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$x_{n+1}=\max\left\{\frac{1}{x_{n-4}},\frac{y_{n-4}}{x_{n-4}}\right\};y_{n+1}=\max\left\{\frac{1}{y_{n-4}},\frac{x_{n-4}}{y_{n-4}}\right\}$ Maksimumlu Fark Denklem Sisteminin Çözümleri

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Özet Aşağıdaki fark denklem sisteminin çözümlerinin davranışları incelenmiştir.

$$x_{n+1}=\max\left\{\frac{1}{x_{n-4}},\frac{y_{n-4}}{x_{n-4}}\right\};y_{n+1}=\max\left\{\frac{1}{y_{n-4}},\frac{x_{n-4}}{y_{n-4}}\right\} \quad (1)$$

Başlangıç şartları pozitif reel sayılardır.

Anahtar sözcükler *Fark Denklemi, Maksimum Operatörü, Yarı Dönmeler.*

$x_{n+1}=\max\left\{\frac{1}{x_{n-4}},\frac{y_{n-4}}{x_{n-4}}\right\};y_{n+1}=\max\left\{\frac{1}{y_{n-4}},\frac{x_{n-4}}{y_{n-4}}\right\}$ Solutions Of The System Of Maximum Difference Equations

Abstract The behaviour of the solutions of the following system of difference equations is examined.

$$x_{n+1}=\max\left\{\frac{1}{x_{n-4}},\frac{y_{n-4}}{x_{n-4}}\right\};y_{n+1}=\max\left\{\frac{1}{y_{n-4}},\frac{x_{n-4}}{y_{n-4}}\right\} \quad (1)$$

Where the initial conditions are positive real numbers.

Keywords *Difference Equation, Maximum Operations, Semicycle.*

1. GİRİŞ

Son zamanlarda, lineere olmayan fark denklemlerinin periyodikliği ile ilgili ilginç çalışmalar yapılmaktadır. Özellikle fark denklem sisteminin periyodikliği, çözümü ve çözümlerin davranışları incelenmektedir. Birçok araştırmacı, son yıllarda özellikle maksimumlu fark denklemleri ve denklem sistemleri ile ilgili araştırma yapmışlardır. Örneğin [1-30].

Tanım 1 :

$$x_{n+1} = f(x_n, x_{n-1}, \ldots, x_{n-s}) \quad n = 0,1,2,\ldots \text{ için} \tag{2}$$

fark denkleminde $\bar{x} = f(\bar{x}, \ldots, \bar{x})$ oluyorsa $\bar{x}$ ye denge noktası denir.

Tanım 2 : $\bar{x}$, (2) denkleminin pozitif bir denge noktası olsun. (2) denkleminin bir $\{x_n\}$ çözümünün bir pozitif yarı dönmesi $\{x_l, x_{l+1}, \ldots, x_m\}$ terimlerinin bir dizisinden oluşur ve bunların hepsi $\bar{x}$ denge noktasına eşit veya büyük bütün terimlerdir. Öyle ki $l \geq 0$ ve $m \leq \infty$ olur ve burada ya $l = 0$ ya da $l > 0$ ve $x_{l-1} < \bar{x}$ ve ya $m = \infty$ ya da $m < \infty$ ve $x_{m+1} < \bar{x}$ dir.

Tanım 3: $\bar{x}$, (2) denkleminin negatif bir denge noktası olsun. (2) denkleminin bir $\{x_n\}$ çözümünün bir negatif yarı dönmesi $\{x_l, x_{l+1}, \ldots, x_m\}$ terimlerinin bir dizisinden oluşur ve bunların hepsi $\bar{x}$ denge noktasından daha küçük terimlerdir. Öyle ki $l \geq 0$ ve $m \leq \infty$ olur ve burada Ya $l = 0$ ya da $l > 0$ ve $x_{l-1} \geq \bar{x}$ veya $m = \infty$ ya da $m < \infty$ ve $x_{m+1} \geq \bar{x}$ dir.

Tanım 4 : $f_1 = 1, f_2 = 1$ ve $n \geq 3$ için $f_n = f_{n-1} + f_{n-2}$ şeklinde tanımlanan sayılara Fibonacci sayıları denir.

2. ANA SONUÇLAR

$$x_{n+1} = \max\left\{\frac{1}{x_{n-4}}, \frac{y_{n-4}}{x_{n-4}}\right\}; y_{n+1} = \max\left\{\frac{1}{y_{n-4}}, \frac{x_{n-4}}{y_{n-4}}\right\} \tag{1}$$

Şimdi (1) denkleminin pozitif denge noktasını bulalım.

$\bar{x} = \max\left\{\frac{1}{\bar{x}}, \frac{\bar{y}}{\bar{x}}\right\}; \bar{y} = \max\left\{\frac{1}{\bar{y}}, \frac{\bar{x}}{\bar{y}}\right\}$ olur. Buradan

$\bar{x} = \frac{1}{\bar{x}}$ veya $\bar{x} = \frac{\bar{y}}{\bar{x}}; \bar{y} = \frac{1}{\bar{y}}$ veya $\bar{y} = \frac{\bar{x}}{\bar{y}}$ elde edilir. $(\bar{x})^2 = 1$ ve $(\bar{y})^2 = 1$ bulunur. Buradan da

$\bar{x} = 1$ ve $\bar{y} = 1$ elde edilir.

