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An Investigation of ChatGPT–Student Interaction In The Learning of Functions

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Abstract: This study investigates the cognitive orientations underlying prompts produced by 10th-grade students during written interactions with ChatGPT while learning functions, as well as the explanatory strategies reflected in ChatGPT's responses. The study aims to identify interaction patterns and associated learning needs. Adopting a qualitative approach, the research was designed as an exploratory case study. Participants were eight students (five female, three male) from a private science high school in southern Türkiye. Data consisted of transcripts of student–ChatGPT interactions recorded during the implementation process. A worksheet on functions structured the learning environment and guided the interactions. Data were analyzed through content analysis using open coding, and student prompts were categorized into five groups. Findings showed that prompts were primarily definition-oriented (41.2%) and representation-oriented (27.5%). ChatGPT's responses reflected three main explanatory strategies: exemplifying (35.3%), visualizing (33.1%), and preventive (31.6%). Overall, the findings indicate that ChatGPT can support conceptual clarification and enrich representational understanding in mathematics learning. However, the effectiveness of this support appears closely related to the quality of students' questioning, verification, and self-regulation behaviors. Accordingly, the study offers recommendations for integrating generative artificial intelligence into mathematics education, emphasizing pedagogically structured usage scenarios and appropriate instructional guidance.
Keywords: ChatGPT, Mathematics Learning, Function Concept, AI-Supported Learning Environment, Interaction

Fonksiyon Öğrenme Sürecinde ChatGPT-Öğrenci Etkileşiminin İncelenmesi

Öz: Bu araştırma, 10. sınıf öğrencilerinin fonksiyonlar konusunu öğrenirken ChatGPT ile kurduğu yazılı etkileşimlerde üretilen promptların bilişsel yönelimini ve ChatGPT yanıtlarının açıklama stratejilerini incelemeyi amaçlamaktadır. Araştırma, etkileşim örüntülerini ve ortaya çıkan öğrenme ihtiyaçlarını betimlemeye odaklanmıştır. Nitel yaklaşımda keşfedici durum çalışması deseninde yürütülen çalışmanın katılımcıları Türkiye'nin güney bölgesinde bulunan bir özel fen lisesinde öğrenim gören 8 öğrencidir (5 kız, 3 erkek). Temel veri kaynağı, uygulama sırasında kaydedilen öğrenci–ChatGPT prompt–yanıt dökümleridir; öğrenme bağlamını yapılandırmak amacıyla fonksiyonlar çalışma kâğıdı kullanılmıştır. Veriler içerik analiziyle açık kodlama yoluyla çözümlenmiş, promptlar beş kategori altında sınıflandırılmıştır. Bulgular, promptların en çok tanıma yönelik (%41,2) ve temsile yönelik (%27,5) olduğunu; ChatGPT yanıtlarının ise örnekleyici (%35,3), görselleştirici (%33,1) ve önleyici (%31,6) biçimlerde dağıldığını göstermiştir. Sonuçlar, ChatGPT'nin kavramsal netleştirme ve temsil zenginleştirme potansiyeline sahip olduğunu; ancak öğrenme çıktılarının öğrencinin soru sorma niteliği ile doğrulama ve öz-düzenleme davranışlarıyla ilişkili olduğunu ortaya koymuştur. Bu sonuçlarına bağlı olarak üretken yapay zekâ araçlarının matematik öğretiminde etkili kullanımı, pedagojik olarak yapılandırılmış kullanım senaryoları ve rehberlik stratejilerine yönelik çeşitli öneriler sunulmuştur.
Anahtar Kelimeler: ChatGPT, Matematik Öğrenme, Fonksiyon Kavramı, Yapay Zekâ Destekli Öğrenme Ortamı, Etkileşim

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Introduction

Artificial intelligence (AI) is a multidisciplinary field that seeks to design and develop computer systems capable of emulating human cognitive processes such as thinking, learning, and problem solving (Zawacki-Richter et al., 2019). In its early stages, AI research primarily focused on rule-based systems and relatively limited algorithmic structures. However, with the rapid advancement of machine learning and natural language processing technologies, AI has evolved into more flexible, interactive, and adaptive systems (Kasneci et al., 2023). This technological progress has created substantial opportunities within the field of education, particularly by enabling the development of personalized learning environments that can accommodate students' individual learning pace and specific learning needs (Luckin et al., 2016). AI-supported systems assume functions such as providing immediate feedback, offering multiple explanatory approaches, and supporting the learning process at an individual level (Hwang & Tu, 2021). This development contributes to the transformation of traditionally teacher-centered instructional processes into more student-centered and interactive structures (Holmes et al., 2019). Systems referred to as large language models (LLMs) are capable of generating context-sensitive responses and establishing interaction-based learning environments through natural language engagement with users (Kasneci et al., 2023).

Chat Generative Pre-trained Transformer (ChatGPT) is an artificial intelligence–based chatbot developed using large language models built upon natural language processing technologies (Kasneci et al., 2023; UNESCO, 2023). ChatGPT has established a pedagogical environment in which students can pose questions related to mathematical concepts, receive immediate explanations, and direct their learning processes through interaction (Lo, 2023). The transformation of artificial intelligence tools from systems that merely transmit information into learning partners that support students' modes of thinking and problem-solving strategies represents one of the most evident indicators of educational change (Hwang & Tu, 2021). The opportunity for students to progress according to their individual learning pace, identify their mistakes, and receive immediate feedback has the potential to transform the mathematics learning process into a more dynamic structure that fosters self-regulated learning (Baidoo-Anu & Owusu Ansah, 2023). Although artificial intelligence offers substantial opportunities for education, the literature also emphasizes several important limitations that should be carefully considered when integrating AI-supported tools into learning environments. Large language models such as ChatGPT generate responses based on probabilistic predictions rather than genuine conceptual understanding, which may result in inaccurate explanations, misleading examples, or mathematically incorrect solutions (Kasneci et al., 2023; UNESCO, 2023). Furthermore, students may experience difficulties evaluating the correctness of AI-generated content, particularly in domains that require high levels of conceptual precision such as mathematics (Wardat et al., 2023). Concerns have also been raised regarding overreliance on AI systems, reduced critical evaluation of information, and the possibility that learners may accept generated responses without sufficient reflection (Bozkurt, 2023). Consequently, contemporary research increasingly emphasizes that AI should not be viewed as a replacement for teachers but rather as a cognitive support tool to complement learning disabilities whose educational effectiveness depends on the quality of learner interaction and teacher guidance (Holmes et al., 2019; Hwang & Tu, 2021).

Within the context of mathematics education, learning disabilities often stem from the density of abstract concepts (functions, limits, etc.) and the necessity of learning mathematical concepts not only at a procedural level but also at a conceptual level (Altun, 2015). Although the concept of function occupies a central position in mathematics instruction and constitutes one of the fundamental components of the high school mathematics curriculum, it remains among the topics with which students experience the greatest difficulty (Özkaya & İşleyen, 2012). Functions require understanding the relationships between variables, transitioning among graphical, algebraic, and verbal representations, and employing abstract thinking skills (Duval, 2006; Sfard, 1991). However, students frequently approach the concept of function merely at a procedural level and encounter difficulties in making sense of the domain, range, and relational structure at a conceptual level (Tall & Vinner, 1981; Özkaya & İşleyen, 2012). Tall and Vinner (1981) argue that learners often develop personal concept images that differ from formal mathematical definitions, leading to persistent misconceptions about functions. Similarly, Sfard (1991) suggests that understanding mathematical concepts requires a transition from operational interpretations toward structural conceptions, a shift that many students find challenging when learning functions. From a representational perspective, Duval (2006) emphasizes that mathematical understanding depends on the ability to coordinate and transform among different semiotic representations. Since functions can be represented graphically, algebraically, numerically, and verbally, students must continuously translate between these representations to construct conceptual understanding. Difficulties in these transformations often result in fragmented knowledge structures and procedural rather than conceptual learning. Consequently, the topic of functions has long been recognized as a critical area for examining students' conceptual development and mathematical reasoning processes. This situation highlights the need for process-oriented and interaction-based instructional approaches that support conceptual understanding in the teaching of functions (Cope et al., 2021). In other words, the concept of function represents a particularly appropriate context for investigating AI-supported learning interactions because it combines conceptual complexity with multiple representational demands.

The interactive affordances of ChatGPT may be particularly relevant in this context. The opportunities provided by AI-supported tools—such as requesting explanations, transitioning between representations, and discussing solution strategies—hold significant pedagogical potential for learning the concept of function (Wardat et al., 2023). In abstract concepts that require representational transformations, ChatGPT can facilitate establishing relationships among graphs, tables, and algebraic expressions, thereby enabling students to recognize their conceptual errors and structure their thinking (Wardat et al., 2023). However, understanding the educational value of these opportunities requires examining not only the correctness of AI-generated answers but also the nature of the questions students formulate and the types of support generated by the AI system during the learning process. The literature indicates that studies examining the effects of ChatGPT in mathematics education have largely adopted outcome-oriented approaches, while process-oriented research investigating students' cognitive behaviors and questioning tendencies during interactions with artificial intelligence remains limited (Bozkurt, 2023; Demircioğlu et al., 2024). Particularly in the Turkish context, the lack of qualitative studies examining the interaction patterns formed by ChatGPT during students' learning processes related to the topic of functions is noteworthy (Karabıyık, 2024).

