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$$x_{n+1} = \max\left\{\frac{1}{x_n}, \frac{y_n}{z_n}\right\}, y_{n+1} = \max\left\{\frac{1}{y_n}, \frac{z_n}{x_n}\right\}, z_{n+1} = \max\left\{\frac{1}{z_n}, \frac{x_n}{y_n}\right\}$$

Maksimumlu Fark Denklem Sisteminin Çözümleri

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Özet: *Aşağıdaki fark denklem sisteminin çözümlerinin periyodikliği ve davranışları incelenmiştir.*

$$x_{n+1} = \max\left\{\frac{1}{x_n}, \frac{y_n}{z_n}\right\}, y_{n+1} = \max\left\{\frac{1}{y_n}, \frac{z_n}{x_n}\right\}, z_{n+1} = \max\left\{\frac{1}{z_n}, \frac{x_n}{y_n}\right\} \quad (1)$$

Başlangıç şartları pozitif reel sayılardır.

Anahtar Kelimeler: *Fark Denklemi, Maksimum Operatörü, Yarı Dönmeler*

$$x_{n+1} = \max\left\{\frac{1}{x_n}, \frac{y_n}{z_n}\right\}, y_{n+1} = \max\left\{\frac{1}{y_n}, \frac{z_n}{x_n}\right\}, z_{n+1} = \max\left\{\frac{1}{z_n}, \frac{x_n}{y_n}\right\}$$

Solutions of the System of Maximum Difference Equations

Abstract: *The behaviour and periodicity of the solutions of the following system of difference equations is examined*

$$x_{n+1} = \max\left\{\frac{1}{x_n}, \frac{y_n}{z_n}\right\}, y_{n+1} = \max\left\{\frac{1}{y_n}, \frac{z_n}{x_n}\right\}, z_{n+1} = \max\left\{\frac{1}{z_n}, \frac{x_n}{y_n}\right\} \quad (1)$$

where the initial conditions are positive real numbers.

Keywords: *Difference Equation, Maximum Operations, Semicycle*

1. GİRİŞ

Son zamanlarda, lineer olmayan fark denklemlerinin periyodikliği ile ilgili ilginç çalışmalar yapılmaktadır. Özellikle fark denklem sisteminin periyodikliği, pozitif ve negatif yarı dönmeleri gibi çözümlerin davranışları incelenmektedir. Birçok araştırmacı, son yıllarda özellikle maksimumlu fark denklemleri ve maksimumlu fark denklem sistemleri ile ilgili araştırma yapmışlardır. Örneğin [1-29].

Tanım 1: $x_{n+1} = f(x_n, x_{n-1}, ..., x_{n-s})$ $n = 0,1,2, \dots$ için (2)
fark denkleminde $\bar{x} = f(\bar{x}, ..., \bar{x})$ oluyorsa $\bar{x}$ ye denge noktası denir.

Tanım 2 : $\bar{x}$, (2) denkleminin pozitif bir denge noktası olsun. (2) denkleminin bir $\{x_n\}$ çözümünün bir pozitif yarı dönmesi $\{x_l, x_{l+1}, ..., x_m\}$ terimlerinin bir dizisinden oluşur ve bunların hepsi $\bar{x}$ denge noktasına eşit veya büyük bütün terimlerdir. Öyle ki $l \geq 0$ ve $\mathrm{m} \leq \infty$ olur ve burada ya $l = 0$ ya da $l > 0$ ve $x_{l-1} < \bar{x}$; ve, ya $m = \infty$ ya da $m < \infty$ ve $x_{m+1} < \bar{x}$ ve $x_{m+1} < \bar{x}$ dir.

Tanım 3: $\bar{x}$, (2) denkleminin negatif bir denge noktası olsun. (2) denkleminin bir $\{x_n\}$ çözümünün bir negatif yarı dönmesi $\{x_l, x_{l+1}, ..., x_m\}$ terimlerinin bir dizisinden oluşur ve bunların hepsi $\bar{x}$ denge noktasından daha küçük terimlerdir. Öyle ki $l \geq 0$ ve $\mathrm{m} \leq \infty$ olur ve burada ya $l = 0$ ya da $l > 0$ ve $x_{l-1} \geq \bar{x}$ veya $m = \infty$ ya da $m < \infty$ ve $x_{m+1} \geq \bar{x}$ dir.

Tanım 4 : Her sayının kendinden öncekiyle toplanması sonucu oluşan sayı dizisine Fibonacci Dizisi denir.

2. ANA SONUÇLAR

$$x_{n+1} = \max\left\{\frac{1}{x_n}, \frac{y_n}{z_n}\right\}, y_{n+1} = \max\left\{\frac{1}{y_n}, \frac{z_n}{x_n}\right\}, z_{n+1} = \max\left\{\frac{1}{z_n}, \frac{x_n}{y_n}\right\}$$

$$\bar{x} = \max\left\{\frac{1}{\bar{x}}, \frac{\bar{y}}{\bar{z}}\right\}, \bar{y} = \max\left\{\frac{1}{\bar{y}}, \frac{\bar{z}}{\bar{x}}\right\}, \bar{z} = \max\left\{\frac{1}{\bar{z}}, \frac{\bar{x}}{\bar{y}}\right\}$$

$$\bar{x} = \frac{1}{x} \Rightarrow \bar{x}^2 = 1 \Rightarrow \bar{x} = 1$$

$$\bar{y} = \frac{1}{y} \Rightarrow \bar{y}^2 = 1 \Rightarrow \bar{y} = 1$$

$$\bar{z} = \frac{1}{z} \Rightarrow \bar{z}^2 = 1 \Rightarrow \bar{z} = 1$$

Lemma 1 : Aşağıdaki başlangıç koşulları göz önüne alındığında,

1. Durum: $1 < x_0 < y_0 < z_0$

x_n çözümü $n \geq 3$ için ; y_n çözümü $n \geq 2$ için , z_n çözümü için $n \geq 4$