Lemma 1 : (1) denklemi için $0<x_{-4}<y_{-4}<1$, $0<x_{-3}<y_{-3}<1$, $0<x_{-2}<y_{-2}<1$, $0<x_{-1}<y_{-1}<1$ ve $0<x_0<y_0<1$ başlangıç şartlarına göre ,
Aşağıdaki ifadeler doğrudur:

a) x_n çözümleri için her pozitif yarı dönme beş terimden oluşur. y_n çözümleri için $n \geq 5$ durumunda her pozitif yarı dönme beş terimden oluşur.

b) x_n çözümleri için her negatif yarı dönme beş terimden oluşur. y_n çözümleri için $n \geq 5$ durumunda her negatif yarı dönme beş terimden oluşur.

c) x_n çözümleri için beş uzunluğundaki her pozitif yarı dönmeyi beş uzunluğundaki negatif yarı dönme takip eder. y_n çözümleri için $n \geq 5$ durumunda beş uzunluğundaki her pozitif yarı dönmeyi beş uzunluğundaki negatif yarı dönme takip eder.

d) x_n çözümleri için beş uzunluğundaki her negatif yarı dönmeyi beş uzunluğundaki pozitif yarı dönme takip eder. y_n çözümleri için $n \geq 5$ durumunda beş uzunluğundaki her negatif yarı dönmeyi beş uzunluğundaki pozitif yarı dönme takip eder

İspat :

$0<x_{-4}<y_{-4}<1$, $0<x_{-3}<y_{-3}<1$, $0<x_{-2}<y_{-2}<1$, $0<x_{-1}<y_{-1}<1$ ve $0<x_0<y_0<1$
Başlangıç şartlarına göre

$$x_1 = \max\left\{\frac{1}{x_{-4}}, \frac{y_{-4}}{x_{-4}}\right\} = \frac{1}{x_{-4}} > \bar{x}$$

$$y_1 = \max\left\{\frac{1}{y_{-4}}, \frac{x_{-4}}{y_{-4}}\right\} = \frac{1}{y_{-4}} > \bar{y}$$

$$x_2 = \max\left\{\frac{1}{x_{-3}}, \frac{y_{-3}}{x_{-3}}\right\} = \frac{1}{x_{-3}} > \bar{x}$$

$$y_2 = \max\left\{\frac{1}{y_{-3}}, \frac{x_{-3}}{y_{-3}}\right\} = \frac{1}{y_{-3}} > \bar{y}$$

$$x_3 = \max\left\{\frac{1}{x_{-2}}, \frac{y_{-2}}{x_{-2}}\right\} = \frac{1}{x_{-2}} > \bar{x}$$

$$y_3 = \max\left\{\frac{1}{y_{-2}}, \frac{x_{-2}}{y_{-2}}\right\} = \frac{1}{y_{-2}} > \bar{y}$$

$$x_4 = \max\left\{\frac{1}{x_{-1}}, \frac{y_{-1}}{x_{-1}}\right\} = \frac{1}{x_{-1}} > \bar{x}$$

$$y_4 = \max\left\{\frac{1}{y_{-1}}, \frac{x_{-1}}{y_{-1}}\right\} = \frac{1}{y_{-1}} > \bar{y}$$

$$x_5 = \max\left\{\frac{1}{x_0}, \frac{y_0}{x_0}\right\} = \frac{1}{x_0} > \bar{x}$$

$$y_5 = \max\left\{\frac{1}{y_0}, \frac{x_0}{y_0}\right\} = \frac{1}{y_0} > \bar{y}$$

$$x_6 = \max\left\{\frac{1}{x_1}, \frac{y_1}{x_1}\right\} = \frac{x_{-4}}{y_{-4}} < \bar{x}$$

$$y_6 = \max\left\{\frac{1}{y_1}, \frac{x_1}{y_1}\right\} = \frac{y_{-4}}{x_{-4}} > \bar{y}$$

$$x_7 = \max\left\{\frac{1}{x_2}, \frac{y_2}{x_2}\right\} = \frac{x_{-3}}{y_{-3}} < \bar{x}$$

$$y_7 = \max\left\{\frac{1}{y_2}, \frac{x_2}{y_2}\right\} = \frac{y_{-3}}{x_{-3}} > \bar{y}$$

$$x_8 = \max\left\{\frac{1}{x_3}, \frac{y_3}{x_3}\right\} = \frac{x_{-2}}{y_{-2}} < \bar{x}$$

$$y_8 = \max\left\{\frac{1}{y_3}, \frac{x_3}{y_3}\right\} = \frac{y_{-2}}{x_{-2}} > \bar{y}$$

$$x_9 = \max\left\{\frac{1}{x_4}, \frac{y_4}{x_4}\right\} = \frac{x_{-1}}{y_{-1}} < \bar{x}$$

$$y_9 = \max\left\{\frac{1}{y_4}, \frac{x_4}{y_4}\right\} = \frac{y_{-1}}{x_{-1}} > \bar{y}$$

$$x_{10} = \max\left\{\frac{1}{x_5}, \frac{y_5}{x_5}\right\} = \frac{x_0}{y_0} < \bar{x}$$

