International Journal of Educational Studies in Mathematics, 2018, 5(4), 123-131

International Journal of Educational Studies in Mathematics

IJESIM

EDUGARDEN

ISSN:2148-5984

# Examining the Changes in Prospective Teachers' Knowledge on Writing Story Problems for Fraction Number Sentences

**Nesrin Sahin[1], James Fetterly[1], Sirin Budak[2], Wesley Martsching[1]**

[1]*University of Central Arkansas, USA*
[2]*Sirin Budak, University of Wisconsin at Stevens Point, USA*

**ABSTRACT**

We investigated how instruction on conceptual understanding of fraction concepts and operations, and instruction on writing story problems changed prospective teachers' (PTs) knowledge on writing story problems for fraction number sentences. We also compared the effect of two instructional approaches, error analysis and direct instruction, on PTs' knowledge for writing story problems. Receiving instruction on fraction concepts and operations, and on writing story problems had a significant effect on PTs' knowledge on writing story problems. However, we did not find any significant difference between the two instructional approaches. In general, writing story problems for fraction addition number sentences was easy for most PTs before they received any instruction. After receiving the instructions PTs showed the highest improvement for fraction subtraction and division number sentences, however writing story problems for fraction multiplication number sentences remained a challenge for most PTs.

**ARTICLE INFO**

***Article History:***
Received: 30.06.2018
Received in revised form: 11.07.2018
Accepted:01.11.2018
Available online:10.11.2018
***Article Type:*** Standard Paper
***Keywords:*** fractions, fraction operations, story problems, elementary mathematics.



## 1. Background

In 2008, the National Mathematics Advisory Panel stated that proficiency with fractions should be a major goal for K-8 mathematics (NMAP, 2008). They stated that proficiency with fractions is foundational for algebra, yet it seems to be severely underdeveloped. Teachers' conceptual knowledge of fraction concepts and how they teach them to their students are important factors for students' conceptual development of fractions.

In the U.S., conventional instruction with fractions is usually procedural or rule-based (NRC, 2001), and U.S. teachers are more likely to emphasize algorithmic processes and less likely to create story problems to help their students understand fractions (An, Kulm, & Wu, 2004). Ozciftci (2007) reported a similar situation in Turkey by stating that instruction on rational numbers emphasizes using procedures and rules rather than focusing on conceptual understanding. A story problem represents the information on a problem in a real life context rather than using mathematical symbols, hence making the problem relatable to student's daily life. Therefore, it is important for teachers to have the ability to create a real life context for the mathematical ideas that they will teach. Students are less likely to make a conceptual error when either a visual model or a story problem context is present in scaffolding (Rittle-Johnson & Koedinger, 2005). Therefore, students need to learn fractions in real-world contexts that are meaningful to them (Cramer & Whitney, 2010). This requires teachers to have the necessary knowledge to develop story problem situations for the concepts that they teach so that students can relate the mathematics that they learn to real life situations (McAllister & Beaver, 2012).

[1] Corresponding author's address: University of Central Arkansas, USA
Telephone: 501-450-5660
e-mail: nesrins@uca.edu

In mathematics education, problem posing is related to both the creation of questions in a mathematical context and to the reformulation of existing ill-structured problems (Pirie, 2002). Posing story problems requires a deeper understanding than the symbolic manipulation of the mathematical content. (Pirie, 2002). However, it is not an easy task, and teachers face problems in drawing meanings from symbolically represented mathematical content for some certain curriculum areas in primary and lower secondary mathematics (Rubenstein &Thompson, 2001). Fraction concepts are one of such curriculum area (Ma, 2010). McAllister and Beaver (2012) examined the story problems created by prospective teachers (PTs) for fraction number sentences and identified 40 distinct errors. Many of their participants reported that they had never written a story problem before, and also many had no idea how to even attempt to write one, particularly for multiplication and division. Students as well as teachers should be challenged with tasks that require them to create story problems for the mathematics that they learn. Having students write story problems allows teachers to assess their students' levels of mathematical understanding (Barlow & Drake, 2008), and student-authored story problems can reveal a variety of students' misconceptions (Alexander & Ambrose, 2010). For example, Dixon et al. (2014) examined the story problems generated by PTs for fraction subtraction, and identified a common misconception, which they described as the incorrect redefinition of the whole.

