

Research Article

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# On the Regularity and Sampling Behavior of the Median Interindividual Difference

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**Abstract:** The Median Interindividual Difference (*MEID*), defined as the median of all positive pairwise differences within a sample, has been proposed as an absolute measure of variability with both asymptotic and bootstrap procedures for interval estimation; however, its finite-sample behavior requires further evaluation. This Monte Carlo simulation examined whether bootstrap and asymptotic approaches show comparable performance in terms of standard errors and confidence interval widths, and whether the sampling distribution of the *MEID* converges to normality under different distributional conditions. For each of 65 conditions (13 sample sizes  $\times$  5 distributions), 1000 samples were generated. Within each condition, mean Wald-type and bootstrap confidence interval widths and log-transformed standard errors were compared using paired t-tests; equality of variability was assessed with the Pitman–Morgan test, and normality of bootstrap-generated sampling distributions was evaluated using the Shapiro–Francia test. Results did not support overall equivalence between asymptotic and bootstrap inference. Practical equivalence emerged under approximately regular continuous distributions and large samples, whereas discrepancies persisted under strong asymmetry. In discrete distributions, the *MEID* converged to a constant as the sample size increased, resulting in near-zero bootstrap variability and limiting the practical usefulness of certain resampling-based confidence interval procedures. Convergence toward normality occurred only in continuous settings and increased with sample size. These findings delineate the conditions under which the *MEID* yields stable, interpretable inference.

**Keywords:** U-quantiles, asymptotic inference, bootstrap methods, degeneracy, discrete distributions

## 1. Introduction

A prior methodological study introduced five measures of variation based on interindividual differences and developed computational procedures using the R environment [1] for their point and interval estimation [2]. These procedures were accompanied by an R script [1], available in a GitHub repository (*R\_script\_for\_IV\_Measures.git*). Four of these measures assess absolute variation, whereas the fifth assesses relative variation. Together, these five statistics constitute a third approach to describing data variability, alternative to both ranges (i.e., the difference between two extreme measures of position) and the mean or median of the distances of data points from a measure of central tendency. Among the absolute measures, the median interindividual difference (*MEID*)—defined as the median of the absolute pairwise differences  $|x_{(i)} - x_{(j)}|$  computed across all distinct pairs of observations in a sample—is of methodological interest because it provides a robust description of variability based on pairwise distances rather than deviations from a measure of central tendency.

To compute the *MEID*, the  $n$  observations in the random sample are first ordered (Equation 1).

$$\begin{aligned}x &= \{x_i\}_{i=1}^n = \{x_1, x_2, \dots, x_n\} \\x_{(1)} &\leq x_{(2)} \leq \dots \leq x_{(n)}\end{aligned}\tag{1}$$

Next, the absolute pairwise differences are calculated by considering only pairs such that  $j > i$ , thereby avoiding duplicate comparisons and ensuring that each pair is included only once (Equation 2).

$$\overbrace{(|x_{(1)} - x_{(2)}|, |x_{(1)} - x_{(3)}|, |x_{(1)} - x_{(4)}|, \dots, |x_{(n-1)} - x_n|)}^{n(n-1)/2} \quad (2)$$

Finally, the median of these pairwise differences is obtained (Equation 3). As a median-based statistic, the *MEID* is especially attractive for ordinal variables and non-normally distributed data, where resistance to extreme values is desirable.

$$\widehat{MEID} = mdn \left( \left\{ |x_{(i)} - x_{(j)}| \right\}_{i=1}^{n-1} \right)_{j>i}^n \quad (3)$$

From a theoretical perspective, the *MEID* can be formulated as a symmetric second-order U-quantile [3]. Under suitable continuity conditions, asymptotic inference can be derived using Hoeffding's central limit theorem [4], allowing the estimation of standard errors and Wald-type confidence intervals. However, these regularity conditions may fail in discrete or bounded distributions, particularly when ties are frequent or when the support is limited [5].

In a previous study [1], specific behavior was observed under certain distributional conditions. For samples drawn from a positively skewed, bounded discrete distribution (binomial distribution with parameters  $n = 10$  and  $p = 0.25$ ), the *MEID* estimator tended to converge to a constant value as the sample size increased, resulting in near-zero bootstrap resampling variability. Under these conditions, the bias-corrected and accelerated (BCa) procedure could not be computed because the jackknife standard errors became zero, yielding confidence intervals with zero width. In contrast, such behavior was not observed for samples drawn from a continuous distribution, namely PERT(0, 8, 10).

These findings raise concerns regarding the practical reliability of the *MEID* under certain distributional conditions. Unlike the other three statistics of absolute variation, the *MEID* may not adequately satisfy the regularity conditions in finite samples when the underlying distribution is discrete or bounded and ties are frequent. Consequently, the asymptotic and bootstrap confidence intervals previously proposed for the *MEID* may exhibit limited inferential usefulness due to the near absence of sampling variability, which suggests the need for a more detailed evaluation of its sampling properties and interval estimation methods. In contrast, the other absolute variation statistics showed satisfactory regularity and inferential performance.

Accordingly, the present simulation study focuses exclusively on the *MEID*. Its objectives are: (1) to examine the finite-sample performance of asymptotic and bootstrap standard errors and confidence intervals under diverse distributional conditions (continuous vs. discrete, symmetric vs. skewed, bounded vs. unbounded), and (2) to analyze the regularity of its sampling distribution, particularly the convergence of the estimator under discrete distributions, departures from normality, and convergence patterns as the sample size increases. By concentrating on these aspects, the study aims to clarify the practical conditions under which the *MEID* provides stable and interpretable inference.

## 2. Materials and Methods

### 2.1 Asymptotic inference for the *MEID*

For a sample of size  $n$ , the  $n(n-1)/2$  positive pairwise differences represent the interindividual differences. The *MEID* corresponds to the median of this dependent set of differences and can be viewed as a symmetric second-order U-quantile based on the kernel  $h(x_i, x_j) = |x_i - x_j|$ , for  $i \neq j$  and  $i$  and  $j \in \{1, \dots, n\}$  [3-4].

The asymptotic standard error of the *MEID* was computed following the procedure previously developed by Moral [2]. First, the density at the estimated *MEID*,  $\hat{f}_h(\widehat{MEID})$ , was obtained using kernel density estimation with a Gaussian kernel (Equation 4).

$$\hat{f}_h(x = \widehat{MEID}) = \frac{1}{nh\sqrt{2\pi}} \sum_{i=1}^n e^{-\frac{1}{2}\left(\frac{x-x_i}{h}\right)^2} \quad (4)$$

The bandwidth ( $h$ ) was selected according to Silverman's robust rule [5], defined as nine-tenths of the minimum of the sample standard deviation and the ratio of the sample interquartile range to that of the standard normal distribution, divided by the fifth root of the sample size (Equation 5).

$$\begin{aligned}
h &= 0.9 \times \min \left( s_{n-1}(x), \frac{q_x(0.75) - q_x(0.25)}{\Phi^{-1}(0.75) - \Phi^{-1}(0.25)} > 0 \right) \times n^{-\frac{1}{5}} \\
&= \frac{0.9}{\sqrt[5]{n}} \times \min \left( s_{n-1}(x), \frac{R_{IQ}(x)}{1.349} > 0 \right)
\end{aligned} \tag{5}$$

Next, the unstandardized empirical influence-function components of the *MEID* ( $\hat{\psi}_i$ ) were computed as shown in Equation 6, where the symbol  $I$  denotes the indicator function [6].

$$\hat{\psi} = \{\hat{\psi}_i\}_{i=1}^n = \left\{ \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n I(|x_i - x_j| \leq \widehat{MEID}) - 0.5 \right\}_{i=1}^n \tag{6}$$

The quantities  $\hat{\psi}_i$  correspond to the unscaled first-order projection terms associated with the U-quantile representation of the *MEID*. The density normalization required for the full influence function is introduced subsequently in the asymptotic variance expression [7].