$$y_{10} = \max\left\{\frac{1}{y_5}, \frac{x_5}{y_5}\right\} = \frac{y_0}{x_0} > \bar{y}$$

$$x_{11} = \max\left\{\frac{1}{x_6}, \frac{y_6}{x_6}\right\} = \left(\frac{y_{-4}}{x_{-4}}\right)^2 > \bar{x}$$

$$y_{11} = \max\left\{\frac{1}{y_6}, \frac{x_6}{y_6}\right\} = \frac{x_{-4}}{y_{-4}} < \bar{y}$$

$$x_{12} = \max\left\{\frac{1}{x_7}, \frac{y_7}{x_7}\right\} = \left(\frac{y_{-3}}{x_{-3}}\right)^2 > \bar{x}$$

$$y_{12} = \max\left\{\frac{1}{y_7}, \frac{x_7}{y_7}\right\} = \frac{x_{-3}}{y_{-3}} < \bar{y}$$

$$x_{13}=\max\left\{\frac{1}{x_8},\frac{y_8}{x_8}\right\}=\left(\frac{y_{-2}}{x_{-2}}\right)^2>\bar{x}$$

$$y_{13}=\max\left\{\frac{1}{y_8},\frac{x_8}{y_8}\right\}=\frac{x_{-2}}{y_{-2}}<\bar{y}$$

$$x_{14}=\max\left\{\frac{1}{x_9},\frac{y_9}{x_9}\right\}=\left(\frac{y_{-1}}{x_{-1}}\right)^2>\bar{x}$$

$$y_{14}=\max\left\{\frac{1}{y_9},\frac{x_9}{y_9}\right\}=\frac{x_{-1}}{y_{-1}}<\bar{y}$$

$$x_{15}=\max\left\{\frac{1}{x_{10}},\frac{y_{10}}{x_{10}}\right\}=\left(\frac{y_0}{x_0}\right)^2>\bar{x}$$

$$y_{15}=\max\left\{\frac{1}{y_{10}},\frac{x_{10}}{y_{10}}\right\}=\frac{x_0}{y_0}<\bar{y}$$

$$\vdots$$

elde edilir.

$x_1>\bar{x}$, $x_2>\bar{x}$, $x_3>\bar{x}$, $x_4>\bar{x}$, $x_5>\bar{x}$, $x_6<\bar{x}$, $x_7<\bar{x}$, $x_8<\bar{x}$, $x_9<\bar{x}$, $x_{10}<\bar{x}$, … buradan da görüldüğü gibi x_n çözümleri PPPPPNNNNNPPPPPNNNNN… şeklindedir.

$y_1>\bar{y}$, $y_2>\bar{y}$, $y_3>\bar{y}$, $y_4>\bar{y}$, $y_5>\bar{y}$, $y_6>\bar{y}$, $y_7>\bar{y}$, $y_8>\bar{y}$, $y_9>\bar{y}$, $y_{10}>\bar{y}$, $y_{11}<\bar{y}$, $y_{12}<\bar{y}$, $y_{13}<\bar{y}$, $y_{14}<\bar{y}$, $y_{15}<\bar{y}$ … buradan da görüldüğü gibi y_n çözümleri $n\geq 5$ için PPPPPNNNNNPPPPPNNNNN … şeklindedir.

x_n çözümleri için her pozitif yarı dönmenin beş terimden oluştuğu görülmektedir.

y_n çözümleri $n\geq 5$ için her pozitif yarı dönmenin beş terimden oluştuğu görülmektedir.

x_n çözümleri için her negatif yarı dönmenin beş terimden oluştuğu görülmektedir.

y_n çözümleri $n\geq 5$ için her negatif yarı dönmenin beş terimden oluştuğu görülmektedir.

Beş uzunluğundaki her pozitif yarı dönmeyi beş uzunluğundaki negatif yarı dönmenin takip ettiği x_n çözümlerinden görülmektedir.

Beş uzunluğundaki her negatif yarı dönmeyi beş uzunluğundaki pozitif yarı dönmenin takip ettiği $n\geq 5$ şartı altındaki y_n çözümlerinden görülmektedir.

Böylece Lemmanın ispatı gösterilmiştir.

Teorem 1 : $(x_n;y_n)$ (1) denkleminin $0<x_{-4}<y_{-4}<1$, $0<x_{-3}<y_{-3}<1$, $0<x_{-2}<y_{-2}<1$, $0<x_{-1}<y_{-1}<1$ ve $0<x_0<y_0<1$ başlangıç şartları altındaki çözümü olsun. , $n=0,1,2,\ldots$ için

$$x_{10n+1}=\left(\frac{1}{x_{-4}}\right)^{f(2n+1)};x_{10n+2}=\left(\frac{1}{x_{-3}}\right)^{f(2n+1)};x_{10n+3}=\left(\frac{1}{x_{-2}}\right)^{f(2n+1)};x_{10n+4}=\left(\frac{1}{x_{-1}}\right)^{f(2n+1)};$$

$$x_{10n+5}=\left(\frac{1}{x_{0}}\right)^{f(2n+1)};x_{10n+6}=\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2n+1)};x_{10n+7}=\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2n+1)};x_{10n+8}=\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2n+1)};$$

$$x_{10n+9}=\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2n+1)};x_{10n+10}=\left(\frac{x_{0}}{y_{0}}\right)^{f(2n+1)}$$

