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Using Theoretical Framework with Sample Activities to Enlighten Prospective Elementary Mathematics Teachers about Technological Pedagogical Content Knowledge (TPACK)

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ABSTRACT

It is important to know which criteria are expected from mathematics teachers since they have difficulty in integrating technology effectively to their lessons and. So, this study aims to make prospective mathematics teachers aware on Technological Pedagogical Content Knowledge (TPACK) by providing them to use the assessment framework in the context of TPACK components for sample geometry activities, and then to identify their levels by the scale called TPACK regarding Geometry Instrument. Case study design from qualitative methods was used in this study. The participants consist of fourth year 90 pre-service elementary mathematics teachers. In procedure, one-hour theoretical information about "Techno-Pedagogical Education" was given, and two-hour micro-teaching including four technology-integrated activities was applied, and then lesson plans, worksheets and GeoGebra files regarding these activities were assessed, evaluated and discussed together by considering the theoretical framework of TPACK level identification. At the end of the implementation, with respect to the results from the scale the total mean value of the instrument ($\bar{X}$ =4.50) is higher than moderate out of six, and also pre-service teachers' content and pedagogical knowledge are higher than the technological content and technological pedagogical knowledge. Therefore, it can be suggested that this study can be extended in a longer term to develop their TPACK levels by providing them an environment that they practice with technology, create their own lesson plans and make micro-teachings.

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1. Introduction

The existence and usage of mathematics, especially geometry can be seen in the world from the nature to the man-made architecture structures. Geometry using in measurements, calculations, product designs, buildings and decorations is connected with the other disciplines like optics, electronics, statistics, biology, pharmacy, physics, chemistry, cryptology and engineering. Considering its significance, learning geometry has an important role in our lives. Therefore, educators, researchers and teachers have been discussing methods to teach geometry. That means, the question of "how to teach it better" has been considered as an issue that content knowledge requires to be used together with pedagogical knowledge. At this point, Shulman (1986) defined pedagogical content knowledge (PCK) for an integration of pedagogical knowledge and content knowledge. For geometry as a content knowledge, PCK is the knowledge to select appropriate material/manipulatives and instruction

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methods/strategies/ techniques in order to make learners comprehend geometrical concepts. (Van de Walle, & Karp, & Bay-Williams, 2007).

Education has been affected by the technology advancement, like other disciplines such as engineering, medicine, trade, science and agriculture. In the classrooms and schools inside the real life environments, technology has become facilitator to make students gain knowledge and skill in this way and prepare them for the future. Accordingly, for mathematics lessons, computerized physical models (manipulatives) from multiple representations and dynamic geometry software for visualization and exploration have been used to help students to make sense of mathematical concepts or problems. Therefore, teachers who are active in instruction process have been expected to have technological content knowledge related to their own fields besides technology knowledge (Van de Wall et all., 2007). In respect to this, Mishra and Koehler (2006) have developed TPCK model by integrating technology to PCK model (Figure 1). TPCK model has seven components that are composed of content (CK), pedagogy (PK), technology (TK) knowledge and their intersections as pedagogical content knowledge (PCK), technological content knowledge (TCK), technological pedagogical knowledge (TPK) and technological pedagogical content knowledge (TPACK). They define these components as follows.

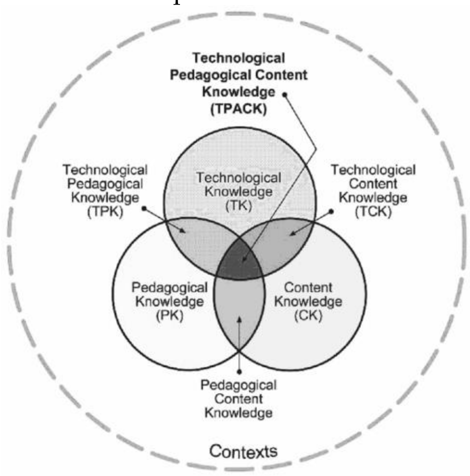


Figure 1. Seven Components of Technological Pedagogical Content Knowledge (Mishra and Koehler, 2006)