2. Durum: $1 < y_0 < z_0 < x_0$

x_n çözümü $n \geq 4$ için ; y_n çözümü $n \geq 3$ için , z_n çözümü için $n \geq 2$

3. Durum: $1 < z_0 < x_0 < y_0$

x_n çözümü $n \geq 2$ için ; y_n çözümü $n \geq 4$ için , z_n çözümü için $n \geq 3$

4. Durum: $0 < x_0 < z_0 < y_0 \leq 0.8$

x_n çözümü $n \geq 5$ için ; y_n çözümü $n \geq 4$ için , z_n çözümü için $n \geq 6$

5. Durum: $0 < y_0 < x_0 < z_0 \leq 0.8$

x_n çözümü $n \geq 6$ için ; y_n çözümü $n \geq 5$ için , z_n çözümü için $n \geq 4$

6. Durum: $0 < z_0 < y_0 < x_0 \leq 0.8$

x_n çözümü $n \geq 4$ için ; y_n çözümü $n \geq 6$ için , z_n çözümü için $n \geq 5$

yukarıdaki başlangıç koşullarına dayanarak (1) denklemi için aşağıdaki ifadeler doğrudur:
(i) Her pozitif yarı dönme iki terimden oluşur.
(ii) Her negatif yarı dönme bir terimden oluşur.
(iii) Bir uzunluğundaki her negatif yarı dönmeyi iki uzunluğundaki pozitif yarı dönme takip eder.

İspat : 1.Durum, x_N, y_N, z_N çözümü $N \geq 0$ ve $1 < x_0 < y_0 < z_0$ için aşağıdaki gibi elde edilir.
Eğer $x_N < y_N < z_N$ ve $x_N > \overline{x}$, $y_N > \overline{y}$, $z_N > \overline{z}$, ise
Bu durumda

$$x_{N+1} = \max\left\{\frac{1}{x_N}, \frac{y_N}{z_N}\right\} = \frac{y_N}{z_N} < \overline{x},$$

$$y_{N+1} = \max\left\{\frac{1}{y_N}, \frac{z_N}{x_N}\right\} = \frac{z_N}{x_N} > \overline{y},$$

$$z_{N+1} = \max\left\{\frac{1}{z_N}, \frac{x_N}{y_N}\right\} = \frac{x_N}{y_N} < \overline{z},$$

$$x_{N+2} = \max\left\{\frac{1}{x_{N+1}}, \frac{y_{N+1}}{z_{N+1}}\right\} = \max\left\{\frac{z_N}{y_N}, \frac{z_N y_N}{x_N^2}\right\} = \frac{z_N y_N}{x_N^2} > \overline{x},$$

$$y_{N+2} = \max\left\{\frac{1}{y_{N+1}}, \frac{z_{N+1}}{x_{N+1}}\right\} = \max\left\{\frac{x_N}{z_N}, \frac{x_N z_N}{y_N^2}\right\} = \frac{x_N z_N}{y_N^2} < \overline{y},$$

$$z_{N+2} = \max\left\{\frac{1}{z_{N+1}}, \frac{x_{N+1}}{y_{N+1}}\right\} = \max\left\{\frac{y_N}{x_N}, \frac{x_N y_N}{z_N^2}\right\} = \frac{y_N}{x_N} > \overline{z},$$

$$x_{N+3} = \max\left\{\frac{1}{x_{N+2}}, \frac{y_{N+2}}{z_{N+2}}\right\} = \max\left\{\frac{x_N^2}{z_N y_N}, \frac{x_N^2 z_N}{y_N^3}\right\} = \frac{x_N^2 z_N}{y_N^3} < \overline{x},$$

$$y_{N+3}=\max\left\{\frac{1}{y_{N+2}},\frac{z_{N+2}}{x_{N+2}}\right\}=\max\left\{\frac{y_N^2}{x_Nz_N},\frac{x_N}{z_N}\right\}=\frac{y_N^2}{x_Nz_N}>\bar{y},$$

$$z_{N+3}=\max\left\{\frac{1}{z_{N+2}},\frac{x_{N+2}}{y_{N+2}}\right\}=\max\left\{\frac{y_N}{x_N},\frac{y_N^3}{x_N^3}\right\}=\frac{y_N^3}{x_N^3}>\bar{z},$$

$$x_{N+4}=\max\left\{\frac{1}{x_{N+3}},\frac{y_{N+3}}{z_{N+3}}\right\}=\max\left\{\frac{y_N^3}{x_N^2z_N},\frac{x_N^2}{z_Ny_N}\right\}=\frac{y_N^3}{x_N^2z_N}>\bar{x},$$

$$y_{N+4}=\max\left\{\frac{1}{y_{N+3}},\frac{z_{N+3}}{x_{N+3}}\right\}=\max\left\{\frac{x_Nz_N}{y_N^2},\frac{y_N^6}{x_N^5z_N}\right\}=\frac{y_N^6}{x_N^5z_N}>\bar{y},$$

$$z_{N+4}=\max\left\{\frac{1}{z_{N+3}},\frac{x_{N+3}}{y_{N+3}}\right\}=\max\left\{\frac{x_N^3}{y_N^3},\frac{x_N^3z_N^2}{y_N^5}\right\}=\frac{x_N^3z_N^2}{y_N^5}<\bar{z},$$

$$x_{N+5}=\max\left\{\frac{1}{x_{N+4}},\frac{y_{N+4}}{z_{N+4}}\right\}=\max\left\{\frac{x_N^2z_N}{y_N^3},\frac{y_N^{11}}{x_N^8z_N^3}\right\}=\frac{y_N^{11}}{x_N^8z_N^3}>\bar{x},$$

$$y_{N+5}=\max\left\{\frac{1}{y_{N+4}},\frac{z_{N+4}}{x_{N+4}}\right\}=\max\left\{\frac{x_N^5z_N}{y_N^6},\frac{x_N^5z_N^3}{y_N^8}\right\}=\frac{x_N^5z_N^3}{y_N^8}<\bar{y},$$

