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Classification of 5D Chaplygin gas models[1]

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Abstract

In this study, we mainly classify some Chaplygin gas unified dark energy models in a five-dimensional (5D) cosmology. It is known that the background evolution for the Chaplygin gas models is equivalent to that for the dark energy interacting with the dark matter. Here, we assume a space-time governed by the five-dimensional Kaluza-Klein framework in order to rewrite energy densities and pressures of some well-known Chaplygin gas proposals. These new formulations can be taken into account to get original cosmological conclusions in further studies.

Keywords: Cosmology, Chaplygin gas, Dark energy, Five dimensions

Introduction

Current cosmological observations have shown that our universe has begun to expand as it did after the big bang (Perlmutter, et al., 1998; Miller, et al., 1999; Bahcall, et al., 1999; Bennett, et al., 2003; Brile, et al. 2003; Spergel, et al., 2003; Tegmark, et al., 2004; Ade, et al., 2016).

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Dark energy is pointed out to be responsible for this mysterious behavior. There are many dark energy models in the literature (see Refs. (Sahni and Starobinsky 2000; Peebles and Ratra 2003; Padmanabhan 2003) for a detailed review of models).

The first, simplest and most traditional dark energy model is the famous cosmological constant, which is described by the equation $\rho+p=0$ and can be used as a perfect fluid. However, this simplest model has some problems such as fine-tuning and cosmic coincidence (Weinberg 1989; Copeland, et al., 2006). Other known dark energy models are as follows: scalar field models (Starobinsky 1998), dark energy densities (Cardone, et al., 2004; Wei and Cai 2008; Urban and Zhitnitsky 2009), unified dark matter-energy formulations (Chimento and Jakubi 1996; Kamenshchik, et al., 2001; Debnath, et al., 2004), using extra dimensional frameworks (Kaluza 1921; Klein 1926; Guo and Zhang 2007; Calcagni 2010) and modified gravity theories (Boisseau, et al., 2000; Capozziello 2002; Cai, et al., 2016).

In the present study, based on the five-dimensional (5B) Kaluza-Klein-type universe model, the Chaplygin gas family, which is one of the unified dark matter-energy proposals, was classified. In this study, the exact expression of energy density which is very important for a cosmological theoretical research is given in 5D form for different Chaplygin gas models. Some of these are original calculations that were obtained for the first time in the literature.

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Chaylygin gas model

This model is generally described by the definition of energy density given below (Kahya, et. al., 2015)

$$p_c = \sum_{i=1}^{\infty} A_i \rho_c^i - \frac{B}{\rho^\alpha}. \quad (1)$$

In the above equation, i, A_i, B and α are constants. In this relation, after assuming $A_i = 0$ and $\alpha = 1$, we get the original Chaplygin gas (OCG) model (Kamenshchik, et al., 2001). The generalized Chaplygin gas (GCG) definition (Bento, et al., 2002; Gorini, et al., 2003; Alam, et al., 2003) is obtained when the case $i=1$ is selected. In the case of another limit, the choice of $i=1$ and $A_1=0$ gives the modified Chaplygin gas (MCG) model (Bento, et al., 2002; Gorini, et al., 2003; Sahni, et al., 2003). For a second order situation, the case $i=1, 2$ is taken. This allows us to reach the extended Chaplygin gas (ECG) model (Pourhassan 2016). On the other hand, there are three other models that can be added to this family. Since these models contain variable parameters, they have different properties than the four models we have introduced above. The first one of these is the variable Chaplygin gas (VCG) model (Lu 2009) and it can be obtained by writing $\alpha=1$ and $B(a)=Ba^{-n}$ (here n and B denote constant parameters) after selecting $A_1=A_2=0$. The latter one is known as the variable modified Chaplygin gas (VMCG) model (Panigrahi and Chatterjee 2016) and it can be defined by assuming $A_1=A_2=0$ and and $B(a)=Ba^{-n}$. The last one is referred to in literature as the variable generalized Chaplygin gas (VGCG) model (Panigrahi and Chatterjee 2017). The energy density expressing this model is obtained from the equation (1) after taking $A_1\neq0, A_2=0, \alpha\neq0$ and $B(a)=Ba^{-n}$.