1. *Content knowledge (CK):* Knowledge about actual subject matter that is to be learned or taught. It includes knowledge of central facts, concepts, theories, and procedures within a given field; knowledge of explanatory frameworks that organize and connect ideas; and knowledge of the rules of evidence and proof.
2. *Pedagogical knowledge (PK):* It refers to the methods and processes of teaching and includes knowledge in classroom management, assessment, lesson plan development and implementation, student learning, student evaluation, and also understanding cognitive, social, developmental theories of learning and how to apply them.
3. *Technology knowledge (TK):* It refers to the knowledge about various technologies, ranging from low-tech technologies such as pencil and paper to digital technologies such as the Internet, digital video, interactive whiteboards, and software programs.
4. *Pedagogical content knowledge (PCK):* It refers to knowledge for developing better teaching practices in the content areas. This knowledge includes knowing what teaching approaches fit the content, how elements of the content can be arranged for better teaching, which teaching strategies incorporate appropriate conceptual representations in order to address learner difficulties and misconceptions and foster meaningful understanding.

5. *Technological content knowledge (TCK):* It refers to the knowledge of how technology can create new representations for specific content. Teachers having this knowledge can provide learners with a specific technology to practice and understand concepts in a specific content area.
6. *Technological pedagogical knowledge (TPK):* It refers to the knowledge of how various technologies can be used in teaching, and to understanding that using technology may change the way teachers teach.
7. *Technological pedagogical content knowledge (TPACK):* It refers to the knowledge required by teachers for integrating technology into their teaching in any content area.

Since today's teachers are expected to have integrated TPACK knowledge, researches, international conferences and thesis about TPACK has been increasing in number since 2006. According to Baran and Canbazoglu-Bilici (2015), studies related to TPACK has been conducted since 2010 in Turkey and these studies mostly preferred to use pre-service teachers including mostly mathematics and science teachers. Also, most of the studies about TPACK are related to adaptation or development and implementation of measurement instruments by different researchers (Bulut, 2012, & Önal, 2016) and designing various technology-integrated application courses to improve prospective teachers' TPACK levels (Bray, & Tangney, 2017; Kabakçı Yurdakul, 2013; İpek vd., 2014; Hosseini, 2015; Alqahtani, vd., 2017) by generally using pedagogic methods like workshop, lesson plan preparation, micro-teaching, project work, blog using, 3D material and ppt preparation and web site creating (Baran, & Canbazoglu-Bilici, 2015).

The study about the development of TPACK levels of pre-service mathematics teachers indicates that the results of TPACK instrument is higher than the results of the lesson plans and their applications assigned by rubrics, which leads to the contradiction between them (Mudzimiri, 2012). In this respect, the research that GeoGebra based lesson plans were assessed by using theoretical framework instead of using surveys shows that most of the participants did not reach to TPACK level and they remained TCK level. That means, many teachers are using it only as a presentation tool, not for making students comprehend, discover or question something although they consider themselves as effective technology users in their lessons. So, it is important for teachers to know which criteria are expected from them to be in TPACK level (Akyüz, 2016). Considering this issue, the framework of TPACK levels adapted to mathematics content can be introduced for mathematics teachers in order to make them conscious when they are applying technology to their lessons. Beside the theoretical information about the expected criteria, the framework can be used within the context of microteaching applications using sample activities since microteaching contributes them to gain experience about implementation, see different types of lessons, instruction' weaknesses and strengths (Belt, 1967), and also develop critical thinking skills by giving feedbacks with observation form (Benton-Kupper, 2001). Therefore, the purpose of the study is to make prospective mathematics teachers aware on TPACK by providing them an environment to use the TPACK assessment framework in the context of the microteachings including sample geometry activities, and then to identify their levels by the instrument. So, this study gives an example of how to use TPACK criteria for implementing dynamic geometry software (Geogebra) into the mathematics lessons with different TPACK levels in order to raise their awareness about effective integration in a short period.