$$z_{N+5}=\max\left\{\frac{1}{z_{N+4}},\frac{x_{N+4}}{y_{N+4}}\right\}=\max\left\{\frac{y_N^5}{x_N^3z_N^2},\frac{x_N^3}{y_N^3}\right\}=\frac{y_N^5}{x_N^3z_N^2}>\bar{z},$$

$$x_{N+6}=\max\left\{\frac{1}{x_{N+5}},\frac{y_{N+5}}{z_{N+5}}\right\}=\max\left\{\frac{x_N^8z_N^3}{y_N^{11}},\frac{x_N^8z_N^5}{y_N^{13}}\right\}=\frac{x_N^8z_N^5}{y_N^{13}}<\bar{x},$$

$$y_{N+6}=\max\left\{\frac{1}{y_{N+5}},\frac{z_{N+5}}{x_{N+5}}\right\}=\max\left\{\frac{y_N^8}{x_N^5z_N^3},\frac{x_N^5z_N}{y_N^6}\right\}=\frac{y_N^8}{x_N^5z_N^3}>\bar{y},$$

$$z_{N+6}=\max\left\{\frac{1}{z_{N+5}},\frac{x_{N+5}}{y_{N+5}}\right\}=\max\left\{\frac{x_N^3z_N^2}{y_N^5},\frac{y_N^{19}}{x_N^{13}z_N^6}\right\}=\frac{y_N^{19}}{x_N^{13}z_N^6}>\bar{z},$$

$$\vdots$$

(i) ve (ii)
Yukarıdaki çözümleri kullanarak aşağıdaki eşitsizlikler elde edilir,

$$x_{N+3}<\bar{x},x_{N+4}>\bar{x},x_{N+5}>\bar{x},x_{N+6}<\bar{x},...,$$
$$y_{N+2}<\bar{y},y_{N+3}>\bar{y},y_{N+4}>\bar{y},y_{N+5}<\bar{y},y_{N+6}>\bar{y},...,$$
$$z_{N+4}<\bar{z},z_{N+5}>\bar{z},z_{N+6}>\bar{z},...,$$

Bu nedenle her pozitif yarı dönme iki terimden oluşur. Her negatif yarı dönme bir terimden oluşur.
(iii)
(I) ve (ii) ispatlarını kullanılarak

$$x_{N+3}<\bar{x}, x_{N+4}>\bar{x}, x_{N+5}>\bar{x}, x_{N+6}<\bar{x}, x_{N+7}>\bar{x}, x_{N+8}>\bar{x}, x_{N+9}<\bar{x},...,$$
$$y_{N+2}<\bar{y}, y_{N+3}>\bar{y}, y_{N+4}>\bar{y}, y_{N+5}<\bar{y}, y_{N+6}>\bar{y}, y_{N+7}>\bar{y}, y_{N+8}<\bar{y}, y_{N+9}>\bar{y},...,$$
$$z_{N+4}<\bar{z}, z_{N+5}>\bar{z}, z_{N+6}>\bar{z}, z_{N+7}<\bar{z}, z_{N+8}>\bar{z}, z_{N+9}>\bar{z},..., \text{olur.}$$

Bir uzunluğundaki negatif yarı dönmeyi iki uzunluğundaki pozitif yarı dönme takip eder.
Aynı şekilde 1. Durumun ispatına benzer şekilde 2. 3. 4. 5.ve 6. durumların ispatını elde edebiliriz.

Lemma 2. Aşağıdaki başlangıç koşulları göz önüne alındığında,

1. Durum: $1<x_0<z_0<y_0$
   x_n çözümü $n\geq 1$ için ; y_n çözümü $n\geq 3$ için , z_n çözümü için $n\geq 2$
2. Durum: $1<y_0<x_0<z_0$
   x_n çözümü $n\geq 2$ için ; y_n çözümü $n\geq 1$ için , z_n çözümü için $n\geq 3$
3. Durum: $1<z_0<y_0<x_0$
   x_n çözümü $n\geq 3$ için ; y_n çözümü $n\geq 2$ için , z_n çözümü için $n\geq 1$
4. Durum: $0<x_0<y_0<z_0\leq 0.8$
   x_n çözümü $n\geq 4$ için ; y_n çözümü $n\geq 3$ için , z_n çözümü için $n\geq 2$
5. Durum: $0<y_0<z_0<x_0\leq 0.8$
   x_n çözümü $n\geq 2$ için ; y_n çözümü $n\geq 4$ için , z_n çözümü için $n\geq 3$
6. Durum: $0<z_0<x_0<y_0\leq 0.8$
   x_n çözümü $n\geq 3$ için ; y_n çözümü $n\geq 2$ için , z_n çözümü için $n\geq 4$

yukarıdaki başlangıç koşullarına dayanarak. (1) denklemi için aşağıdaki ifadeler elde edilir:
(i) Her pozitif yarı dönme iki terimden oluşur;
(ii) Her negatif yarı dönme bir terimden oluşur;
(iii) İki uzunluğundaki her pozitif yarı dönmeyi bir uzunluğundaki negatif yarı dönme takip eder.

İspat. Lemma 2 nin ispatı Lemma 1 in ispatına benzer şekilde gösterilebilir.

