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Gelecek elimizde...

Solutions of the Maximum Difference Equation

$$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$$

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Abstract: *The behaviour and periodicity of the solutions of the following system of difference equations is examined*

$$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$$

where the initial conditions are positive real numbers.

Keywords: *Difference Equation, Maximum Operations, Semicycle, Periodicity*

$$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; \; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$$

Maksimumlu Fark Denkleminin Çözümleri

Özet: *Aşağıdaki fark denklem sisteminin çözümlerinin periyodikliği ve davranışları incelenmiştir.*

$$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$$

Başlangıç şartları pozitif reel sayılardır.

Anahtar Kelimeler: *Fark Denklemi, Maksimum Operatörü, Yarı Dönmeler, Periyodiklik.*

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1. INTRODUCTION

Recently, there has been a great interest in studying the periodic nature of nonlinear difference equations. Although difference equations are relatively simple in form, it is, unfortunately, extremely difficult to understand thoroughly the periodic behavior of their solutions. The periodic nature of nonlinear difference equations of the max type has been investigated by many authors. See for example [1-30].

Definition 1.1. Let I be an interval of real numbers and let $f : I^{s+1} \to I$ be a continuously differentiable function where s is a non-negative integer. Consider the difference equation

$$x_{n+1} = f(x_n, x_{n-1}, ..., x_{n-s}) \text{ for } n = 0,1,2,\ldots \tag{2}$$

with the initial values $x_{-s},..., x_0 \in I$. A point $\overline{x}$ called an equilibrium point of the (2) if $\overline{x} = f(\overline{x},...,\overline{x})$.

Definition 1.2. A positive semicycle of a solutions $\{x_n\}_{n=-s}^{\infty}$ of the (2) consist of a string of terms $\{x_l, x_{l+1},..., x_m\}$ all greater than or equal to equilibrium $\overline{x}$ with $l \geq -s$ and $m \leq \infty$ such that either $l = -s$ or $l > -s$ ve $x_{l-1} < \overline{x}$ and either $m = \infty$ or $m < \infty$ and $x_{m+1} < \overline{x}$.

Definition 1.3. A negative semicycle of a solutions $\{x_n\}_{n=-s}^{\infty}$ of (2) consist of a string of terms $\{x_l, x_{l+1},..., x_m\}$ all less than or equal to equilibrium $\overline{x}$ with $l \geq -s$ and $m \leq \infty$ such that either $l = -s$ or $l > -s$ and $x_{l-1} \geq \overline{x}$ and either $m = \infty$ or $m \leq \infty$ and $x_{m+1} \geq \overline{x}$.

2. MAIN RESULT

Let $\overline{x}$ and $\overline{y}$ be the unique positive equilibrium of the (1), then clearly

$$\overline{x} = \max\left\{\frac{1}{\overline{x}}, \frac{\overline{y}}{\overline{x}}\right\}; \overline{y} = \min\left\{\frac{1}{\overline{y}}, \frac{\overline{x}}{\overline{y}}\right\}$$

$$\overline{x} = \frac{1}{\overline{x}} \Rightarrow \overline{x}^2 = 1 \Rightarrow \overline{x} = 1 \quad \text{and} \quad \overline{x} = \frac{\overline{y}}{\overline{x}} \Rightarrow \overline{x}^2 = \overline{y},$$

$$\overline{y} = \frac{1}{\overline{y}} \Rightarrow \overline{y}^2 = 1 \Rightarrow \overline{y} = 1 \quad \text{and} \quad \overline{y} = \frac{\overline{x}}{\overline{y}} \Rightarrow \overline{y}^2 = \overline{x},$$

We can obtain $\overline{x} = 1$ and $\overline{y} = 1$.

Lemma 2.1. Assume that

$$0 < x_{-1} < x_0 < y_{-1} < y_0 < A, 0 < x_{-1} < x_0 < y_0 < y_{-1} < A, 0 < x_{-1} < y_{-1} < x_0 < y_0 < A,$$
$$0 < x_{-1} < y_{-1} < y_0 < x_0 < A, 0 < x_{-1} < y_0 < y_{-1} < x_0 < A, 0 < x_{-1} < y_0 < x_0 < y_{-1} < A,$$
$$0 < y_0 < x_0 < y_{-1} < x_{-1} < A, 0 < y_0 < x_0 < x_{-1} < y_{-1} < A, 0 < y_0 < y_{-1} < x_0 < x_{-1} < A,$$
$$0 < y_0 < y_{-1} < x_{-1} < x_0 < A, 0 < y_0 < x_{-1} < y_{-1} < x_0 < A, 0 < y_0 < x_{-1} < x_0 < y_{-1} < A,$$
$$0 < y_{-1} < x_0 < y_0 < x_{-1} < A, 0 < y_{-1} < y_0 < x_0 < x_{-1} < A, 0 < y_{-1} < y_0 < x_{-1} < x_0 < A,$$