Building upon the aforementioned gap in the literature, this study focuses on examining the nature of the interaction established between students and ChatGPT during the learning process of the topic of functions. This interaction is based on three complementary theoretical perspectives: (i) constructivist views of conceptual development in mathematics (Tall & Vinner, 1981), (ii) semiotic representation theory (Duval, 2006), and (iii) cognitive load and worked-example theory in learning processes (Clark & Paivio, 1991; Sweller, 1988). These frameworks collectively inform how students construct meaning in interaction with AI-based systems and how generative responses may function as cognitive support tools. Within the scope of this research, ChatGPT is not conceptualized as a tool that replaces the teacher; rather, it is approached as a learning partner that supports students' thinking processes and makes their cognitive orientations visible through the questions (prompts) they generate (Hwang & Tu, 2021). The study aims to contribute to the existing literature by offering both theoretical and practical insights into how AI-supported learning environments are reflected in mathematics learning processes from an interactional perspective (Bozkurt, 2023; Kasneci et al., 2023). In addition, the findings are expected to provide teachers with concrete examples illustrating how ChatGPT can be utilized as a cognitive guidance tool within classroom and hybrid instructional settings, thereby presenting a practice-oriented framework (Lo, 2023). In this respect, the study seeks to contribute to both academic discussions and educational practice by generating implications for the integration of AI-supported tools into instructional designs and for the effective structuring of teacher guidance. In line with this objective, the study aimed to address the following research questions:

1. In the learning process related to the topic of functions, what types of cognitively oriented questions (prompts) do students pose to ChatGPT?

2. In the learning process related to the topic of functions, what types of pedagogically functional responses does ChatGPT provide to students' questions?

This study is based on the assumption that the written expressions and question formats produced by students during their interaction with ChatGPT provide meaningful indicators for understanding cognitive orientations and support needs within the learning process. The notion of cognitively oriented questions refers to student-generated questions that reflect cognitive processes associated with understanding, interpreting, connecting, explaining, justifying, or evaluating mathematical ideas. Rather than focusing solely on obtaining a correct answer, these questions reveal learners' attempts to construct meaning, resolve conceptual uncertainty, and develop understanding of the topic. From this perspective, students' questions are considered indicators of their cognitive orientations during learning. Correspondingly, instead of conceptualizing ChatGPT outputs as cognitively oriented responses, this study interprets them as cognition-supporting responses that provide explanations, examples, representational connections, feedback, or pedagogically functional guidance intended to facilitate students' understanding. This distinction is important because cognitive orientation is fundamentally a characteristic of learners' thinking processes, whereas AI-generated responses function as external supports that may contribute to those processes.

Method

Research Design

This study was conducted within the framework of a qualitative research approach, as it aims to explore in depth the structure of the interaction established between students and ChatGPT during the learning process of functions, as well as the cognitive processes that emerge throughout this interaction. Qualitative research allows for the examination of phenomena within their natural contexts, particularly in situations where individuals' experiences, interaction patterns, and meaning-making processes cannot be adequately explained through quantitative variables alone (Creswell, 2013;

Yıldırım & Şimşek, 2018). In this respect, qualitative approaches provide a robust theoretical framework for studies seeking to analyze the internal structure of participants' experiences and interpretations (Merriam, 2009). In the present study, the context-based examination of both the questions generated by students during their interaction with ChatGPT and the structure of the responses provided by the system required the adoption of a qualitative methodological perspective.

Within this qualitative framework, the study employed a case study design. Case studies enable researchers to examine a contemporary phenomenon holistically and in depth within its real-life context, particularly when the boundaries between the phenomenon and its context are not clearly defined (Yin, 2018). The case investigated in this study consists of the interaction patterns established between students and ChatGPT, an artificial intelligence–based large language model, during the process of learning the topic of functions. Since the research sought to analyze both students' questioning patterns and the response-generation structures of ChatGPT within a natural learning environment, the case study design was considered an appropriate methodological choice for addressing the research objectives.

Participants

The study group consisted of eight 10th-grade students enrolled in a private science high school located in the southern region of Türkiye. Among the participants, five were female and three were male. The participants were selected through purposeful sampling, a strategy commonly employed in qualitative research to obtain rich and in-depth data relevant to the research problem (Yıldırım & Şimşek, 2018). This sampling approach was preferred because it enables the identification of participants who are most capable of reflecting the phenomenon under investigation and providing detailed insights into the interaction process examined in the study. The participant selection process was guided by three criteria that were directly related to the purpose of the research. First, students were required to participate voluntarily in all stages of the study. Second, participants were expected to have prior experience using ChatGPT or similar AI-based conversational systems. This criterion was established to minimize difficulties arising from unfamiliarity with the technology and to ensure that students could focus on the learning process rather than on learning how to use the tool itself. Third, participants were required to demonstrate sufficient cognitive readiness for engaging in learning activities related to the concept of functions. Cognitive readiness in this study referred to students' prior exposure to the topic of functions within the mathematics curriculum, their ability to interpret mathematical representations such as graphs and algebraic expressions at a basic level, and their capacity to formulate questions and sustain dialogue during interaction-based learning activities. Information regarding these characteristics was obtained through consultation with the mathematics teacher and a review of students' recent mathematics performance records. This criterion was considered important because the study aimed to examine the nature of students' interactions with ChatGPT while learning functions; therefore, participants needed to possess the foundational knowledge required to meaningfully engage in discussions related to the topic. In accordance with these criteria, the selected students participated in the ChatGPT interaction process, and all data collection procedures were carried out with this group. Prior to the implementation of the study, participants were informed about the purpose, scope, and procedures regarding the use of the collected data. To ensure confidentiality, students were represented using coded identifiers during the analysis process, and no personal information was included in the reporting of the findings. Furthermore, the necessary ethics committee approval and institutional permissions were obtained before the study was conducted.

Data Collection Procedures

In the study, the primary data collection tool consisted of the written interaction records generated by students during their engagement with ChatGPT. These records included the questions (prompts) that students directed to ChatGPT regarding the topic of functions and the responses produced by the system. All interactions were digitally recorded throughout the implementation process and transcribed for analysis. These prompt–response records constituted the main dataset of the study.

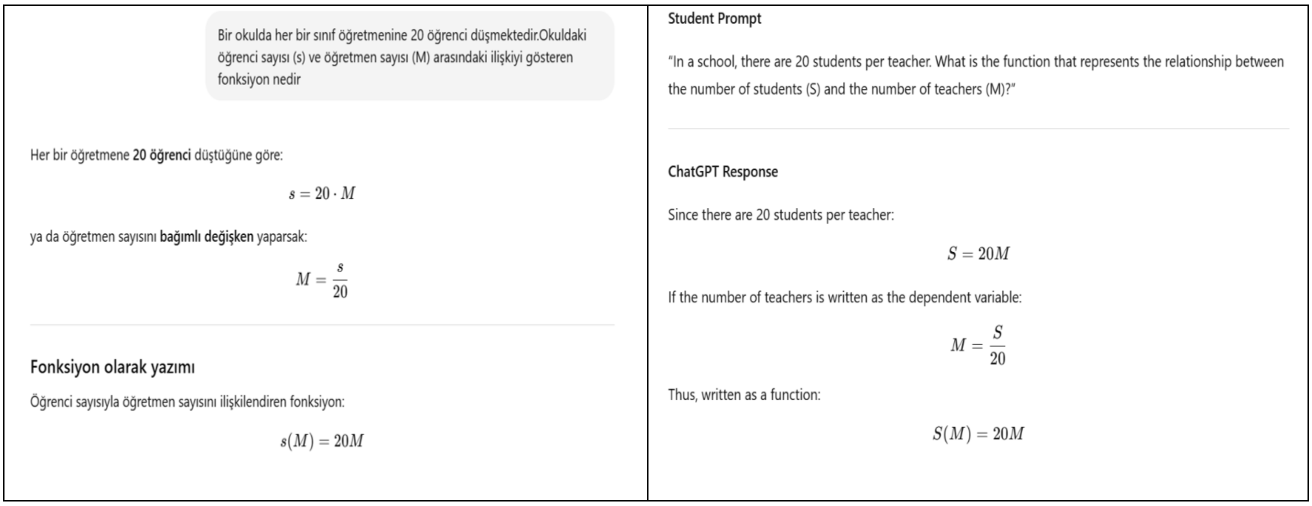
The interactions were conducted using ChatGPT (GPT-4) and were carried out in Turkish, which was the participants' native language and the language of instruction in their mathematics classes. Students were encouraged to interact with ChatGPT naturally and to formulate questions according to their individual learning needs while working on tasks related to the concept of functions. In addition to the interaction records, a worksheet related to the topic of functions was used during the implementation process. The worksheet was not considered a data collection instrument and was not included in the data analysis. Rather, it served as a contextual support tool designed to structure the learning environment, provide a common mathematical context for all participants, and guide students' interactions with ChatGPT. The worksheet included activities and questions related to the concept of functions and was used solely to facilitate the interaction process. Accordingly, the analyses reported in this study were based exclusively on the written prompt–response records obtained from student–ChatGPT interactions.

Procedure

In the study, a two-stage pilot implementation was conducted prior to the main application in order to test the functionality of the data collection process, evaluate the alignment of the data to be obtained from student–ChatGPT interactions with the research objectives, and clarify the implementation procedures. In qualitative research, preliminary studies conducted before the main implementation constitute an important phase in terms of testing the feasibility of the data collection process, becoming familiar with the research setting, and identifying potential methodological issues (Creswell, 2013; Merriam & Tisdell, 2016). In this context, the pilot implementation was carried out with two students who were not included in the main study group, and the records obtained during the pilot process were not included in the analysis. The screenshots and ChatGPT outputs collected during the pilot were documented to justify methodological decisions for the main implementation and to support the traceability of the process.

In the first stage of the pilot, in order to observe how students used ChatGPT in a natural manner, no guidance, constraints, or initial prompts were provided; instead, students were given access to the worksheet and ChatGPT, and the process was left open-ended. At this stage, although students were expected to attempt to solve the problem situations in the worksheet through interaction with ChatGPT, it was observed that in some interactions students directly copied the question and entered it into ChatGPT, and ChatGPT generated a ready-made solution by solving the problem on behalf of the student. This situation was evaluated as a methodological risk, as it tended to reduce the student's active cognitive engagement in the solution process and had the potential to weaken the data related to "student-generated questioning" and the "interaction-based thinking process," which constituted the focus of the study.