Teorem 1. (x_n, y_n, z_n) (1) denkleminin $0<x_0<y_0<z_0\leq 0.8$ koşulu için çözümü olsun. Bu durumda

$$(3)\quad \begin{cases} x_{3n+2}=\dfrac{z_0^{f(3n+2)}}{x_0^{f(3n)}y_0^{f(3n+1)}}, x_{3n+3}=\dfrac{x_0^{f(3n)}y_0^{f(3n+1)}}{z_0^{f(3n+2)}}, x_{3n+4}=\dfrac{z_0^{f(3n+2)}}{x_0^{f(3n)}y_0^{f(3n+1)}}, n=0,1,2,... \\ y_{3n+1}=\dfrac{z_0^{f(3n+1)}}{x_0^{f(3n-1)}y_0^{f(3n)}}, y_{3n+2}=\dfrac{x_0^{f(3n-1)}y_0^{f(3n)}}{z_0^{f(3n+1)}}, y_{3n+3}=\dfrac{z_0^{f(3n+1)}}{x_0^{f(3n-1)}y_0^{f(3n)}}, n=0,1,2,... \\ z_{3n}=\dfrac{z_0^{f(3n)}}{x_0^{f(3n-2)}y_0^{f(3n-1)}}, z_{3n+1}=\dfrac{x_0^{f(3n-2)}y_0^{f(3n-1)}}{z_0^{f(3n)}}, z_{3n+2}=\dfrac{z_0^{f(3n)}}{x_0^{f(3n-2)}y_0^{f(3n-1)}}, n=0,1,2,... \end{cases}$$

İspat. Tümevarım yöntemi (3) denkleminin ispatı için kullanılabilir.
$n=0$ için

$$x_1=\max\left\{\frac{1}{x_0},\frac{y_0}{z_0}\right\}=\frac{1}{x_0},$$

$$y_1=\max\left\{\frac{1}{y_0},\frac{z_0}{x_0}\right\}=\frac{1}{y_0},$$

$$z_1=\max\left\{\frac{1}{z_0},\frac{x_0}{y_0}\right\}=\frac{1}{z_0},$$

doğrudur.

(3) denklemi $n=k$ için doğru olduğunu kabul edelim.

$$x_{3k+2}=\frac{z_0^{f(3k+2)}}{x_0^{f(3k)}y_0^{f(3k+1)}},x_{3k+3}=\frac{x_0^{f(3k)}y_0^{f(3k+1)}}{z_0^{f(3k+2)}},x_{3k+4}=\frac{z_0^{f(3k+2)}}{x_0^{f(3k)}y_0^{f(3k+1)}},k=0,1,2,...$$

$$y_{3k+1}=\frac{z_0^{f(3k+1)}}{x_0^{f(3k-1)}y_0^{f(3k)}},y_{3k+2}=\frac{x_0^{f(3k-1)}y_0^{f(3k)}}{z_0^{f(3n+1)}},y_{3k+3}=\frac{z_0^{f(3k+1)}}{x_0^{f(3k-1)}y_0^{f(3k)}},k=0,1,2,...$$

$$z_{3k}=\frac{z_0^{f(3k)}}{x_0^{f(3k-2)}y_0^{f(3k-1)}},z_{3k+1}=\frac{x_0^{f(3k-2)}y_0^{f(3k-1)}}{z_0^{f(3k)}},z_{3k+2}=\frac{z_0^{f(3k)}}{x_0^{f(3k-2)}y_0^{f(3k-1)}},k=0,1,2,...$$

(3) denkleminin $n=k+1$ için doğru olduğunu gösterelim.

$$z_{3k+3}=\max\left\{\frac{1}{z_{3k+2}},\frac{x_{3k+2}}{y_{3k+2}}\right\}=\max\left\{\frac{x_0^{f(3k-2)}y_0^{f(3k-1)}}{z_0^{f(3k)}},\frac{z_0^{f(3k+2)}}{x_0^{f(3k)}y_0^{3k+1}}\cdot\frac{z_0^{f(3k+1)}}{x_0^{f(3k-1)}y_0^{(3k)}}\right\}$$

$$=\max\left\{\frac{x_0^{f(3k-2)}y_0^{f(3k-1)}}{z_0^{f(3k)}},\frac{z_0^{f(3k+3)}}{x_0^{f(3k+1)}y_0^{(3k+2)}}\right\}=\frac{z_0^{f(3k+3)}}{x_0^{f(3k+1)}y_0^{(3k+2)}}$$

$$z_{3k+4}=\max\left\{\frac{1}{z_{3k+3}},\frac{x_{3k+3}}{y_{3k+3}}\right\}=\max\left\{\frac{x_0^{f(3k+1)}y_0^{f(3k+2)}}{z_0^{f(3k+3)}},\frac{x_0^{f(3k)}y_0^{f(3k+1)}}{z_0^{f(3k+2)}}\cdot\frac{x_0^{f(3k-1)}y_0^{(3k)}}{z_0^{f(3k+1)}}\right\}$$

$$=\max\left\{\frac{x_0^{f(3k+1)}y_0^{f(3k+2)}}{z_0^{f(3k+3)}},\frac{x_0^{f(3k+1)}y_0^{f(3k+2)}}{z_0^{f(3k+3)}}\right\}=\frac{x_0^{f(3k+1)}y_0^{f(3k+2)}}{z_0^{f(3k+3)}}$$

$$y_{3k+4}=\max\left\{\frac{1}{y_{3k+3}},\frac{z_{3k+3}}{x_{3k+3}}\right\}=\max\left\{\frac{x_0^{f(3k-1)}y_0^{f(3k)}}{z_0^{f(3k+1)}},\frac{z_0^{f(3k+3)}}{x_0^{f(3k+1)}y_0^{f(3k+2)}}\cdot\frac{z_0^{f(3k+2)}}{x_0^{f(3k)}y_0^{(3k+1)}}\right\}$$

$$=\max\left\{\frac{x_0^{f(3k-1)}y_0^{f(3k)}}{z_0^{f(3k+1)}},\frac{z_0^{f(3k+4)}}{x_0^{f(3k+2)}y_0^{f(3k+3)}}\right\}=\frac{z_0^{f(3k+4)}}{x_0^{f(3k+2)}y_0^{f(3k+3)}}$$

$$x_{3k+5}=\max\left\{\frac{1}{z_{3k+4}},\frac{x_{3k+4}}{y_{3k+4}}\right\}=\max\left\{\frac{x_0^{f(3k)}y_0^{f(3k+1)}}{z_0^{f(3k+2)}},\frac{z_0^{f(3k+4)}}{x_0^{f(3k+3)}y_0^{f(3k+2)}}\cdot\frac{z_0^{f(3k+3)}}{x_0^{f(3k+1)}y_0^{(3k+2)}}\right\}$$