$$0<y_{-1}<x_{-1}<y_0<x_0<A,0<y_{-1}<x_0<x_{-1}<y_0<A,0<y_{-1}<x_{-1}<x_0<y_0<A,$$
$$0<x_0<y_{-1}<y_0<x_{-1}<A,0<x_0<y_0<x_{-1}<y_{-1}<A,0<x_0<y_0<y_{-1}<x_{-1}<A,$$
$$0<x_0<x_{-1}<y_0<y_{-1}<A,0<x_0<x_{-1}<y_{-1}<y_0<A,0<x_0<y_{-1}<x_{-1}<y_0<A,$$

Then the following statements are true for the solutions of the (1) :
(x_n, y_n) is the solution, solution x_n, for $n \geq 0$ and solution y_n, for $n \geq 0$;

a) Every positive semicycle consists of three terms.
b) Every negative semicycle consists of two terms.
c) Every positive semicycle of length three is followed by a negative semicycle of length two.
d) Every negative semicycle of length two is followed by a positive semicycle of three.

Proof.

$$x_1=\max\left\{\frac{1}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{1}{x_{-1}}>\bar{x},$$
$$y_1=\max\left\{\frac{1}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{1}{y_{-1}}>\bar{y},$$
$$x_2=\max\left\{\frac{1}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{\frac{1}{x_0},\frac{1}{x_0y_{-1}}\right\}=\frac{1}{x_0y_{-1}}>\bar{x},$$
$$y_2=\max\left\{\frac{1}{y_0},\frac{x_1}{y_0}\right\}=\max\left\{\frac{1}{y_0},\frac{1}{y_0x_{-1}}\right\}=\frac{1}{y_0x_{-1}}>\bar{y},$$
$$x_3=\max\left\{\frac{1}{x_1},\frac{y_2}{x_1}\right\}=\max\left\{x_{-1},\frac{1}{y_0}\right\}=\frac{1}{y_0}>\bar{x},$$
$$y_3=\max\left\{\frac{1}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{y_{-1},\frac{1}{x_0}\right\}=\frac{1}{x_0}>\bar{y},$$
$$x_4=\max\left\{\frac{1}{x_2},\frac{y_3}{x_2}\right\}=\max\left\{x_0y_{-1},y_{-1}\right\}=y_{-1}<\bar{x},$$
$$y_4=\max\left\{\frac{1}{y_2},\frac{x_3}{y_2}\right\}=\max\left\{y_0x_{-1},x_{-1}\right\}=x_{-1}<\bar{y},$$
$$x_5=\max\left\{\frac{1}{x_3},\frac{y_4}{x_3}\right\}=\max\left\{y_0,y_0x_{-1}\right\}=y_0<\bar{x},$$
$$y_5=\max\left\{\frac{1}{y_3},\frac{x_4}{y_3}\right\}=\max\left\{x_0,x_0y_{-1}\right\}=x_0<\bar{y},$$
$$x_6=\max\left\{\frac{1}{x_4},\frac{y_5}{x_4}\right\}=\max\left\{\frac{1}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{1}{y_{-1}}>\bar{x},$$
$$y_6=\max\left\{\frac{1}{y_4},\frac{x_5}{y_4}\right\}=\max\left\{\frac{1}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{1}{x_{-1}}>\bar{y},$$
$$x_7=\max\left\{\frac{1}{x_5},\frac{y_6}{x_5}\right\}=\max\left\{\frac{1}{y_0},\frac{1}{y_0x_{-1}}\right\}=\frac{1}{y_0x_{-1}}>\bar{x},$$

$$y_7 = \max\left\{\frac{1}{y_5}, \frac{x_6}{y_5}\right\} = \max\left\{\frac{1}{x_0}, \frac{1}{x_0 y_{-1}}\right\} = \frac{1}{x_0 y_{-1}} > \bar{y},$$

$$x_8 = \max\left\{\frac{1}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{y_{-1}, \frac{1}{x_0}\right\} = \frac{1}{x_0} > \bar{x},$$

$$y_8 = \max\left\{\frac{1}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0} > \bar{y},$$

$$y_9 = \max\left\{\frac{1}{y_7}, \frac{x_8}{y_7}\right\} = \max\left\{x_0 y_{-1}, y_{-1}\right\} = y_{-1} < \bar{y},$$

$$x_{10} = \max\left\{\frac{1}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{x_0, x_0 y_{-1}\right\} = x_0 < \bar{x},$$

$$y_{10} = \max\left\{\frac{1}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{y_0, x_{-1} y_0\right\} = y_0 < \bar{y},$$