In this study, we examined the changes in PTs' knowledge on writing story problems for fraction number sentences. This study sought to answer the following research questions:

1. Was there a mean difference between the number of correct story problems written by PTs before and after receiving instruction on fraction concepts and operations?
2. Was there a mean difference between the number of correct story problems written by PTs before and after receiving instruction on writing story problems for fraction number sentences?
3. Was there a mean difference between the number of correct story problems written by PTs who received instruction that focused on error analysis and who received direct instruction on writing story problems for fraction number sentences?
4. What common errors did PTs make when they write story problems for specified fraction number sentences?

## 2.Methods

### 2.1.Participants and Data Collection

This quasi-experimental study used quantitative data to examine the changes in PTs' knowledge on writing story problems for fraction number sentences. Participants were 65 PTs who were enrolled in two sections of mathematics content course designed for elementary and middle school teachers. One section was chosen as the treatment group and the other section served as the control group. PTs in both groups were given two number sentences for each of the four basic operations for which the first number sentence included two fractions (e.g. a/b*c/d), and the second number sentence included two mixed numbers (e.g. m a/d*n c/d)), totaling eight number sentences. They were asked to write a story problem for each number sentence at three data collection points: (a) in August, before they received any instruction, (b) in October, after they received instruction on fraction concepts and operations, and (c) in November, after they received instruction on writing story problems for fraction number sentences.

### 2.2.Class Instruction

For both the treatment and control groups, the first part of the class instruction focused on mathematical sense making of fraction concepts and operations using different strategies with emphasis on modeling strategies. Class instruction usually started with PTs working on contextualized problems in their small groups, and then continued with sharing variety of solution strategies as a whole class. When both groups finished fraction concepts and operations, the groups

received different types of instruction on writing story problems for specified fraction number sentences. About six, 50 minute long class meetings were devoted to writing story problems in the treatment group, and three 75-minute long class meetings were devoted in the control group. The treatment group received instruction that used an error analysis approach, and the control group received direct instruction for writing story problems. In the treatment group, we provided our PTs with story problems that included variety of errors, which are presented in table 1. The codes in table 1 were created by the first and second authors of this study using data collected from our PTs enrolled into the same course in previous semesters.

Table 1. Error Code with Explanation

| Error Code | Name | Sample problem / Explanation |
|---|---|---|
| 1 | Lack of unit | Lucy has 2/3 of candies. She made goodie bags that hold ½. How many goodie bags can she make? |
| 2 | Undefined whole | Beck and Mary have some leftover pizzas. Beck ate ½ of her pizza and Mary ate 2/3 of her pizza. How much did they eat together? |
| 3 | Different size wholes | If you ate 1/2 of a pie you had and then ate 2/3 of a mini pizza from pizza hut. How much did you eat altogether? |
| 4 | Structurally correct but contextually not logical problems | I had 2 2/3 pies and I ate 1 1/2 of it. How much did I eat? (One can not eat more than what they have started with) |
| 5 | Key word | ¾ goes into 7/8 how many times? |
| 6 | Two step subtraction problems that involves multiplication in the first step | I have 5/6 gallon of lemonade. I drank 1/3 of the lemonade. How much lemonade do I have left? |
| 7 | Addition Problem | When written for a subtraction or a multiplication or a division number sentence. |
| 8 | Subtraction Problem | When written for an addition or a multiplication or a division number sentence.. |
| 9 | Multiplication Problem | When written for an addition or a subtraction, or a division number sentence.. |
| 10 | Division Problem | When written for an addition or a subtraction, or a multiplication number sentence. |
| 11 | Other (Problem does not make sense, includes more than one error, or problem incomplete) | Wade is using 1/3 of a deck of cards for a magic trick. If 3/5 of the cards he is using are red, then how many are black? |
| 999 | missing | When PTs did not attempt to write a story problem |

PTs in the treatment group first were asked to identify the errors in their small groups. While determining the errors, PTs were encouraged to use modeling strategies to solve the problems if needed. Next, the errors and how to change the story problems to eliminate the errors were discussed as a whole class. Then, PTs wrote their own story problems for specified fraction number sentences and we shared several problems and discussed the errors, if any, as a whole class.