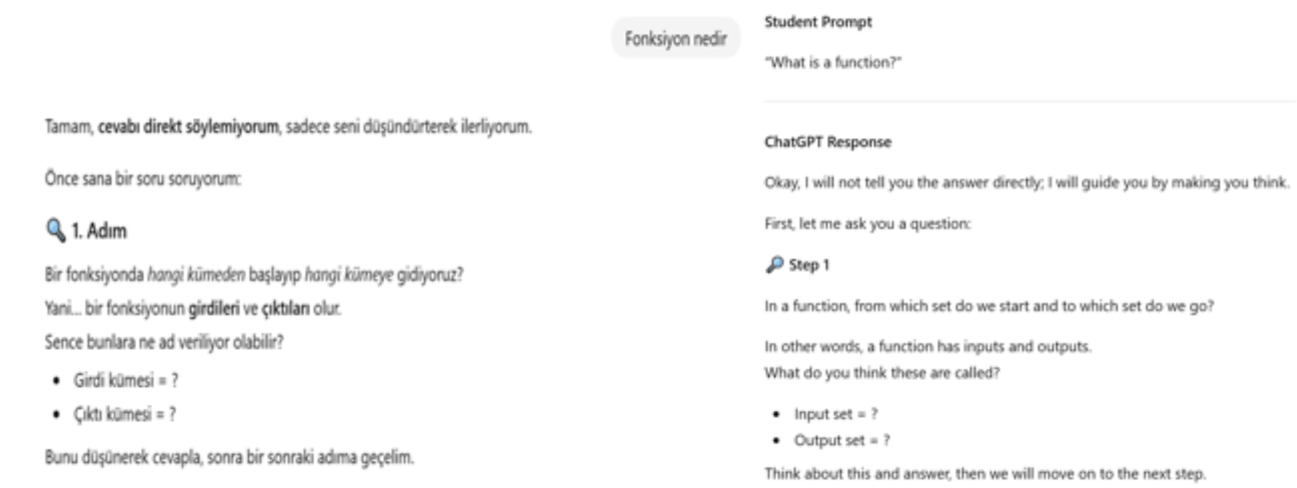
Figure 1.
Pilot Implementation Stage 1: Student-ChatGPT Interaction(Original and English Transcription)



The example presented in Figure 1 makes visible a usage pattern in which the student transferred the question as it was and ChatGPT generated a direct solution; therefore, it reveals the necessity of establishing certain frameworks in the main implementation to preserve the quality of the data.

In the second stage of the pilot implementation, an initial prompt assigning a role to ChatGPT and aiming to prevent the system from providing direct answers was tested in order to reduce the tendency observed in the first stage of having the problem solved directly. At this stage, before the student initiated the interaction, the following instruction was added to the system: "In this application, consider me as a 10th-grade student; do not provide the direct answer to my questions, but try to guide me toward the solution." However, this implementation demonstrated that such a prompt could excessively constrain ChatGPT's response-generation behavior and disrupt the flow of interaction. Therefore, the role-assigned initial prompt was evaluated as interfering with the student's natural interaction process and restricting the flow of data.

Figure 2.
Pilot Implementation Stage 2: Example of a Constrained Response Following a Role-Assigned Prompt (Original and English Transcription)



The example presented in Figure 2 demonstrates that the role-assigned initial prompt could, in certain cases, significantly restrict ChatGPT's ability to generate responses to the student. Based on this finding, it was decided that such artificial directives would not be used in the main implementation; instead, clear limitations aimed at preserving data quality without disrupting the flow of interaction were adopted.

As a result of the pilot findings, an explicit limitation was introduced in the main implementation, instructing students not to copy and paste the questions from the worksheet directly into ChatGPT and not to upload images of the questions. Instead, students were expected to reconstruct the problem situation in their own words when presenting it to ChatGPT and to request conceptual explanations, hints, and guidance rather than directly demanding the final answer during the solution process. This framework was evaluated as supporting students' active engagement in the solution process and strengthening the quality of interaction-based qualitative data targeted by the study.

In line with this methodological framework, the main implementation was conducted in the school's computer laboratory, with each student assigned to an individual computer, and care was taken to ensure that students' interactions with ChatGPT were carried out in the same physical environment and under similar conditions. The established limitations were explained to students in a clear and comprehensible manner; in particular, it was emphasized that questions from the worksheet should not be copied, images should not be uploaded, and the problem situation should be reconstructed in the student's own words before being presented to ChatGPT. This orientation was structured to encourage students to benefit from artificial intelligence through conceptual explanations, hints, and guidance rather than direct answer generation.

Students were provided with a worksheet on the topic of functions that included open-ended problems (Appendix 1); while working on these questions, they were expected to engage in written interaction with ChatGPT to advance their solution processes. The worksheet was used as a contextual tool to structure the learning environment in which students interacted with ChatGPT and to initiate the interaction process. Within this framework, students articulated each problem situation in their own words when presenting it to ChatGPT; during the solution process, they requested conceptual explanations when needed, asked for hints, sought guidance regarding possible next steps, or sustained the interaction by questioning the accuracy of the generated responses.

During the implementation process, the researcher personally conducted the procedure. The researcher circulated in the classroom during the activity and observed students' screens at regular intervals to ensure that interaction patterns aligned with the study instructions. When instances of direct copying or non-engaged input were observed, students were reminded of the study requirement in a neutral and non-directive manner, without providing any mathematical or content-related assistance. These observations indicated that students generally complied with the instruction to reformulate their questions, and no systematic deviation from this expectation was identified during the session. The researcher's role throughout the implementation was limited to technical facilitation and procedural monitoring. This included ensuring access to ChatGPT, resolving minor technical issues (e.g., login or connectivity problems), and maintaining the overall flow of the session. No guidance was provided regarding mathematical reasoning, problem-solving strategies, or the formulation of prompts, in order to preserve the authenticity of the interaction data. The implementation was completed in a two-hour session. The decision to conduct the study within a single two-hour session was based on the exploratory nature of the research design and the aim of capturing naturalistic interaction patterns between students and ChatGPT within a focused learning episode. Given that the study did not aim to examine long-term learning outcomes but rather the immediate nature of questioning behaviors and AI-generated responses, a single-session design was considered sufficient to generate rich interaction data within a controlled instructional context.

The research was conducted in accordance with the ethics approval dated 03.11.2025 and numbered 230, granted by the Mersin University Faculty of Education Ethics Committee. Prior to the implementation phase, participants were informed about the purpose and scope of the study, the data collection procedures, the principle of voluntary participation, and the confidentiality of the research process. In this context, student voluntary participation forms as well as parental informed consent forms were obtained. Furthermore, the necessary permissions to conduct the implementation at the institution where the research took place were secured from the relevant Directorate of National Education. All procedures throughout the research process were carried out in accordance with established research and publication ethics principles. The collected data were stored in compliance with confidentiality principles, and participants were represented using coded identifiers during the reporting process, ensuring that no personally identifiable information was disclosed. The raw interaction transcripts obtained through this process were subsequently organized and prepared for the analysis stage in alignment with the objectives of the study.

Data Analysis

In the study, the written interaction records generated by students during their learning process of functions with ChatGPT were analyzed using a content analysis approach (Creswell, 2013). The primary unit of analysis consisted of prompt–response pairs formed by the questions (prompts) directed by students to ChatGPT and the responses provided by ChatGPT to these questions. The analysis process included the holistic reading of the data, the identification of meaningful units of expression, the generation of codes through open coding, and the formation of inductive categories by grouping similar codes together (Punch, 2013; Baş & Akturan, 2008).

Student prompts and ChatGPT responses were coded independently by two experts in mathematics education in order to strengthen the conceptualization of the data. In cases of disagreement between coders, discussions were conducted until consensus was reached, and the final code–category structure was established. To ensure the reliability of the coding process, the inter-coder agreement rate was calculated. Using the formula proposed by Miles and Huberman (2014) ([Agreement / (Agreement + Disagreement) × 100]), the inter-coder agreement was determined to be 90%. Based on the finalized coding scheme, the resulting categories were presented in tables, accompanied by their corresponding frequency and percentage values.

In line with the objectives of the study, the analysis was conducted along two focal points. In the first focus, the prompts directed by students to ChatGPT were classified according to their cognitive orientations; in the second focus, ChatGPT's responses were categorized according to types of explanation.

In the classification of student prompts, five cognitive categories were identified: definition-oriented, representation-oriented, context extension, self-regulation, and verification. The categories, their definitions, and representative example statements are presented in Table 1.

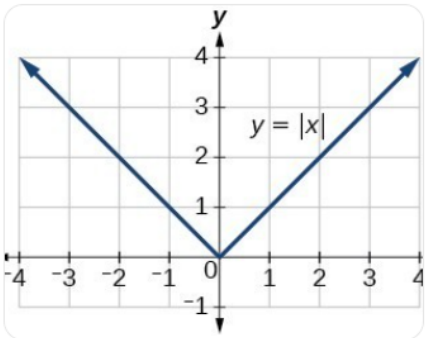
Table 1.
Classification of Student Questions According to Cognitive Categories in ChatGPT–Student Interactions

Category	Definition	Example
Definition-Oriented	It describes instances in which the student recalls, identifies, or reformulates a concept, term, or fundamental rule. This level corresponds to the foundational stage of knowledge access.	"What is a function?" – This is the most frequently asked question when students experience difficulty defining the concept of function.
Representation-Oriented	It describes instances in which the student translates or coordinates a mathematical concept across graphical, tabular, verbal, and algebraic representations. The coordination of multiple representations enhances conceptual understanding in mathematics.	"How can I express this relationship as a formula?" – An actual statement used by students during a graph-to-equation transformation.
Context Extension	It refers to instances where students deviate from the intended conceptual focus of the task either by shifting to an unrelated topic (topic drift) or by extending prior knowledge	"A negative number always means going downward." – An authentic example of a student

	inappropriately, resulting in misconceptions or overgeneralizations.	misconception related to incorrect contextual transfer.
Self-Regulation	It describes the student's regulation of their own learning process, including error recognition, requests for clarification or correction, and the reconstruction of their solution. This reflects metacognitive monitoring.	"I think I made a mistake; let me check it again." – An example of metacognitive monitoring during problem solving.
Verification	It describes instances in which the student evaluates the validity of their solution, critically examines its justification, or confirms it through an alternative strategy. This process is central to mathematical reasoning.	"Does this answer really make sense?" – An expression used by students during verification and justification processes.