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Consequently, we have obtained:

$$x_1 > \bar{x},\ x_2 > \bar{x},\ x_3 > \bar{x},\ x_4 < \bar{x},\ x_5 < \bar{x},\ x_6 > \bar{x},\ x_7 > \bar{x},\ x_8 > \bar{x},\ x_9 < \bar{x},\ x_{10} < \bar{x}, \ldots$$

$$y_1 > \bar{y},\ y_2 > \bar{y},\ y_3 > \bar{y},\ y_4 < \bar{y},\ y_5 < \bar{y},\ y_6 > \bar{y},\ y_7 > \bar{y},\ y_8 > \bar{y},\ y_9 < \bar{y},\ y_{10} < \bar{y}, \ldots$$

Hence, the solution is x_n for $n \geq 0$ and solution y_n for $n \geq 0$;

Every positive semicycle consists of three terms,

a) Every negative semicycle consists of two terms,

b) Using the proof a);

The solution is x_n for $n \geq 0$ and solution y_n for $n \geq 0$ every positive semicycle consists of three terms, every negative semicycle consists of two terms;

c) Using the proof a);

Therefore, the solution x_n for $n \geq 0$ and solution y_n for $n \geq 0$ every negative semicycle of length two is followed by a positive semicycle of length three.

Lemma 2.2. Assume that

$$A < x_0 < y_{-1} < y_0 < x_{-1} < 1, A < x_0 < y_0 < x_{-1} < y_{-1} < 1, A < x_0 < y_0 < y_{-1} < x_{-1} < 1,$$

$$A < y_0 < x_{-1} < x_0 < y_{-1} < 1,\ A < y_0 < x_0 < y_{-1} < x_{-1} < 1, A < x_{-1} < x_0 < y_{-1} < y_0 < 1,$$

$$A < x_{-1} < x_0 < y_0 < y_{-1} < 1, A < x_{-1} < y_0 < x_0 < y_{-1} < 1, A < x_0 < x_{-1} < y_{-1} < y_0 < 1,$$

$$A < x_0 < x_{-1} < y_0 < y_{-1} < 1, A < x_0 < y_{-1} < x_{-1} < y_0 < 1, A < x_{-1} < y_{-1} < x_0 < y_0 < 1,$$

$$A < x_{-1} < y_{-1} < y_0 < x_0 < 1, A < x_{-1} < y_0 < y_{-1} < x_0 < 1, A < y_{-1} < x_{-1} < x_0 < y_0 < 1,$$

$$A < y_{-1} < x_{-1} < y_0 < x_0 < 1, A < y_{-1} < x_0 < x_{-1} < y_0 < 1, A < y_0 < x_{-1} < y_{-1} < x_0 < 1,$$

$$A < y_0 < y_{-1} < x_{-1} < x_0 < 1, A < y_0 < y_{-1} < x_0 < x_{-1} < 1, A < y_{-1} < x_0 < y_0 < x_{-1} < 1,$$
$$A < y_{-1} < y_0 < x_{-1} < x_0 < 1, A < y_{-1} < y_0 < x_0 < x_{-1} < 1, A < y_0 < x_0 < x_{-1} < y_{-1} < 1,$$

Then the following statements are true for the solutions of the (1) :

x_n, y_n is the solution, solution x_n, for $n \geq 1$and solution y_n, for $n \geq 1$;

a) Every position semicycle consists of three terms.
b) Every negative semicycle consists of three terms.
c) Every positive semicycle of length three is followed by a negative semicycle of length three.
d) Every negative semicycle of length three is followed by a positive semicycle of three.

Proof. Similarly, we can obtain the proof of Lemma 2.2 as in the proof of Lemma 2.1.