In the control group, direct teaching approach was used. PTs in the control group were asked to write story problems. Then, we strategically chose story problems that included errors for each operation so that we could go through all error types that were presented in table 1. We explained the errors in the story problems, and had a whole class discussion about how to change the story problems to eliminate the errors.

### 2.3.Data Analysis

Two researchers independently coded the story problems generated by our PTs at each data collection point into two categories as correct and incorrect. The percent agreement method used for inter-rater reliability and it was calculated to be 0.86. The researchers met twice to discuss the discrepancies and came to an agreement on the final coding. Descriptive statistics were obtained for each type of operation at each data collection point. A two-way repeated measures Anova was conducted to

compare the differences between the treatment and control groups, and to compare the differences among three data collection points. The two researchers also coded the data for errors based on the error-codes presented in table 1.

**3.Results**

The line plots in figures 1 and 2 show proportions of correct story problems obtained by the PTs in the treatment and control groups for each operation separately at each data collection point. Table 2 displays the proportions for each group at each data collection point as well as the averages for both groups in a table.

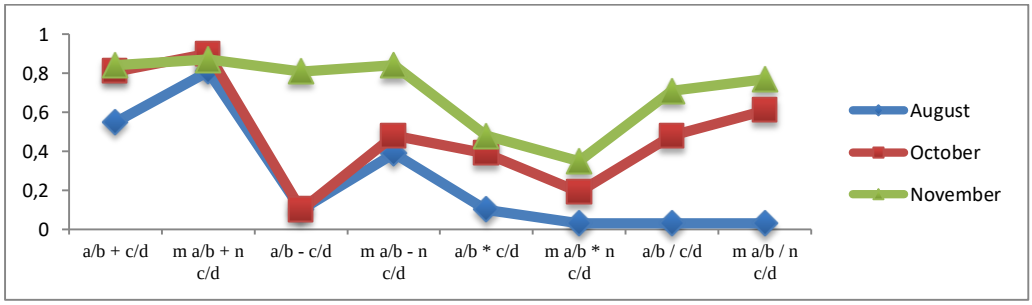


Figure1: The proportions of correct story problems for each operation at each data collection point in the treatment group

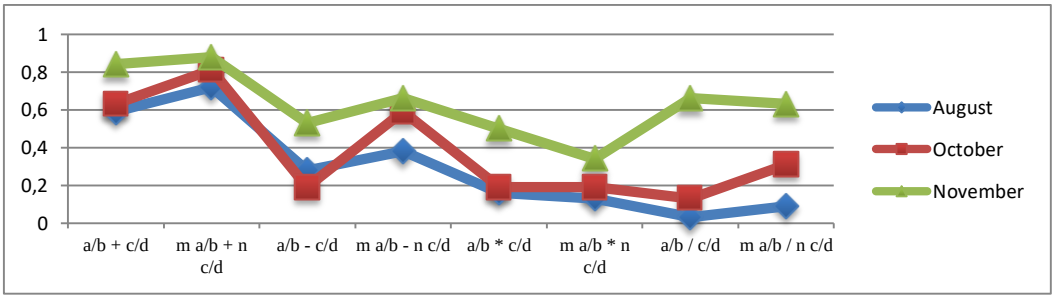


Figure 2: The proportions of correct story problems for each operation at each data collection point in the control group

The descriptive statistics presented above and below show that for both groups, the highest proportion (on average 57% and 77% for fractions and mixed numbers, respectively) of correct story problems written by PTs was for the addition number sentences in August, before they received instruction on fraction concepts and operations. The proportion of correct story problems for subtraction number sentences was very low (on average 19% and 39% for fractions and mixed numbers, respectively) and the proportions were the lowest for multiplication and division number sentences. This revealed that writing story problems for multiplication and division number sentences were the most challenging tasks for PTs in August.