In the classification of ChatGPT's responses, three categories were identified: exemplifying, visualizing, and preventive/corrective. The responses were categorized based on patterns reflecting the nature of mathematical explanation. The definitions of the categories and representative examples are presented in Table 2.

Table 2.
Classification of ChatGPT Responses According to Types of Explanation in ChatGPT–Student Interactions

Category	Definition	Example
Exemplifying Responses	ChatGPT's explanation of a mathematical concept by concretizing it through examples. Example-based explanations support the student's conceptual meaning-making process by making the fundamental properties of the concept visible.	"Example: 1 → 4 2 → 4 3 → 5 (Here, 1 and 2 map to the same value → not one-to-one)"
Visualizing Responses	It describes ChatGPT's presentation of a mathematical concept through graphical, tabular, verbal, and algebraic representations, as well as its coordination across these forms. The use of multiple representations facilitates the recognition of conceptual connections and enhances mathematical understanding.	
Preventive/Corrective Responses	It describes ChatGPT's capacity to anticipate possible misconceptions or to address existing errors, thereby facilitating the accurate conceptual construction. Corrective feedback mitigates misconceptions and strengthens both the validity and retention of learning.	✗ In the diagram: If two different elements of A map to the same element of B → it is not one-to-one.

Finally, in the reporting process, the excerpts selected for each category were presented verbatim without altering the meaning of the data; thus, it was aimed to ground the findings in the data and to support the transparency and traceability of the analysis process (Miles & Huberman, 2014).

Results

The findings derived from the prompt–response pairs based on the ChatGPT interaction records, which constitute the primary data of the study, are presented. Frequency and percentage values for each category are provided, and are supported by direct example screenshots.

The frequency and percentage distributions of the prompts generated by students during their interactions with ChatGPT regarding the topic of functions are presented in Table 3.

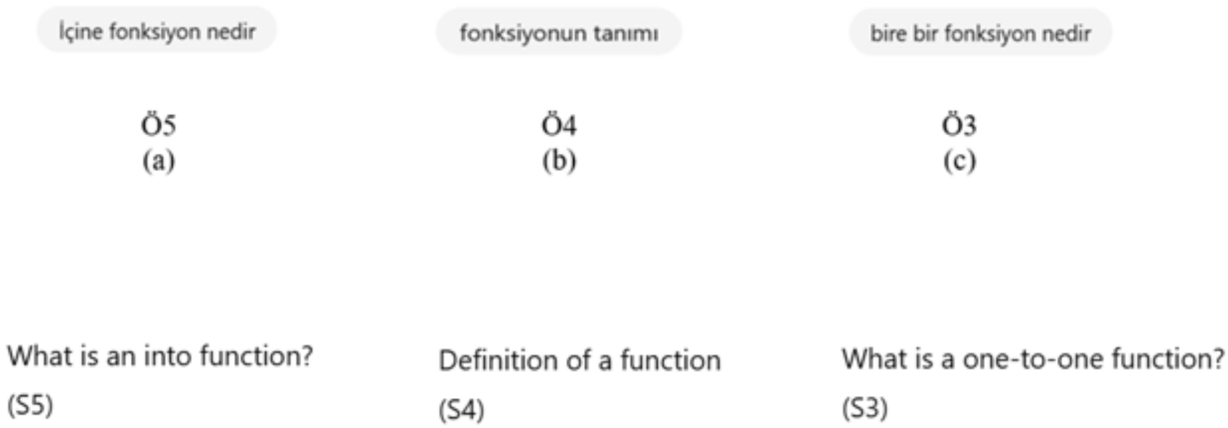
Table 3.
Distribution of ChatGPT–Student Prompts According to Cognitive Categories

Category	*f*	*%*
Definition-Oriented	*21*	*41,2*
Representation-Oriented	*14*	*27,5*
Context Extension	*7*	*13,7*
Verification	*6*	*11,8*
Self-Regulation	*3*	*5,9*
Total	*51*	*100*

When Table 3 is examined, it is observed that definition-oriented prompts constitute the highest proportion (f = 21; 41.2%). This is followed by representation-oriented prompts (f = 14; 27.5%). While the categories of context extension (f = 7; 13.7%) and verification (f = 6; 11.8%) are represented at a moderate level, the lowest proportion is found in the self-regulation category (f = 3; 5.9%).

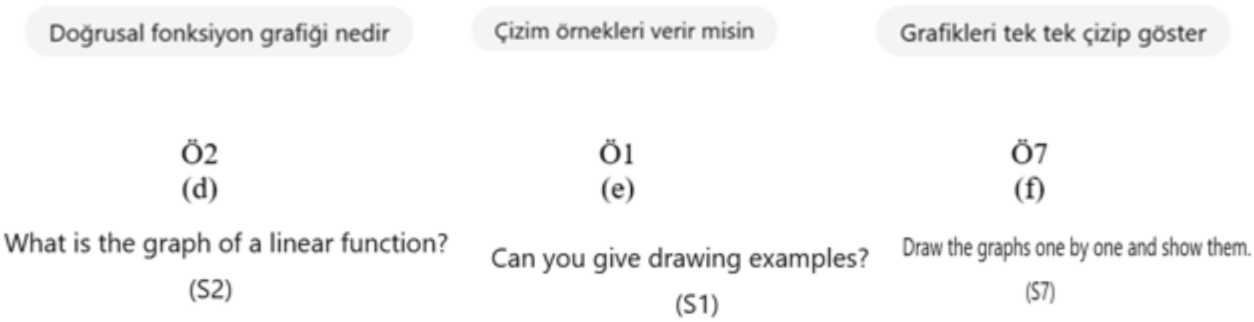
Definition-oriented prompts encompass students' questions aimed at requesting basic definitions or properties related to the concept of function and types of functions. In this category, students sought explanations in the form of "what is…?" by using brief and direct expressions indicative of introductory-level conceptual inquiry. Examples of this pattern can be observed in prompts such as "What is a function?", "Definition of function," and "What is a one-to-one function?" (Figure 3a–c).

Figure 3.
Examples of Student Prompts in the Definition-Oriented Category:Original Turkish and English Transcription (a) S5, (b) S4, (c) S3.



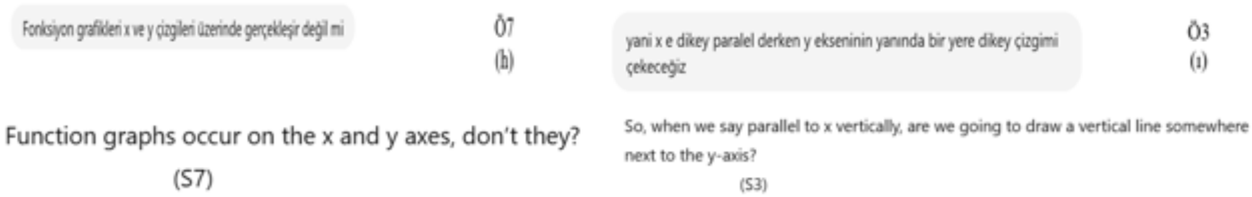
Representation-oriented prompts include students' requests related to the production or presentation of graph/drawing-based representations of functions. In this category, students not only requested explanations regarding what a graph is, but also used representation-focused directives such as asking for drawing examples and requesting that graphs be drawn step by step or individually. Prompts such as "What is the graph of a linear function?", "Can you provide drawing examples?", and "Draw and show the graphs one by one" exemplify this category (Figure 4d–f).

Figure 4.
Examples of Student Prompts in the Representation-Oriented Category: Original Turkish and English Transcription (d) S2, (e) S1, (f) S7.



Verification prompts encompass the expressions directed by students to check their own reasoning or understanding developed during the interaction, to seek confirmation, and to eliminate uncertainty. In such prompts, students frequently used confirmation patterns such as "…right?" and reformulated their understanding based on previous explanations using statements beginning with "so…". Additionally, instances in which students directly requested clarification of a symbol or concept in ChatGPT's response, such as "What do you mean by … here?", were also evaluated within this category. Examples of this pattern are presented in Figure 5 (Figure 5h, 3i).

Figure 5.
Examples of Verification-Oriented Student Prompts: Original Turkish and English Transcription (h) S7, (ı) S3.

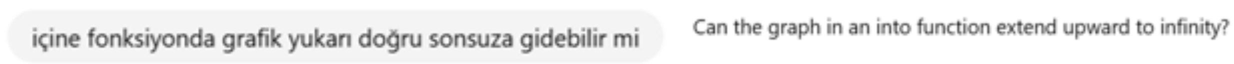


Context extension prompts encompass instances in which students posed questions that were not directly related to the topic or context of functions during the interaction. In this category, students shifted from the targeted subject area to a different mathematical content area by asking a new question. For example, prompts such as "How is the area of a square calculated?" and "How are fractional equations formed?" indicate requests that diverge from the context of functions (Figure 6k–m).

Figure 6.
Examples of Context Extension Prompts: Original Turkish and English Transcription (k) S6, (l) S4, (m) S8.