2. Method

2.1. Research Design

Case study design from qualitative research approaches was used in this study. This design enables to examine one or more than one cases, environment or program and to collect qualitative and quantitative data (Gay, Mills, & Airasian, 2012, McMillan & Schumacher, 2010). The selected design is appropriate for this study since it aims to show a sample application model to be followed in order to make participants conscious about TPACK levels, and also to identify their levels quantitatively.

2.2. Sample

The sample of this study consists of fourth year 90 pre-service elementary mathematics teachers in the fall term of 2017-2018 academic year of a big university in Turkey. Since the study needs participants taken the courses related to content, pedagogy and technology knowledge before to understand this application process in a limited time, forth year students were selected as a sample by using purposive sampling method. This method allows choosing participants that can provide the needed information based on a research purpose and previous information about population (Fraenkel, Wallen, & Hyun, 2012).

2.3. Data Collection Tools

The survey called "TPACK regarding Geometry Instrument" developed by Bulut (2012) was used as a data collection tool at the end of the study to identify participants TPACK levels and to take their views about the implementation. This instrument containing seven subcomponents in the context of TPACK has 51items with six point Likert-type scale that has the interval between "absolutely agree" and "absolutely disagree". Although there exist TPACK surveys or scales measuring general attitudes towards TPACK or attitudes on specific fields like mathematics, science or language, this instrument focuses on only geometry as a content knowledge. Also, in this study the reason why it was chosen is its compatibility with the application process including activities for geometry objectives. Furthermore, the findings were analysed by using the descriptive analysis.

2.4. Procedure

As shown in Table 1., firstly one-hour theoretical information about "Techno-Pedagogical Education" was given, and two-hour micro teaching including four technology-integrated activities prepared by researcher was applied, and then lesson plans, worksheets and GeoGebra files regarding these activities were assessed, evaluated and discussed together by considering the theoretical framework of TPACK level identification. Furthermore, at the end of the implementation, "TPACK regarding Geometry Instrument" the survey was distributed them to identify their own levels and express their views about the application.

Table 1. Stages of the Process

Theoretical Information	Definitions and examples of CK, PK, PCK, TK, TCK, TPK, TPACK was given.
Micro-teaching Application	Four activities, their worksheets, Geogebra files was applied. The objectives belongs to the activities used: 1.*Activity Objective:* "Students should be able to determine that the ratio of the circumference of a circle to its diameter is a constant value by measuring." 2.*Activity Objective:* "to form parallelogram area formula; solve related problems." 3.*Activity Objective:* " to calculate the length of circle and arc." 4.*Activity Objective:* "to make connection between percentage and fraction/decimal representation of same value and translate them to each other."
Assessment of Activities	The lesson plans, worksheets and GeoGebra files regarding these activities was assessed by using theoretical framework developed by Akyüz (2016).
Survey	*TPACK regarding Geometry Instrument* was distributed to students.

2.4.1 Theoretical Information:

In the stage of "Theoretical Information", CK, PK, PCK, TK, TPK, TCK and TPACK was defined with examples respectively.

- For PK, assessment methods on student performance, classroom management, different teaching methods were reminded.

- For PCK, geometry objectives on curriculum, common misconceptions and multiple representations, some manipulatives (base-ten blocks, geoboard, symmetry mirror, fraction bars, algebra tiles etc.) were mentioned briefly. Also, points to consider in preparing a worksheet through the objective were explained by showing some sections from sample worksheets. According to them, some formal cases in organizing and designing worksheets were emphasized as follows; "*creating tables*" can enable students to find patterns by making comparison and discover related formulas; "*leaving blanks with an appropriate length* (dotted/undotted)" can allow them to make their own definitions, explanations and to express their own reasoning; "*putting graph, isometric, doted paper background*" below questions can enable drawing properly.
- For TPK, social media (twitter, etc.) and computer applications (kahoot, edmodo etc.) was introduced for the usage in classroom assessment and discussions.
- For TCK, applications or software in the geometry field like GeoGebra, Geometry's Sketchpad, Cabri, Mozaic Digital Teaching, Tinkercad, Sketchup 3D, NCTM Illuminations was introduced by examples and videos. The examples constructed by dynamic geometry or 3D software were demonstrated, and the usage of them in real life problems, mathematical modeling and teaching geometrical conceptions was discussed together.