For a symmetric U-quantile estimator of order  $m$  with target quantile  $\theta$ , the asymptotic variance ( $\sigma_\theta^2$ ) is determined by the first-order Hoeffding projection and takes the general form presented in Equation 7, where  $\zeta_1$  denotes the variance of the first-order projection term and  $f(\theta)$  is the density evaluated at the target quantile  $\theta$  [3–4, 8–9].

$$\sigma_\theta^2 = \frac{m^2(\zeta_1)}{f(\theta)^2} \tag{7}$$

Because the *MEID* is based on a second-order symmetric kernel ( $m = 2$ ), the asymptotic variance incorporates the multiplicative factor  $m^2 = 4$ . Empirically,  $\zeta_1$  was estimated by the sample variance of the empirical projection components  $\hat{\psi}_i$ , yielding the asymptotic standard error shown in Equation 8.

$$ase(\widehat{MEID}) = 2 \times \sqrt{\frac{s^2(\hat{\psi})}{n(\hat{f}_n(\widehat{MEID}))^2}} = \sqrt{\frac{4 \times s^2(\hat{\psi})}{n(\hat{f}_n(\widehat{MEID}))^2}} \tag{8}$$

Based on this standard error, a Wald-type confidence interval was constructed as shown in Equation 9:

$$P\left(\widehat{MEID} - z_{1-\frac{\alpha}{2}} ase(\widehat{MEID}) \leq MEID \leq \widehat{MEID} + z_{1-\frac{\alpha}{2}} ase(\widehat{MEID})\right) = 1 - \alpha \tag{9}$$

Under standard regularity conditions, this estimator is asymptotically justified by Hoeffding's central limit theorem for second-order U-statistics, as well as by related U-quantile theory [3–4]. However, these regularity conditions may fail in discrete or bounded distributions, particularly when ties are frequent or when the support is limited [10].

It should be noted that the asymptotic variance accounts for the dependency induced by the pairwise kernel structure through the first-order projection of the associated U-process [8–9]. Although the adjustment associated with the second-order U-quantile structure was implemented in the accompanying R code of the original formulation [2], its role was not explicitly detailed in the analytical derivation. The present formulation makes this dependency correction explicit to ensure consistency between the theoretical expression and the computational implementation.

## 2.2. Monte Carlo simulation design

One thousand random samples were generated for each of 13 sample sizes (30, 40, 50 to 500 in increments of 50, and 1000). The selected sample sizes were intended to examine both finite-sample behavior and asymptotic convergence. Small and moderate sample sizes (30–100) were included because inferential irregularities are typically more pronounced in this range, whereas larger sample sizes (150–1000) allowed the assessment of the extent to which the sampling distribution of the *MEID* approaches asymptotic normality as the sample size increases.

Samples were generated from five probability distributions selected to represent a broad range of distributional conditions relevant to the theoretical regularity of the *MEID*. Three continuous distributions were considered: (a) a symmetric mesokurtic distribution (normal distribution with parameters  $\mu = 0$  and  $\sigma = 1$ ), included as a benchmark reference distribution satisfying standard regularity assumptions; (b) a symmetric platykurtic bounded distribution (arcsine distribution with support  $(0, 1)$ ), included to examine the effect of bounded support under continuity; and (c) a positively

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skewed, leptokurtic distribution (gamma distribution with parameters  $\alpha = 10$  and  $\beta = 1$ ), selected to evaluate the effect of skewness and heavy tails on interval estimation and sampling variability.

In addition, two discrete distributions were examined because discreteness and ties are theoretically expected to affect the regularity conditions underlying asymptotic and bootstrap inference for the *MEID*. The binomial distribution with parameters  $n = 10$  and  $p = 0.25$  was specifically included because a previous methodological study [1] detected degeneracy in the bootstrap sampling distribution of the *MEID* under this condition. The Poisson distribution with parameter  $\lambda = 5$  was incorporated as a second discrete distribution with unbounded support and moderate asymmetry to determine whether the observed irregularities generalize beyond the bounded binomial case.

The selected distributions therefore allowed the simultaneous examination of continuity versus discreteness, bounded versus unbounded support, and symmetric versus skewed shapes, which are the main theoretical factors expected to influence the regularity and inferential stability of the *MEID*.

In total, 65,000 random samples were generated (13 sample sizes  $\times$  5 distributions  $\times$  1000 replications). To ensure reproducibility, the pseudo-random number generator in R was initialized with a fixed seed (123) at the beginning of each simulation condition (i.e., each combination of distribution and sample size). The seed was not reset during the Monte Carlo replications or bootstrap resampling steps; therefore, all samples and resamples within each condition were generated sequentially from the same random-number stream.

For each condition and each of the 1000 Monte Carlo samples, the bootstrap standard error and the width of the 95% bootstrap confidence interval for the *MEID* were computed using the bias-corrected and accelerated (BCa) method with 1000 bootstrap resamples [11]. In addition, the asymptotic standard error and the width of the corresponding 95% Wald-type confidence interval were calculated.

Separate scripts were used to compute standard errors and confidence intervals using the BCa, percentile, and jackknife resampling methods. Although the same initial seed value (123) was specified in all scripts, differences in intermediate computational processes resulted in different sequences of resampled datasets across methods. Consequently, minor discrepancies appeared in the average asymptotic standard errors and Wald-type confidence interval widths reported in different tables.

### 2.3. Statistical analysis

To address the first objective, the mean widths of the 95% bootstrap and Wald-type confidence intervals were compared across the 65 conditions using paired t tests. Effect size was estimated using Cohen's paired-samples  $d$ , defined as  $d = t / \sqrt{n}$ , where  $t$  denotes the test statistic and  $n$  the number of paired observations. Values of  $d < 0.20$  were interpreted as trivial, 0.20–0.49 as small, 0.50–0.79 as medium, and  $\geq 0.80$  as large [12].