Self-regulation prompts include questions generated by students with the aim of monitoring and regulating their own learning processes, testing prior knowledge, ensuring conceptual coherence, and examining boundary conditions related to the topic. In such prompts, the student not only seeks to define the concept but also attempts to structure their understanding by posing control questions such as "What happens under which condition?" The prompt "In an injective function, can the graph increase upward indefinitely?" exemplifies this pattern (Figure 7).

Figure 7.
Example of a Self-Regulation Prompt: Original Turkish and English Transcription



When the overall distribution is examined, it is observed that student prompts predominantly focused on requesting definitions and explanations of fundamental concepts, while requests related to representation production and graphical presentation also held a notable place. Prompts characterized by verification and context extension were observed in more limited numbers, and self-regulation-oriented prompts were identified as having the lowest frequency. The findings indicate that student interactions diversified across different cognitive orientations.

The frequency and percentage distributions of ChatGPT's responses to student prompts according to explanatory categories are presented in Table 4.

Table 4.
Distribution of ChatGPT Responses According to Types of Explanation

Category	f	%
Exemplifying Responses	78	35,3
Visualizing Responses	73	33,1
Preventive Responses	70	31,6
Total	221	100

Exemplifying explanations encompass responses in which ChatGPT explains a concept or rule through concrete examples, value sets, mapping lists, and numerical representations. In such responses, the explanation is supported by direct example functions, example mappings, and list-format representations.

The exemplification of types of graphs that represent functions, the demonstration of one-to-one functions through element–image mappings, and the definition of a function in the form of ordered pair lists constitute examples of this category (Figure 8a–c).

Figure 8.
Examples of ChatGPT Responses in the Exemplifying Explanation Category: Original Turkish and English Transcription (a), (b), (c).

✔ **Fonksiyon olan grafik örnekleri**

- Doğru ($y = 2x + 1$)
- Parabol ($y = x^2$)
- Mutlak değer ($y = |x|$)
- Üstel fonksiyon ($y = 2^x$)

Hepsi dik çizgiyle yalnızca **1 kez kesilir**.

(a)

Examples of graphs that are functions

- Line ($y = 2x + 1$)
- Parabola ($y = x^2$)
- Absolute value ($y = |x|$)
- Exponential function ($y = 2^x$)

All of them are intersected only once by a vertical line.

(a)

En basit örnek

A = {1,2,3}
B = {4,5,6}

f(1) = 4
f(2) = 5
f(3) = 6

Hiçbir y tekrar edilmedi
Her x, B'de farklı yere gitti
✔ **Birebir fonksiyon**

(b)

The simplest example

A = {1, 2, 3}
B = {4, 5, 6}

f(1) = 4
f(2) = 5
f(3) = 6

No y-value is repeated.
Each x goes to a different value in B.
One-to-one function.

(b)

ÖRNEK 1 — Liste biçiminde fonksiyon

$$f = \{(1,3), (2,5), (4,7)\}$$

Bu fonksiyon **geçerli bir fonksiyondur** çünkü her x → tek y.

(c)

EXAMPLE 1 — Function in list form

f = {(1,3), (2,5), (4,7)}

This function is a valid function
because each x corresponds to exactly one y.

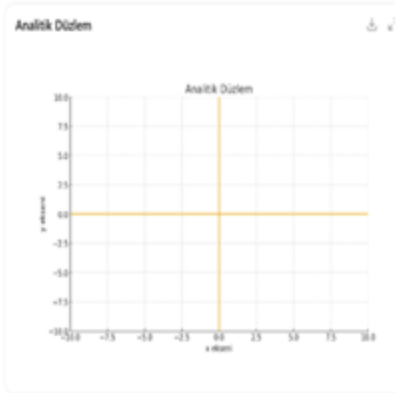
(c)

Visualizing explanations include responses in which ChatGPT employs visual representations such as graphs, analytic plane drawings, function graph generation, and mapping diagrams. In these responses, conceptual explanations are presented through direct graphical representations or visual schematic formats.

The drawing of a parabola graph, the representation of functions on the analytic plane, and the demonstration of a function through input–output mapping diagrams were evaluated within this category (Figure 9d–f).

Figure 9.
Examples of ChatGPT Responses in the Visualizing Explanation Category: Original Turkish and English Transcription.

(d)



(e)

(f)

Into Function: $f(x) = x^2$

(d)

Analytical Plane

(e)

1. Function (Each element in A maps to exactly one element)

Explanation:

Each element in set A maps to exactly one element — this is a function.

(f)

Preventive explanations encompass responses in which ChatGPT explicitly distinguishes situations that may lead to conceptual misconceptions, presents function and non-function cases in a comparative manner, and includes warnings against potential errors. In such responses, alongside correct examples, non-function graph examples and non–one-to-one function cases are particularly emphasized.

The listing of graph types that do not represent functions, the demonstration of non-function cases through the vertical line test, and the explanation of a non–one-to-one function example constitute illustrations of this category (Figure 10g–ı).

Figure 10.

Examples of ChatGPT Responses in the Preventive Explanation Category: Original Turkish and English Transcription.

(g)

(h)

(ı)

Examples of graphs that are NOT functions

- Circle
- Ellipse
- "C"-shaped curve
- Rotating curves
- Closed shapes that create peaks or valleys at the same x-value

In these cases, a vertical line intersects the graph at two or more points.

(g)

NOT FUNCTION GRAPHS

1. Circle

A vertical line intersects the circle at two points — not a function.

(h)

2. Example of a function that is NOT one-to-one

$f(x) = x^2$

Why is it not 1–1?

- $f(2) = 4$
- $f(-2) = 4$

Different x-values map to the same y-value.
Therefore, it is not one-to-one.

(ı)

When the overall distribution is examined, it is observed that ChatGPT's responses diversified through the use of different types of explanations. The responses were found to be concretized through examples, supported by graph- and diagram-based visual representations, and presented in a balanced manner by distinguishing contrasting or inverse properties.

Discussion, Conclusion and Recommendations

In this study, the prompt–response pairs derived from students' interactions with ChatGPT regarding the topic of functions were discussed along two primary dimensions: the cognitive orientations of the prompts produced by students and the types of explanations provided in ChatGPT's responses. The initial findings indicate that students most frequently generated definition-oriented prompts. This suggests that, when dealing with an abstract and multi-representational mathematical concept such as function, students tend to prioritize clarifying definitions and fundamental properties. Within the framework of the revised Bloom's taxonomy, such requests may be associated with cognitive processes primarily related to the levels of remembering and understanding (Dao & Le, 2023; Anderson & Krathwohl, 2001). Therefore, it can be argued that students often seek alignment between the formal definition and their mental concept image of the function concept, and that definition-based questions reflect this tension (Tall & Vinner,1981; Vinner & Dreyfus, 1989). In this respect, the predominance of definition-oriented prompts suggests that students position ChatGPT as a tool for conceptual clarification in the initial stage of interaction.

The fact that representation-oriented prompts ranked second is consistent with the nature of learning functions. Understanding the concept of function requires the ability to transition among verbal, algebraic, and graphical representations (Mirin, 2024). Students' requests for graphs and drawings may be interpreted as efforts to construct the concept across different representational systems. The complementary and constructive roles of multiple representations in learning (Ainsworth, 2006) indicate that coordination among representations strengthens conceptual understanding. Studies on functions also emphasize that graphical interpretation and representational transformations lie at the core of functional understanding (Carlson et al., 2002). In this context, students' requests for representational production from ChatGPT demonstrate that artificial intelligence is being used as a representational support tool.

Prompts in the verification category indicate that students did not unconditionally accept ChatGPT's responses; rather, they sought confirmation in specific instances. Questioning and confirmation-seeking behaviors in the learning process are associated with cognitive monitoring and comprehension control (Van Loon & Roebers, 2024). Considering the importance of justification and verification practices in mathematics education (Stylianides, 2007), such prompts suggest an orientation toward regulating one's own thinking. Although they appear less frequent, verification prompts indicate that the interaction was capable of acquiring a critical dimension.

The limited representation of self-regulation prompts suggests that students employed metacognitive regulation strategies to a restricted extent during their interaction with ChatGPT. Self-regulation refers to the set of processes through which individuals set goals, monitor their progress, and adjust their strategies accordingly (Zimmerman, 2000). The decisive role of metacognitive control in mathematical problem solving has been particularly emphasized (Zepeda & Nokes-Malach, 2023). Therefore, even though self-regulation prompts were few in number, their presence is pedagogically meaningful; however, the broader emergence of such prompts may be considered unlikely to occur spontaneously and should be supported through instructional guidance and task design.

Prompts categorized as context extension may be associated with the phenomenon of topic drift observed in chat-based digital environments. In interaction-based environments, shifts in focus and topic branching are identified as natural risks (Herring, 2010). The broad response-generation capacity of artificial intelligence systems may make it difficult for students to maintain the existing conceptual frame. This finding indicates the importance of explicitly structuring task boundaries and problem contexts in AI-supported learning processes (UNESCO, 2023). In additon, the presence of contextual expansion indicates two distinct patterns: (i) conceptual misinterpretations arising from inappropriate transfer of prior knowledge and (ii) partial or complete shifts away from the intended mathematical focus. While some instances reflect misconceptions within the domain of functions, others represent topic drift, where the interaction diverges into unrelated conversational directions. This dual nature suggests that deviations are not solely conceptual errors but may also be influenced by the open-ended structure of ChatGPT-based interaction environments.