$$=\max\left\{\frac{x_0^{f(3k)}y_0^{f(3k+1)}}{z_0^{f(3k+2)}},\frac{z_0^{f(3k+5)}}{x_0^{f(3k+3)}y_0^{(3k+4)}}\right\}=\frac{z_0^{f(3k+5)}}{x_0^{f(3k+3)}y_0^{(3k+4)}}$$

$$y_{3k+5}=\max\left\{\frac{1}{y_{3k+4}},\frac{z_{3k+4}}{x_{3k+4}}\right\}=\max\left\{\frac{x_0^{f(3k+2)}y_0^{f(3k+3)}}{z_0^{f(3k+4)}},\frac{x_0^{f(3k+1)}y_0^{f(3k+2)}}{z_0^{f(3k+3)}}\cdot\frac{x_0^{f(3k)}y_0^{f(3k+1)}}{z_0^{f(3k+2)}}\right\}$$

$$=\max\left\{\frac{x_0^{f(3k+2)}y_0^{f(3k+3)}}{z_0^{f(3k+4)}},\frac{x_0^{f(3k+2)}y_0^{f(3k+3)}}{z_0^{f(3k+4)}}\right\}=\frac{x_0^{f(3k+2)}y_0^{f(3k+3)}}{z_0^{f(3k+4)}}$$

$$z_{3k+5}=\max\left\{\frac{1}{z_{3k+4}},\frac{x_{3k+4}}{y_{3k+4}}\right\}=\max\left\{\frac{z_0^{f(3k+3)}}{x_0^{f(3k-1)}y_0^{f(3k+2)}},\frac{x_0^{f(3k+2)}}{x_0^{f(3k)}y_0^{f(3k+1)}}\cdot\frac{x_0^{f(3k+2)}y_0^{f(3k+3)}}{z_0^{f(3k+4)}}\right\}$$

$$=\max\left\{\frac{z_0^{f(3k+3)}}{x_0^{f(3k+1)}y_0^{f(3k+2)}},\frac{x_0^{f(3k+1)}y_0^{f(3k+2)}}{z_0^{f(3k+3)}}\right\}=\frac{z_0^{f(3k+3)}}{x_0^{f(3k+1)}y_0^{f(3k+2)}}$$

$$x_{3k+6}=\max\left\{\frac{1}{x_{3k+5}},\frac{y_{3k+5}}{z_{3k+5}}\right\}=\max\left\{\frac{x_0^{f(3k+3)}y_0^{f(3k+4)}}{z_0^{f(3k+5)}},\frac{x_0^{f(3k+2)}y_0^{f(3k+3)}}{z_0^{f(3k+4)}}\cdot\frac{x_0^{f(3k+1)}y_0^{f(3k+2)}}{z_0^{f(3k+3)}}\right\}$$

$$=\max\left\{\frac{x_0^{f(3k+3)}y_0^{f(3k+4)}}{z_0^{f(3k+5)}},\frac{x_0^{f(3k+3)}y_0^{f(3k+4)}}{z_0^{f(3k+5)}}\right\}=\frac{x_0^{f(3k+3)}y_0^{f(3k+4)}}{z_0^{f(3k+5)}}$$

$$y_{3k+6}=\max\left\{\frac{1}{y_{3k+5}},\frac{z_{3k+5}}{x_{3k+5}}\right\}=\max\left\{\frac{z_0^{f(3k+4)}}{x_0^{f(3k+2)}y_0^{f(3k+3)}},\frac{z_0^{f(3k+3)}}{x_0^{f(3k+1)}y_0^{f(3k+2)}}\cdot\frac{x_0^{f(3k+3)}y_0^{f(3k+4)}}{z_0^{f(3k+5)}}\right\}$$

$$=\max\left\{\frac{z_0^{f(3k+4)}}{x_0^{f(3k+2)}y_0^{f(3k+3)}},\frac{x_0^{f(3k+2)}y_0^{f(3k+3)}}{z_0^{f(3k+4)}}\right\}=\frac{z_0^{f(3k+4)}}{x_0^{f(3k+2)}y_0^{f(3k+3)}}$$

$$z_{3k+6}=\max\left\{\frac{1}{z_{3k+5}},\frac{x_{3k+5}}{y_{3k+5}}\right\}=\max\left\{\frac{x_0^{f(3k+1)}y_0^{f(3k+2)}}{z_0^{f(3k+3)}},\frac{z_0^{f(3k+5)}}{x_0^{f(3k+3)}y_0^{f(3k+4)}}\cdot\frac{z_0^{f(3k+4)}}{x_0^{f(3k+2)}y_0^{f(3k+3)}}\right\}$$

$$=\max\left\{\frac{x_0^{f(3k+1)}y_0^{f(3k+2)}}{z_0^{f(3k+3)}},\frac{z_0^{f(3k+6)}}{x_0^{f(3k+4)}y_0^{f(3k+5)}}\right\}=\frac{z_0^{f(3k+6)}}{x_0^{f(3k+4)}y_0^{f(3k+5)}}$$

$$x_{3k+7}=\max\left\{\frac{1}{x_{3k+6}},\frac{y_{3k+6}}{z_{3k+6}}\right\}=\max\left\{\frac{z_0^{f(3k+5)}}{x_0^{f(3k+3)}y_0^{f(3k+4)}},\frac{z_0^{f(3k+3)}}{x_0^{f(3k+2)}y_0^{f(3k+3)}}\cdot\frac{x_0^{f(3k+4)}y_0^{f(3k+5)}}{z_0^{f(3k+6)}}\right\}$$

$$=\max\left\{\frac{z_0^{f(3k+5)}}{x_0^{f(3k+3)}y_0^{f(3k+4)}},\frac{x_0^{f(3k+3)}y_0^{f(3k+4)}}{z_0^{f(3k+5)}}\right\}=\frac{z_0^{f(3k+5)}}{x_0^{f(3k+3)}y_0^{f(3k+4)}}$$

