

EFFECT OF CRITICAL SPEEDS AND SHAPE CHANGES ON OPERATING CONDITIONS OF SINGLE DISC ROTOR SYSTEM

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ABSTRACT

Rotors have a widespread usage in today's machine technology. Rotors can be used in electric motors, gas turbines, pumps and other mechanical systems. If the rotor system parameters are defined, it is possible to estimate the vibration frequencies. Any deformation from the rotor system components or wear lead to high frequency vibrations. These vibration parameters can be determined theoretically and experimentally. Vibration characteristics such as frequency, amplitude, speed and acceleration of vibration are the most important factors in the performance of a machine in operating conditions and in diagnosing mechanical problems.

In this study; the critical speeds of a single disk rotor system have been calculated and compared to the theoretical and critical speed values determined by the Dynrot software. Examples of such dynamic behavioral mechanical systems (servo, hydraulic, pneumatic motor shaft, cutter disk-shaft systems, etc.) between industrial field power, hydraulic pneumatic power train and shaft-disk systems have been similarly applied to the methods used in this work . In particular, it has been concluded that the software used in this study would be useful in characterizing the dynamic behavior of more complex rotors such as vacuum pumps, gas turbines, rotary, diffusion and turbomolecular pumps.

Keywords: Rotor dynamic, vibration, predictive maintenance

INTRODUCTION

Unbalance in rotating machines has been increasing day by day with the development of high-speed machines. Unbalance in rotating elements of machines is the most important source of vibration. Vibrations resulting from unbalance can cause wear and tear on the machine elements, as well as fatigue and breakage in the machine elements can cause the decrease machine performance and the loss of power if the vibrations affect other elements of the machine. In addition, high level noises result in unbalance. To avoid negative effects such as this and such, unbalances in machinery must be eliminated or balanced. According to DIN / ISO 1925, unbalance is defined as the condition that occurs when centrifugal forces on a track generate vibrational forces or motions on the bearings. Another definition can be given as the separation of the center of mass from the axis of rotation of the rotor due to the irregularity of the mass distribution of an rotor. Balancing is a process of correcting or removing unwanted inertia forces and moments in a rotating machine. In other words, it tries to align the center of mass with the axis of rotation of the rotor (Belek H. T., 2002).

Rotor unbalance is the most common cause of machine vibrations. Most of problems with rotating machine elements can be solved by eliminating rotor balancing and axial misalignment. Even a small amount of unbalance in machine elements rotating at high speed can cause serious problems. Rotors are used in many engineering applications such as pumps, fans, propellers and turbo machines. Vibration, caused by unbalance, can lead to damage to machine critical parts such as bearings, seals, gears and couplings. In practice, rotors can never be perfectly balanced due to manufacturing errors such as porosity in the cast, uneven density of the material, production tolerances and material gain or loss during operation (Eshleman, R.A., 1969). In determining unbalance response, some analytical methods such as the transfer method have been applied in the literature (Mitchell, L., Mellen, D.D.M., 1995). In addition, the unbalanced parts of the rotor rotate at the same speed as the rotor, so that the force due to the unbalance is synchronized (Bently, D. E., 2002). Rao proposed analytical closed form expressions for large and small axis radii of the unbalance response orbital for the single shaft rotor-bed system (Gupta, K.D. Gupta K. A., 1993). Rao et al., 1996 and Shiau et al. 1998, have reported that vibration signals, which are useful tools for studying advanced machine mechanical failures and at the same time detecting mechanical faults, can be widely used (Ramachandran, K.P., 2004).

It should always be taken into consideration that very serious faults can occur in the balancing process or in particular in the control of the residual unbalance, from the rotating system components connected to the rotating body or from the elements supporting the rotating body without their own bearings. The reasons for the occurrence of some faults are:

- Unbalance effects arising from rotating or supporting elements,
- Faults resulting from the components of the rotating or supporting elements,
- Faults originating from gaps between the rotating or supporting elements and rotating masses,
- Eccentric faults from the junction of the rotating masses with the journal with respect to the shaft axis.

In this study; the critical speeds of a single disk rotor system have been calculated and compared to the theoretical and critical speed values determined by the Dynrot software. Examples of such dynamic behavioral mechanical systems (servo, hydraulic, pneumatic motor shaft, cutter disk-shaft systems, etc.) between industrial field power, hydraulic pneumatic power train and shaft-disk systems have been similarly applied to the methods used in this work. It has been concluded that the software used in this study would be useful especially when the dynamic behaviors of rotors in more complex structures (eg vacuum pumps, gas turbines, diffusion and turbomolecular pumps) were characterized.