Variability in the standard errors obtained by the two methods was compared using the Pitman–Morgan test for equality of variances in paired samples [13–14], as implemented in R [15]. The null hypothesis was  $H_0: \sigma^2(SE_{asymp}) = \sigma^2(SE_{boot})$ . Rejection of  $H_0$  indicates differential dispersion between the asymptotic and bootstrap estimators across Monte Carlo replications. Thus, this test evaluates whether one estimation method produces more variable (i.e., less stable) standard errors than the other when both are computed from the same simulated samples. The Pitman–Morgan procedure was selected because the observations are intrinsically paired and, given the large number of Monte Carlo replications, the test is expected to perform adequately even under moderate deviations from normality. In addition, log-transformed standard errors were compared using a paired Student's t test ( $H_0: E[\ln(SE_{asymp})] = E[\ln(SE_{boot})]$ , equivalently  $H_0: E[\ln(SE_{asymp} / SE_{boot})] = 0$ , i.e., a geometric mean ratio equal to one). Whereas the Pitman–Morgan test evaluates differences in dispersion, this second analysis assesses systematic differences in the magnitude of the standard errors produced by the two methods [16]. Rejection of the null hypothesis indicates that one method tends to yield consistently larger or smaller standard errors than the other across Monte Carlo replications. The logarithmic transformation was applied because standard errors are strictly positive and may exhibit right skewness, particularly for bootstrap-based estimators; the transformation stabilizes the variance and reduces skewness in the paired observations prior to comparison. These two analyses provide complementary information on

the relative performance of the methods. Effect sizes were computed as Cohen’s *d* for paired samples, with both tests based on a studentized difference.

For the discrete distributions (binomial and Poisson), jackknife standard errors, jackknife type-T confidence interval widths [17], and bootstrap percentile confidence interval widths were additionally computed across the 13 sample sizes due to convergence difficulties in obtaining BCa intervals for larger samples. The same comparisons (asymptotic vs. jackknife) were conducted for both confidence interval widths and standard errors using paired *t* tests and the Pitman–Morgan test [13–14].

To address the second objective, the goodness of fit to normality of the bootstrap sampling distributions of the *MEID* was assessed using the Shapiro–Francia *W'* test [18–19]. For each of the 65 conditions, the proportion of Monte Carlo samples in which the null hypothesis of normality was not rejected at  $\alpha = 0.05$  was calculated.

For data analysis, several R packages were used: the R package *boot* [20] to generate bootstrap samples; *mc2d* [21] to draw samples from the PERT distribution; *moments* [22] to perform the D’Agostino skewness test, the Anscombe–Glynn kurtosis test, and the D’Agostino–Belanger–D’Agostino normality test; *nortest* [23] to conduct the Shapiro–Francia normality test; and *PairedData* [15] to compute the Pitman–Morgan test for equal variances in paired data. Other tests, such as Student’s *t*-test, were implemented using base R functions [1].

### 3. Results

#### 3.1 Comparisons between asymptotic and bootstrap standard errors and Wald-type and BCa confidence interval widths

The average point estimates of the *MEID* were highly stable across sample sizes for the three continuous distributions. Mean estimates across conditions were 0.9573 for the standard normal distribution, 0.3607 for the arcsine distribution, and 2.9320 for the gamma distribution, without a clear monotonic pattern as the sample size increased (Table 1).

For the two discrete distributions, the estimates showed a decreasing monotonic pattern across sample sizes. In the binomial distribution, the estimates equaled 1 for sample sizes of 250 or more observations, whereas in the Poisson distribution they equaled 2 for sample sizes of 200 or more observations. Overall variability across sample sizes remained small (Table 1).

**Table 1.** Average Point Estimates of *MEID*.

Sample size	Normal N(0, 1)	Arcsine (0, 1)	Gamma ( $\alpha=10, \beta=0.5$ )	Binomial ( $n=10, p=1/4$ )	Poisson ( $\lambda = 5$ )
30	0.9645	0.3620	2.9426	1.1650	2.11
40	0.9625	0.3608	2.9657	1.13	2.0810
50	0.9605	0.3629	2.9431	1.0860	2.0560
100	0.9611	0.3611	2.9390	1.0185	2.0075
150	0.9558	0.3607	2.9222	1.002	2.002
200	0.9556	0.3607	2.9344	1.004	2
250	0.9565	0.3606	2.9183	1	2
300	0.9555	0.3602	2.9138	1	2
350	0.9515	0.3601	2.9287	1	2
400	0.9563	0.3601	2.9279	1	2
450	0.9564	0.3599	2.9269	1	2
500	0.9539	0.3601	2.9275	1	2
1000	0.9552	0.3600	2.9253	1	2
Average	0.9573	0.3607	2.9320	1.0312	2.0197

Elaborated by the author.

For the three continuous distributions, the widths of the Wald-type intervals were consistently smaller than those obtained with the BCa method, although the differences decreased as the sample size increased. In the normal distribution, the associated effect sizes were initially small and became trivial for sample sizes of 400 or more observations; for  $n = 1000$ , the difference was no longer statistically significant. A similar pattern was observed for the gamma distribution, although the difference

remained statistically significant at  $n = 1000$  despite a trivial effect size. For the arcsine distribution, effect sizes ranged from medium to small across sample sizes (Table 2).

**Table 2.** Paired comparison of Wald-Type and BCa Confidence Interval Widths for the *MEID* in Samples Drawn from Continuous Probability Distributions.

<i>n</i>	CI_width		Paired t-test			CI_width		Paired t-test		
	W-type	Bca	<i>t</i>	<i>p</i>	<i>d</i>	W-type	Bca	<i>t</i>	<i>p</i>	<i>d</i>
Normal (0, 1)						Arcsine $\equiv$ Beta( $\alpha = 0.5, \beta = 0.5$ )				
30	0.559	0.575	-10.28	<.001	-0.33	0.152	0.178	-19.05	<.001	-0.60
40	0.476	0.491	-13.86	<.001	-0.44	0.119	0.130	-14.53	<.001	-0.46
50	0.421	0.430	-11.97	<.001	-0.38	0.101	0.108	-14.26	<.001	-0.45
100	0.292	0.296	-11.14	<.001	-0.35	0.062	0.066	-19.55	<.001	-0.62
150	0.238	0.240	-9.14	<.001	-0.29	0.049	0.051	-17.17	<.001	-0.54
200	0.205	0.206	-5.92	<.001	-0.19	0.041	0.043	-18.09	<.001	-0.57
250	0.182	0.183	-8.30	<.001	-0.26	0.036	0.037	-16.62	<.001	-0.53
300	0.165	0.166	-7.00	<.001	-0.22	0.033	0.033	-15.72	<.001	-0.50
350	0.153	0.154	-7.04	<.001	-0.22	0.030	0.031	-16.25	<.001	-0.51
400	0.143	0.144	-5.77	<.001	-0.18	0.028	0.029	-16.66	<.001	-0.53
450	0.135	0.136	-5.42	<.001	-0.17	0.026	0.027	-13.80	<.001	-0.44
500	0.128	0.129	-5.40	<.001	-0.17	0.025	0.025	-13.70	<.001	-0.43
1000	0.090	0.091	-1.89	0.060	-0.06	0.017	0.017	-11.57	<.001	-0.37
<b>Gamma(<math>\alpha = 10, \beta = 1</math>)</b>										
30	1.760	1.833	-13.15	<.001	-0.42					
40	1.509	1.554	-11.72	<.001	-0.37					
50	1.340	1.375	-13.18	<.001	-0.42					
100	0.920	0.934	-11.89	<.001	-0.38					
150	0.746	0.755	-9.87	<.001	-0.31					
200	0.640	0.646	-8.81	<.001	-0.28					
250	0.572	0.577	-7.17	<.001	-0.23					
300	0.522	0.527	-8.99	<.001	-0.28					
350	0.484	0.488	-6.87	<.001	-0.22					
400	0.449	0.452	-6.45	<.001	-0.20					
450	0.426	0.429	-5.55	<.001	-0.18					
500	0.404	0.406	-5.89	<.001	-0.19					
1000	0.284	0.285	-3.10	0.002	-0.10					

Note. *CI\_width* = mean width of the Wald-type (W-type) and bias-corrected and accelerated (BCa) confidence intervals, *t* = t-test statistic, *p* = two-tailed *p* value, and *d* = Cohen's *d* effect size statistic. Degrees of freedom = 999. *MEID* = Median Interindividual Difference. Elaborated by the author.