Theorem 2.1. *Let A=1 and* $(x_n.y_n)$ *be the solution of the (1) for*

$$0 < x_{-1} < x_0 < y_{-1} < y_0 < A, 0 < x_{-1} < x_0 < y_0 < y_{-1} < A, 0 < x_{-1} < y_{-1} < x_0 < y_0 < A,$$
$$0 < x_{-1} < y_{-1} < y_0 < x_0 < A, 0 < x_{-1} < y_0 < y_{-1} < x_0 < A, 0 < x_{-1} < y_0 < x_0 < y_{-1} < A,$$
$$0 < y_0 < x_0 < y_{-1} < x_{-1} < A, 0 < y_0 < x_0 < x_{-1} < y_{-1} < A, 0 < y_0 < y_{-1} < x_0 < x_{-1} < A,$$
$$0 < y_0 < y_{-1} < x_{-1} < x_0 < A, 0 < y_0 < x_{-1} < y_{-1} < x_0 < A, 0 < y_0 < x_{-1} < x_0 < y_{-1} < A,$$
$$0 < y_{-1} < x_0 < y_0 < x_{-1} < A, 0 < y_{-1} < y_0 < x_0 < x_{-1} < A, 0 < y_{-1} < y_0 < x_{-1} < x_0 < A,$$
$$0 < y_{-1} < x_{-1} < y_0 < x_0 < A, 0 < y_{-1} < x_0 < x_{-1} < y_0 < A, 0 < y_{-1} < x_{-1} < x_0 < y_0 < A,$$
$$0 < x_0 < y_{-1} < y_0 < x_{-1} < A, 0 < x_0 < y_0 < x_{-1} < y_{-1} < A, 0 < x_0 < y_0 < y_{-1} < x_{-1} < A,$$
$$0 < x_0 < x_{-1} < y_0 < y_{-1} < A, 0 < x_0 < x_{-1} < y_{-1} < y_0 < A, 0 < x_0 < y_{-1} < x_{-1} < y_0 < A$$

Then every (x_n, y_n) *is periodic with period ten*:

$$x(n) = \left\{ \frac{1}{x_{-1}}; \frac{1}{x_0 y_{-1}}; \frac{1}{y_0}; y_{-1}; y_0; \frac{1}{y_{-1}}; \frac{1}{y_0 x_{-1}}; \frac{1}{x_0}; x_{-1}; x_0; \quad \ldots \right\},$$
$$y(n) = \left\{ \frac{1}{y_{-1}}; \frac{1}{y_0 x_{-1}}; \frac{1}{x_0}; x_{-1}; x_0; \frac{1}{x_{-1}}; \frac{1}{x_0 y_{-1}}; \frac{1}{y_0}; y_{-1}; y_0; \quad \ldots \right\}.$$

Proof.

$$x_1 = \max\left\{ \frac{1}{x_{-1}}, \frac{y_0}{x_{-1}} \right\} = \frac{1}{x_{-1}},$$
$$y_1 = \max\left\{ \frac{1}{y_{-1}}, \frac{x_0}{y_{-1}} \right\} = \frac{1}{y_{-1}},$$
$$x_2 = \max\left\{ \frac{1}{x_0}, \frac{y_1}{x_0} \right\} = \max\left\{ \frac{1}{x_0}, \frac{1}{x_0 y_{-1}} \right\} = \frac{1}{x_0 y_{-1}},$$
$$y_2 = \max\left\{ \frac{1}{y_0}, \frac{x_1}{y_0} \right\} = \max\left\{ \frac{1}{y_0}, \frac{1}{y_0 x_{-1}} \right\} = \frac{1}{y_0 x_{-1}},$$

$$x_3 = \max\left\{\frac{1}{x_1}, \frac{y_2}{x_1}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0},$$

$$x_4 = \max\left\{\frac{1}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{x_0 y_{-1}, y_{-1}\right\} = y_{-1},$$

$$y_4 = \max\left\{\frac{1}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{y_0 x_{-1}, x_{-1}\right\} = x_{-1},$$

$$x_5 = \max\left\{\frac{1}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{y_0, y_0 x_{-1}\right\} = y_0,$$

$$y_5 = \max\left\{\frac{1}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{x_0, x_0 y_{-1}\right\} = x_0,$$

$$x_6 = \max\left\{\frac{1}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{1}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{1}{y_{-1}},$$

$$y_6 = \max\left\{\frac{1}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{1}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{1}{x_{-1}},$$

$$x_7 = \max\left\{\frac{1}{x_5}, \frac{y_6}{x_5}\right\} = \max\left\{\frac{1}{y_0}, \frac{1}{y_0 x_{-1}}\right\} = \frac{1}{y_0 x_{-1}},$$

$$y_7 = \max\left\{\frac{1}{y_5}, \frac{x_6}{y_5}\right\} = \max\left\{\frac{1}{x_0}, \frac{1}{x_0 y_{-1}}\right\} = \frac{1}{x_0 y_{-1}},$$