DYNAMIC ANALYSIS OF THE SINGLE DISC ROTOR

Description Of The Physical Model

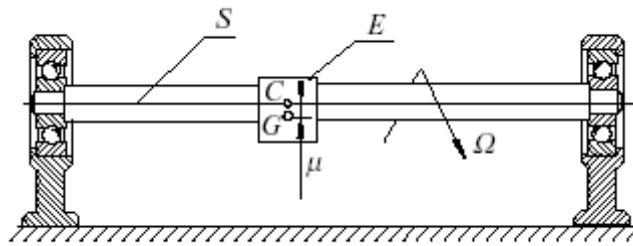


Figure 1. The Physical Model (Krodkiwski, J.M., 2000).

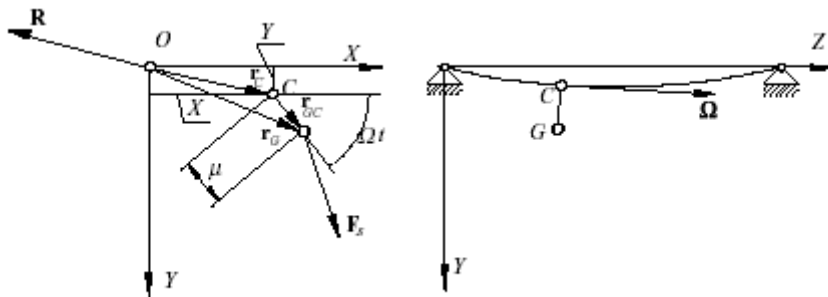


Figure 2. Indication of the absolute position of the center of gravity G (Krodkiwski, J.M., 2000).

Let us consider rotor shown in Fig. 2.1 assuming that it rotates with a constant angular speed Ω . The shaft S of the rotor is supported rigidly at its ends. Assume that the shaft can be considered massless and flexible whereas the element E can be approximated by a particle of mass m . This particle is attached to the shaft at the centre of gravity G of the element E . The centre of gravity G is displaced by μ from the geometrical centre of the shaft cross-section C . The distance μ represents imbalance of the element E and can be considered as a small magnitude. To analyze motion of this system let us introduce the inertial system of coordinates XYZ as it is shown in Fig. 2.2. The instantaneous position of the centre C is determined by the position vector r_C . The centre of gravity G rotates with the angular velocity Ω respect to this centre. Since the angular velocity is constant, the relative instantaneous position of the centre of gravity G is determined by the angle Ωt and the imbalance μ (vector r_{GC}). The absolute position of the centre of gravity G , in Fig. 2.2 is denoted by r_G . The vector F_s represents the static resultant force acting on the element E . R stands for the interaction force between the element considered and the shaft (Krodkiwski, J.M., 2000).

Mathematical Model

Motion of the centre of gravity G is governed by the Newton's second law,

$$m\ddot{r}_G = R + F_s \quad (2.1)$$

Equation 2.2 is obtained from Fig. 2.2

$$\begin{aligned} r_G &= I(X + \mu \cos \Omega t) + J(Y + \mu \sin \Omega t) \\ R &= -IkX - JkY \\ F_s &= I F_{Xs} + J F_{Ys} \end{aligned} \quad (2.2)$$

In the above formula k stands for stiffness of the shaft at the point C and XY are its coordinate pairs. Substituting Eq's. 2.2 into Eq. 2.1, results in the following set of differential equations.

$$m(\ddot{Y} - \mu \Omega^2 \sin \Omega t) = -kY + F_{Ys} \quad (2.3)$$

or after reorganization

$$\begin{aligned} m\ddot{X} + kX &= F_{Xs} + m\mu \Omega^2 \cos \Omega t \\ m\ddot{Y} + kY &= F_{Ys} + m\mu \Omega^2 \sin \Omega t \end{aligned} \quad (2.4)$$

The particular solution of the non-homogeneous Eq. 2.5

$$\begin{aligned} m\ddot{X} + kX &= F_{Xs} \\ m\ddot{Y} + kY &= F_{Ys} \end{aligned} \quad (2.5)$$

yields the equilibrium position (X_s ; Y_s). Upon assuming the particular position in form

$$\begin{aligned} X &= X_s \\ Y &= Y_s \end{aligned} \quad (2.6)$$

one may obtain the following formulae for coordinates of the equilibrium position which are usually referred to as the *static deflection* of the shaft.

$$X_s = \frac{F_{Xs}}{k} \quad Y_s = \frac{F_{Ys}}{k} \quad (2.7)$$

The total deflection of the shaft X, Y are sum of the static deflection X_s ; Y_s and the *dynamic deflection* x, y (see Fig. 2.3).