Table 2: The proportions of correct story problems for each operation at each data collection point in both groups, and the averages for both groups.

| | | Number Sentences | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Time | Condition | $\frac{a}{b}+\frac{c}{d}$ | $m\frac{a}{b}+n\frac{c}{d}$ | $\frac{a}{b}-\frac{c}{d}$ | $m\frac{a}{b}-n\frac{c}{d}$ | $\frac{a}{b}\times\frac{c}{d}$ | $m\frac{a}{b}\times n\frac{c}{d}$ | $\frac{a}{b}\div\frac{c}{d}$ | $m\frac{a}{b}\div n\frac{c}{d}$ |
| August | Treatment | .55 | .81 | .1 | .39 | .1 | .03 | .03 | .03 |
| | Control | .59 | .72 | .28 | .38 | .16 | .13 | .03 | .09 |
| | Average | .57 | .77 | .19 | .39 | .13 | .08 | .03 | .06 |
| October | Treatment | .81 | .9 | .1 | .48 | .39 | .19 | .48 | .61 |
| | Control | .63 | .81 | .19 | .59 | .19 | .19 | .13 | .31 |
| | Average | .72 | .86 | .15 | .54 | .29 | .19 | .31 | .46 |
| November | Treatment | .84 | .87 | .81 | .84 | .48 | .35 | ,71 | .77 |
| | Control | .84 | .88 | .53 | .66 | .5 | .34 | .66 | .63 |
| | Average | .84 | .88 | .67 | .75 | .49 | .35 | .69 | .70 |

Comparisons of proportions at different data collection points revealed that, in general, there was an increase in the proportions of correct story problems written for all four types of number sentences in October, after PTs received instruction on fraction concepts and operations. There was also a significant amount of increase in the proportions of correct story problems written for all four types of number sentences in November, after PTs received instruction on writing story problems.

Prospective teachers in both groups showed the highest improvement in writing story problems for subtraction and division number sentences from August to November. The increase in the number of correct story problems written by PTs in the treatment group were higher than it was in the control group. The least improvement from August to November was seen on story problems for multiplication number sentences in both groups.

To compare the within and between group differences, we used two-way repeated measures ANOVA. The Mauchly's Test of Sphericity was not significant. Table 3 displays the mean number of correct story problems in August, October and November for treatment and control groups.

Table 3. Descriptive Statistics

| | T / C | Mean | Std. D. | N |
|---|---|---|---|---|
| August | T | 2.03 | 1.251 | 31 |
| | C | 2.29 | 1.643 | 34 |
| | Total | 2.17 | 1.464 | 65 |
| October | T | 3.97 | 1.871 | 31 |
| | C | 3.03 | 1.586 | 34 |
| | Total | 3.48 | 1.778 | 65 |
| November | T | 5.68 | 1.777 | 31 |
| | C | 4.74 | 2.151 | 34 |
| | Total | 5.18 | 2.022 | 65 |

Table 4 displays that there is a statistically significant effect of instruction (instruction on fraction concepts and operations, and instruction on writing story problems) on their scores for writing correct story problems ($F_2$= 78.21, $p < .001$). Approximately 55% of the variance in score can be accounted for by repeated trials. Table 5 shows that there is not a significant effect of condition (i.e. no statistically significant difference between the treatment and control groups) ($F_2$=2.178, $p > .05$). Finally, table 6 shows that instruction on fraction concepts and operations, and instruction on writing story problems had a significant effect on PTs' knowledge of writing story problems. PTs obtained significantly lower scores in August than they obtained in October, and their scores from October were significantly lower than the scores they obtained in November.