$$x_8 = \max\left\{\frac{1}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{y_{-1}, \frac{1}{x_0}\right\} = \frac{1}{x_0},$$

$$y_8 = \max\left\{\frac{1}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0},$$

$$x_9 = \max\left\{\frac{1}{x_7}, \frac{y_8}{x_7}\right\} = \max\left\{x_{-1} y_0, x_{-1}\right\} = x_{-1},$$

$$y_9 = \max\left\{\frac{1}{y_7}, \frac{x_8}{y_7}\right\} = \max\left\{x_0 y_{-1}, y_{-1}\right\} = y_{-1},$$

$$x_{10} = \max\left\{\frac{1}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{x_0, x_0 y_{-1}\right\} = x_0,$$

$$y_{10} = \max\left\{\frac{1}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{y_0, x_{-1} y_0\right\} = y_0,$$

$$\vdots$$

If $n \geq 0$, then

$$x(n)=\left\{\frac{1}{x_{-1}};\frac{1}{x_0y_{-1}};\frac{1}{y_0};y_{-1};y_0;\frac{1}{y_{-1}};\frac{1}{y_0x_{-1}};\frac{1}{x_0};x_{-1};x_0;\ ...\right\},$$
$$y(n)=\left\{\frac{1}{y_{-1}};\frac{1}{y_0x_{-1}};\frac{1}{x_0};x_{-1};x_0;\frac{1}{x_{-1}};\frac{1}{x_0y_{-1}};\frac{1}{y_0};y_{-1};y_0;\quad ...\right\}.$$

Theorem 2.2. *Let* $A<1$ *and* (x_n,y_n) *be the solution of the system of difference equations (1) for*

$$A<x_0<y_{-1}<y_0<x_{-1}<1,\ A<x_0<y_0<x_{-1}<y_{-1}<1,\ A<x_0<y_0<y_{-1}<x_{-1}<1,$$
$$A<y_0<x_{-1}<x_0<y_{-1}<1,\ A<y_0<x_0<y_{-1}<x_{-1}<1,\ A<x_{-1}<x_0<y_{-1}<y_0<1,$$
$$A<x_{-1}<x_0<y_0<y_{-1}<1,\ A<x_{-1}<y_0<x_0<y_{-1}<1,\ A<x_0<x_{-1}<y_{-1}<y_0<1,$$
$$A<x_0<x_{-1}<y_0<y_{-1}<1,\ A<x_0<y_{-1}<x_{-1}<y_0<1,\ A<x_{-1}<y_{-1}<x_0<y_0<1,$$
$$A<x_{-1}<y_{-1}<y_0<x_0<1,\ A<x_{-1}<y_0<y_{-1}<x_0<1,\ A<y_{-1}<x_{-1}<x_0<y_0<1,$$
$$A<y_{-1}<x_{-1}<y_0<x_0<1,\ A<y_{-1}<x_0<x_{-1}<y_0<1,\ A<y_0<x_{-1}<y_{-1}<x_0<1,$$
$$A<y_0<y_{-1}<x_{-1}<x_0<1,\ A<y_0<y_{-1}<x_0<x_{-1}<1,\ A<y_{-1}<x_0<y_0<x_{-1}<1,$$
$$A<y_{-1}<y_0<x_{-1}<x_0<1,\ A<y_{-1}<y_0<x_0<x_{-1}<1,\ A<y_0<x_0<x_{-1}<y_{-1}<1$$

Then every (x_n,y_n) is periodic with period six:

$$x(n)=\left\{\frac{y_0}{x_{-1}};\frac{1}{y_{-1}};\frac{1}{y_0};\frac{y_{-1}}{x_0};x_{-1};x_0;\quad ...\right\},$$
$$y(n)=\left\{\frac{x_0}{y_{-1}};\frac{1}{x_{-1}};\frac{1}{x_0};\frac{x_{-1}}{y_0};y_{-1};y_0;\quad ...\right\}.$$

Proof.