$$y_{10n+1}=\left(\frac{1}{y_{-4}}\right)^{f(2n)};y_{10n+2}=\left(\frac{1}{y_{-3}}\right)^{f(2n)};y_{10n+3}=\left(\frac{1}{y_{-2}}\right)^{f(2n)};y_{10n+4}=\left(\frac{1}{y_{-1}}\right)^{f(2n)};$$

$$y_{10n+5}=\left(\frac{1}{y_{0}}\right)^{f(2n)};y_{10n+6}=\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2n+2)};y_{10n+7}=\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2n+2)};y_{10n+8}=\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2n+2)};$$

$$y_{10n+9}=\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2n+2)};y_{10n+10}=\left(\frac{y_{0}}{x_{0}}\right)^{f(2n+2)}$$

çözümler elde edilir.

İspat :
Bu teoremin ispatını tümevarım yöntemiyle gösterelim.

$$x_1=\max\left\{\frac{1}{x_{-4}},\frac{y_{-4}}{x_{-4}}\right\}=\frac{1}{x_{-4}}>\bar{x}$$

$$y_1=\max\left\{\frac{1}{y_{-4}},\frac{x_{-4}}{y_{-4}}\right\}=\frac{1}{y_{-4}}>\bar{y}$$

$$x_2=\max\left\{\frac{1}{x_{-3}},\frac{y_{-3}}{x_{-3}}\right\}=\frac{1}{x_{-3}}>\bar{x}$$

$$y_2=\max\left\{\frac{1}{y_{-3}},\frac{x_{-3}}{y_{-3}}\right\}=\frac{1}{y_{-3}}>\bar{y}$$

$$x_3=\max\left\{\frac{1}{x_{-2}},\frac{y_{-2}}{x_{-2}}\right\}=\frac{1}{x_{-2}}>\bar{x}$$

$$y_3=\max\left\{\frac{1}{y_{-2}},\frac{x_{-2}}{y_{-2}}\right\}=\frac{1}{y_{-2}}>\bar{y}$$

$$x_4=\max\left\{\frac{1}{x_{-1}},\frac{y_{-1}}{x_{-1}}\right\}=\frac{1}{x_{-1}}>\bar{x}$$

$$y_4=\max\left\{\frac{1}{y_{-1}},\frac{x_{-1}}{y_{-1}}\right\}=\frac{1}{y_{-1}}>\bar{y}$$

$$x_5 = \max\left\{\frac{1}{x_0}, \frac{y_0}{x_0}\right\} = \frac{1}{x_0} > \bar{x}$$

$$y_5 = \max\left\{\frac{1}{y_0}, \frac{x_0}{y_0}\right\} = \frac{1}{y_0} > \bar{y}$$

$$x_6 = \max\left\{\frac{1}{x_1}, \frac{y_1}{x_1}\right\} = \frac{x_{-4}}{y_{-4}} < \bar{x}$$

$$y_6 = \max\left\{\frac{1}{y_1}, \frac{x_1}{y_1}\right\} = \frac{y_{-4}}{x_{-4}} > \bar{y}$$

$$x_7 = \max\left\{\frac{1}{x_2}, \frac{y_2}{x_2}\right\} = \frac{x_{-3}}{y_{-3}} < \bar{x}$$

$$y_7 = \max\left\{\frac{1}{y_2}, \frac{x_2}{y_2}\right\} = \frac{y_{-3}}{x_{-3}} > \bar{y}$$

$$x_8 = \max\left\{\frac{1}{x_3}, \frac{y_3}{x_3}\right\} = \frac{x_{-2}}{y_{-2}} < \bar{x}$$

$$y_8 = \max\left\{\frac{1}{y_3}, \frac{x_3}{y_3}\right\} = \frac{y_{-2}}{x_{-2}} > \bar{y}$$

$$x_9 = \max\left\{\frac{1}{x_4}, \frac{y_4}{x_4}\right\} = \frac{x_{-1}}{y_{-1}} < \bar{x}$$

$$y_9 = \max\left\{\frac{1}{y_4}, \frac{x_4}{y_4}\right\} = \frac{y_{-1}}{x_{-1}} > \bar{y}$$

$$x_{10} = \max\left\{\frac{1}{x_5}, \frac{y_5}{x_5}\right\} = \frac{x_0}{y_0} < \bar{x}$$

$$y_{10} = \max\left\{\frac{1}{y_5}, \frac{x_5}{y_5}\right\} = \frac{y_0}{x_0} > \bar{y}$$

$$x_{11} = \max\left\{\frac{1}{x_6}, \frac{y_6}{x_6}\right\} = \left(\frac{y_{-4}}{x_{-4}}\right)^2 > \bar{x}$$

$$y_{11} = \max\left\{\frac{1}{y_6}, \frac{x_6}{y_6}\right\} = \frac{x_{-4}}{y_{-4}} < \bar{y}$$

$$x_{12} = \max\left\{\frac{1}{x_7}, \frac{y_7}{x_7}\right\} = \left(\frac{y_{-3}}{x_{-3}}\right)^2 > \bar{x}$$

$$y_{12} = \max\left\{\frac{1}{y_7}, \frac{x_7}{y_7}\right\} = \frac{x_{-3}}{y_{-3}} < \bar{y}$$