For the binomial distribution, the BCa method produced increasingly wide confidence intervals for all sample sizes except  $n = 30$ . This increase was associated with progressively larger upper limits, which became infinite for sample sizes of 250 or more observations. Consequently, the mean interval width was unavailable (NA) under these conditions (Table 3).

Percentile intervals were wider than Wald-type intervals, although the differences decreased across sample sizes and became nonsignificant for sample sizes between 400 and 500. However, as the sample size increased, the *MEID* point estimate increasingly converged to a constant value, resulting in near-zero bootstrap variability. Consequently, the number of valid bootstrap interval estimates progressively decreased, and for  $n = 1000$ , no valid percentile interval estimates were obtained (Table 3).

A similar pattern was observed for the Poisson distribution. BCa interval widths increased across sample sizes, and for sample sizes of 250 or more observations, no valid BCa interval estimates were obtained. Except for  $n = 30$ , percentile intervals were narrower than Wald-type intervals, with nonsignificant differences for sample sizes of 40 and 50. Nevertheless, the convergence of the *MEID* estimate to a constant value with increasing sample size produced near-zero bootstrap variability, progressively reducing the number of valid percentile estimates, and no valid estimates were obtained for  $n = 1000$  (Table 3).

Very large *t* and *d* values were associated with near-zero variance in the paired differences.

**Table 3.** Paired Comparisons of Wald-type and BCa or Bootstrap-Percentile Confidence Interval Widths for the *MEID* in Samples Drawn from Discrete Probability Distributions.

<i>n</i>	CI_width		Paired t-test				CI_width		Paired t-test			
	Wald	Bca	<i>t</i>	<i>df</i>	<i>p</i>	<i>d</i>	Wald	Perc	<i>t</i>	<i>df</i>	<i>p</i>	<i>d</i>
Binomial( <i>n</i> = 10, <i>p</i> = 0.25)												
30	0.471	0.531	6.79	321	<.001	0.38	0.473	0.791	-23.64	976	<.001	-0.76
40	0.344	0.808	-6.78	160	<.001	-0.54	0.357	0.750	-23.97	950	<.001	-0.78
50	0.264	0.925	-14.64	92	<.001	-1.53	0.273	0.733	-32.57	945	<.001	-1.06
100	0.124	1	-8.73	18	<.001	-2.06	0.126	0.536	-24.20	848	<.001	-0.83
150	0.083	1	-64.86	1	0.010	-64.86	0.083	0.427	-18.76	727	<.001	-0.70
200	0.063	1	-387.2	3	<.001	-223.6	0.063	0.280	-12.10	625	<.001	-0.48
250	0.051	NA	NA	NA	NA	NA	0.051	0.192	-7.78	479	<.001	-0.36
300	0.044	NA	NA	NA	NA	NA	0.044	0.128	-4.92	382	<.001	-0.25
350	0.038	NA	NA	NA	NA	NA	0.038	0.098	-3.42	284	0.001	-0.20
400	0.034	NA	NA	NA	NA	NA	0.034	0.046	-0.85	193	0.399	-0.06
450	0.030	NA	NA	NA	NA	NA	0.030	0.027	0.29	150	0.775	0.02
500	0.027	NA	NA	NA	NA	NA	0.028	0.049	-0.98	102	0.329	-0.10
1000	0.015	NA	NA	NA	NA	NA	0.015	0	NA	NA	NA	NA
Poisson( $\lambda$ = 5)												
30	1.267	0.707	35.97	974	<.001	1.15	1.273	1.294	-1.49	999	0.135	-0.05
40	1.056	0.510	33.43	929	<.001	1.10	1.047	1.043	0.36	999	0.721	0.01
50	0.890	0.410	27.61	822	<.001	0.96	0.896	0.876	1.69	999	0.091	0.05
100	0.499	0.144	16.68	332	<.001	0.92	0.499	0.388	8.07	931	<.001	0.26
150	0.354	0.044	13.91	90	<.001	1.47	0.354	0.273	6.42	677	<.001	0.25
200	0.272	0	44.59	20	<.001	9.97	0.274	0.210	4.69	443	<.001	0.22
250	0.223	0	42.06	4	<.001	21.03	0.223	0.088	9.85	306	<.001	0.56
300	0.191	0	6264.3	1	<.001	6264.3	0.190	0.056	9.51	196	<.001	0.68
350	0.166	NA	NA	NA	NA	NA	0.166	0.025	11.50	121	<.001	1.05
400	0.148	NA	NA	NA	NA	NA	0.148	0.017	8.53	59	<.001	1.11
450	0.133	NA	NA	NA	NA	NA	0.132	0	4.83	36	<.001	0.80
500	0.121	NA	NA	NA	NA	NA	0.122	0.042	2.26	23	0.033	0.47
1000	0.067	NA	NA	NA	NA	NA	0.066	NA	NA	NA	NA	NA

Note. CI\_width = mean width of the Wald-type (Wald) and bias-corrected and accelerated (BCa) or bootstrap percentile (Perc) confidence intervals, *t* = t-test statistic (NA = Not Available), *p* = two-tailed *p* value, *df* = degrees of freedom, and *d* = Cohen's *d* effect size statistic. *MEID* = Median Interindividual Difference. Elaborated by the author.

Jackknife t-type intervals were also examined [17]. For the binomial distribution, all jackknife confidence intervals were valid, unlike those obtained with the bootstrap methods. For small sample sizes (*n* = 30–50), the jackknife intervals were significantly wider than the Wald-type intervals, although the associated effect sizes were small. For larger sample sizes, the jackknife intervals became narrower, and their mean widths were equal to zero for sample sizes of 350 or more observations (Table 4).

For larger samples, the jackknife intervals were significantly narrower, and the interval widths converged to zero for sample sizes of 350 or more observations, consistent with the convergence of the *MEID* estimate to a constant value and the resulting near-zero variability. Very large *t* and *d* values were associated with near-zero variance in the paired differences (Table 4).

**Table 4.** Paired Comparisons of Wald-Type and Jackknife-t-Type Confidence Interval Widths for the *MEID* in Samples Drawn from Discrete Probability Distributions.