2.4.2. Micro-Teaching Application

In this process, four worksheets and four lesson plans was distributed to participants separated into groups of two or three. Each of the worksheets includes one activity aimed at teaching one geometry objective. After completing these activities as researcher acting like a teacher and pre-service teachers in the role of elementary students, lesson plans, worksheets, Geogebra files and the micro-teaching regarding these activities were analysed, assessed and discussed together by considering the theoretical framework of TPACK level identification in Table 2.

Table 2. Theoretical framework for TPACK level identification (Akyüz, 2016)

CK	Using Dynamic Geometry Software (DGS) to draw figures without dynamic features (same activity can be done by programs that have not dynamic features)
TK	Designing activities with DGS and these activities are using dynamic features of DGS effectively **(**same activity can not be done by programs like Powerpoint)
TCK	Activities including questions that support observation and justification of mathematical rules by using dynamic features Creating figures not only by the help of tools but also by using mathematical features
TPK	Instruction of weak mathematical content by the help of technology and it has pedagogical elements (asking different solutions, explanations) Linking activities to daily life
TPACK	Asking questions leading to generalizations by assembling technology with a rich mathematical content and a pedagogical approach, and content development providing students to discover mathematical rules Questioning why it is true and whether it is true for every case besides justifying one rule dynamically

2.4.3. *Assessments and Evaluation of Sample Activity*

The activity aimed at teaching of the six-grade geometry objective *"students should be able to determine that the ratio of the circumference of a circle to its diameter is a constant value by measuring".* The worksheet of this activity was given in Figure 2 and its lesson plan was given at Appendix 1. After applying this activity to participants in micro-teaching, evaluations were discussed among them by considering TPACK components. According to the framework, starting the lesson by asking question (can you measure the circumference of circle with any size by using only a straight ruler?) to draw students' attentions and giving them pause for thought point out teacher's pedagogical knowledge. In the first question, after giving example that draws equilateral triangles inside and outside of the circle, teacher ask them to draw square in a similar way and make predictions about the circumference of the circle. At this point, giving example (drawing with triangles), relating with their previous knowledge (using regular polygons) and asking them to make estimation (approximate circumference) show the pedagogical knowledge. Then, teacher asks students to think about what can be done to make estimations more close to the circumference and opens Geogebra 1 file that enables drawing regular polygons more than four sides inside and outside of the circle easily and allows students to see the transition from polygons to circle by dragging slide with "n" the number of polygon sides as its screenshot shown in Figure 3. At this point, if the Geogebra file used dynamic feature of DGS is used to only demonstration tool, it will be lack of pedagogical knowledge and remain at TCK level. However, giving some time students to discover it themselves, putting some blanks in worksheet to write their explanations what they have explored, allowing them to express their reasoning, and creating discussion environments for different approaches lead TCK level to be TPACK. Also, asking to make calculation of circumference by using rope and comparing the result with their estimations are appropriate in terms of pedagogical content knowledge.

In the section B, students were asked to construct four circles having different radii by using Geogebra and record circumferences they obtained to the table in the worksheet. So, they were expected to notice the relationship between the diameter and circumference. At this point, for calculations using software after physical object (rope) and giving hint (look at the ratio of circumference to diameter) were found pedagogically appropriate. Also, as its screenshot given in Figure 4, *Geogebra2* file illustrating the dynamic opening of the circumference of the circle let students observe different values in diameter and circumference of circles more by dragging the slider. So, students are expected to discover the ratio 3,14 and see the symbol of pi number. If Geogebra file is used with probing questions contributing to observation and justification the number pi as presentation tool like video, it can be considered as TCK level. However, questions for generalizing and exploration, classroom discussion environment created for brainstorming and the worksheet containing the table, blanks for explanations can be considered as pedagogical sides, which leads to TPACK level. Also, by asking to relate the last expression ($\frac{P1}{R} < \pi < \frac{P2}{R}$) to the first one ($P_1 < C < P_2$) in the last question and giving them historical information about pi and Archimedes method that they used in this activity can be taken into account as pedagogical approach. However, integrating daily life examples can develop this activity more.