In conclusion, within the scope of this study, since a substantial portion of the prompts clustered around requests for definitions/properties and representations/graphs, brief instructional templates that enable students to produce these two types of prompts more systematically may be employed in classroom ChatGPT use (e.g., "write the definition + ask for an example + request a non-function counterexample"). Given that verification and self-regulation prompts were observed at low frequencies, mandatory control steps that direct students toward "control/confirmation" types of prompts may be incorporated into the interaction process (e.g., "How would you verify the correctness of the response?" "Generate a non-function example"). As instances of context extension were identified, explicit boundaries aimed at preserving the task context may be established in classroom activities conducted with ChatGPT (e.g., clear directives such as "produce responses only within the context of functions"; brief refocusing prompts when the topic deviates). An important methodological consideration in this regard is the potential influence of the worksheet structure on students' question-generation patterns. Since the worksheet included tasks aligned with an instructional approach focusing on functions, it may have indirectly shaped the distribution of question types, particularly by encouraging the prominence of recognition- and representation-oriented questions. Accordingly, the findings should be interpreted not solely as reflections of students' spontaneous cognitive orientations, but rather as interaction patterns emerging within a structured task environment. This limitation should be taken into account when considering the generalizability of the results.

Another finding obtained indicates that ChatGPT responses were clustered under example-based, visualizing, and preventive explanation types, demonstrating that the model was capable of generating explanation types analogous to different instructional strategies. The predominance of example-based explanations aligns with previous findings suggesting that worked examples help reduce cognitive load and facilitate the construction of mental schemas (Ngu, 2025). The concretization of abstract mathematical concepts through examples may contribute to a clearer delineation of conceptual boundaries.

Visualizing explanations may be associated with the dual coding approach, which posits that presenting textual information together with visual representations can support learning (Clark & Paivio, 1991). However, the contribution of multiple representations to learning depends on their functional use. Given the central role of graphical representations in the teaching of functions (Ainsworth, 2006), visualizing responses can be considered an explanatory strategy that facilitates conceptual understanding and meaning-making.

Preventive explanations indicate that the model was able to signal potential misconceptions in advance. This situation may be associated with the scaffolding approach, which refers to supporting the learner at critical thresholds (Van de Pol et al., 2015). It is well established that addressing misconceptions at an early stage strengthens learning (Butterfuss& Kendeou, 2020). The presence of preventive explanations suggests that generative artificial intelligence carries not only an explanatory but also a guiding potential.

In conclusion, it is demonstrated that ChatGPT has the potential to generate explanations aimed at conceptual clarification, representational enrichment, and the prevention of misconceptions in the teaching of functions. Moreover, since these response types were distributed at comparable rates, teachers may standardize students' "ways of reading responses" by explicitly identifying and labeling example-based/visualizing/preventive explanations separately in the classroom (e.g., "Does this response provide an example, visualize, or warn?"). Although ChatGPT demonstrated multiple pedagogically relevant response types, it should also be acknowledged that large language models may occasionally produce incorrect or misleading mathematical explanations. This limitation is particularly important in mathematical domains requiring formal accuracy and conceptual rigor. Therefore, the educational value of ChatGPT should be interpreted as conditional upon user verification and instructional scaffolding rather than as inherently reliable.

Within the scope of this study, the findings based on interaction records are limited to the description of prompt and response contents; no inferences were made regarding students' learning levels, achievement, or other data sources. In future studies, the relationship between prompt categories and response types (e.g., which type of prompt more frequently triggered which type of response) may be systematically examined. The distribution of categories may be comparatively investigated by replicating the study across different school types, grade levels, and mathematical topics. By preserving the same coding scheme, studies may be conducted with larger samples, and the stability of coding categories as well as inter-rater agreement may be retested across different contexts.

Contributors

No individuals or institutions have contributed to this article.

Ethics Committee Approval Information

This program was ethically approved by the Mersin University Faculty of Education Ethics Committee with decision number 230 on November 3, 2025.

Conflict of Interest

The authors declare that there is no conflict of interest related to this work.

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Genişletilmiş Özet

Giriş

Yapay zekâ (YZ), insanın düşünme, öğrenme ve problem çözme gibi bilişsel süreçlerini taklit edebilen sistemlerin geliştirilmesini hedefleyen çok disiplinli bir alandır. Son yıllarda makine öğrenmesi ve doğal dil işleme alanlarındaki ilerlemeler, YZ'nin eğitim ortamlarında daha etkileşimli ve kişiselleştirilebilir biçimlerde kullanılmasını mümkün kılmıştır. Bu dönüşüm, özellikle büyük dil modellerine (BDM) dayalı sohbet tabanlı sistemlerin, öğrenenlerin anlık geri bildirim alması, farklı açıklama biçimlerine erişmesi ve öğrenme sürecini kendi hızında sürdürmesi açısından yeni olanaklar sunduğunu göstermektedir. Bu sistemler arasında yer alan ChatGPT, doğal dil üzerinden iletişim kurabilmesi ve bağlama duyarlı yanıtlar üretebilmesi nedeniyle öğrencilerin kavramlara ilişkin soru sorma, açıklama isteme ve öğrenme sürecini etkileşim yoluyla yönlendirme pratiklerini görünür kılan bir öğrenme ortamı oluşturmuştur.

Matematik eğitimi bağlamında ChatGPT gibi üretken YZ araçlarının önemi, yalnızca "doğru cevabı verme" kapasitesinden değil; öğrencinin kavramsal anlamayı yapılandırmasına yardımcı olabilecek açıklama, örnekleme, temsil sunma ve hata risklerine karşı uyarıda bulunma gibi öğretimsel işlevler üretebilmesinden kaynaklanmaktadır. Bununla birlikte matematik öğrenme güçlükleri çoğu zaman soyut kavramların yoğunluğu, temsiller arası geçiş gereksinimi ve kavramsal anlamanın işlem temelli yaklaşımlarla sınırlı kalması gibi nedenlerle ortaya çıkmaktadır. Lise matematik programının merkezi kavramlarından biri olan fonksiyon, değişkenler arasındaki ilişkiyi yorumlama, grafiksel–cebirsel–sözel temsilleri koordine edebilme ve soyut düşünme becerilerini gerektirmesi bakımından öğrencilerin sıklıkla zorlandığı bir konu alanıdır. Öğrenciler fonksiyon kavramını kimi zaman yalnızca "formül" ya da "grafik" ile özdeşleştirebilmekte; tanım kümesi–değer kümesi ilişkisi, birebirlik ve fonksiyon olma koşulları gibi kavramsal sınırları anlamlandırmada güçlük yaşayabilmektedir. Bu nedenle fonksiyon öğretiminde, öğrencinin düşünmesini görünür kılan ve temsiller arası koordinasyonu destekleyen süreç odaklı yaklaşımlara ihtiyaç duyulmaktadır.

Alanyazında ChatGPT'nin matematik öğretimindeki rolüne ilişkin çalışmalar artmakla birlikte, araştırmaların önemli bir bölümünün sonuç/başarı odaklı ilerlediği; öğrencilerin YZ ile etkileşim sırasında sergiledikleri bilişsel yönelimleri, soru sorma eğilimlerini ve etkileşim örüntülerini süreç temelli olarak inceleyen nitel çalışmaların görece sınırlı kaldığı görülmektedir. Özellikle Türkiye bağlamında, ChatGPT'nin fonksiyon öğrenme sürecinde öğrenciyle nasıl bir etkileşim yapısı kurduğuna ilişkin ayrıntılı betimlemeler ve bu etkileşimi anlamlandırmaya dönük kategorik çerçeveler yeterince açıklanmamıştır. Bu çalışmanın amacı, 10. sınıf öğrencilerinin fonksiyonlar konusunu öğrenirken ChatGPT ile kurduğu yazılı etkileşimlerde üretilen promptların bilişsel yönelimini ve ChatGPT'nin bu promptlara verdiği yanıtların açıklama stratejilerini inceleyerek etkileşim örüntülerini ve ortaya çıkan öğrenme ihtiyaçlarını betimlemektir. Bu doğrultuda araştırmada (i) öğrencilerin ChatGPT'ye bilişsel açıdan ne tür sorular yönelttiği ve (ii) ChatGPT'nin bu sorulara bilişsel açıdan ne tür yanıtlar ürettiği sorularına yanıt aranmıştır.

Yöntem

Bu araştırma, 10. sınıf öğrencilerinin fonksiyonlar konusunu öğrenme sürecinde ChatGPT ile kurdukları yazılı etkileşimin yapısını ve bu etkileşim sırasında ortaya çıkan bilişsel yönelimleri incelemeyi amaçladığından nitel araştırma yaklaşımıyla yürütülmüştür. Çalışmada, etkileşim örüntülerinin doğal öğrenme bağlamı içinde ayrıntılı ve bütüncül biçimde ortaya konulmasına olanak tanıması nedeniyle durum çalışması deseni benimsenmiştir. İncelenen olguya ilişkin literatürün sınırlı olması ve etkileşim biçimlerinin önceden kestirilememesi nedeniyle araştırma, keşfedici durum çalışması olarak desenlenmiştir. Bu bağlamda çalışma, ChatGPT'nin "öğretmenin yerine geçen" bir sistem olarak değil; öğrencinin soru üretimi yoluyla düşünme süreçlerini görünür kılan bir öğrenme ortağı olarak konumlandırıldığı bir etkileşim ortamını incelemektedir.

Araştırmanın çalışma grubunu, Türkiye'nin güney bölgesinde yer alan bir özel fen lisesinde öğrenim gören 10. sınıf düzeyindeki 8 öğrenci (5 kız, 3 erkek) oluşturmaktadır. Katılımcılar amaçlı örnekleme yaklaşımıyla belirlenmiş; gönüllü katılım göstermeleri, daha önce ChatGPT kullanmış olmaları ve etkileşim temelli öğrenme sürecine katılım için bilişsel hazırbulunuşluklarının yeterli olması temel ölçütler olarak dikkate alınmıştır. Katılımcı gizliliğini sağlamak amacıyla öğrenciler analiz sürecinde kodlarla temsil edilmiştir.