$$\begin{aligned} X &= X_s + \chi \\ Y &= Y_s + y \end{aligned} \quad (2.8)$$

Introduction of Eq. 2.8 into the mathematical model 2.4 in equations which govern the

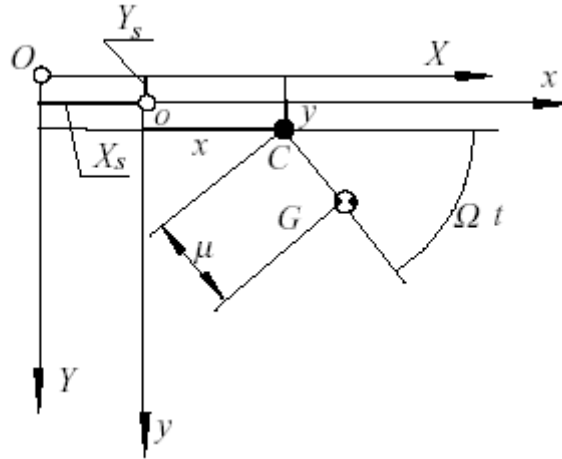


Figure 3. Dynamic deflections xy .

$$m\ddot{y} + ky = m\mu\Omega^2 \sin \Omega t \quad (2.9)$$

or

$$\ddot{\chi} + \omega^2 \chi = q \cos \Omega t \quad (2.10)$$

$$\ddot{y} + \omega^2 y = q \sin \Omega t \quad (2.11)$$

where

$$w = \sqrt{\frac{k}{m}} \quad q = \mu\Omega^2 \quad (2.12)$$

Upon multiplying the equation 2.11 by the imaginary unit i and adding the equations 2.10 and 2.11, one may obtain the equations of motion of the rotor in the following form

$$\ddot{z} + \omega^2 z = qe^{i\Omega t} \quad (2.13)$$

where

$$z = \chi + iy \quad (2.14)$$

The above equation governs motion of the rotor in the stationary system of coordinates xyz .

Let us introduce the rotating system of coordinates $x_R ; y_R ; z_R$; shown in Fig. 2.4. Axis z_R

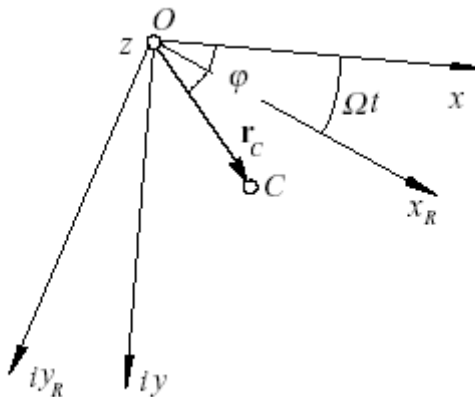


Figure 4. The rotating system of coordinates $x_R ; y_R ; z_R$

coincides axis z and axes x_R and y_R rotates with the constant angular velocity Ω . In terms of the complex notations, position of the point C in the stationary system of coordinates xyz is

$$z = |z|e^{i\varphi} \quad (2.15)$$

and in the rotating system of coordinates is

$$z_R = |z_R| e^{i(\varphi - \Omega t)} = |z_R| e^{i\varphi} e^{-i\Omega t} \quad (2.16)$$

Substituting Eq. 2.15 into Eq. 2.16 yields the relationship between coordinates of the same point in the stationary ($x; iy$) and the rotating ($x_R; y_R$) system of coordinates.

$$z_R = z e^{-i\Omega t} \quad (2.17)$$

The inverse transformation is

$$z = z_R e^{+i\Omega t} \quad (2.18)$$

Differentiating of Eq. 2.18 with respect to time one can obtain

$$\begin{aligned} \dot{z} &= \dot{z}_R e^{i\Omega t} + z_R i\Omega e^{i\Omega t} \\ \ddot{z} &= \ddot{z}_R e^{i\Omega t} + 2\dot{z}_R i\Omega e^{i\Omega t} - z_R \Omega^2 e^{+i\Omega t} \end{aligned} \quad (2.19)$$

Substituting Eq. 2.19 into the mathematical model equation 2.10 gives equation of motion of the rotor in terms of the rotating system of coordinates (Krodkiwski, J.M., 2000).

$$\ddot{z}_R + 2\dot{z}_R i\Omega + z_R (\omega^2 - \Omega^2) = q \quad (2.20)$$

THEORETICAL MODEL RESULTS AND EVALUATIONS

The single disc shaft system (De Laval Rotor) with its geometric and mechanical characteristics is modeled by the software and this rotor system is operated under different operating conditions (260, 800 and 1200 rpm). Several analyzes have been performed for the rotor system during these different cases of operation. These are critical speeds and natural frequency analyzes. In addition, these values are also compared with the theoretically calculated values.