$$x_{13} = \max\left\{\frac{1}{x_8}, \frac{y_8}{x_8}\right\} = \left(\frac{y_{-2}}{x_{-2}}\right)^2 > \bar{x}$$

$$y_{13} = \max\left\{\frac{1}{y_8}, \frac{x_8}{y_8}\right\} = \frac{x_{-2}}{y_{-2}} < \bar{y}$$

$$x_{14} = \max\left\{\frac{1}{x_9}, \frac{y_9}{x_9}\right\} = \left(\frac{y_{-1}}{x_{-1}}\right)^2 > \bar{x}$$

$$y_{14} = \max\left\{\frac{1}{y_9}, \frac{x_9}{y_9}\right\} = \frac{x_{-1}}{y_{-1}} < \bar{y}$$

$$x_{15} = \max\left\{\frac{1}{x_{10}}, \frac{y_{10}}{x_{10}}\right\} = \left(\frac{y_0}{x_0}\right)^2 > \bar{x}$$

$$y_{15} = \max\left\{\frac{1}{y_{10}}, \frac{x_{10}}{y_{10}}\right\} = \frac{x_0}{y_0} < \bar{y}$$

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n = 0 için doğrudur. n = k için doğru olduğunu kabul edelim.

$$x_{10k+1} = \left(\frac{1}{x_{-4}}\right)^{f(2k+1)}; x_{1kn+2} = \left(\frac{1}{x_{-3}}\right)^{f(2k+1)}; x_{10k+3} = \left(\frac{1}{x_{-2}}\right)^{f(2k+1)}; x_{10k+4} = \left(\frac{1}{x_{-1}}\right)^{f(2k+1)};$$

$$x_{10k+5} = \left(\frac{1}{x_0}\right)^{f(2k+1)}; x_{10k+6} = \left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+1)}; x_{10k+7} = \left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+1)}; x_{10k+8} = \left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+1)};$$

$$x_{10k+9} = \left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+1)}; x_{10k+10} = \left(\frac{x_0}{y_0}\right)^{f(2k+1)}$$

$$y_{10k+1} = \left(\frac{1}{y_{-4}}\right)^{f(2k)}; y_{10k+2} = \left(\frac{1}{y_{-3}}\right)^{f(2k)}; y_{10k+3} = \left(\frac{1}{y_{-2}}\right)^{f(2k)}; y_{10k+4} = \left(\frac{1}{y_{-1}}\right)^{f(2k)};$$

$$y_{10k+5} = \left(\frac{1}{y_0}\right)^{f(2k)}; y_{10k+6} = \left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+2)}; y_{10k+7} = \left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}; y_{10k+8} = \left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)};$$

$$y_{10k+9} = \left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)}; y_{10k+10} = \left(\frac{y_0}{x_0}\right)^{f(2k+2)}$$

n = k+1 için doğru olduğunu gösterelim.

$$x_{10k+11}=\max\left\{\frac{1}{x_{10k+6}},\frac{y_{10k+6}}{x_{10k+6}}\right\}=\max\left\{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+1)},\frac{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+2)}}{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+1)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+1)},\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+2)}\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+1)}\right\}$$

$$=\max\left\{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+1)},\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+3)}\right\}$$

$$=\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+3)}$$

$$y_{10k+11}=\max\left\{\frac{1}{y_{10k+6}},\frac{x_{10k+6}}{y_{10k+6}}\right\}=\max\left\{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+2)},\frac{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+1)}}{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+2)},\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+1)}\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+2)},\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+3)}\right\}$$

$$=\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+2)}$$

$$x_{10k+12}=\max\left\{\frac{1}{x_{10k+7}},\frac{y_{10k+7}}{x_{10k+7}}\right\}=\max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+1)},\frac{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}}{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+1)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+1)},\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+1)}\right\}$$

$$=\max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+1)},\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}\right\}$$

$$=\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}$$

$$y_{10k+12}=\max\left\{\frac{1}{y_{10k+7}},\frac{x_{10k+7}}{y_{10k+7}}\right\}=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)},\frac{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+1)}}{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)},\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+1)}\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)},\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)}\right\}$$

$$=\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)}$$

$$x_{10k+13} = \max\left\{\frac{1}{x_{10k+8}}, \frac{y_{10k+8}}{x_{10k+8}}\right\} = \max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+1)}, \frac{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)}}{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+1)}}\right\}$$

$$= \max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+1)}, \left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)}\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+1)}\right\}$$

$$= \max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+1)}, \left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}\right\}$$

$$= \left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}$$

$$y_{10k+13} = \max\left\{\frac{1}{y_{10k+8}}, \frac{x_{10k+8}}{y_{10k+8}}\right\} = \max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}, \frac{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+1)}}{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)}}\right\}$$

$$= \max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}, \left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+1)}\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}\right\}$$

$$= \max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}, \left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)}\right\}$$

$$= \left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}$$

$$x_{10k+14}=\max\left\{\frac{1}{x_{10k+9}},\frac{y_{10k+9}}{x_{10k+9}}\right\}=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+1)},\frac{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)}}{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+1)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+1)},\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)}\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+1)}\right\}$$

$$=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+1)},\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}\right\}$$