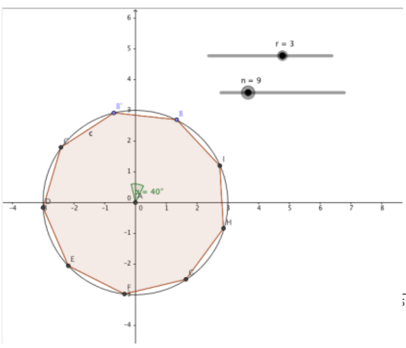


Figure 3. GeoGebra File 1

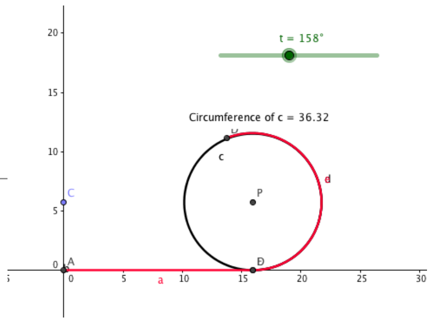


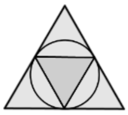
Figure 4. GeoGebra File 2

WORKSHEET

Part A

Can you measure the circumference of circle with any size by using only a straight ruler?

Example:



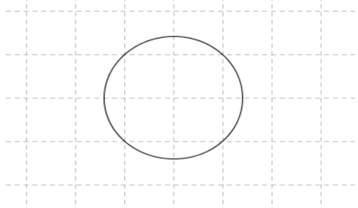
If the perimeter of inside triangle = P_1

the perimeter of outside triangle = P_2

the circumference of a circle = C, then

$P_1 < C < P_2$

1.) Draw your circle below. Like the example, draw two squares inside and outside of the circle. Measure perimeters of squares and make estimation about the circumference of the circle.



Your estimation:

2.) Open the "**Geogebra1**" file. Drag the slider n through left and right. What did you notice when you changed the sides of polygons? What can you say about the circumference of the circle when n is increasing?

……..

3.) Now, you have also a rope. Find your circle's circumference. Is this result corresponding to your estimation that you made in the question 2.

Part B

4.) Construct four circles that have different radii by using the GeoGebra and the option "circle with center and given radius". Find their circumferences. Then record what you obtained in the table below.

	Diameter	Circumference
Circle 1		
Circle 2		
Circle 3		
Circle 4		

5.) According to the table above, is there a relationship between the diameter and circumference? If yes, explain your reasoning (hint: look at the ratio). Use the Geogebra 2 to justify your answer.

………

………

6.) If you select the diameter 1 by using Geogebra, what is shown for the circumference of the circle? So, decide to what it equals?

7.) Remember that we found $P_1 < C < P_2$ at the beginning. So, can we write an equation like $\frac{P1}{R} < \pi < \frac{P2}{R}$. Explain your reasoning.

Figure 5. Worksheet of The Sample Activity

3. Findings

Descriptive analysis of the prospective mathematics teachers for TPACK regarding Geometry Instrument was given in Table 3.

Table 3. Descriptive Analysis in terms of TPACK components

Components	Number of items	N	$\bar{X}$	SD
CK	7	90	4.79	1.026
PK	8	90	4.78	.491
PCK	8	90	4.70	.680
TK	7	90	4.21	.863
TPK	7	90	4.53	.773
TCK	5	90	4.03	.923
TPACK	9	90	4.32	.818

As shown in Table 3., pre-service mathematics teachers' perception on content knowledge CK ($\bar{X}$ =4.79) is the highest mean value while technological content knowledge TCK ($\bar{X}$ =4.03) is the lowest mean value. Also, it can be seen that the mean values of pedagogical knowledge PK ($\bar{X}$ =4.78) and content knowledge CK ($\bar{X}$ =4.79) are close to each other. Moreover, it can be seen that their perceptions on technological content knowledge TCK ($\bar{X}$ =4.03) and technological knowledge TK ($\bar{X}$ =4.21) are less than technological pedagogical knowledge TPK ($\bar{X}$ =4.53). The total mean value of the instrument ($\bar{X}$ =4.50) is higher than moderate out of six.