Theoretical Analysis

Model System

In this section, the analysis of the critical speeds of the rotor system in the loaded and unloaded operating conditions has been carried out.

$L = 600$ mm. (distance between the beds), $d = 32$ mm (shaft diameter), $M = 2902$ grams (disk mass), shaft weight = 4529 grams, $E = 207.109$ Pa (for steel).

3.1.2. Critical Speed of the Shaft

The critical speed in the case of the unloaded operation of the shaft in the rotor system (Zajackowski, J., 1997).

$$\begin{aligned} \omega_s &= 9,87 \cdot \sqrt{\frac{E \cdot I}{M \cdot L^3}} \\ I &= \frac{\pi d^4}{64} = \frac{\pi \cdot (0,032)^4}{64} = 5,14 \cdot 10^{-8} \text{ m}^4 \end{aligned} \quad (3.1)$$

(3.2)

$$\begin{aligned} E &= 207.10^9 \text{ Pa} \quad (\text{Steel}) \\ M &= 4,529 \text{ kg} \\ L &= 0.6 \text{ m} \end{aligned}$$

$$\omega_s = 9,87 \cdot \sqrt{\frac{207.10^9 \cdot 5,14 \cdot 10^{-8}}{4,529 \cdot (0.6)^3}} = 9,87 \cdot \sqrt{\frac{10654.29}{0.978}} = 9,87 \times 104.374$$

$$\omega_s = 1030.173 \text{ rad/s} \quad (\text{Critical speed when unloaded})$$

Single Disc System

The critical speed of the rotor system of De Laval in the case of loaded operation (Zajaczkowski, J., 1997),

$$\omega_1 = \sqrt{\frac{3.E.I.L}{M.a^2.b^2}} \quad (3.3)$$

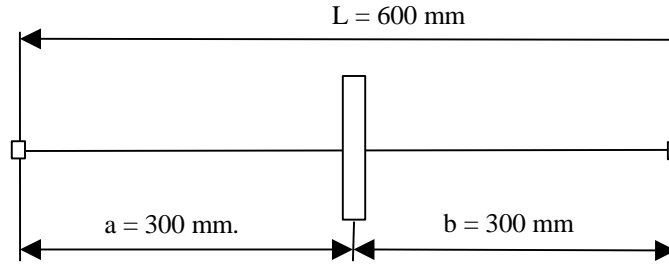


Figure 5. Single disc loading.

$$\omega_1 = \sqrt{\frac{3.207 \cdot 10^9 \cdot 5,14 \cdot 10^{-8} \cdot 0.6}{2,902 \cdot (0,3)^2 \cdot (0,3)^2}}$$

$$\omega_1 = \sqrt{\frac{19177.722}{2,902 \cdot (0,3)^2 \cdot (0,3)^2}} = \sqrt{\frac{19177.22}{0.0235}}$$

$$\omega_1 = 903.355 \text{ rad/s} \quad (\text{Critical speed for single disk center})$$

Other critical speeds which calculated at the first critical speed 4, 9, 16, 25, may float on the floors (Zajaczkowski, J., 1997).

From Dunkerley's Equation;

$$\frac{1}{\omega_n^2} = \frac{1}{\omega_s^2} + \frac{1}{\omega_1^2} \quad (3.4)$$

$$\frac{1}{\omega_n^2} = \frac{1}{(1030.173)^2} + \frac{1}{(903.355)^2}$$

$$\frac{1}{\omega_n^2} = 2.167 \cdot 10^{-6}$$

$$\omega_n^2 = 461467.4665$$

$$\omega_n = 679.314 \text{ rad/s}$$

The critical speed of the complete system (shaft + single disk) without neglecting the mass of the shaft.

Analyses Applied to Software Modeled Rotor Model

A modeling method based on the finite element method for rotor dynamics has been used in the modeling of the dynamic system that is the basis of operation. Rolling bearings used in the system are modeled software using spring, damping elements and bearings for dynamic analysis in Dynrot. Because bearings used in software do not allow certain calculation types such as critical speed (Figure 3.2) (Genta, G., 1998).