Araştırmada temel veri kaynağı, uygulama sırasında kaydedilen öğrenci–ChatGPT prompt–yanıt dökümleridir. Etkileşim bağlamını yapılandırmak amacıyla fonksiyonlar konusuna yönelik bir çalışma kâğıdı kullanılmış; öğrencilerin bu görevler üzerinde çalışırken ChatGPT ile yazılı etkileşime girmeleri sağlanmıştır. Asıl uygulama öncesinde iki öğrenciyle iki aşamalı bir ön uygulama gerçekleştirilmiştir. İlk aşamada yönlendirmesiz kullanımda öğrencilerin soruları doğrudan kopyalayıp çözdürme eğilimi gözlenmiş; ikinci aşamada denenen rol atamalı başlangıç promptunun ise yanıt üretimini aşırı sınırladığı görülmüştür. Bu bulgular doğrultusunda asıl uygulamada, öğrencilerin soruları kopyalayıp yapıştırmaması ve soru görseli yüklememesi; problem durumunu kendi ifadeleriyle yeniden yapılandırarak kavramsal açıklama, ipucu ve yönlendirme talep etmesi yönünde açık sınırlılıklar getirilmiştir. Uygulama okul bilgisayar laboratuvarında, her öğrenci bir bilgisayarda olacak şekilde yaklaşık iki saatlik oturumda yürütülmüş; araştırmacı yalnızca süreç ve teknik işleyiş düzeyinde destek sunmuş, içerik düzeyinde müdahalede bulunmamıştır.

Veriler içerik analiziyle çözümlenmiştir. Prompt–yanıt çiftleri bütüncül olarak okunmuş, açık kodlama ile anlamlı birimler etiketlenmiş ve benzer kodlar birleştirilerek kategoriler oluşturulmuştur. Kodlama, matematik eğitimi alanında uzman iki bağımsız kodlayıcı tarafından yürütülmüş; Miles ve Huberman formülüyle hesaplanan kodlayıcılar arası uyum %90 olarak belirlenmiştir. Araştırma, Mersin Üniversitesi Eğitim Bilimleri Etik Kurulu onayı doğrultusunda yürütülmüş; öğrenci gönüllü katılım ve veli onam süreçleri tamamlanmıştır.

Bulgular

Bu araştırmada elde edilen bulgular, fonksiyon öğrenme sürecinde öğrencilerin ChatGPT'ye yönelttikleri promptların bilişsel yönelimleri ile ChatGPT'nin ürettiği yanıtların açıklama stratejileri olmak üzere iki eksende ele alınmıştır. Veriler, 51 prompt ve bu promptlara karşılık üretilen 221 yanıt biriminden oluşan prompt–yanıt çiftleri üzerinden analiz edilmiştir. Bulgular, öğrencilerin ChatGPT'yi ağırlıklı olarak kavramsal netleştirme ve temsil destekleyici bir öğrenme kaynağı olarak kullandığını; buna karşın doğrulama ve öz-düzenleme temelli etkileşimlerin daha sınırlı kaldığını göstermektedir.

Öğrenci promptlarının dağılımı incelendiğinde, en yüksek oranı tanıma yönelik promptların oluşturduğu görülmüştür (%41,2; f=21). Bu bulgu, fonksiyon gibi soyut ve çok temsilli bir kavramda öğrencilerin öncelikle tanım, temel özellik ve kavramsal sınırları netleştirme ihtiyacı hissettiklerine işaret etmektedir. Tanıma yönelimli istemler, öğrencinin "kavrama giriş" düzeyinde açıklama talep ettiğini; dolayısıyla bilişsel süreçlerin daha çok hatırlama ve anlama basamaklarında yoğunlaştığını düşündürmektedir. Bu durum, öğrencilerin fonksiyon kavramında sıklıkla görülen "resmî tanım–kavram imgesi" gerilimini azaltmak için ChatGPT'yi hızlı bir açıklama ve kavramsal çerçeve kurma aracı olarak konumlandırdığını destekler. Bu yönelim, ChatGPT kullanımının başlangıç aşamasında öğrencinin bilgiye erişimini kolaylaştırsa da etkileşimin daha üst düzey akıl yürütmeye taşınabilmesi için öğretimsel yapılandırmaya gereksinim olduğunu göstermektedir.

İkinci sırada yer alan temsile yönelik promptlar (%27,5; f=14), fonksiyon öğreniminde temsil dönüşümlerinin belirleyici rolüyle tutarlı bir görünüm sergilemiştir. Öğrencilerin grafik çizimi istemeleri, grafiği adım adım üretmeye yönelik yönlendirme talep etmeleri veya farklı temsiller arasında geçişi kolaylaştıracak açıklamalar istemeleri; ChatGPT'nin "temsil üretici/temsil arası köprü kurucu" bir araç olarak kullanıldığını düşündürmektedir. Fonksiyon kavramının grafiksel, cebirsel ve sözel temsillerle birlikte anlam kazandığı dikkate alındığında, bu bulgu ChatGPT'nin temsiller arası koordinasyonu destekleyebilecek bir potansiyeli görünür kılmaktadır. Bununla birlikte temsil istemlerinin niteliği, öğrencinin yalnızca "çizim göster" düzeyinde kalabildiğinde yüzeysel bir görselleştirme talebine dönüşebilmekte; temsilin kavramsal işlevi üzerine düşünmeyi zorunlu kılmadığında öğrenme çıktısı sınırlı kalabilmektedir. Bu nedenle temsil odaklı istemlerin, "neden bu grafik?", "hangi özellik bu temsilde görünür?" gibi kavramsal gerekçelendirmeyi tetikleyecek biçimde yapılandırılması önemlidir.

Bağlam dışına çıkma (%13,7; f=7) ve doğrulama (%11,8; f=6) kategorilerinin orta düzeyde temsil edilmesi, etkileşim sürecinin iki yönlü bir risk ve fırsat barındırdığını göstermektedir. Bağlam dışına çıkma örnekleri, sohbet tabanlı ortamlarda konu sapmasının doğal bir olgu olduğunu ve görev sınırlarının korunmasının kritik olduğunu düşündürmektedir. Bu bulgu, sınıf içi ChatGPT kullanımında problem bağlamını sürdürecek yönergelerin ve odaklayıcı öğretmen rehberliğinin gerekliliğini destekler. Doğrulama istemleri ise öğrencilerin ChatGPT yanıtlarını bütünüyle otorite olarak kabul etmediğini; bazı durumlarda teyit, açıklama ve yeniden ifade etme ihtiyacı duyduğunu göstermektedir. Matematiksel gerekçelendirme ve doğrulama pratikleri açısından bu bulgu pedagojik olarak değerlidir; ancak oranların düşük olması, doğrulamanın sistematik bir alışkanlık hâline gelmediğini düşündürmektedir.

Öz-düzenleme promptlarının en düşük düzeyde kalması (%5,9; f=3), öğrencilerin ChatGPT ile etkileşimde metabilişsel izleme ve strateji düzenleme davranışlarını sınırlı ölçüde sergilediğine işaret etmektedir. Oysa öz-düzenleme, öğrencinin öğrenme sürecini yönetmesi, hatasını fark etmesi ve çözüm stratejisini yeniden yapılandırması bakımından kritik bir bileşendir. Bu bulgu, ChatGPT'nin tek başına öz-düzenlemeyi otomatik olarak üretmediğini; bunun görev tasarımıyla desteklenmesi gerektiğini göstermektedir. Örneğin öğrencilerin her etkileşim sonunda "yanıtın doğruluğunu nasıl kontrol ettin?", "hangi adımda zorlandın?" gibi zorunlu kontrol sorularına yanıt vermesi, etkileşimi daha metabilişsel bir düzeye taşıyabilir.

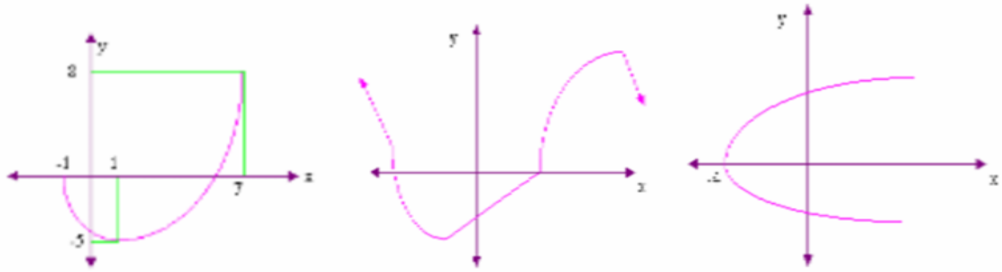
ChatGPT yanıtları açısından bulgular, yanıtların örnekleyici (%35,3; f=78), görselleştirici (%33,1; f=73) ve önleyici/düzeltici (%31,6; f=70) biçimlerde birbirine yakın oranlarda dağıldığını göstermiştir. Örnekleyici yanıtların güçlü biçimde temsil edilmesi, kavramın örnekler üzerinden somutlaştırılmasıyla bilişsel yükün azalabileceğine ve kavramsal sınırların daha görünür hâle gelebileceğine işaret eder. Görselleştirici yanıtlar, özellikle fonksiyon öğretiminde merkezi bir rolü olan grafiksel temsillerin öğrenmeyi destekleme potansiyelini yansıtmaktadır. Önleyici/düzeltici yanıtların anlamlı düzeyde bulunması ise modelin olası kavram yanılgılarını ayırt ederek uyarı üretme kapasitesini ortaya koymakta; bu durum, pedagojik açıdan “yapı iskelesi” işlevine benzer bir destek sunabildiğini düşündürmektedir. Ancak bu yanıt türlerinin öğrenmeye etkisi, öğrencinin istem niteliği ve yanıtı nasıl işlediğiyle yakından ilişkilidir.