In addition to them, according to participants' views about this study expressed at the end of the instrument, it was stated that they found this study beneficial, they gained awareness about TPACK, they have felt insufficient in technological knowledge, they want to learn DGS, they asked courses teaching DGS like Geogebra, they have seen the importance of pedagogy, they have learned the technology integration criteria in mathematics lesson, it helped them to assemble their knowledge they have learned for four years, they asked an extended version of this study.

4. Discussion & Conclusion

Technology has become a part of education and usage of dynamic geometry software (DGS) has become common in mathematics education. However, using DGS in lessons does not show that teachers can integrate it effectively since they mostly use it only as a presentation tool or visual demonstration vehicle instead of exploration and justification material. As a result, knowing the expected TPACK level criterions in mathematics can enable mathematics teachers to plan their lessons by integrating it in terms of these criterions (Akyüz, 2016). So, in this study the theoretical framework regarding TPACK levels was used with four sample activities based geometry which have Geogebra files, worksheets and lesson plans to make participants gain awareness about TPACK by assessing micro teachings applied, and also the instrument was used to identify their levels.

The worksheet, geogebra file screenshots and lesson plan of one activity among four activities were presented in this study. The description of the activity applied to participants as microteaching was given with interpretations done in classroom by discussing with respect to TPACK levels framework. According to the framework adapted by Akyüz (2016), the geogebra files can be demonstrated to students as an powerpoint presentation (CK level), the dynamic features of this software can be used (TK level), questions that leads students to make observations, justifications of mathematical rules by using dynamic features can be added (TCK level), different solutions and explanations can be asked to students (TPK level), and by the help of the dynamic features of geogebra and leading questions students can discover mathematical rules by generalizing and explain their reasoning (TPACK level). Therefore, the sample activity was examined in terms of its usage of geogebra files (for only demonstration or dynamic features), worksheet questions that lead to justification, estimation, discovering, generalization and explanation of reasoning and the structure of the worksheet whether it contains blanks, tables or visuals to help students to express their thoughts, make generalization and explain their reasoning.

After completing the four microteachings in which one researcher played a role as a teacher and participants as a student with appropriate grade level, TPACK regarding Geometry Instruments that contain one open-ended question and 51 items were distributed to each participant. With respect to the results from the scale used at the end of the study, pre-service teachers' content (CK) and pedagogical knowledge (PK) are higher than the technological content (TCK) and technological pedagogical knowledge (TPACK). This may result from that they have taken the courses related to content (mathematics) and pedagogical (educational sciences) knowledge, however they have not take technology related courses except Mathematica tuturials that was used for advanced subjects such as functions, limits. Their technological content knowledge (TCK) has the least score among them, which may result from the fact that the most of the participants did not familiar with the dynamic geometry software like Geogebra and they did not have an opportunity to prepare models for elementary students.

In addition, as the participants stated their views in open-ended question, they have found this study beneficial; they felt insufficient in technology; want to learn DGS; want to integrate it into mathematics lessons with respect to the criterion; pointed out the importance of the pedagogy; asked an extended version of the study that helped them to assemble their four years knowledge.