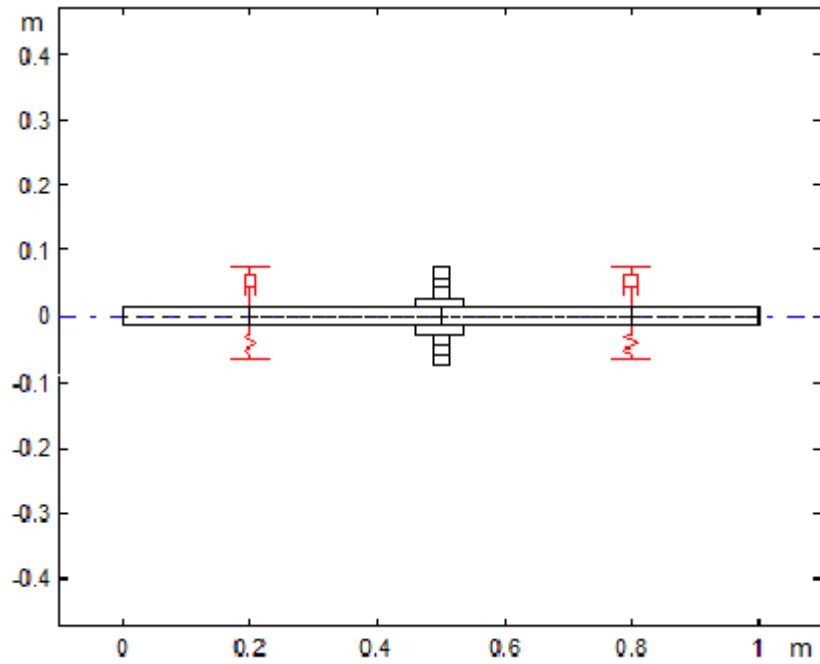


Figure 6. The modeled shape of the rotor system.

Analysis of critical speeds

In Table 3.1, the critical velocity values obtained by the analytical method using equation 3.3 are given together with the critical velocity values obtained by the software. There are differences between the critical speed values obtained from the software and analytical calculation. The reason is the modeling of rolling bearings of the rotor system with a spring and a damping element as an assumption in the software. When modeling is desired as a bearing, restrictions arise in calculations. Therefore, instead of a roller bearing, a spring and a damping element are preferred.

Table 1. Different critical speed values.

	Critical speed values calculated from Equation (3.1)	Critical speed values calculated by software
1 st Critical speed	903.355 rad/s	585.548989 rad/s
2 nd Critical speed	3613.420 rad/s	1666.509126 rad/s
3 rd Critical speed	8130.195 rad/s	1696.511142 rad/s
4 th Critical speed	14453.68 rad/s	3379.765629 rad/s
5 th Critical speed	22583.875 rad/s	4688.035919 rad/s

At the 5 different critical speeds shown in the figures below, the first mode shape of the impeller is due to the operation of the rotor. The horizontal axis in the graph shows the modeled amplitude of the rotor, and the vertical axis shows the variation in amplitude.

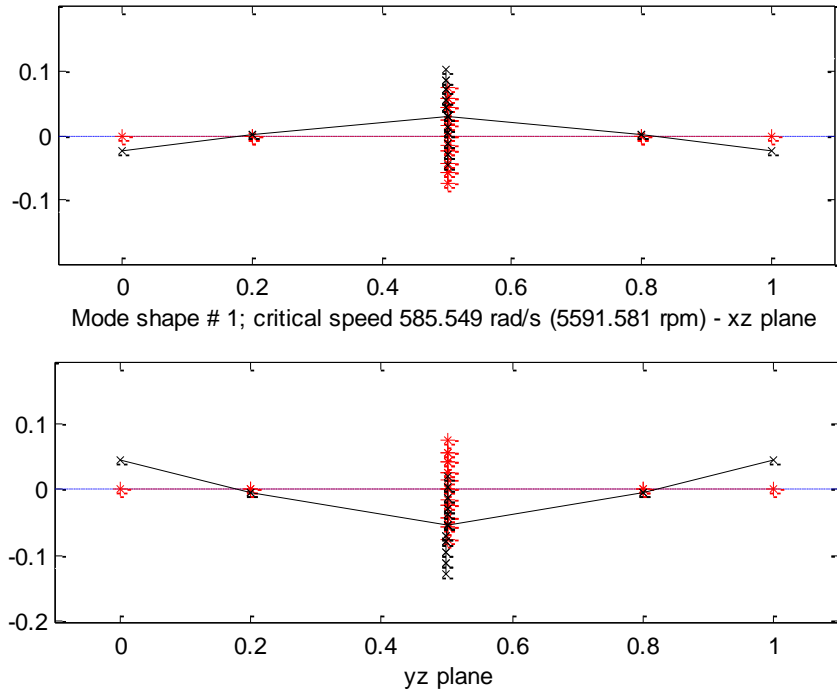


Figure 7. Deformation occurred in the rotor system at the first critical speed.