Table 4. Tests of Within-Subjects Effects

Measure: MEASURE_1

| Source | | Type III Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared | Noncent. Parameter | Observed Power[a] |
|---|---|---|---|---|---|---|---|---|---|
| Instruction | Sphericity Assumed | 301.836 | 2 | 150.918 | 78.21 | .000 | 0.554 | 156.419 | 1 |
| | Greenhouse-Geisser | 301.836 | 1.842 | 163.827 | 78.21 | .000 | 0.554 | 144.094 | 1 |
| | Huynh-Feldt | 301.836 | 1.925 | 156.76 | 78.21 | .000 | 0.554 | 150.59 | 1 |
| | Lower-bound | 301.836 | 1 | 301.836 | 78.21 | .000 | 0.554 | 78.21 | 1 |
| Instruction* Condition | Sphericity Assumed | 15.621 | 2 | 7.811 | 4.048 | .02 | 0.06 | 8.095 | 0.712 |
| | Greenhouse-Geisser | 15.621 | 1.842 | 8.479 | 4.048 | .023 | 0.06 | 7.457 | 0.686 |
| | Huynh-Feldt | 15.621 | 1.925 | 8.113 | 4.048 | 0.021 | 0.06 | 7.794 | 0.7 |
| | Lower-bound | 15.621 | 1 | 15.621 | 4.048 | 0.049 | 0.06 | 4.048 | 0.508 |

Table 4. contines

| | | | | |
|---|---|---|---|---|
| Error(Instruction) | Sphericity Assumed | 243.138 | 126 | 1.93 |
| | Greenhouse-Geisser | 243.138 | 116.072 | 2.095 |
| | Huynh-Feldt | 243.138 | 121.304 | 2.004 |
| | Lower-bound | 243.138 | 63 | 3.859 |

a Computed using alpha =

Table 5. Tests of Between-Subjects Effects

Measure: MEASURE_1

Transformed Variable: Average

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. | Partial Eta Squared | Noncent. Parameter | Observed Power[a] |
|---|---|---|---|---|---|---|---|---|
| Intercept | 2553.73 | 1 | 2553.73 | 490.176 | 0 | 0.886 | 490.176 | 1 |
| Condition | 14.161 | 1 | 14.161 | 2.718 | 0.104 | 0.041 | 2.718 | 0.368 |
| Error | 328.219 | 63 | 5.21 | | | | | |

[a] Computed using alpha =

Table 6. Pairwise Comparisons

Measure: MEASURE_1

| (I) Instruction | (J) Instruction | Mean Difference (I-J) | Std. Error | Sig.b | 95% Confidence Interval for Difference[b] | |
|---|---|---|---|---|---|---|
| | | | | | Lower Bound | Upper Bound |
| August | October | -1.335* | 0.205 | .000 | -1.841 | -0.83 |
| | | -3.043* | 0.258 | .000 | -3.677 | -2.409 |
| October | August | 1.335* | 0.205 | .000 | 0.83 | 1.841 |
| | November | -1.708* | 0.264 | .000 | -2.358 | -1.058 |
| November | August | 3.043* | 0.258 | .000 | 2.409 | 3.677 |
| | October | 1.708* | 0.264 | .000 | 1.058 | 2.358 |

Based on estimated marginal means, * The mean difference is significant at the [b] Adjustment for multiple comparisons: Bonferroni.

### 3.1.Common Errors Found in Story Problems Generated by PTs

Based on the error codes presented in table 1, we coded the errors found in the story problems written by our PTs in November, which was after the PTs received instruction on writing story problems. In the following section we discussed the errors that were recognizable, hence, not the errors coded in the "other" category according to table 1.

Both the treatment and the control group performed similar on writing story problems for addition number sentences. The average proportion of the correct story problems in the treatment and the control group were 84% for adding two fractions, and 88% for adding two mixed numbers. Hence, there was relatively low number of errors. Among those, the most common one was lacking a unit for one of the fractions given in the number sentence. The story problem in table 7 represents this type of error that we identified for addition number sentences.