$$x_1=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{y_0}{x_{-1}},$$
$$y_1=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{y_0}{x_{-1}},$$
$$x_2=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{\frac{A}{x_0},\frac{1}{y_{-1}}\right\}=\frac{1}{y_{-1}},$$
$$y_2=\max\left\{\frac{A}{y_0},\frac{x_1}{y_0}\right\}=\max\left\{\frac{A}{y_0},\frac{1}{x_{-1}}\right\}=\frac{1}{x_{-1}},$$
$$x_3=\max\left\{\frac{A}{x_1},\frac{y_2}{x_1}\right\}=\max\left\{\frac{Ax_{-1}}{y_0},\frac{1}{y_0}\right\}=\frac{1}{y_0},$$
$$y_3=\max\left\{\frac{1}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{\frac{Ay_{-1}}{x_0},\frac{1}{x_0}\right\}=\frac{1}{x_0},$$

$$x_4 = \max\left\{\frac{1}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{Ay_{-1}, \frac{y_{-1}}{x_0}\right\} = \frac{y_{-1}}{x_0},$$

$$y_4 = \max\left\{\frac{A}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{Ax_{-1}, \frac{x_{-1}}{y_0}\right\} = \frac{x_{-1}}{y_0},$$

$$x_5 = \max\left\{\frac{A}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{Ay_0, x_{-1}\right\} = x_{-1},$$

$$y_5 = \max\left\{\frac{A}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{Ax_0, y_{-1}\right\} = y_{-1},$$

$$x_6 = \max\left\{\frac{A}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{Ax_0}{y_{-1}}, x_0\right\} = x_0,$$

$$y_6 = \max\left\{\frac{A}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{Ay_0}{x_{-1}}, y_0\right\} = y_0,$$

$$\vdots$$

If $n \geq 1$, then:

$$x(n) = \left\{\frac{y_0}{x_{-1}}; \frac{1}{y_{-1}}; \frac{1}{y_0}; \frac{y_{-1}}{x_0}; x_{-1}; x_0; \quad \ldots\right\},$$

$$y(n) = \left\{\frac{x_0}{y_{-1}}; \frac{1}{x_{-1}}; \frac{1}{x_0}; \frac{x_{-1}}{y_0}; y_{-1}; y_0; \quad \ldots\right\}.$$

3. EXAMPLES

Example 3.1. If the initial conditions are selected as follows:

$$x[-1] = 0.3; x[0] = 0.4; y[-1] = 0.8; y[0] = 0.9; A = 1;$$

The following solutions are obtained:

$$x(n) = \begin{cases} 3.3333333, 3.125, 1.111111112, 0.8, 0.8999999999, 1.25, 3.703703703704, 2.5, 0.3, 0.4, 3.3333333335, \\ 3.125, 1.111111112, 0.8, 0.89999999, 1.25, 3.703703703703704, 2.5, 0.3, 0.4, \ldots \end{cases}$$

$$y(n) = \begin{cases} 1.25, 3.703703703703704, 2.5, 0.3, 0.4, 3.3333333335, 3.125, 1.11111111112, 0.8, 0.89999999, 1.25, \\ 3.703703703703704, 2.5, 0.3, 0.4, 3.3333333, 3.125, 1.111111112, 0.8, 0.89999999999, \ldots \end{cases}$$

The graph of the solutions is given below.