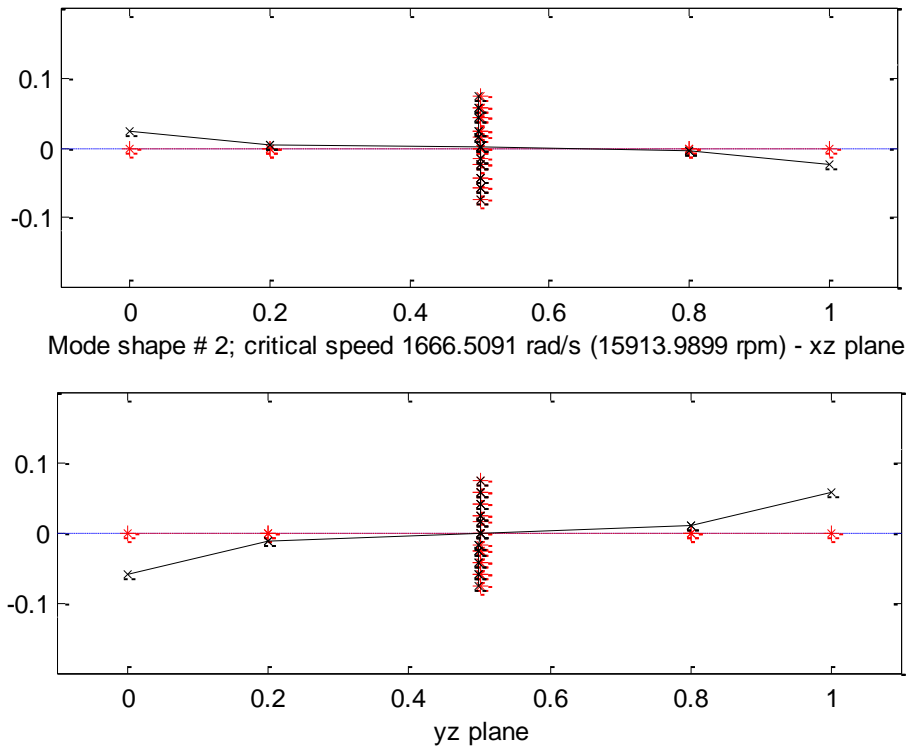


Figure 8. Shape change in the rotor system at the second critical speed.

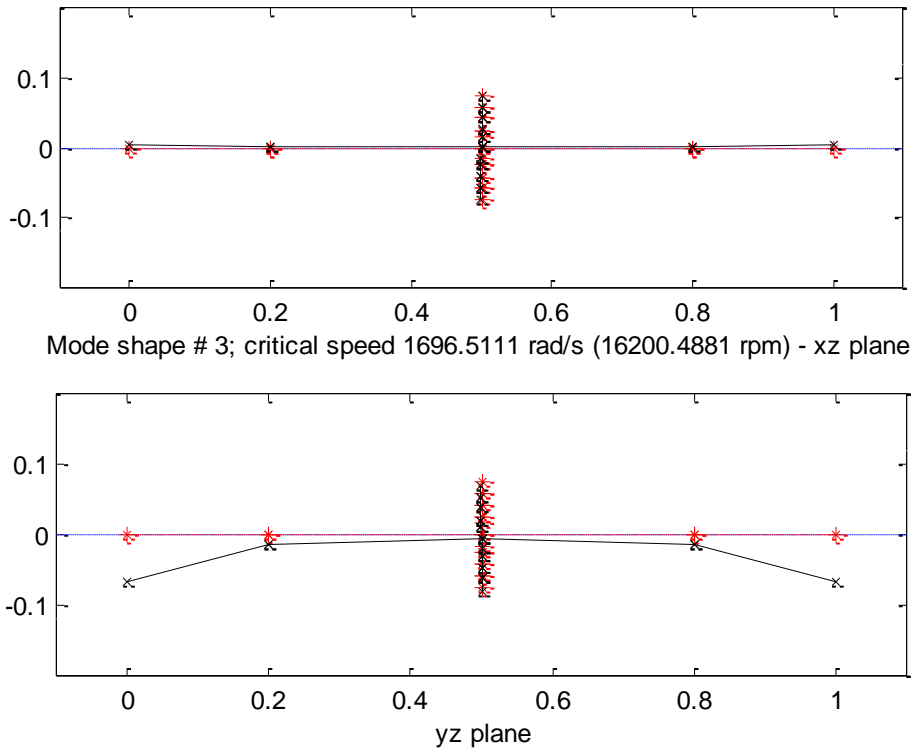


Figure 9. Shape change in the rotor system at the third critical speed.