Teorem 2.

a) (x_n, y_n, z_n) (1) denkleminin $1<x_0<z_0<y_0$ şartı için çözümü olsun .
Bu durumda

$$x_{3n+2}=\frac{y_0^{f(3n+1)}z_0^{f(3n+2)}}{x_0^{f(3n+3)}},x_{3n+3}=\frac{x_0^{f(3n+3)}}{y_0^{f(3n+1)}z_0^{f(3n+2)}},x_{3n+4}=\frac{y_0^{f(3n+1)}z_0^{f(3n+2)}}{x_0^{f(3n+3)}},n=0,1,2,...$$

$$y_{3n+1}=\frac{y_0^{f(3n)}z_0^{f(3n+1)}}{x_0^{f(3n+2)}},y_{3n+2}=\frac{x_0^{f(3n+2)}}{y_0^{f(3n)}z_0^{f(3n+1)}},y_{3n+3}=\frac{y_0^{f(3n)}z_0^{f(3n+1)}}{x_0^{f(3n+2)}},n=1,2,3,...$$

$$z_{3n+3}=\frac{y_0^{f(3n+2)}z_0^{f(3n+3)}}{x_0^{f(3n+4)}},z_{3n+4}=\frac{x_0^{f(3n+4)}}{y_0^{f(3n+2)}z_0^{f(3n+3)}},z_{3n+5}=\frac{y_0^{f(3n+2)}z_0^{f(3n+3)}}{x_0^{f(3n+4)}},n=0,1,2,...$$

elde edilir.

b) (x_n,y_n,z_n) (1) denkleminin $1<y_0<x_0<z_0$ şartı için çözümü olsun.
Bu durumda

$$x_{3n+3}=\frac{z_0^{f(3n+2)}x_0^{f(3n+3)}}{y_0^{f(3n+4)}},x_{3n+4}=\frac{y_0^{f(3n+4)}}{z_0^{f(3n+2)}x_0^{f(3n+3)}},x_{3n+5}=\frac{y_0^{f(3n+2)}z_0^{f(3n+3)}}{x_0^{f(3n+4)}},n=0,1,2,...$$

$$y_{3n+2}=\frac{z_0^{f(3n+1)}z_0^{f(3n+2)}}{x_0^{f(3n+3)}},y_{3n+3}=\frac{y_0^{f(3n+3)}}{z_0^{f(3n+1)}x_0^{f(3n+2)}},y_{3n+4}=\frac{z_0^{f(3n+1)}x_0^{f(3n+2)}}{y_0^{f(3n+3)}},n=0,1,2,...$$

$$z_{3n+1}=\frac{z_0^{f(3n)}z_0^{f(3n+1)}}{y_0^{f(3n+2)}},z_{3n+2}=\frac{y_0^{f(3n+2)}}{z_0^{f(3n)}x_0^{f(3n+1)}},z_{3n+3}=\frac{z_0^{f(3n)}x_0^{f(3n+1)}}{y_0^{f(3n+2)}},n=1,2,3,...$$

elde edilir.

c) (x_n,y_n,z_n) (2) denkleminin $1<y_0<z_0<x_0$ şartı için çözümü olsun.
Bu durumda

$$x_{3n+1}=\frac{x_0^{f(3n)}y_0^{f(3n+1)}}{z_0^{f(3n+2)}},x_{3n+2}=\frac{z_0^{f(3n+2)}}{x_0^{f(3n)}y_0^{f(3n+1)}},y_{3n+3}=\frac{z_0^{f(3n+5)-2}}{x_0^{f(3n+3)-2}y_0^{f(3n+4)}},n=1,2,3,...$$

$$y_{3n}=\frac{x_0^{f(3n-1)}y_0^{f(3n)}}{z_0^{f(3n+1)}},y_{3n+1}=\frac{z_0^{f(3n+1)}}{x_0^{f(3n-1)}y_0^{f(3n)}},y_{3n+2}=\frac{z_0^{f(3n+4)-2}}{x_0^{f(3n+2)-2}y_0^{f(3n+3)}},n=1,2,3,...$$

$$z_{3n}=\frac{z_0^{f(3n)}}{x_0^{f(3n-2)}y_0^{f(3n-1)}},z_{3n+1}=\frac{z_0^{f(3n+3)-2}}{x_0^{f(3n+1)-2}y_0^{f(3n+2)}},n=1,2,3,...$$

$$z_{3n+2}=\frac{x_0^{f(3n+1)}y_0^{f(3n+2)}}{z_0^{f(3n+3)}},n=0,1,2,...$$

elde edilir.

d) (x_n,y_n,z_n) (2) denkleminin $1<z_0<x_0<y_0$ şartı için çözümü olsun.
Bu durumda

$$x_{3n}=\frac{x_0^{f(3n)}}{z_0^{f(3n-1)}y_0^{f(3n-2)}},\ x_{3n+1}=\frac{x_0^{f(3n+3)-2}}{y_0^{f(3n+1)-2}z_0^{f(3n+2)}},\ n=1,2,3,...$$

$$x_{3n+2}=\frac{y_0^{f(3n+1)}z_0^{f(3n+2)}}{x_0^{f(3n+3)}},\ n=0,1,2,,...$$

$$y_{3n+1}=\frac{y_0^{f(3n)}z_0^{f(3n+1)}}{x_0^{f(3n+2)}},\ y_{3n+2}=\frac{x_0^{f(3n+2)}}{y_0^{f(3n)}z_0^{f(3n+1)}},\ y_{3n+3}=\frac{x_0^{f(3n+5)-2}}{y_0^{f(3n+3)-2}z_0^{f(3n+4)}},\ n=,1,2,3,...$$