<i>n</i>	Binomial( <i>n</i> = 10, <i>p</i> = 0.25)					Poisson( $\lambda$ = 5)				
	CI_width		Paired t-test			CI_width		Paired t-test		
	W-type	Jack	<i>t</i>	<i>p</i>	<i>d</i>	W-type	Jack	<i>t</i>	<i>p</i>	<i>d</i>
30	0.4683	1.3592	-8.89	<.001	-0.28	1.2687	1.3265	-0.60	0.549	-0.02
40	0.3514	0.9813	-6.65	<.001	-0.21	1.0388	0.8725	1.85	0.065	0.06
50	0.2659	0.8361	-6.17	<.001	-0.20	0.8820	0.6804	2.38	0.018	0.08
100	0.1248	0.3552	-3.13	0.002	-0.10	0.5005	0.1061	9.43	<.001	0.30
150	0.0825	0.0643	0.52	0.604	0.02	0.3519	0	194.31	<.001	6.15

200	0.0634	0.0475	0.47	0.636	0.01	0.2719	0.0274	8.95	<.001	0.28
250	0.0513	0	442.14	<.001	13.99	0.2228	0	213.89	<.001	6.77
300	0.0436	0.0339	0.29	0.774	0.01	0.1911	0.0055	33.67	<.001	1.07
350	0.0378	0	679.60	<.001	21.50	0.1661	0	240.03	<.001	7.59
400	0.0336	0	708.36	<.001	22.41	0.1477	0	247.84	<.001	7.84
450	0.0302	0	763.24	<.001	24.15	0.1333	0	257.26	<.001	8.14
500	0.0276	0	790.83	<.001	25.02	0.1217	0	272.72	<.001	8.63
1000	0.0148	0	1100.23	<.001	34.81	0.0665	0	357.34	<.001	11.31

Note. *CI\_width* = mean width of the Wald-type (W-type) and jackknife t-type confidence intervals (Jack), *t* = t-test statistic, *p* = two-tailed *p* value, and *d* = Cohen's *d* effect size statistic. Degrees of freedom = 999. *MEID* = Median Interindividual Difference. Elaborated by the author.

For the normal and gamma distributions, asymptotic and bootstrap standard errors were highly similar across sample sizes. Most mean differences were either nonsignificant or associated with trivial effect sizes. The variability of the asymptotic standard errors was generally greater than that of the bootstrap estimates for medium and large sample sizes, although the associated effect sizes were small. For the arcsine distribution, bootstrap standard errors consistently exhibited larger means and greater variability than asymptotic standard errors across all sample sizes. Effect sizes for the mean differences decreased with increasing sample size, whereas those associated with variability increased from trivial or small to moderate values (Table 5).

**Table 5.** Comparison of Asymptotic and Bootstrap Standard Errors of *MEID* for Continuous Distributions.

<i>n</i>	<i>SE</i>		Paired t-test for mean_log_ratio					Pitman–Morgan test			
	Asymp	Boot	m_ln	<i>t</i>	<i>df</i>	<i>p</i>	<i>d</i>	<i>rSD</i>	<i>t</i>	<i>p</i>	<i>d</i>
<b>Normal (0, 1)</b>											
30	0.143	0.143	-0.0029	-1.57	999	0.116	-0.05	0.10	3.17	0.002	0.10
40	0.121	0.122	-0.0038	-2.51	999	0.012	-0.08	0.06	1.76	0.078	0.06
50	0.107	0.107	0.0020	1.59	999	0.113	0.05	0.07	2.17	0.031	0.07
100	0.074	0.074	0.0011	1.15	999	0.251	0.04	-0.07	-2.34	0.019	-0.07
150	0.061	0.061	0.0004	0.42	999	0.674	0.01	-0.12	-3.82	<.001	-0.12
200	0.052	0.052	0.0023	2.90	999	0.004	0.09	-0.16	-5.22	<.001	-0.17
250	0.046	0.046	0.0004	0.46	999	0.647	0.01	-0.12	-3.96	<.001	-0.13
300	0.042	0.042	0.0025	3.31	999	0.001	0.10	-0.18	-5.85	<.001	-0.19
350	0.039	0.039	0.0014	1.94	999	0.053	0.06	-0.20	-6.47	<.001	-0.20
400	0.037	0.037	0.0017	2.24	999	0.025	0.07	-0.21	-6.71	<.001	-0.21
450	0.035	0.035	0.0012	1.58	999	0.114	0.05	-0.29	-9.62	<.001	-0.30
500	0.033	0.033	0.0005	0.63	999	0.528	0.02	-0.24	-7.67	<.001	-0.24
1000	0.023	0.023	0.0027	3.68	999	<.001	0.12	-0.29	-9.63	<.001	-0.30
<b>Arcsine <math>\equiv</math> Beta(<math>\alpha = 0.5, \beta = 0.5</math>)</b>											
30	0.039	0.046	-0.1680	-28.37	999	<.001	-0.90	-0.28	-9.15	<.001	-0.29
40	0.030	0.034	-0.1283	-26.75	999	<.001	-0.85	-0.12	-3.67	<.001	-0.12
50	0.026	0.029	-0.1164	-29.16	999	<.001	-0.92	-0.15	-4.92	<.001	-0.16
100	0.016	0.017	-0.0691	-28.37	999	<.001	-0.90	-0.16	-5.16	<.001	-0.16
150	0.012	0.013	-0.0403	-23.08	999	<.001	-0.73	-0.28	-9.04	<.001	-0.29
200	0.011	0.011	-0.0403	-23.08	999	<.001	-0.73	-0.28	-9.19	<.001	-0.29
250	0.009	0.010	-0.0305	-19.15	999	<.001	-0.61	-0.24	-7.70	<.001	-0.24
300	0.008	0.009	-0.0236	-16.33	999	<.001	-0.52	-0.34	-11.39	<.001	-0.36
350	0.008	0.008	-0.0213	-15.08	999	<.001	-0.48	-0.39	-13.31	<.001	-0.42
400	0.007	0.007	-0.0208	-15.45	999	<.001	-0.49	-0.36	-12.31	<.001	-0.39
450	0.007	0.007	-0.0165	-13.34	999	<.001	-0.42	-0.39	-13.30	<.001	-0.42
500	0.006	0.006	-0.0157	-12.33	999	<.001	-0.39	-0.42	-14.76	<.001	-0.47
1000	0.004	0.004	-0.0080	-7.93	999	<.001	-0.25	-0.49	-17.81	<.001	-0.56
<b>Gamma(<math>\alpha = 10, \beta = 1</math>)</b>											
30	0.449	0.451	-0.0057	-2.94	999	0.003	-0.09	0.10	3.29	<.001	0.10
40	0.385	0.385	-0.0014	-0.98	999	0.329	-0.03	0.04	1.26	0.210	0.04
50	0.342	0.342	0.0005	0.41	999	0.680	0.01	-0.04	-1.28	0.200	-0.04
100	0.235	0.234	0.0024	2.48	999	0.013	0.08	-0.14	-4.32	<.001	-0.14
150	0.190	0.190	0.0009	1.05	999	0.296	0.03	-0.10	-3.28	0.001	-0.10

