to skill-based outcomes such as PO_2 and PO_3. In contrast, SWE308 shows relatively stronger associations with competence-related outcomes, including PO_4 and PO_5, indicating its specialized role within the curriculum. The network also reveals program outcomes that rely on a limited subset of courses, suggesting potential vulnerability in outcome coverage and opportunities for curricular rebalancing. Overall, this visualization provides an intuitive yet quantitatively grounded perspective on how micro-level assessment contributions propagate through courses to shape program-level outcome attainment, thereby supporting data-informed curriculum evaluation and continuous improvement processes.

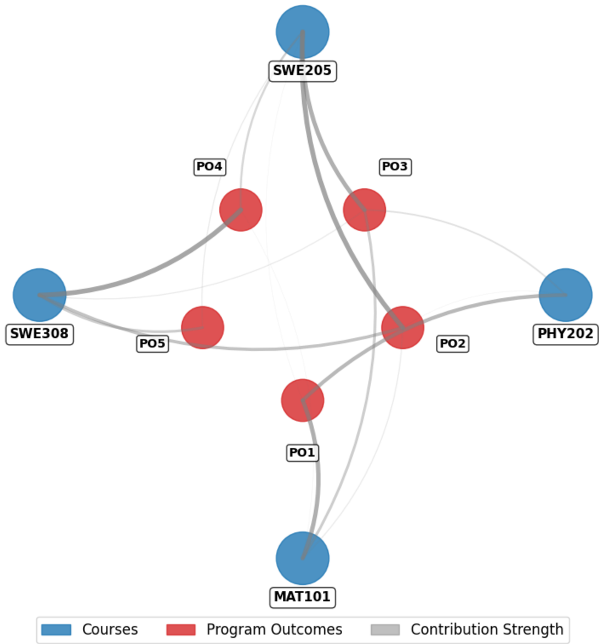

Figure 2. Course-Program Outcome Contribution Network

Table 14 presents the student's assessment scores obtained from the courses included in the curriculum, the contribution weights of these assessment components to the course grades, and the resulting course-level weighted averages (WA). The results indicate that the student's overall academic performance remains within a moderate-to-high range across all courses, with WA values varying between 69.4 and 85. In particular, the relatively high or balanced weighted averages observed in PHY202 and SWE308 suggest a generally satisfactory level of achievement when evaluated using traditional course-based assessment approaches.

Table 14. Student assessment scores, assessment weights, and weighted averages.

Course	Midterm Exam		Homework		Project		Final Exam		WA
	Score	$E_w^{(C,m)}$	Score	$E_w^{(C,h)}$	Score	$E_w^{(C,p)}$	Score	$E_w^{(C,f)}$	
MAT101	76	40%	–	–	–	–	65	60%	**69.4**
PHY202	85	30%	–	–	–	–	85	70%	**85**
SWE205	75	30%	95	20%	–	–	58	50%	**70.5**
SWE308	65	25%	–	–	85	25%	68	50%	**71.5**

However, when these aggregated course-level results are compared with the program outcome–based performance indicators reported in Table 13, it becomes evident that student performance is not uniformly distributed across program outcomes.

Notably, despite the similarity of WA values across courses, the performance indicator calculated for PO_5 (0.5697) is considerably lower than those of the other program outcomes, revealing that course-level weighted averages may obscure weaknesses in specific competency domains. This comparison demonstrates that conventional WA-based evaluation lacks discriminative power at the program outcome level, whereas the proposed PIM provides a more sensitive and goal-oriented performance analysis by leveraging micro-level assessment data.

4.3. Sensitivity Analysis and Model Validation

In order to evaluate the robustness and validity of the proposed PIM, both sensitivity analysis and model validation procedures were conducted. Sensitivity analysis examines how variations in key model parameters influence the resulting performance indicators, while model validation assesses the logical and mathematical consistency of the model outputs. In this study, a threshold-based sensitivity analysis was performed by varying the threshold parameter τ, which determines the minimum strength of LO–PO relationships included in the computation. In addition, the analytical consistency of the model was examined through boundary-condition validation to verify that the model produces meaningful results under extreme performance scenarios. These analyses collectively provide evidence regarding the stability, interpretability, and reliability of the proposed evaluation framework.

4.3.1. Threshold Sensitivity Analysis

Table 15 summarizes the resulting PI values obtained under different threshold values τ. The threshold parameter determines the minimum strength of LO–PO relationships that are included in the computation; therefore, increasing τ progressively filters out weaker relationships from the model.

Table 15. Performance indicators obtained under different threshold parameter values (τ).

τ	PI_1	PI_2	PI_3	PI_4	PI_5
0	67.77%	71.04%	67.17%	71.45%	56.97%
1	67.77%	71.04%	67.17%	71.45%	56.97%
2	67.77%	71.04%	66.88%	70.81%	48.89%
3	69.05%	69.93%	70.80%	70.81%	0.00%
4	65.28%	68.79%	78.57%	70.81%	0.00%
5	60.86%	78.43%	78.57%	0.00%	0.00%

The results show that the performance indicators remain identical for $\tau = 0$ and $\tau = 1$. This behavior is expected because LO–PO relationships with value 0 do not contribute to the weighting mechanism of the model. Consequently, excluding these zero-valued relationships does not alter the computed PI values. When the threshold is increased to $\tau = 2$, minor changes appear in several program outcomes. In particular, PI_3 and PI_4 exhibit slight decreases, while PI_5 shows a more noticeable decline from 56.97% to 48.89%. This observation suggests that PI_5 relies partially on weaker LO–PO relationships, which are removed once the threshold increases. At $\tau = 3$, the model begins to exhibit more pronounced structural changes. PI_5 drops to 0%, indicating that all relationships contributing to this outcome have strengths lower than 3. In other words, PI_5 in the current dataset is primarily supported by weak or moderate LO–PO connections. Meanwhile, PI_1 and PI_3 show slight increases, implying that when weaker relationships are excluded, the remaining stronger contributions produce a higher normalized indicator.