$$=\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}$$

$$y_{10k+14}=\max\left\{\frac{1}{y_{10k+9}},\frac{x_{10k+9}}{y_{10k+9}}\right\}=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)},\frac{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+1)}}{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)},\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+1)}\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)},\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)}\right\}$$

$$=\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)}$$

$$x_{10k+15}=\max\left\{\frac{1}{x_{10k+10}},\frac{y_{10k+8}}{x_{10k+10}}\right\}=\max\left\{\left(\frac{y_0}{x_0}\right)^{f(2k+1)},\frac{\left(\frac{y_0}{x_0}\right)^{f(2k+2)}}{\left(\frac{x_0}{y_0}\right)^{f(2k+1)}}\right\}$$

$$=\max\left\{\left(\frac{y_0}{x_0}\right)^{f(2k+2)},\left(\frac{y_0}{x_0}\right)^{f(2k+2)}\left(\frac{y_0}{x_0}\right)^{f(2k+1)}\right\}$$

$$=\max\left\{\left(\frac{y_0}{x_0}\right)^{f(2k+2)},\left(\frac{y_0}{x_0}\right)^{f(2k+3)}\right\}$$

$$=\left(\frac{y_0}{x_0}\right)^{f(2k+3)}$$

$$y_{10k+15}=\max\left\{\frac{1}{y_{10k+10}},\frac{x_{10k+10}}{y_{10k+10}}\right\}=\max\left\{\left(\frac{x_0}{y_0}\right)^{f(2k+2)},\frac{\left(\frac{x_0}{y_0}\right)^{f(2k+1)}}{\left(\frac{y_0}{x_0}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{x_0}{y_0}\right)^{f(2k+2)},\left(\frac{x_0}{y_0}\right)^{f(2k+1)}\left(\frac{x_0}{y_0}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{x_0}{y_0}\right)^{f(2k+2)},\left(\frac{x_0}{y_0}\right)^{f(2k+3)}\right\}$$

$$=\left(\frac{x_0}{y_0}\right)^{f(2k+2)}$$

$$x_{10k+16}=\max\left\{\frac{1}{x_{10k+11}},\frac{y_{10k+11}}{x_{10k+11}}\right\}=\max\left\{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+3)},\frac{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+2)}}{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+3)},\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+2)}\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+3)},\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+3)}$$

$$y_{10k+16}=\max\left\{\frac{1}{y_{10k+11}},\frac{x_{10k+11}}{y_{10k+11}}\right\}=\max\left\{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+2)},\frac{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+3)}}{\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+2)},\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+3)}\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+2)},\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2k+4)}$$

$$x_{10k+17}=\max\left\{\frac{1}{x_{10k+12}},\frac{y_{10k+12}}{x_{10k+12}}\right\}=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)},\frac{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)}}{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)},\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)}\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)},\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+3)}$$

$$y_{10k+17}=\max\left\{\frac{1}{y_{10k+12}},\frac{x_{10k+12}}{y_{10k+12}}\right\}=\max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)},\frac{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}}{\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)},\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+3)}\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+2)},\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2k+4)}$$

$$x_{10k+18}=\max\left\{\frac{1}{x_{10k+13}},\frac{y_{10k+13}}{x_{10k+13}}\right\}=\max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)},\frac{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}}{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)},\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)},\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+3)}$$

$$y_{10k+18}=\max\left\{\frac{1}{y_{10k+13}},\frac{x_{10k+13}}{y_{10k+13}}\right\}=\max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)},\frac{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}}{\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)},\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+3)}\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+2)},\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2k+4)}$$

$$x_{10k+19}=\max\left\{\frac{1}{x_{10k+14}},\frac{y_{10k+14}}{x_{10k+14}}\right\}=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)},\frac{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)}}{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)},\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)}\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)},\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+3)}$$

$$y_{10k+19}=\max\left\{\frac{1}{y_{10k+14}},\frac{x_{10k+14}}{y_{10k+14}}\right\}=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)},\frac{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}}{\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)},\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+3)}\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+2)},\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2k+4)}$$

$$x_{10k+20}=\max\left\{\frac{1}{x_{10k+15}},\frac{y_{10k+15}}{x_{10k+15}}\right\}=\max\left\{\left(\frac{x_0}{y_0}\right)^{f(2k+3)},\frac{\left(\frac{x_0}{y_0}\right)^{f(2k+2)}}{\left(\frac{y_0}{x_0}\right)^{f(2k+3)}}\right\}$$

$$=\max\left\{\left(\frac{x_0}{y_0}\right)^{f(2k+3)},\left(\frac{x_0}{y_0}\right)^{f(2k+2)}\left(\frac{x_0}{y_0}\right)^{f(2k+3)}\right\}$$

$$=\max\left\{\left(\frac{x_0}{y_0}\right)^{f(2k+3)},\left(\frac{x_0}{y_0}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{x_0}{y_0}\right)^{f(2k+3)}$$

$$y_{10k+20}=\max\left\{\frac{1}{y_{10k+15}},\frac{x_{10k+15}}{y_{10k+15}}\right\}=\max\left\{\left(\frac{y_0}{x_0}\right)^{f(2k+2)},\frac{\left(\frac{y_0}{x_0}\right)^{f(2k+3)}}{\left(\frac{x_0}{y_0}\right)^{f(2k+2)}}\right\}$$

$$=\max\left\{\left(\frac{y_0}{x_0}\right)^{f(2k+2)},\left(\frac{y_0}{x_0}\right)^{f(2k+3)}\left(\frac{y_0}{x_0}\right)^{f(2k+2)}\right\}$$

$$=\max\left\{\left(\frac{y_0}{x_0}\right)^{f(2k+2)},\left(\frac{y_0}{x_0}\right)^{f(2k+4)}\right\}$$

$$=\left(\frac{y_0}{x_0}\right)^{f(2k+4)}$$

Böylece teoeremin doğruluğu ispatlanmış oldu.