Furthermore, that the total mean value of the TPACK instrument ($\bar{X}$ =4.50) is higher than moderate out of six is parallel to the researches done in literature (Baran, & Canbazoglu-Bilici, 2015; Bulut, 2012). However, as Mudzimiri (2012) stated, the instrument results and the scores from rubrics assessing the applications can be unmatching. Teachers cannot reach to TPACK level although they know how to integrate (Akyüz, 2016). In other words, like the existence of differences between scale scores and lesson plans, there may be conflict between the lesson plan and its implementation from pedagogical aspects. Therefore, it can be suggested that this study aimed to arouse awareness about TPACK can be extended in longer term to develop their levels like the courses designed (Akyüz, 2016; Bray, & Tangney, 2017; Kabakçı Yurdakul, 2013; İpek vd., 2014; Hosseini, 2015; Alqahtani, vd., 2017). However, it is important to use TPACK level framework to enable them be aware of the integration criteria for mathematics content. So, in the lights of the findings participants want to take courses regarding TCK, TPK and TPACK, so it can be suggested that courses that focuses on teaching DGS like GeoGebra for TCK, courses focusing on using blog, twitter, kahoot for TPK, and courses focusing on teaching how to integrate them for TPACK can be given.

Therefore, microteaching applications that include the parts of lesson planning, videotaping and feedbacks (Belt, 1967; Benton-Kupper, 2001) can be extended by observation forms focusing on specific content and technology integration. The applications including lesson plans constructed by pre-service teachers and their micro-teachings can be examined and evaluated in the further studies instead of instruments. So, creating an active learning environment that pre-service teachers are forming lesson plans and worksheets by using various software and programs, applying them and giving feedbacks by using frameworks and taking feedbacks from their peers and the researchers can provide them to reach and develop the level of TPACK.

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Appendix-1

LESSON PLAN

Your Name:	Rabia		
Learning Area:	Geometry/Circle		
Grade Level:	6th	Duration:	80 mins
Materials:	Activity sheets, Geogebra File		
Objectives:	(6.3.3.3) Students should be able to determine that the ratio of the circumference of a circle to its diameter is a constant value by measuring.		
Prerequisite Knowledge:	Students can identify the regular polygons and use the inequality		
Teaching Method(s):	Discovering, questioninq		

START

- ✓ To draw students' attention, teacher starts the lesson by asking this question: "Can you measure the circumference of circle with any size by using only a straight ruler?"
- ✓ Then, teacher distributes rulers to each student.
- ✓ After giving enough time to think about it, teacher asks what kind of polygons can be measured by the ruler to remind them polygons that they know such as triangle, square.

MIDDLE

- ✓ Teacher wants them to look at the example in activity sheet and wants one/two students to tell aloud what it means.
- ✓ Like the given example, teacher asks to do first question. So, students will make estimation about the circumference of the circle by drawing squares inside and outside of the circle.
- ✓ After they all completed, teacher asks to open the "Geogebra1" file and drag the slider n through left and right. Then, s/he asks them to respond the questions. "What did you notice when you changed the sides of polygons? What can you say about the circumference of the circle when n is increasing? "
- ✓ Then teacher distributes strings and circles to each them and wants to measure its circumference by using only both string and ruler. Also, s/he wants them to compare their results by their estimation made in the question 2.
- ✓ For the 5th question, teacher asks them to construct four circles that have different radii by using the GeoGebra and the option "circle with center and given radius" and find their circumferences by using length tool. Then, s/he asks to record what they obtained in the table.
- ✓ Also, s/he allows them to use "Geogebra2"file to explore the relationship and encourages to use their tables in previous question. By giving hint "look at the ratio of circumference to diameter", s/he makes students to discover the ratio 3,14.
- ✓ Then, s/he asks them to find the circumference of the circle with 1 diameter by using Geogebra. Thus, they can see the symbol of pi number. Then teacher wants them to decide what pi equals to. So, students can discover that pi is 3,14.
- ✓ For the last question, teacher asks them to make connection between the expressions that $P_1 < C < P_2$ is given at the beginning and $\frac{P1}{R} < \pi < \frac{P2}{R}$ and explain their reasoning.

CLOSING

- ✓ Then, teacher want students to summarizes and s/he informs that they started by inside&outside polygons of the circle to make prediction like Archimedes approaching to finding pi and they examined the relation of the diameter and circumference of the circle. So, they found pi as a ratio of the circumference of the circle to its diameter.
- ✓ As a conclusion to alert students about the importance of number pi, teacher writes on the board that 3.14159 26535 89793 23846 is and tells about pi and gives real life examples. https://www.youtube.com/watch?v=zIHGOx5LOO