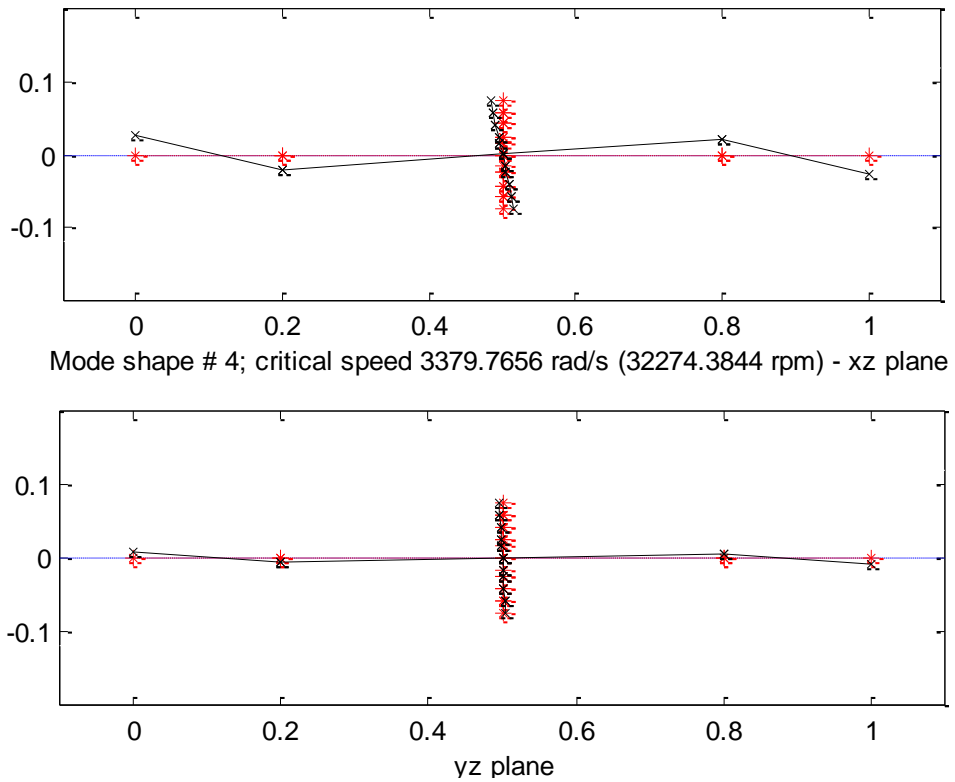


Figure 10. Deformation occurred in the rotor system at the fourth critical speed.

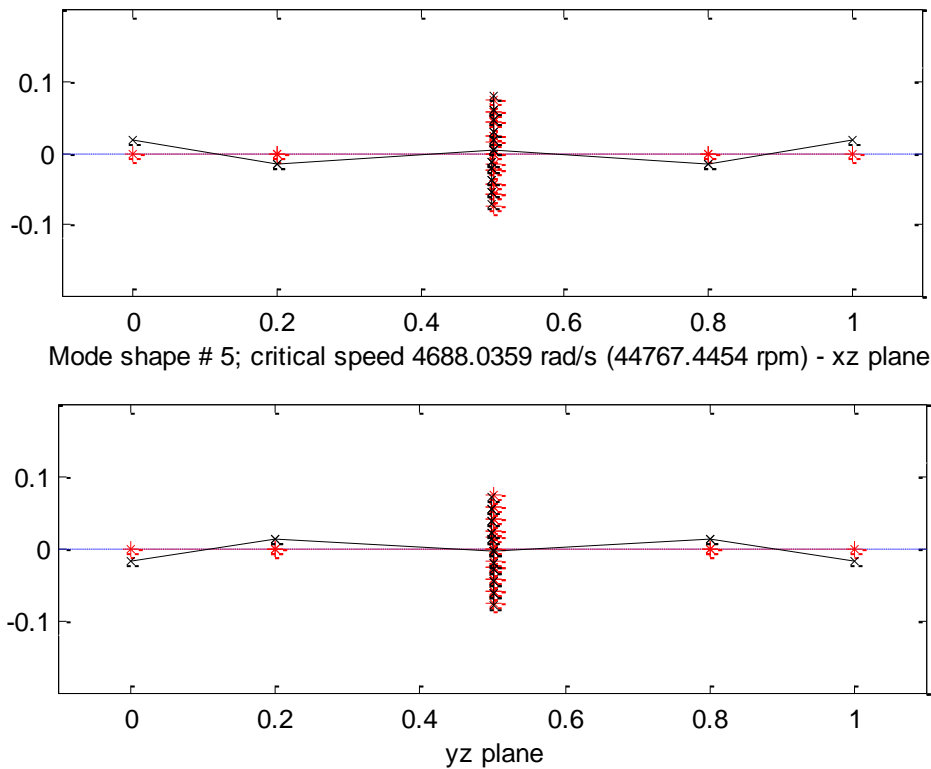


Figure 11. Deformation occurred in the rotor system at the fifth critical speed.