$$z_{3n}=\frac{y_0^{f(3n-1)}z_0^{f(3n)}}{x_0^{f(3n+1)}},\ z_{3n+1}=\frac{x_0^{f(3n+1)}}{y_0^{f(3n-1)}z_0^{f(3n)}},\ z_{3n+2}=\frac{x_0^{f(3n+4)-2}}{y_0^{f(3n+2)-2}z_0^{f(3n+3)}},\ n=1,2,3,...$$

e) (x_n, y_n, z_n) (1) denkleminin için $1<z_0<y_0<x_0$ şartı için çözümü olsun.
Bu durumda

$$x_{3n+1}=\frac{x_0^{f(3n)}y_0^{f(3n+1)}}{z_0^{f(3n+2)}},\ x_{3n+2}=\frac{z_0^{f(3n+2)}}{x_0^{f(3n)}y_0^{f(3n+1)}},\ x_{3n+3}=\frac{x_0^{f(3n)}y_0^{f(3n+1)}}{z_0^{f(3n+2)}}\ n=1,2,3,...$$

$$y_{3n+3}=\frac{x_0^{f(3n+2)}y_0^{f(3n+3)}}{z_0^{f(3n+4)}},\ y_{3n+4}=\frac{z_0^{f(3n+4)}}{x_0^{f(3n+2)}y_0^{f(3n+3)}},\ y_{3n+5}=\frac{x_0^{f(3n+2)}y_0^{f(3n+3)}}{z_0^{f(3n+4)}}\ n=0,1,2,...$$

$$z_{3n+2}=\frac{x_0^{f(3n+1)}y_0^{f(3n+2)}}{z_0^{f(3n+3)}},\ z_{3n+3}=\frac{z_0^{f(3n+3)}}{x_0^{f(3n+1)}y_0^{f(3n+2)}},\ z_{3n+4}=\frac{x_0^{f(3n+1)}y_0^{f(3n+2)}}{z_0^{f(3n+3)}}\ n=0,1,2,...$$

elde edilir.

f) (x_n, y_n, z_n) (1) denkleminin $1<z_0<y_0<x_0$ şartı için çözümü olsun.
Bu durumda

$$x_{3n+2}=\frac{z_0^{f(3n+2)}}{x_0^{f(3n)}y_0^{f(3n+1)}},\ x_{3n+3}=\frac{x_0^{f(3n)}y_0^{f(3n+1)}}{z_0^{f(3n+2)}},\ x_{3n+4}=\frac{z_0^{f(3n+2)}}{x_0^{f(3n)}y_0^{f(3n+1)}},\ n=1,2,3,...$$

$$y_{3n+1}=\frac{z_0^{f(3n+1)}}{x_0^{f(3n-1)}y_0^{f(3n)}},\ y_{3n+2}=\frac{x_0^{f(3n-1)}y_0^{f(3n)}}{z_0^{f(3n+1)}},\ y_{3n+3}=\frac{z_0^{f(3n+1)}}{x_0^{f(3n-1)}y_0^{f(3n)}},\ n=1,2,3,...$$

$$z_{3n}=\frac{z_0^{f(3n)}}{x_0^{f(3n-2)}y_0^{f(3n-1)}},\ z_{3n+1}=\frac{x_0^{f(3n-2)}y_0^{f(3n-1)}}{z_0^{f(3n)}},\ z_{3n+2}=\frac{z_0^{f(3n)}}{x_0^{f(3n-2)}y_0^{f(3n-1)}},\ n=1,2,3,...$$

elde edilir.

g) (x_n, y_n, z_n) (2) denkleminin $0<x_0<z_0<y_0\le 0,8$ şartı için çözümü olsun.
Bu durumda

$$x_{3n+2}=\frac{z_0^{f(3n+2)}}{x_0^{f(3n)}y_0^{f(3n+1)}},x_{3n+3}=\frac{x_0^{f(3n)}y_0^{f(3n+1)}}{z_0^{f(3n+2)}},x_{3n+4}=\frac{x_0^{f(3n+3)-2}y_0^{f(3n+4)}}{z_0^{f(3n+5)-2}},n=1,2,3,...$$
$$y_{3n+1}=\frac{z_0^{f(3n+1)}}{x_0^{f(3n-1)}y_0^{f(3n)}},y_{3n+2}=\frac{x_0^{f(3n-1)}y_0^{f(3n)}}{z_0^{f(3n+1)}},y_{3n+3}=\frac{x_0^{f(3n+2)-2}y_0^{f(3n+3)}}{z_0^{f(3n+4)-2}},n=1,2,3,...$$
$$z_{3n}=\frac{z_0^{f(3n)}}{x_0^{f(3n-2)}y_0^{f(3n-1)}},z_{3n+1}=\frac{x_0^{f(3n-2)}y_0^{f(3n-1)}}{z_0^{f(3n)}},z_{3n+2}=\frac{x_0^{f(3n+1)-2}y_0^{f(3n+2)}}{z_0^{f(3n+3)-2}},n=1,2,3,...$$

elde edilir.

h) (x_n,y_n,z_n) (1) denkleminin $0<y_0<x_0<z_0\le 0.8$ şartı için çözümü olsun.
Bu durumda

$$x_{3n}=\frac{x_0^{f(3n)}}{y_0^{f(3n-2)}z_0^{f(3n-1)}},x_{3n+1}=\frac{y_0^{f(3n-2)}z_0^{f(3n-1)}}{x_0^{f(3n)}},x_{3n+2}=\frac{y_0^{f(3n+1)-2}z_0^{f(3n+2)}}{x_0^{f(3n+3)-2}},n=1,2,3,...$$
$$y_{3n+2}=\frac{x_0^{f(3n+2)}}{y_0^{f(3n)}z_0^{f(3n+1)}},y_{3n+3}=\frac{y_0^{f(3n)}z_0^{f(3n+1)}}{x_0^{f(3n+2)}},y_{3n+4}=\frac{y_0^{f(3n+3)-2}z_0^{f(3n+4)}}{x_0^{f(3n+5)-2}},n=1,2,3,...$$
$$z_{3n+1}=\frac{x_0^{f(3n+1)}}{y_0^{f(3n-1)}z_0^{f(3n)}},z_{3n+2}=\frac{y_0^{f(3n-1)}z_0^{f(3n)}}{x_0^{f(3n+1)}},z_{3n+3}=\frac{y_0^{f(3n+2)-2}z_0^{f(3n+3)}}{x_0^{f(3n+4)-2}},n=1,2,3,...$$

elde edilir.