Teorem 2 : (1) denklem sistemi $0<x_{-4}<y_{-4}<1$, $0<x_{-3}<y_{-3}<1$, $0<x_{-2}<y_{-2}<1$, $0<x_{-1}<y_{-1}<1$ ve $0<x_0<y_0<1$ başlangıç şartlarına göre

a)

$$\lim_{n\to\infty}x_{10n+1}=\infty;\ \lim_{n\to\infty}x_{10n+2}=\infty;\ \lim_{n\to\infty}x_{10n+3}=\infty;\ \lim_{n\to\infty}x_{10n+4}=\infty;\ \lim_{n\to\infty}x_{10n+5}=\infty;$$

$$\lim_{n\to\infty}x_{10n+6}=0;\ \lim_{n\to\infty}x_{10n+7}=0;\ \lim_{n\to\infty}x_{10n+8}=0;\ \lim_{n\to\infty}x_{10n+9}=0;\ \lim_{n\to\infty}x_{10n+10}=0$$

$$\lim_{n\to\infty} y_{10n+1} = 0;\ \lim_{n\to\infty} y_{10n+2} = 0;\ \lim_{n\to\infty} y_{10n+3} = 0;\ \lim_{n\to\infty} y_{10n+4} = 0;\ \lim_{n\to\infty} y_{10n+5} = 0;$$

b) $$\lim_{n\to\infty} y_{10n+6} = \infty;\ \lim_{n\to\infty} y_{10n+7} = \infty;\ \lim_{n\to\infty} y_{10n+8} = \infty;\ \lim_{n\to\infty} y_{10n+9} = \infty;\ \lim_{n\to\infty} y_{10n+10} = \infty$$

olur.

İspat: a)

$0 < x_{-4} < y_{-4} < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+1} = \lim_{n\to\infty}\left(\frac{1}{x_{-4}}\right)^{f(2n+1)} = \left(\frac{1}{x_{-4}}\right)^{f(\infty)} = \left(\frac{1}{x_{-4}}\right)^{\infty} = \infty$$

elde edilir.

$0 < x_{-3} < y_{-3} < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+2} = \lim_{n\to\infty}\left(\frac{1}{x_{-3}}\right)^{f(2n+1)} = \left(\frac{1}{x_{-3}}\right)^{f(\infty)} = \left(\frac{1}{x_{-3}}\right)^{\infty} = \infty$$

elde edilir.

$0 < x_{-2} < y_{-2} < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+3} = \lim_{n\to\infty}\left(\frac{1}{x_{-2}}\right)^{f(2n+1)} = \left(\frac{1}{x_{-2}}\right)^{f(\infty)} = \left(\frac{1}{x_{-2}}\right)^{\infty} = \infty$$

elde edilir.

$0 < x_{-1} < y_{-1} < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+4} = \lim_{n\to\infty}\left(\frac{1}{x_{-1}}\right)^{f(2n+1)} = \left(\frac{1}{x_{-1}}\right)^{f(\infty)} = \left(\frac{1}{x_{-1}}\right)^{\infty} = \infty$$

elde edilir.

$0 < x_0 < y_0 < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+5} = \lim_{n\to\infty}\left(\frac{1}{x_0}\right)^{f(2n+1)} = \left(\frac{1}{x_0}\right)^{f(\infty)} = \left(\frac{1}{x_0}\right)^{\infty} = \infty$$

elde edilir.

$0 < x_{-4} < y_{-4} < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+6} = \lim_{n\to\infty}\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2n+1)} = \left(\frac{x_{-4}}{y_{-4}}\right)^{f(\infty)} = \left(\frac{x_{-4}}{y_{-4}}\right)^{\infty} = 0 \quad ,$$

elde edilir.

$0 < x_{-3} < y_{-3} < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+7} = \lim_{n\to\infty}\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2n+1)} = \left(\frac{x_{-3}}{y_{-3}}\right)^{f(\infty)} = \left(\frac{x_{-3}}{y_{-3}}\right)^{\infty} = 0 \quad ,$$

elde edilir.

$0 < x_{-2} < y_{-2} < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+8} = \lim_{n\to\infty}\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2n+1)} = \left(\frac{x_{-2}}{y_{-2}}\right)^{f(\infty)} = \left(\frac{x_{-2}}{y_{-2}}\right)^{\infty} = 0 \quad ,$$

elde edilir.

$0 < x_{-1} < y_{-1} < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+9} = \lim_{n\to\infty}\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2n+1)} = \left(\frac{x_{-1}}{y_{-1}}\right)^{f(\infty)} = \left(\frac{x_{-1}}{y_{-1}}\right)^{\infty} = 0 \quad ,$$

elde edilir.