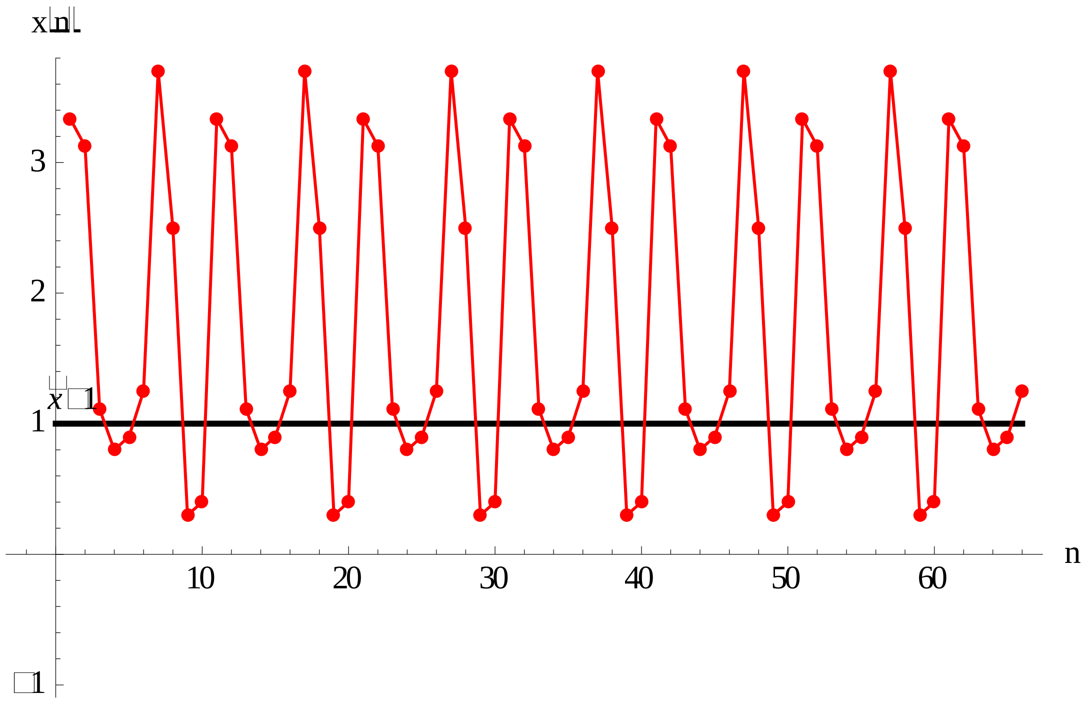

Figure 1. $x(n)$ graph of the solutions

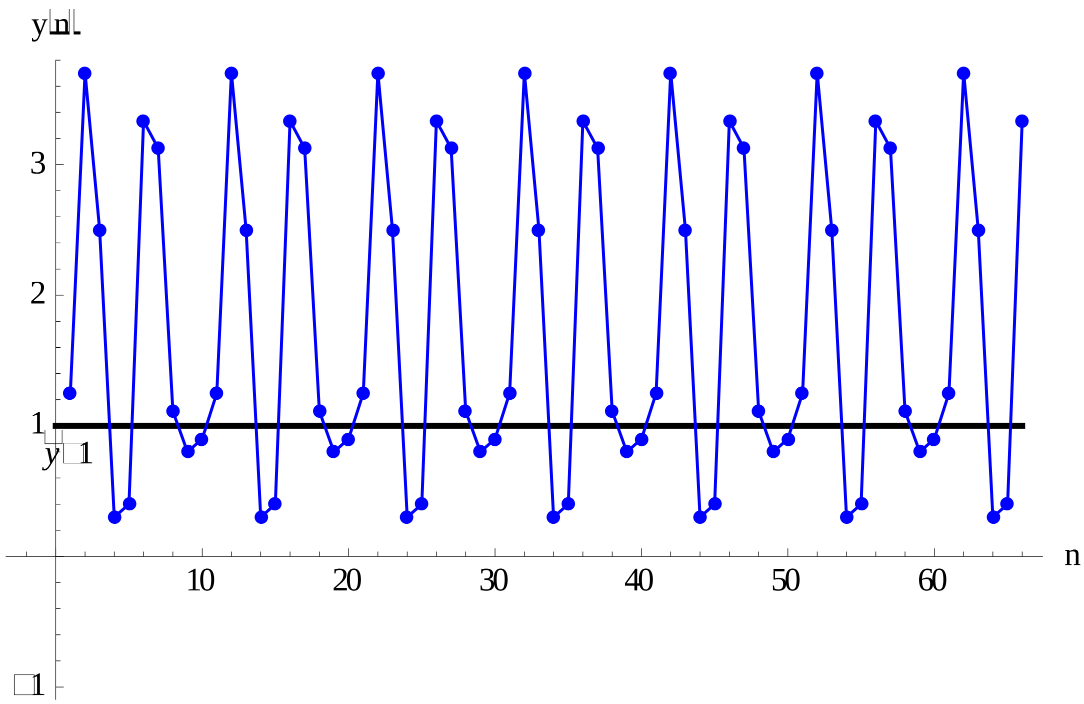

Figure 2. $y(n)$ graph of the solutions

Example 3.2. If the initial conditions are selected as follows:

$$A = 0.1\ x[-1] = 0.9;\ \ x[0] = 0.3;\ y[-1] = 0.5;\ \ y[0] = 0.7$$

The following solutions are obtained:

x(n)={0.777778, 2.,1.42857,1.66667, 0.9, 0.3, 0.777778, 2.,1.42857,1.66667, 0.9, 0.3, ...}
y(n)={0.6,1.11111,3.33333,1.28571,0.5, 0.7, 0.6,1.11111,3.33333,1.28571,0.5, 0.7, ...}
The graph of the solutions is given below.