Sonuç

Sonuç olarak çalışma, ChatGPT’nin fonksiyon öğretiminde kavramsal netleştirme, temsil zenginleştirme ve hata risklerini önleyici açıklamalar üretme açısından güçlü bir potansiyel taşıdığını göstermektedir. Bu nedenle öğretimsel uygulamalarda, tanıma ve temsil istemlerini sistematikleştiren kısa prompt şablonlarının kullanılması; doğrulama ve öz-düzenlemeyi zorunlu adımlar hâline getiren etkinlik tasarımlarının geliştirilmesi önerilebilir. Çalışmanın bulguları, yalnızca etkileşim kayıtlarının içerik analiziyle sınırlı olup öğrenci başarısı ya da öğrenme düzeyine ilişkin ölçümler içermemektedir. Gelecek araştırmalarda prompt türleri ile yanıt stratejileri arasındaki ilişkiler (hangi promptun hangi yanıt türünü tetiklediği), farklı okul türleri ve matematik konularında karşılaştırmalı olarak incelenebilir; daha geniş örneklemlerle kod kategorilerinin kararlılığı ve geçerliği yeniden test edilebilir.

APPENDIX 1

1. Aşağıdaki grafiklerin fonksiyon olup-olmadığını araştırınız.



( )Evet / ( )Hayır
Çünkü, ...
....
....
....
....
....

1. Investigate whether the graphs below represent a function.

( ) Yes / ( ) No

Because, ...

....

....

2. Aşağıdaki bağıntılardan hangileri fonksiyondur, araştırınız.

$\mathbf{f : Z \rightarrow Z \quad f(x) = \frac{x+2}{2}}$

$\mathbf{f : N \rightarrow N \quad f(x) = x - 2}$

$\mathbf{f : R^{+} \rightarrow R \quad f(x) = \sqrt{x}}$

$\mathbf{f : Z \rightarrow N \quad f(x) = x^2}$

2. Investigate which of the following relations are functions.

a) $f : \mathbb{Z} \rightarrow \mathbb{Z}$, $f(x) = \dfrac{x+2}{2}$
b) $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(x) = x - 2$
c) $f : \mathbb{R}^{+} \rightarrow \mathbb{R}$, $f(x) = \sqrt{x}$
d) $f : \mathbb{Z} \rightarrow \mathbb{N}$, $f(x) = x^2$

3. Aşağıda verilen ifadelerin fonksiyon olarak kuralını belirleyiniz.

i. Karenin bir kenar uzunluğu (x) ve alanı (A) arasındaki ilişkiyi gösteren fonksiyon.

ii. Bir okulda her bir sınıf öğretmenine 20 öğrenci düşmektedir. Okuldaki öğrenci sayısı (s) ve öğretmen sayısı (M) arasındaki ilişkiyi gösteren fonksiyon.

3. Determine the rule of the function for each statement given below.

i. **The function that represents the relationship between the side length of a square (x) and its area (A).**

ii. **In a school, there are 20 students for each classroom teacher. The function that represents the relationship between the number of students (s) and the number of teachers (M).**

4. Aşağıdakilerden hangisi tek ya da çift fonksiyondur? Bulunuz.

a) $\mathbf{f(x) = x^4 + x^2 + 1}$

b) $\mathbf{f(x) = x^3 - 6x}$

4. Determine which of the following is an odd function or an even function.

a) $f(x) = x^4 + x^2 + 1$

b) $f(x) = x^3 - 6x$

5. Aşağıdaki fonksiyonlardan hangisi veya hangileri A'dan B'ye 1-1 fonksiyondur?

I) $f=\{(1,6),(2,6),(3,4)\}$

II) $f=\{(1,1),(1,4),(2,6),(3,5)\}$

III $f=\{(1,6),(2,1),(3,4)\}$

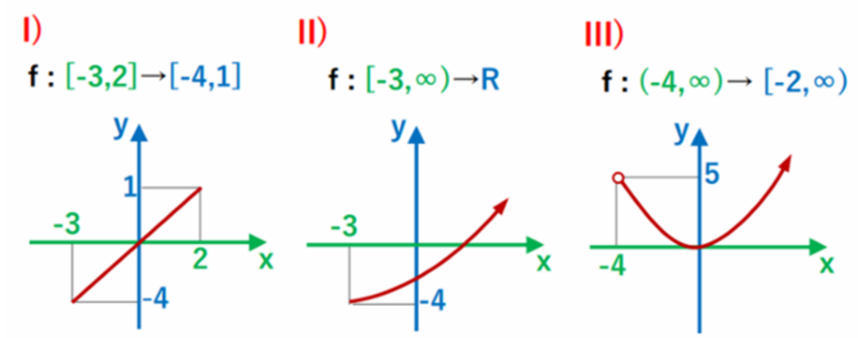
5. Which of the following functions (if any) is a one-to-one function from A to B?

I) $f = \{(1,6),(2,6),(3,4)\}$

II) $f = \{(1,1),(1,4),(2,6),(3,5)\}$

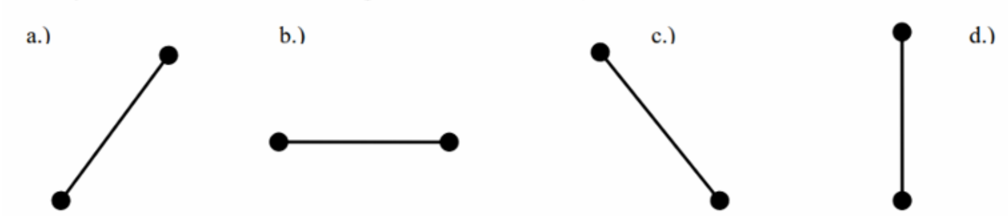
III) $f = \{(1,6),(2,1),(3,4)\}$

6. Aşağıda grafiği verilmiş fonksiyonların hangilerinin örten hangilerinin içine olduklarını bulunuz.



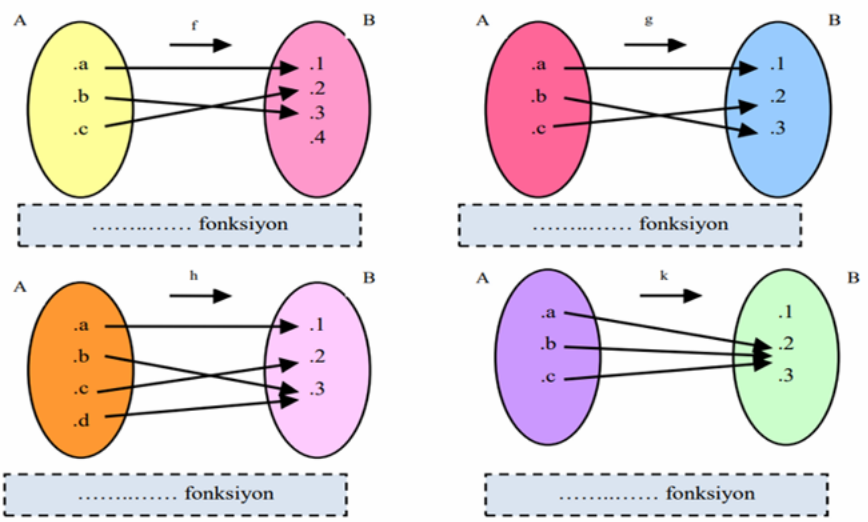
6. **For the functions whose graphs are given below, determine which functions are surjective and which are injective.**

7. Aşağıdaki şekillerin birkaçını kullanarak analitik düzlemde bir fonksiyonun grafiğini oluşturunuz. Oluşturduğunuz fonksiyonun grafiğinin noktalarını belirleyerek, fonksiyonun kuralı, tanım ve değer kümelerini tartışınız.



7. **Using some of the shapes below, create the graph of a function on the coordinate plane. Identify the points on the graph you created, and discuss the rule of the function, its domain, and its range.**

8. Aşağıda şema ile verilen fonksiyonların türlerini belirtiniz. Fonksiyonun neden bu türe ait olduğunu gerekçeleri ile açıklayınız.



8. **Identify the types of the functions given by the mapping diagrams below. Explain your reasons for why each function belongs to that type.**

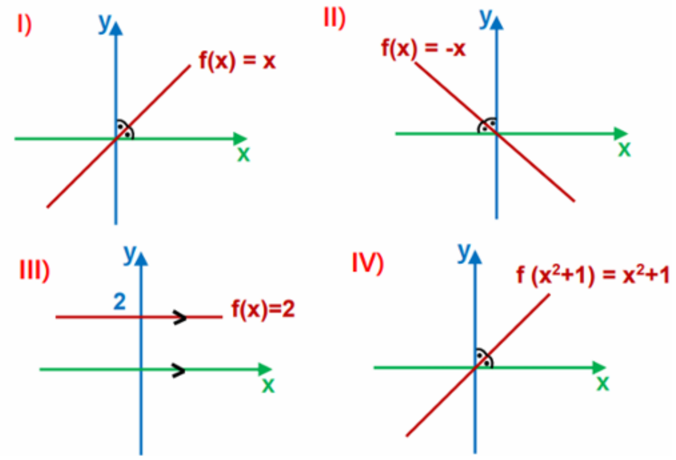
9. Aşağıdaki her bir durumda tabloda verilen değerler bir fonksiyon belirtir mi? Neden? Belirtiyorsa fonksiyonun kuralını belirleyip, fonksiyonu liste biçiminde yazınız. Fonksiyonun tablo ve listesini yorumlayınız.

(a)	x	9	4	1	0	1	4	9
	y	-3	-2	-1	0	1	2	3

(c)	x	0	1	2	3	4	5	6
	y	7	7	7	7	7	7	7

9. In each case below, do the values given in the table define a function? Why? If they do, determine the rule of the function and write the function in set (ordered-pair) form. Interpret (comment on) the table and the set representation of the function.

10. Aşağıdakilerden hangisi ya da hangileri birim fonksiyon grafiğidir?



10. Which of the following is/are the graph(s) of the identity function?