200	0.163	0.163	0.0003	0.34	999	0.734	0.01	-0.18	-5.63	<.001	-0.18
250	0.146	0.146	0.0024	2.97	999	0.003	0.09	-0.13	-4.27	<.001	-0.14
300	0.133	0.133	0.0012	1.63	999	0.103	0.05	-0.18	-5.86	<.001	-0.19
350	0.124	0.124	0.0006	0.77	999	0.440	0.02	-0.22	-7.16	<.001	-0.23
400	0.115	0.115	0.0008	1.04	999	0.299	0.03	-0.21	-6.69	<.001	-0.21
450	0.109	0.109	0.0021	2.76	999	0.006	0.09	-0.27	-8.97	<.001	-0.28
500	0.103	0.103	0.0018	2.41	999	0.016	0.08	-0.17	-5.56	<.001	-0.18
1000	0.073	0.073	0.0012	1.77	999	0.078	0.06	-0.29	-9.44	<.001	-0.30

Note.  $n$  = sample size. Paired t test:  $SE$  = asymptotic (Asymp) and bootstrap (Boot) standard errors,  $m\_ln$  = mean log ratio of asymptotic (numerator) to bootstrap (denominator) standard errors;  $t$  = test statistic,  $p$  = two-tailed  $p$  value under the t distribution with degrees of freedom ( $df$  = 999 with continuous distributions and  $\leq 999$  on discrete distributions) for  $H_0: E[\ln(SE_{Asymp} / SE_{Boot})] = 0$ , and  $d$  = Cohen's  $d$  effect size. Pitman-Morgan test:  $r_{SD}$  = Pearson correlation between the sum and the difference of paired data,  $t$  = test statistic,  $p$  = two-tailed  $p$  value under the t distribution with 998 degrees of freedom for  $H_0: \sigma^2(SE_{Asymp}) = \sigma^2(SE_{Boot})$ , and  $d$  = Cohen's  $d$  effect size. Elaborated by the author.

In the discrete distributions, different patterns were observed. For the binomial distribution, both the means and the variability of the bootstrap standard errors were consistently greater than those of the asymptotic estimates, with effect sizes increasing as the sample size increased. However, as the sample size became very large, the *MEID* estimate converged to a constant value, and the bootstrap resamples consequently exhibited near-zero variability. For  $n = 1000$ , the bootstrap standard error was therefore equal to zero, producing undefined logarithmic differences and extremely large test statistics due to the absence of variance in the paired comparisons (Table 6).

For the Poisson distribution, bootstrap standard errors were larger than asymptotic ones in small samples but smaller in medium and large samples. However, the variability of the bootstrap standard errors remained greater across all sample sizes, with very large effect sizes in intermediate and large samples. As with the binomial distribution, convergence of the *MEID* estimate to a constant value in very large samples led to near-zero bootstrap variability. Consequently, for  $n = 1000$ , the bootstrap standard error was equal to zero, yielding extremely large test statistics associated with the near-zero variance of the paired differences (Table 6).

**Table 6.** Comparison of Asymptotic and Bootstrap Standard Errors of *MEID* for Discrete Distributions.

$n$	$SE$		Paired t-test for mean_log_ratio				Pitman-Morgan test				
	Asymp	Boot	$m\_ln$	$t$	$df$	$p$	$d$	$r_{SD}$	$t$	$p$	$d$
<b>Binomial(<math>n = 10, p = 0.25</math>)</b>											
30	0.120	0.303	-0.8422	-38.42	963	<.001	-1.24	-0.81	-43.42	<.001	-1.37
40	0.088	0.282	-1.0699	-48.28	966	<.001	-1.55	-0.90	-64.27	<.001	-2.03
50	0.067	0.251	-1.2077	-50.40	934	<.001	-1.65	-0.95	-94.48	<.001	-2.99
100	0.032	0.169	-1.5722	-56.11	837	<.001	-1.94	-0.98	-163.54	<.001	-5.18
150	0.021	0.117	-1.7109	-57.65	734	<.001	-2.13	-1	-716.59	<.001	-22.68
200	0.016	0.077	-1.7515	-56.09	610	<.001	-2.27	-1	-813.48	<.001	-25.75
250	0.013	0.053	-1.8295	-54.71	486	<.001	-2.48	-1	-1826.18	<.001	-57.81
300	0.011	0.033	-1.8043	-53.31	380	<.001	-2.73	-1	-1697.86	<.001	-53.74
350	0.010	0.021	-1.8078	-45.69	270	<.001	-2.78	-1	-1340.03	<.001	-42.42
400	0.009	0.015	-1.9051	-44.77	203	<.001	-3.14	-1	-1470.56	<.001	-46.55
450	0.008	0.009	-1.8609	-38.79	146	<.001	-3.21	-1	-1529.98	<.001	-48.43
500	0.007	0.007	-2.042	-39.36	113	<.001	-3.70	-1	-1456.10	<.001	-46.09
1000	0.004	0	NA	NA	NA	NA	NA	-0.98	-145.38	<.001	-4.60
<b>Poisson(<math>\lambda = 5</math>)</b>											
30	0.323	0.407	-0.2213	-29.92	999	<.001	-0.95	-0.36	-12.15	<.001	-0.38
40	0.269	0.345	-0.2118	-22.47	999	<.001	-0.71	-0.66	-27.71	<.001	-0.88
50	0.227	0.299	-0.2043	-17.63	999	<.001	-0.56	-0.80	-41.61	<.001	-1.32
100	0.127	0.146	0.0687	3.13	932	0.002	0.10	-0.95	-95.71	<.001	-3.03
150	0.090	0.082	0.0788	2.85	687	0.005	0.11	-0.97	-120.28	<.001	-3.81
200	0.070	0.043	0.0360	1.12	443	0.263	0.05	-0.96	-115.98	<.001	-3.67
250	0.057	0.025	-0.0861	-2.31	293	0.021	-0.14	-0.96	-105.03	<.001	-3.32
300	0.049	0.013	-0.0229	-0.60	204	0.549	-0.04	-0.93	-79.16	<.001	-2.51
350	0.042	0.007	-0.0645	-1.20	108	0.232	-0.12	-0.91	-68.73	<.001	-2.18
400	0.038	0.004	-0.1191	-2.32	78	0.023	-0.26	-0.88	-57.66	<.001	-1.83
450	0.034	0.002	-0.1433	-2.39	44	0.021	-0.36	-0.74	-35.18	<.001	-1.11

500	0.031	0.001	-0.1408	-2.36	28	0.026	-0.45	-0.63	-25.86	<.001	-0.82
1000	0.017	0	NA	NA	NA	NA	NA	1	2.1×10 <sup>9</sup>	<.001	>999

Note.  $n$  = sample size;  $SE$  = asymptotic (Asymp) and bootstrap (Boot) standard errors;  $m\_ln$  = mean log ratio of asymptotic to bootstrap standard errors;  $r_{SD}$  = Pearson correlation between the sum and difference of paired values;  $t$  = test statistic;  $p$  = two-tailed p value; and  $d$  = Cohen's  $d$  effect size. For the paired t test,  $H_0: E[\ln(SE_{asymp}/SE_{boot})] = 0$ . For the Pitman–Morgan test,  $H_0: \sigma^2(SE_{asymp}) = \sigma^2(SE_{boot})$ . NA = Not Available. Degrees of freedom were 999 for continuous distributions and  $\leq 999$  for discrete distributions in the paired t test, and 998 in the Pitman–Morgan test.  $MEID$  = Median Interindividual Difference. Elaborated by the author.