ı) (x_n,y_n,z_n) (1) denkleminin $0<y_0<z_0<x_0\le 0.8$ şartı için çözümü olsun.
Bu durumda

$$x_{3n+3}=\frac{x_0^{f(3n+3)}}{y_0^{f(3n+1)}z_0^{f(3n+2)}},x_{3n+4}=\frac{y_0^{f(3n+1)}z_0^{f(3n+2)}}{x_0^{f(3n+3)}},x_{3n+5}=\frac{x_0^{f(3n+3)}}{y_0^{f(3n+1)}z_0^{f(3n+2)}},n=0,1,2,...$$
$$y_{3n+2}=\frac{x_0^{f(3n+2)}}{y_0^{f(3n)}z_0^{f(3n+1)}},y_{3n+3}=\frac{y_0^{f(3n)}z_0^{f(3n+1)}}{x_0^{f(3n+2)}},y_{3n+4}=\frac{x_0^{f(3n+2)}}{y_0^{f(3n)}z_0^{f(3n+1)}},n=1,2,3,...$$
$$z_{3n+1}=\frac{x_0^{f(3n+1)}}{y_0^{f(3n-1)}z_0^{f(3n)}},z_{3n+2}=\frac{y_0^{f(3n-1)}z_0^{f(3n)}}{x_0^{f(3n+1)}},z_{3n+3}=\frac{x_0^{f(3n+1)}}{y_0^{f(3n-1)}z_0^{f(3n)}},n=1,2,3,...$$

elde edilir.

i) (x_n,y_n,z_n) (1) denkleminin $0<z_0<x_0<y_0\le 0.8$ şartı için çözümü olsun.
Bu durumda

$$x_{3n+1}=\frac{y_0^{f(3n+1)}}{z_0^{f(3n-1)}x_0^{f(3n)}},\ x_{3n+2}=\frac{z_0^{f(3n-1)}x_0^{f(3n)}}{y_0^{f(3n+1)}},\ x_{3n+3}=\frac{y_0^{f(3n+1)}}{z_0^{f(3n-1)}x_0^{f(3n)}},\ n=1,2,3,\ldots$$
$$y_{3n+3}=\frac{y_0^{f(3n+3)}}{z_0^{f(3n+1)}x_0^{f(3n+2)}},\ y_{3n+4}=\frac{z_0^{f(3n+1)}x_0^{f(3n+2)}}{y_0^{f(3n+3)}},\ y_{3n+5}=\frac{y_0^{f(3n+3)}}{z_0^{f(3n+1)}x_0^{f(3n+2)}},\ n=0,1,2,,\ldots$$
$$z_{3n+2}=\frac{y_0^{f(3n+2)}}{z_0^{f(3n)}x_0^{f(3n+1)}},\ z_{3n+3}=\frac{z_0^{f(3n)}x_0^{f(3n+1)}}{y_0^{f(3n+2)}},\ z_{3n+4}=\frac{y_0^{f(3n+2)}}{z_0^{f(3n)}x_0^{f(3n+1)}},\ n=1,2,3,\ldots$$

elde edilir.

j) (x_n, y_n, z_n) (1) denkleminin $0<z_0<y_0<x_0\le 0.8$ şartı için çözümü olsun.
Bu durumda

$$x_{3n+1}=\frac{y_0^{f(3n+1)}}{z_0^{f(3n-1)}x_0^{f(3n)}},\ x_{3n+2}=\frac{z_0^{f(3n-1)}x_0^{f(3n)}}{y_0^{f(3n+1)}},\ x_{3n+3}=\frac{z_0^{f(3n+2)-2}x_0^{f(3n+3)}}{y_0^{f(3n+4)-2}},\ n=1,2,3,\ldots$$
$$y_{3n}=\frac{y_0^{f(3n)}}{z_0^{f(3n-2)}x_0^{f(3n-1)}},\ y_{3n+1}=\frac{z_0^{f(3n-2)}x_0^{f(3n-1)}}{y_0^{f(3n)}},\ y_{3n+2}=\frac{z_0^{f(3n+1)-2}x_0^{f(3n+2)}}{y_0^{f(3n+3)-2}},\ n=1,2,3,\ldots$$
$$z_{3n+2}=\frac{y_0^{f(3n+2)}}{z_0^{f(3n)}x_0^{f(3n+1)}},\ z_{3n+3}=\frac{z_0^{f(3n)}x_0^{f(3n+1)}}{y_0^{f(3n+2)}},\ z_{3n+4}=\frac{z_0^{f(3n+3)-2}x_0^{f(3n+4)}}{y_0^{f(3n+5)-2}},\ n=1,2,3,\ldots$$

elde edilir.

İspat. Teorem 2 nin ispatını teorem 1 e benzer şekilde elde ederiz.

3. TARTIŞMA VE SONUÇ

Bu çalışmada, $x_{-1}; x_0; y_{-1}; y_0$ başlangıç şartları pozitif reel sayılar olmak üzere, $x_{n+1}=\max\left\{\frac{1}{x_n},\frac{y_n}{z_n}\right\}, y_{n+1}=\max\left\{\frac{1}{y_n},\frac{z_n}{x_n}\right\}, z_{n+1}=\max\left\{\frac{1}{z_n},\frac{x_n}{y_n}\right\}$ maksimumlu fark denklem sisteminin çözümlerinin davranışları incelenmiştir. Bu fark denklem sisteminde katsayıları değiştirilerek yeni maksimumlu fark denklem sistemleri oluşturulabilir. Oluşturulacak yeni maksimumlu fark denklem sisteminin çözüm davranışları incelenebilir.

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