5D Chaplygin gas family

For the theoretical calculations in this study, it is assumed that the universe consists of Chaplygin gas, dark radiation and barionic matter. 5D Kaluza-Klein type Friedmann-Robertson-Walker space-time model is depicted by the line-element written as follows (Ozel, et al., 2010):

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\varphi^2) + (1-kr^2)dx_5^2\right] \quad (2)$$

In this equation $a(t)$ is known as the cosmic scale multiplier, and k is the curvature parameter, where $k=0$, $k=+1$ and $k=-1$ indicate flat, open and closed universe forms respectively. The energy-momentum distribution of this type universe model is assumed to be expressed by the following mathematical expression:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu} \quad (3)$$

where μ and ν indices take the values (0, 1, 2, 3, 4). On the other hand, the relation

$$\rho = \rho_b + \rho_r + \rho_c \quad (4)$$

shows the total energy density while

$$p = p_b + p_r + p_c \quad (5)$$

describes the total pressure (the indices b, r and c are respectively barionic, dark radiation and Chaplygin gas abbreviations). Additionally, u_μ is the 5-velocity vector and it satisfies the constraint $u_\mu u^\mu = 1$.
Einstein field equations are written as follows:

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (6)$$

In the above equation, $R_{\mu\nu}$, R, $g_{\mu\nu}$ and G are known as Ricci tensor, Ricci scalar, metric tensor and mass-gravitational constant, respectively. When the equations (2), (3), (4) and (5) are used in the equation (6), the expressions known as Friedmann equations are written as follows:

$$H^2 + \frac{k}{a^2} = \frac{4\pi G}{3}\rho, \quad (7)$$

$$2H^2 + \dot{H} + \frac{k}{a^2} = -\frac{8\pi G}{3}p. \quad (8)$$

When writing Friedmann equations, $H = \frac{\dot{a}}{a}$ is defined as the Hubble expansion parameter.

On the other hand, following the conservation equation $T^{\mu\nu}_{;\nu} = 0$ or the Friedmann equations written in (7) and (8), the following continuity equations are obtained:

$$\dot{\rho}_b + 4H\rho_b = 0, \qquad (9)$$

$$3\dot{\rho}_r + 16H\rho_r = 0, \qquad (10)$$

$$\dot{\rho}_c + 4H(1+\omega_c)\rho_c = 0. \qquad (11)$$

In the above continuity equations, $\omega_b=0$, $\omega_r=1/3$ and $\omega_c=p_c/\rho_c$ are used for the state-equation parameter ($\omega=p/\rho$). The differential equations (9) and (10) are easy to solve. After the necessary mathematical operations are performed, one can find the follwing results

$$\rho_b = \rho_b^0 a^{-4}, \qquad (12)$$

$$\rho_r = \rho_r^0 a^{-\frac{16}{3}}. \qquad (13)$$

where ρ_b^0 and ρ_r^0 indicate the initial values. Moreover, we can define the following dimensionless parameters

$$\Omega_b = \frac{8\pi G}{3H^2}\rho_b, \qquad (14)$$

$$\Omega_r = \frac{8\pi G}{3H^2}\rho_r \qquad (15)$$

$$\Omega_c = \frac{8\pi G}{3H^2}\rho_c, \qquad (16)$$

$$\Omega_k = -\frac{k}{a^2H^2}. \qquad (17)$$

Consequently, it is possible to write the Friedmann equation (7) in a much more aesthetic and simple way when the above dimensionless parameters are defined:

$$\Omega_b + \Omega_r + \Omega_c + \Omega_k = 1. \qquad (18)$$

We can now focus on equation (11) in order to obtain energy density expression clearly in the Chaplygin gas model. Accordingly, when we use the equations (1) and (11) together, a solution for ρ_c can be obtained. Solutions for the Chaplygin gas family are given in Table 1.

Table 1. Relations that represent Chaplygin gas models.