Shape changes occurred at the calculated critical speeds of the rotor system modeled in Figs. 3.3a-e are graphically shown. When the graphics are examined; the amplitude value in the radial direction at the first critical speed (585.548989 rad / s) is higher than the other modes. The critical speed value in the first mode is considerably lower than the critical speed in the other modes. Therefore, we can interpret the fact that the system does not operate at a stable speed at low speeds and that the harmonics of the 1st critical speed of the system are not approached at other running speeds because the radial amplitude value of the shape modification is high. As the operating speed of the system increases, the amplitude value in the radial direction decreases. In other words, as the speed of operation increases, the system becomes more stable and it becomes a normal rotation.

Campbell Diagram

As a result of the analyzes made, the graph in Figure 3.4 is obtained. The horizontal axis in the graph shows the change in the running speed and the vertical axis shows the change in the natural frequency. First, the maximum and minimum speeds of the system were determined (min speed 260 rpm and max speed 1200 rpm), and 10 rs and + 20 natural frequencies were found in the plus and minus directions at the rotation speeds of 1 s . The graph in Figure 3.4 also shows the change in natural frequencies in these positive and negative directions. Although it appears that the lines in the graph are straight, this is due to the fact that there is not a lot of variation in the natural frequencies found at the end of the rotation speeds. This change can be seen comfortably as the graph is enlarged. This change can also be interpreted by the fact that during the operation of the system at different speeds, the change in the bed (spring-damping element) system occurs at a lesser rate, which affects the natural frequencies. From this, it is clear that the natural frequency of the system depends on the angular velocity. However, the increase in natural frequencies in the positive direction and decrease in the negative plane until the increase in speed, ie 260 rpm 1200 rpm, shows that the change in rotation direction can affect the change in natural speed as well as the working speed.

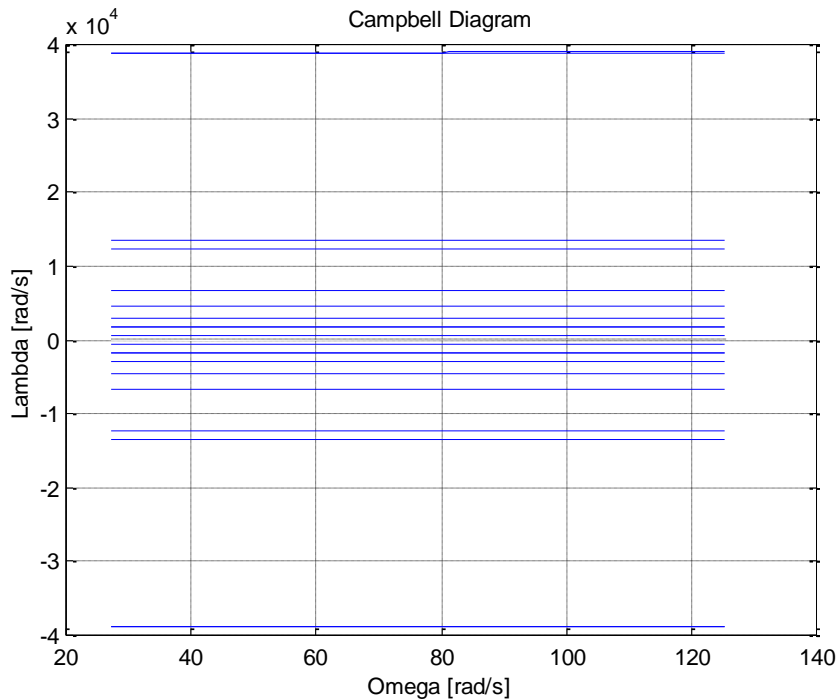


Figure 12. Campbell diagram.

CONCLUSIONS AND RECOMMENDATIONS

Vibration characteristics are the most important factors in the performance of a machine in operating conditions and in diagnosing mechanical problems. These characteristics are frequency, amplitude, speed and acceleration of vibration.

In this study, five critical speed values are calculated for the rotor system and it is observed that the critical speed values determined by theoretical and software are different from each other. Further studies can be done to increase speeds and change the dimensions. It is conceivable that the methods used in this study can be applied in a similar way to mechanical systems with dynamic behavior such as industrial field power, hydraulic pneumatic drive and shaft-disk systems (servo, hydraulic, pneumatic motor shaft). In particular, it has been concluded that the software used in this study would be useful in characterizing the dynamic behavior of more complex rotors (eg vacuum pumps, gas turbines, diffusion and turbomolecular pumps).

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