For the binomial distribution, the means and variability of the jackknife standard errors were significantly greater than those of the asymptotic standard errors for sample sizes up to 100, with extremely large effect sizes. For  $n = 150$  and 200, the jackknife standard errors were smaller than the asymptotic ones, while effect sizes remained extremely large because of the reduced variability. From  $n = 250$  onward, the jackknife standard errors were equal to zero in nearly all simulations, except at  $n = 300$ , where one nonzero value was observed. This pattern reflects the convergence of the  $MEID$  estimate to a constant value in large samples under the binomial distribution, resulting in near-zero resampling variability. Consequently, mean comparisons became undefined, and the Pitman–Morgan correlations approached  $\pm 1$  due to the absence of variance in the paired differences (Table 7).

A similar pattern was observed for the Poisson distribution, beginning at  $n = 150$ . At  $n = 200$  and 300, the jackknife standard errors were close to zero and showed minimal variability, producing extremely large variability statistics due to the near-zero variance of the paired differences. For the remaining sample sizes, the jackknife standard errors were equal to zero, and the resulting Pitman–Morgan correlations were equal or close to  $\pm 1$  (Table 7).

**Table 7.** Comparison of Asymptotic and Jackknife Standard Errors of  $MEID$  Using Paired t Test of the Mean Log Ratio and the Pitman–Morgan Test.

$n$	$SE$		Paired t-test for mean_log_ratio					Pitman–Morgan test			
	Asym	Jack	$m\_ln$	$t$	$df$	$p$	$d$	$r_{SD}$	$t$	$p$	$d$
<b>Binomial(<math>n = 10, p = 0.25</math>)</b>											
30	0.1195	0.3323	-2.5333	-52.59	160	<.001	-4.16	-.9910	-233.67	<.001	-7.40
40	0.0897	0.2426	-2.8551	-37.99	97	<.001	-3.86	-.9715	-129.37	<.001	-4.10
50	0.0678	0.2080	-3.0706	-40.61	79	<.001	-4.57	-.9976	-453.86	<.001	-14.37
100	0.0318	0.0895	-3.8638	-37.33	23	<.001	-7.78	-.9997	-1396.78	<.001	-44.21
150	0.0211	0.0163	-4.4857	-12.45	3	.001	-7.19	-.9998	-1488.27	<.001	-47.11
200	0.0162	0.0120	-5.1325	-27.41	1	.023	-27.41	-.9999	-2327.13	<.001	-73.66
250	0.0131	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
300	0.0111	0.0086	NA	NA	NA	NA	NA	-1	-5457.20	<.001	-172.74
350	0.0096	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
400	0.0086	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
450	0.0077	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
500	0.007	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
1000	0.0038	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
<b>Poisson(<math>\lambda = 5</math>)</b>											
30	0.3237	0.3243	-1.5671	-37.93	172	0	-2.89	-.983	-167.27	<.001	-5.29
40	0.2650	0.2157	-1.9823	-39.83	91	0	-4.17	-.989	-209.48	<.001	-6.63
50	0.2250	0.1693	-2.2084	-40.27	63	0	-5.07	-.992	-250.87	<.001	-7.94
100	0.1277	0.0267	-3.0156	-12.52	6	0	-5.11	-.991	-230.12	<.001	-7.28
150	0.0898	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
200	0.0694	0.0070	NA	NA	NA	NA	NA	-.995	-320.61	<.001	-10.15
250	0.0568	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
300	0.0488	0.0014	NA	NA	NA	NA	NA	-.956	-103.19	<.001	-3.27
350	0.0424	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
400	0.0377	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
450	0.034	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
500	0.0311	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>
1000	0.0170	0	NA	NA	NA	NA	NA	1	2.12×10 <sup>9</sup>	<.001	6.71×10 <sup>7</sup>

Note.  $n$  = sample size;  $SE$  = asymptotic (Asym) and jackknife (Jack) standard errors;  $m\_ln$  = mean log ratio of asymptotic to jackknife standard errors;  $r$  = Pearson correlation between the sum and difference of paired values;  $t$  = test statistic;  $p$  = two-tailed p value; and  $d$  = Cohen's  $d$  effect size. For the paired t test,  $H_0: E[\ln(SE_{asymp}/SE_{jack})] = 0$ . For the Pitman–Morgan test,  $H_0: \sigma^2(SE_{asymp}) = \sigma^2(SE_{jack})$ . NA = Not Available.  $MEID$  = Median Interindividual Difference. Elaborated by the author.

### 3.2 Convergence to Normality in the Bootstrap-Generated Sampling Distribution

The bootstrap-generated sampling distribution of the *MEID* showed increasing conformity to normality for the three continuous distributions as the sample size increased. Convergence was more pronounced for the normal and gamma distributions, for which the proportion of samples satisfying the Shapiro–Francia normality criterion exceeded 90% at  $n = 1000$ . The arcsine distribution also showed increasing conformity to normality, although at a slower rate across sample sizes (Table 8).

In contrast, no convergence toward normality was observed for the discrete distributions. For the binomial distribution, the null hypothesis of normality was rejected in all samples across all sample sizes. A similar pattern was observed for the Poisson distribution, for which no samples satisfied the normality criterion; results for  $n = 1000$  were unavailable because the bootstrap estimates were degenerate (Table 8).

**Table 8.** Percentage of samples for which the null hypothesis of normality is NOT rejected at  $\alpha = 0.05$  for five distributions using Shapiro-Francia  $W'$  test

Sample size	Normal N(0, 1)	Arcsine (0, 1)	Gamma ( $\alpha = 10, \beta = 1$ )	BN ( $n = 10, p = 0.25$ )	Poisson ( $\lambda = 5$ )
30	0.6	0	0.5	0	0
40	3.9	0.2	3.8	0	0
50	9.7	0.1	5.1	0	0
100	43.6	3.9	41.6	0	0
150	64.4	15.2	62.8	0	0
200	77.2	22.6	73.4	0	0
250	81.7	32.4	78.2	0	0
300	85.9	45.9	84.2	0	0
350	87.1	49.9	86.7	0	0
400	89.7	58.8	87.6	0	0
450	89	62.7	89.9	0	0
500	90.8	68.1	90.8	0	0
1000	94	84.2	92.2	0	NA

Note. *MEID* = Median Interindividual Difference. NA = Not Available. Elaborated by the author.

## 4. Discussion

The first objective was to evaluate the equivalence between Wald-type and BCa confidence interval widths, as well as between asymptotic and bootstrap standard errors, for the *MEID*. The results do not support overall equivalence in terms of interval width and estimated variability. These discrepancies depended strongly on sample size and on the structural properties of the underlying distribution.

In continuous distributions, Wald-type intervals were systematically narrower than BCa intervals. Under normality and moderate skewness (gamma distribution), these differences decreased as the sample size increased, reaching trivial effect sizes in large samples, although statistical significance persisted at some intermediate sizes. This pattern suggests that, in large simulation samples, statistically significant results may reflect negligible practical differences between methods. In contrast, under strong asymmetry and bounded support (arcsine distribution), convergence toward practical similarity was slower, and effect sizes were initially small to medium. In these settings, the BCa method yielded systematically more conservative intervals. Overall, approximate similarity between asymptotic and bootstrap interval widths for the *MEID* can be assumed only in large samples drawn from continuous distributions that satisfy regularity conditions.