Table 7: Sample problem representing the error in addition story problems

| Number Sentence | Sample Problem with Error |
|---|---|
| ¼ + 2/3 =? | "I have 1/4 cup of sugar. I added 2/3 to that. How much do I have?" |

For subtraction number sentences, the treatment group performed better than the control group. In the treatment group 81% of the story problems written for subtracting fractions, and 84% of the story problems written for subtracting mixed numbers were correct, whereas in the control group the percentages were 53% and 66%, respectively. The most common error was writing a two-step subtraction problem that involved multiplication in the first step. The following problem in table 8 represents this type of error.

Table 8: Sample problems representing the error in subtraction story problems

| Number Sentence | Sample Problems with Error |
|---|---|
| 5/6 -1/3 =? | "I have 5/6 of a pizza and ate 1/3 of it. How much pizza did I have left?" |
| 2 5/6 – 1 1/3 =? | "Bob has 2 5/6 pizzas leftover. He ate 1 1/3 of his leftover pizzas for lunch. How much pizza does he have left?" |

Dixon et al. (2014) identified this error as an incorrect redefinition of the whole, since "5/6 of a pizza" and "1/3 of it" do not refer to the same size whole. Besides not matching the specified number sentence, the second problem also does not make sense logically, because 2 5/6 – 1 1/3* 2 25/6 would result a negative number.

The prospective teachers in both groups struggled most with writing story problems for multiplication number sentences. On average, 49% of the story problems written for multiplying fractions were correct, and 35% of the problems written for multiplying mixed numbers were correct. The most prevalent type of error was the "other" type of error. Besides the "other" category, the most common errors for multiplying two fractions were to write a two-step subtraction problem for which multiplication was involved in the first step, and to write a division problem, respectively. Following story problems in table 9 represent the most common errors for multiplication number sentences.

Table 9: Sample problems representing the errors in fraction multiplication story problems

| Number Sentence | Sample Problems Each with a Different Type of Error |
|---|---|
| 1/3 x 3/5 =? | "Zack has 3/5 of a pizza leftover. Olivia ate 1/3 of his leftovers. How much is left?"<br>"Mary Claire needs 1/3 cup of flour to make a cake. She has 3/5 cup of flour. How many cakes can she make?" |

For multiplying two mixed numbers, -besides the "other" category- the most common errors were to write a division problem, to write a two-step subtraction problem involving multiplication in the first step (not logical scenario), and to write a problem where the problem was structurally correct but contextually not logical, respectively. Following story problems in table 10 represent the common errors for fraction multiplication number sentences.

Tabo l0: Sample problems representing the errors in mixed number multiplication story problems

| Number Sentence | Sample Problems Each with a Different Type of Error |
|---|---|
| 2 2/3 x 1 ½ =? | "It takes 1 1/2 cups of sauce to make one serving of pasta. James has 2 2/3 cups of sauce. How many servings of pasta can James make using all the sauce?"<br>"Justin has 2 2/3 pizzas leftover. He ate 1 1/2 of the leftovers. How much is left?"<br>"I had 2 2/3 pies and I ate 1 1/2 of it. How much did I eat?" |

For division number sentences, the treatment group performed better than the control group. In the treatment group 71% of the story problems written for dividing fractions, and 77% of the story problems written for dividing mixed numbers were correct, whereas the percentages were 66% and 63% for the control group, respectively. Similar to the multiplication, the errors coded in "other" category were the most common error type in student generated story problems for division number sentences. Besides the other category, the most common error for division was lacking a unit for one of the fractions in the number sentence. The story problem in table 11 represents this type of problem.

Table 11: Sample problem representing the error in division story problems

| Number Sentence | Sample problem with Error |
|---|---|
| 7/8 ÷3/4 =? | "I have 7/8 pounds of M&M's. I want each bag to be filled with 3/4. How many bags will I need?" |

**4. Discussion / Implications**

The number of studies conducted to investigate PTs' knowledge of writing story problems for fraction number sentences is very limited. Those studies have identified that writing story problems for fraction numbers sentences was challenging for many PTs. In our study we have investigated how instruction affects PTs' performances on writing story problems for fraction number sentences. The PTs showed a significant increase in their mean number of correct story problems for fraction number sentences after receiving instruction on fraction concepts and operations. Furthermore, their scores continued to improve significantly after they received instruction on writing story problems. Therefore, we recommend that mathematics educators should include tasks about writing story problems for specified fraction number sentences in their content courses. Initially, most PTs were able to write a correct story problem for addition number sentences. Only a few PTs were able to write story problems that represented the specified subtraction number sentences, and even fewer numbers of PTs were able to do so for multiplication and division number sentences.