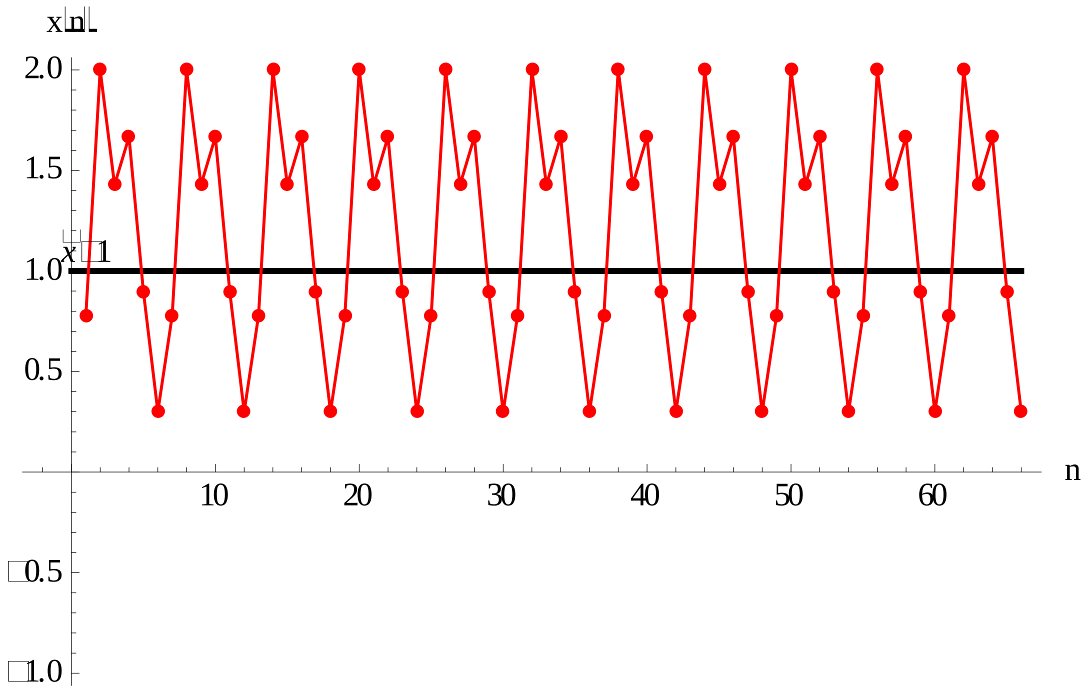

Figure 3. *x*(*n*) graph of the solutions

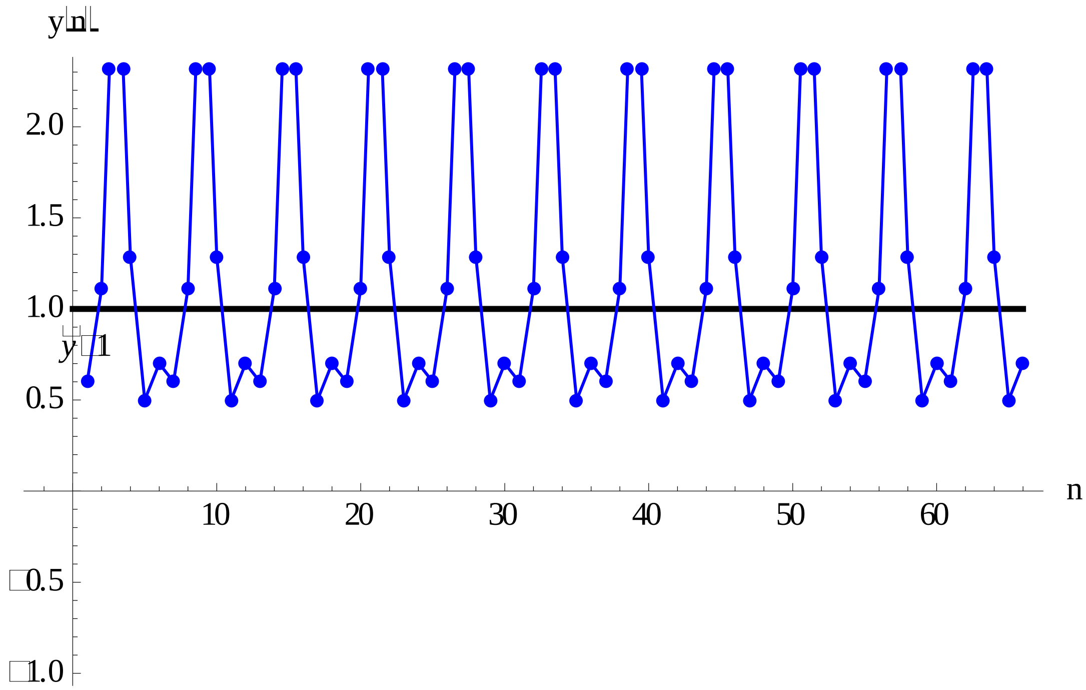

Figure 4. *y*(*n*) graph of the solutions

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