Model	Free Parameter	Pressure (p_c)	Energy density (ρ_c)
OCG	$A_i = 0, \alpha = 1, B \neq 0$	$-B/\rho_c$	$\sqrt{B + \frac{\lambda}{a^8}}$
GCG	*i=1*, $A_1 \neq 0, B \neq 0, \alpha \neq 0$	$A_1\rho_c - \frac{B}{\rho_c^\alpha}$	$\left(\frac{B+\frac{\eta}{a^{4(1+A_1)(1+\alpha)}}}{1+A_1}\right)^{\frac{1}{1+\alpha}}$
MCG	*i=1*, $A_1 = 0, B \neq 0, \alpha \neq 0$	$-\frac{B}{\rho_c^\alpha}$	$\left(B + \frac{\xi}{a^{4(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}$
ECG	*(i=1, 2)*, $A_1 \neq 0, A_2 \neq 0$, $B = Ba^{-n}, \alpha = 1$	$A_1\rho_c + A_2\rho_c^2 - \frac{B}{\rho_c}$	$\frac{1+\zeta e^{3\pi}a^{20B}+\sqrt{5\zeta e^{3\pi}a^{20B}-1}}{\zeta e^{3\pi}a^{20B}-1}$
VCG	*(i=1, 2)*, $A_1 = A_2 = 0$, $B = Ba^{-n}, \alpha = 1$	$-\frac{Ba^{-n}}{\rho_c}$	$\frac{8}{8-n}\frac{B}{a^n} - \frac{\varepsilon}{a^8}$ (Salti, et al., 2018a)
VMCG	*(i=1, 2)*, $A_1 = A_2 = 0$, $B = Ba^{-n}, \alpha \neq 0$	$-\frac{Ba^{-n}}{\rho_c^\alpha}$	$\frac{+\alpha)}{\alpha)-n}\frac{B}{a^n} - \frac{\sigma}{a^{4(1+\alpha)}}$ (Salti, et al., 2018b)
VCCG	*(i=1, 2)*, $A_1 \neq 0, A_2 = 0$, $B = Ba^{-n}, \alpha \neq 0$	$A_1\rho_c - \frac{Ba^{-n}}{\rho_c^\alpha}$	$\frac{4(1+\alpha)}{4(1+\alpha)(1+A_1)-n}\frac{B}{a^n} - \frac{\delta}{a^{4(1+A_1)(1+\alpha)}}$

While obtaining the solutions given in Table 1, some constants have been defined and accepted. Here, the condition $0 < \alpha \leq 1$ is provided and it is assumed to be $tan^{-1}(\rho_c + 1) \approx \frac{\pi}{2}$, $A_1 = \frac{B}{2} - 1$ and $A_2 = \frac{B}{2}$ for the ECG description. In order to obtain meaningful results by using the ECG model, it should be $\zeta e^{3\pi} a^{20B} > 1$. There is a relationship between the current energy density (ρ_g) and the cosmological energy density (ρ_k) in the form of $\rho_g = 1.31\rho_k$ (Kahya, et. al., 2015; Pourhassan 2016). Accordingly, when the case $a=1$ considered, the value $\rho_c = 1.31$ is obtained. In the light of these conditions and results, the following results are obtained for the free parameters written in Table 1:

$$\lambda = (1{,}31)^2 - B, \tag{19}$$

$$\eta = B\left[\frac{(1{,}31)^{1+\alpha}}{2} - 1\right], \tag{20}$$

$$\xi = (1{,}31)^{1+\alpha} - B, \tag{21}$$

$$\zeta_1 = e^{-3\pi}, \text{ ya da } \zeta_2 = 66e^{-3\pi}, \tag{22}$$

$$\varepsilon = 1{,}31 - \frac{8B}{8-n}, \tag{23}$$

$$\sigma = 1{,}31 - \frac{4B(1+\alpha)}{4(1+\alpha)-n}, \tag{24}$$

$$\delta = 1{,}31 - \frac{4B(1+\alpha)}{2B(1+\alpha)-n}. \tag{25}$$

When we look at the mathematical expressions given above, it is seen that all the results consist of the free parameters A, B, α and n which are given in the equation (1). These are the free parameters predicted by the theory used in defining the model, and by using observational data, it is necessary to determine the most appropriate values for the theory-observation alignment.

Conclusion

In this study, the energy intensities which describe the 7 different theoretical models of the Chaplygin gas family are obtained from the 5D universe model. These results depend on the free parameters predicted by the theory and the appropriate values of these parameters can be determined by numerical analysis using observational data. For these analyzes, in the first step, the energy density is subsituted in the Friedmann equation (7) in order to get an clear statement for the cosmic Hubble parameter:

$$H = \sqrt{\frac{4\pi G}{3}\rho_c - \frac{k}{a^2}} \tag{26}$$

Since it is possible to reach the numerical observation data for the Hubble expansion parameter, one can obtain the most suitable proposal (most compatible with the observational data) by determining the free parameters in the models. Moreover, it is possible to compare the models with different methods by performing different analyzes including calculation of some cosmological parameters such as the state-equation parameter, deceleration parameter, luminance distance, sound speed and some thermodynamical quantities. In addition to these, new and original research results can be reached by focusing on redefining scalar field models. These problems are planned to be made in the future and will form the basis of other studies.

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