After receiving instruction on fraction concepts and operations, and specifically on writing story problems, they improved their knowledge of writing story problems more for subtraction and division number sentences than they did for multiplication number sentences. Many of our student stated in class that among the four operations, multiplication number sentences was the most challenging ones for them to write a story problem for.

In this study, we also examined the two types of instruction (error analysis approach and direct instruction) on writing story problems, but did not find any significant differences between these two approaches. The PTs receiving the error analysis approach had a higher proportion of correct story problems than the PTs had in the control group for subtraction and division number sentences, but the two groups obtained similar proportions of correct story problems for multiplication and addition number sentences.

In conclusion, although PTs showed a decent amount of increase in their knowledge for creating story problems for subtraction and division number sentences, creating a story problem for a fraction multiplication number sentence remained a challenge for most PTs in our study. In our future studies we would like to investigate why multiplication number sentence was the most challenging task to write a story problem for, and in what ways we can improve our instruction to be able to help our PTs to improve their knowledge on writing story problems specifically for multiplication number sentences.

**5. Limitations**

We encouraged our PTs to put forth their best effort when they wrote story problems for specified fraction number sentences at each data collection point, however we can never be sure whether they did so or not.

**References**

Alexander, C. M., & Ambrose, R. C. (2010). Digesting Student-Authored Story Problems. *Mathematics Teaching in the Middle School*, *16*(1), 27-33.

Ann, S., Kulm, G. & Wu, Z. (2004). The Pedagogical content knowledge of middle school mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145-172.

Barlow, A. T., & Drake, J. M. (2008). Division by a fraction: Assessing understanding through problem writing. *Mathematics Teaching in the Middle School*, *13*(6), 326-332.

Cramer, K., & Whitney, S. (2010). Learning rational number concepts and skills in elementary school classrooms. *Teaching and learning mathematics: Translating research for elementary school teachers*, 15-22.

Dixon, J. K., Andreasen, J. B., Avila, C. L., Bawatneh, Z., Deichert, D. L., Howse, T. D., & Turner, M. S. (2014). Redefining the whole: Common errors in elementary preservice teachers' self-authored word problems for fraction subtraction. *Investigations in Mathematics Learning*, *7*(1), 1-22.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Routledge.

McAllister, C. J., & Beaver, C. (2012). Identification of error types in preservice teachers' attempts to create fraction story problems for specified operations. *School Science and Mathematics*, *112*(2), 88-98.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

National Research Council, & Mathematics Learning Study Committee. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

Özçiftçi, R. (2007). Rasyonel sayıların öğretimindeki hatalar ve alınması gereken tedbirler (Doctoral dissertation, Selçuk Üniversitesi Fen Bilimleri Enstitüsü).

Pirie, S. (2002). Problem posing: What can it tell us about students' mathematical understanding? In D. Mewborn, P. Sztajn, E. White, H. Wiegel, R. Bryant & K. Nooney (Eds.), Psychology of Mathematics Education North America (PME-NA), Athens, GA (pp. 927–958, vol 1-4). Columbus, OH: Eric Clearinghouse for Science, Mathematics, and Environmental Education.

Rittle-Johnson, B., & Koedinger, K. R. (2005). Designing knowledge scaffolds to support mathematical problem solving. *Cognition and Instruction*, *23*(3), 313-349.

Rubenstein, R. N., & Thompson, D. R. (2001). Learning mathematical symbolism: Challenges and instructional strategies. *The Mathematics Teacher*, *94*(4), 265.