$0 < x_0 < y_0 < 1$ olduğu için

$$\lim_{n\to\infty} x_{10n+10} = \lim_{n\to\infty}\left(\frac{x_0}{y_0}\right)^{f(2n+1)} = \left(\frac{x_0}{y_0}\right)^{f(\infty)} = \left(\frac{x_0}{y_0}\right)^{\infty} = 0 \quad .$$

elde edilir.

b)

$0 < x_{-4} < y_{-4} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+1} = \lim_{n\to\infty}\left(\frac{x_{-4}}{y_{-4}}\right)^{f(2n)} = \left(\frac{x_{-4}}{y_{-4}}\right)^{f(\infty)} = \left(\frac{x_{-4}}{y_{-4}}\right)^{\infty} = 0 \quad ,$$

elde edilir.

$0 < x_{-3} < y_{-3} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+2} = \lim_{n\to\infty}\left(\frac{x_{-3}}{y_{-3}}\right)^{f(2n)} = \left(\frac{x_{-3}}{y_{-3}}\right)^{f(\infty)} = \left(\frac{x_{-3}}{y_{-3}}\right)^{\infty} = 0 \quad ,$$

elde edilir.

$0 < x_{-2} < y_{-2} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+3} = \lim_{n\to\infty}\left(\frac{x_{-2}}{y_{-2}}\right)^{f(2n)} = \left(\frac{x_{-2}}{y_{-2}}\right)^{f(\infty)} = \left(\frac{x_{-2}}{y_{-2}}\right)^{\infty} = 0 \quad ,$$

elde edilir.

$0 < x_{-1} < y_{-1} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+4} = \lim_{n\to\infty}\left(\frac{x_{-1}}{y_{-1}}\right)^{f(2n)} = \left(\frac{x_{-1}}{y_{-1}}\right)^{f(\infty)} = \left(\frac{x_{-1}}{y_{-1}}\right)^{\infty} = 0 \quad ,$$

elde edilir.

$0 < x_{0} < y_{0} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+5} = \lim_{n\to\infty}\left(\frac{x_{0}}{y_{0}}\right)^{f(2n)} = \left(\frac{x_{0}}{y_{0}}\right)^{f(\infty)} = \left(\frac{x_{0}}{y_{0}}\right)^{\infty} = 0 ,$$

elde edilir.

$0 < x_{-4} < y_{-4} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+6} = \lim_{n\to\infty}\left(\frac{y_{-4}}{x_{-4}}\right)^{f(2n+2)} = \left(\frac{y_{-4}}{x_{-4}}\right)^{f(\infty)} = \left(\frac{y_{-4}}{x_{-4}}\right)^{\infty} = \infty \quad ,$$

elde edilir.

$0 < x_{-3} < y_{-3} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+7} = \lim_{n\to\infty}\left(\frac{y_{-3}}{x_{-3}}\right)^{f(2n+2)} = \left(\frac{y_{-3}}{x_{-3}}\right)^{f(\infty)} = \left(\frac{y_{-3}}{x_{-3}}\right)^{\infty} = \infty \quad ,$$

elde edilir.

$0 < x_{-2} < y_{-2} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+8} = \lim_{n\to\infty}\left(\frac{y_{-2}}{x_{-2}}\right)^{f(2n+2)} = \left(\frac{y_{-2}}{x_{-2}}\right)^{f(\infty)} = \left(\frac{y_{-2}}{x_{-2}}\right)^{\infty} = \infty \quad ,$$

elde edilir.

$0 < x_{-1} < y_{-1} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+9} = \lim_{n\to\infty}\left(\frac{y_{-1}}{x_{-1}}\right)^{f(2n+2)} = \left(\frac{y_{-1}}{x_{-1}}\right)^{f(\infty)} = \left(\frac{y_{-1}}{x_{-1}}\right)^{\infty} = \infty \quad ,$$

elde edilir.

$0 < x_{0} < y_{0} < 1$ olduğu için

$$\lim_{n\to\infty} y_{10n+10} = \lim_{n\to\infty}\left(\frac{y_{0}}{x_{0}}\right)^{f(2n+2)} = \left(\frac{y_{0}}{x_{0}}\right)^{f(\infty)} = \left(\frac{y_{0}}{x_{0}}\right)^{\infty} = \infty \quad .$$

elde edilir.

3. TARTIŞMA VE SONUÇ

Bu çalışmada, $x_{-4};x_{-3};x_{-2};x_{-1};x_0;y_{-4};y_{-3};y_{-2};y_{-1};y_0$ başlangıç şartları sıfırdan farklı reel sayılar olmak üzere, $x_{n+1}=\max\left\{\frac{1}{x_{n-4}},\frac{y_{n-4}}{x_{n-4}}\right\}; y_{n+1}=\max\left\{\frac{1}{y_{n-4}},\frac{x_{n-4}}{y_{n-4}}\right\}$ maksimumlu fark denklem sisteminin çözümlerinin davranışları incelenmiştir. Bu fark denklem sisteminde katsayılar değiştirilerek yeni maksimumlu fark denklem sistemleri oluşturulabilir. Oluşturulacak yeni maksimumulu fark denklem sisteminin çözüm davranışları incelenebilir.

KAYNAKLAR

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