For higher thresholds ($\tau = 4$ and $\tau = 5$), the filtering effect becomes even more pronounced. Only strong and very strong LO–PO relationships remain in the model. As a result, PI_3 increases significantly, reaching 78.57%, indicating that this outcome is strongly supported by high-strength LO–PO relationships. In contrast, PI_4 and PI_5 lose all contributing relationships at $\tau = 5$, leading their indicators to drop to 0%.

Overall, the analysis demonstrates that the proposed PIM exhibits relatively stable behavior for moderate threshold values ($\tau \leq 2$). Larger threshold values progressively reduce the coverage of the LO–PO mapping and may eliminate certain program outcomes entirely if they rely primarily on lower-strength relationships. These findings confirm that while the model is robust under realistic threshold settings, the selection of τ plays an important role in determining the level of outcome filtering applied in the analysis.

4.3.2. Analytical Validation Through Boundary Conditions

In addition to the threshold sensitivity analysis presented earlier, the mathematical consistency of the proposed Performance Indicator Model was examined through boundary-case validation. Boundary conditions represent extreme but theoretically meaningful scenarios that a valid performance measurement model should handle correctly.

First, consider the case in which a student achieves full marks in all assessment questions contributing to a given program outcome. In this situation, all question performance values reach their maximum level, and the weighted aggregation in the proposed model yields a normalized performance indicator equal to one. Therefore, the model satisfies the condition $PI_p = 1$, indicating complete attainment of the corresponding program outcome.

Second, if a student receives zero scores in all assessment questions contributing to a program outcome, all corresponding weighted contributions become zero. Consequently, the resulting performance indicator becomes $PI_p = 0$, correctly representing the absence of outcome attainment.

Third, when the threshold parameter τ filters out all LO–PO relationships associated with a particular program outcome, no assessment contributions remain for that outcome. In such cases the model produces a null contribution, which is reported as a zero attainment value. This behavior reflects the loss of coverage for that outcome under strict threshold conditions.

These boundary-case evaluations demonstrate that the proposed model behaves consistently under extreme conditions and produces normalized indicators within a meaningful range.

5. Threats to Validity and Model Limitations

Despite its methodological rigor, the proposed Performance Indicator Model (PIM) is subject to certain threats to validity and practical limitations that should be acknowledged. From a construct validity perspective, the model relies on predefined learning outcome–program outcome (LO–PO) relationship matrices and assessment weights specified by academic staff. Although these matrices follow established MÜDEK/ABET practices, the assignment of relationship strengths inevitably involves expert judgment, which may introduce subjectivity. Variations in instructors' interpretations of contribution levels or in the formulation of learning outcomes can influence the resulting performance indicators, even when the same mathematical framework is applied.

Regarding internal validity, the model assumes that each exam question measures a single learning outcome accurately and exclusively. In practice, particularly for open-ended or design-oriented questions, a question may partially address multiple competencies. The one-to-one mapping constraint adopted in this study simplifies the computational structure but may lead to an underrepresentation of intersecting skills such as problem-solving or communication. Furthermore, the model assumes consistency in assessment difficulty across questions and courses, an assumption that may not fully hold in heterogeneous assessment environments.

Finally, with respect to outcome validity, the model focuses exclusively on direct assessment data derived from exam questions and graded components. Indirect assessment methods—such as student surveys, alumni feedback, and employer evaluations—are intentionally excluded to preserve objectivity and mathematical clarity. However, this design choice limits the model's ability to capture perceived learning gains and long-term competencies. Consequently, the performance indicators produced by the PIM should be interpreted as evidence of demonstrated academic achievement rather than holistic educational impact. Future research may address this limitation by integrating indirect assessment data within a hybrid evaluation framework.

6. Conclusion

This study presented a novel Performance Indicator Model (PIM) for the assessment of program outcomes through a micro-level, question-based evaluation framework. Unlike conventional assessment approaches that rely on course-level averages, the proposed model integrates proportional question scores with curriculum-specific weighting factors, including ECTS credits, assessment contribution ratios, and LO–PO relationship strengths. This integration enables the computation of normalized, traceable, and PO-specific performance indicators that accurately reflect student achievement across different competency domains.

The application scenario demonstrated that the proposed model can reveal substantial discrepancies between traditional weighted averages and PO-based performance indicators. While course-level grades may suggest satisfactory overall achievement, the PIM exposed hidden weaknesses in specific program outcomes that would otherwise remain undetected. This finding confirms the necessity of micro-level assessment models for accreditation-oriented evaluation and continuous improvement processes.

From an accreditation and quality assurance perspective, the PIM offers a transparent, repeatable, and mathematically robust mechanism that aligns with MÜDEK/ABET requirements. The model is adaptable to different disciplines and curricula, provided that LO–PO mappings and assessment structures are available. Future studies may extend the model by incorporating indirect assessment data, cohort-level analytics, or longitudinal performance tracking to further enhance its decision-support capabilities.

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This work is original.

2. Author Contributions:
Concept: AE,EY; ***Conceptualization:*** AE,EY; ***Literature Search:*** AE,EY; ***Data Collection:*** AE,EY; ***Data Processing:*** AE,EY; ***Analysis:*** AE,EY; ***Writing – original draft:*** AE,EY; ***Writing – review & editing:*** AE,EY.

3. Ethics approval:
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6. GenAI Usage Statement:
Generative AI tools were used only for language editing and translation purposes.

7. Sustainable Development Goals:

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