Regarding standard errors, asymptotic and bootstrap estimates were very similar under normality and moderate skewness, with trivial mean differences for most sample sizes. However, in the arcsine distribution, bootstrap standard errors were consistently larger and more variable than asymptotic ones, reflecting increased sensitivity to boundary concentration and skewness. Thus, when continuity and smoothness conditions are reasonably satisfied, asymptotic inference for the *MEID* appears broadly consistent with bootstrap-based estimation in moderate to large samples. Under marked skewness and

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boundedness, bootstrap procedures capture additional variability and cannot be considered strictly comparable to asymptotic estimates.

The behavior of the *MEID* changes qualitatively in discrete distributions. In the binomial and Poisson cases, the BCa method frequently produced infinite upper confidence limits or undefined interval estimates. The percentile bootstrap also yielded progressively fewer valid interval estimates as the sample size increased, because the *MEID* estimate converged toward discrete constants and the variability of the bootstrap distribution approached zero. These results indicate structural differences between asymptotic and resampling-based procedures rather than simple quantitative discrepancies in interval width.

The jackknife procedure avoided some of the numerical difficulties observed with BCa intervals but revealed the same underlying structural phenomenon: in moderate and large samples, confidence interval widths and standard errors converged to zero. Variability tests consequently produced extremely large statistics because the variance of the paired differences approached zero. This pattern reflects the convergence of the *MEID* toward population constants (1 in the binomial distribution and 2 in the Poisson distribution), leaving little or no variability to be detected by resampling procedures.

This behavior follows directly from the discrete structure of the two distributions and from the definition of the *MEID* as the median of all pairwise absolute differences. In the binomial distribution with parameters  $n = 10$  and  $p = 0.25$ , the probability mass is highly concentrated around the values 2 and 3, making an absolute difference of 1 the most frequent and ultimately the median pairwise difference as the sample size increases. Similarly, in the Poisson distribution with  $\lambda = 5$ , most observations cluster around neighboring integer values near the mean, and the distribution of pairwise absolute differences becomes centered around 2. As the sample size increases, the empirical distribution of pairwise differences stabilizes around these discrete values, causing the *MEID* to converge to fixed integers and reducing the variability detectable by resampling procedures.

In the binomial distribution, bootstrap standard errors exceeded asymptotic ones in small samples, whereas the variability estimated by both methods approached zero in large samples. In the Poisson distribution, the pattern was non-monotonic, with bootstrap variability systematically larger and particularly inflated in intermediate and large samples. At  $n = 1000$ , the bootstrap standard error was equal to zero because the *MEID* estimate had effectively converged to a constant value. Therefore, asymptotic and resampling-based procedures exhibit markedly different behavior in discrete contexts. Although the asymptotic method remained computationally stable, the intrinsic variability of the *MEID* approached zero under these structural conditions. Future research could examine whether alternative resampling approaches for discrete data with extensive ties, such as smoothed bootstrap or data jittering procedures, improve the stability of interval estimation for the *MEID* under these conditions. Consequently, the observed convergence of the *MEID* toward discrete constants in the binomial and Poisson cases should be interpreted as an empirical result requiring further theoretical investigation.

With respect to the second objective, convergence toward normality of the sampling distribution of the *MEID* was slow in continuous distributions and virtually absent in discrete ones. In continuous settings, approximate normality emerged gradually as the sample size increased, but convergence was slower under strong skewness and boundary concentration (arcsine distribution). In discrete distributions, the sampling distribution converged toward highly concentrated discrete forms rather than toward a smooth normal distribution. This behavior suggests that the *MEID* is highly sensitive to discreteness, ties, and lack of smoothness, and that the regularity conditions underlying Hoeffding-type central limit results for U-quantiles are not effectively satisfied in such contexts [3–4, 6].

## 5. Limitations

Several limitations should be acknowledged. Only 13 sample sizes were considered, ranging from 30 to 1000. The lower bound corresponds to the minimum commonly recommended for bootstrap procedures [24–25], and the upper bound represents a conventional threshold for large-sample studies [26]. Additionally, only five well-known univariate distributions were examined—three continuous and two discrete—chosen to vary in boundedness, skewness, and kurtosis while maintaining finite moments. Therefore, these results should not be generalized to other distributional forms without further study.

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The study focused primarily on interval widths, standard errors, and numerical behavior rather than on empirical coverage probabilities. Consequently, the adequacy of the interval estimation procedures cannot be evaluated solely in terms of interval narrowness or numerical stability. Future studies should jointly examine interval width, coverage probability, and the frequency of valid interval estimation. Each condition was based on 1000 Monte Carlo replications and 1000 bootstrap resamples per sample, representing commonly recommended minimum values [27–28]. Greater numerical stability might be achieved with larger numbers of replications and bootstrap resamples, particularly under conditions in which the resampling procedures produced undefined quantities or near-zero variability. Finally, the study was based primarily on simulation results and did not include formal theoretical derivations for the asymptotic behavior of the *MEID* under discrete distributions [29]. Consequently, the observed convergence of the *MEID* toward discrete constants in the binomial and Poisson cases should be interpreted as an empirical result requiring further theoretical investigation.

## 6. Conclusions

Practical similarity between Wald-type and bootstrap inference for the *MEID* was observed primarily in sufficiently large samples drawn from regular continuous distributions, particularly under moderate skewness and smooth distributional conditions. Under strong asymmetry, bootstrap procedures tended to yield more conservative estimates of uncertainty, and differences between methods remained more pronounced.

In the discrete distributions examined, the *MEID* showed empirical convergence toward fixed constants, causing the variability of the bootstrap distribution to approach zero and leading to marked discrepancies between asymptotic and resampling-based procedures. Overall, the behavior of the *MEID* depended strongly on sample size, discreteness, skewness, and distributional regularity. Approximate normality of its sampling distribution emerged gradually in continuous settings but was not observed in the discrete distributions considered in this study.

The results also suggest that the adequacy of interval estimation procedures for the *MEID* should be evaluated jointly in terms of interval width, empirical coverage, and numerical stability.

## 7. Suggestions

Further research should examine the behavior of the *MEID* under a wider range of distributional forms, degrees of discreteness, and sample sizes in order to clarify the structural conditions under which the estimator converges toward discrete constants and resampling variability approaches zero. Studies based on empirical data or on simulated data generated under alternative distributional assumptions would also help evaluate its performance under more realistic data-generating conditions [30]. Future studies could also investigate alternative resampling strategies for discrete data with extensive ties, such as smoothed bootstrap or data jittering procedures, in order to assess whether they improve the numerical stability of *MEID* interval estimation under these conditions [31].

In particular, the performance of the *MEID* with ordinal variables deserves further attention because of its median-based formulation and its potential usefulness in non-normal settings [32]. Finally, the *MEID* may serve as a useful descriptive measure of variability in applied quantitative research, provided that distributional regularity and the potential reduction of variability under discrete conditions are carefully considered [33].

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