



Variable Structure PID Control for Multivariable Attitude Stabilization Under Cross-Axis Coupling in Helicopter Systems

Yaqine SAADA¹, Fadi ALYOUSSEF²

¹ Aerospace Engineering, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

s202442640@kfupm.edu.sa—ORCID> [0009-0009-4659-2228](https://orcid.org/0009-0009-4659-2228)

² Interdisciplinary Research Center for Aviation and Space Exploration, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

fadi.youssef@kfupm.edu.sa—ORCID> [0000-0003-1878-1549](https://orcid.org/0000-0003-1878-1549)

Article Information

Article Types	Research Article
Received	08 March 2026
Accepted	28 April 2026

Year: 2026 | **Volume:** 3 | **Issue:** 1 | **Pages:** 30-37

Cite as: Y. Saada and F. Alyoussef, "Variable structure PID control for multivariable attitude stabilization under cross-axis coupling in helicopter systems," *Hendese Journal of Technical Sciences and Engineering*, Vol. 3, No. 1, pp. 30–37, 2026, doi: 10.5281/zenodo.19899924.

Corresponding Author: Fadi ALYOUSSEF



Variable Structure PID Control for Multivariable Attitude Stabilization Under Cross-Axis Coupling in Helicopter Systems

Yaqine SAADA¹ , Fadi ALYOUSSEF*² 

¹Aerospace Engineering, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

²Interdisciplinary Research Center for Aviation and Space Exploration, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

(Received: 08.03.2026, Accepted: 28.04.2026, Published Online: 30.04.2026)

Keywords

Variable structure PID, Multivariable attitude control, Cross-axis coupling, Gain switching, MIMO systems

ABSTRACT

Cross-axis torque coupling degrades transient performance in multivariable rotary wing attitude systems when fixed gain PID controllers are tuned under single loop assumptions. Variable structure PID control introduces state dependent switching between predefined gain sets and offers a low order approach for modifying transient behavior without increasing controller complexity. This study evaluates fixed gain and variable structure PID control on a coupled pitch yaw helicopter platform modeled as a linearized two input two output system with explicit torque interaction. Switching logic selects aggressive or damping gains based on the sign of the error and its derivative. In this paper, closed loop responses are assessed using overshoot, peak deviation, settling behavior, and integral of absolute error under identical reference commands and actuator conditions. The switching configuration reduces transient amplification and cumulative tracking deviation in the axis most influenced by coupling while maintaining comparable convergence characteristics in the less affected axis. The observed axis dependent behavior aligns with asymmetry in the plant coupling structure. These results indicate that state dependent gain modulation within a classical PID framework improves disturbance rejection in coupled attitude dynamics and provides a practical control strategy for multivariable UAV and eVTOL stabilization problems.



Published by Muş Alparslan University, Muş, Türkiye
This is an open access article under the CC BY-NC license

*Corresponding Author: fadi.youssef@kfupm.edu.sa

Volume: 3

Issue: 1

Cite as: Y. Saada and F. Alyoussef, "Variable structure PID control for multivariable attitude stabilization under cross-axis coupling in helicopter systems," *Hendese Journal of Technical Sciences and Engineering*, Vol. 3, No. 1, pp. 30–37, 2026, doi: 10.5281/zenodo.19899924.

Doi: 10.5281/zenodo.19899924

1. INTRODUCTION

Precise multi-axis attitude control is necessary for modern unmanned aerial vehicles and eVTOL platforms to keep stability and tracking performance under unpredictable flying conditions and disturbances. As system complexity increases, interactions between rotational degrees of freedom and actuator constraints impose fundamental challenges on control design. In fact, strong coupling between rotational axes exists due to reaction torques and aerodynamic interference. In this paper, we mainly focus on a pitch yaw model in which these interactions remain under linearized operating conditions and significantly affect transient response. When controllers are designed using independent single loop assumptions, coupling appears in the closed loop as a structured disturbance. This behavior leads to overshooting as well as increased control effort, and sensitivity to gain tuning during transient maneuvers [1]. Laboratory helicopter platforms provide a practical benchmark for studying multivariable control under coupling. For helicopter systems with multiple degrees of freedom, pitch actuation induces yaw motion through reaction torque while yaw inputs alter pitch dynamics through aerodynamic coupling [1]. These effects occur even for small amplitude commands and intensify near actuator saturation [2]. Decentralized control structures experience performance degradation when cross-axis interactions are not explicitly addressed [1], [3].

PID control remains widely used in aerospace and electromechanical systems due to structural simplicity and transparency. Fixed gain PID designs rely on a compromise between transient speed and damping. In coupled systems, this compromise becomes restrictive. For one, integral action amplifies cross-axis disturbances and increases susceptibility to windup. Also, PID controllers do not have a good response for aero dynamic systems with poles on the imaginary axis [4], [5], [6]. Advanced controllers such as sliding mode controllers might overcome the disadvantages of PID controller. However, these controllers might have chattering, which can harm actuators [7], [8], [9].

Variable structure PID (VS-PID) control introduces state-dependent switching between multiple gain sets. Switching based on the error trajectory enables different controller behavior depending on whether the system state moves toward or away from equilibrium. Scientific papers show improved disturbance rejection and reduced oscillatory behavior compared to fixed gain PID control while preserving implementation simplicity [10]. Such an approach avoids extensive offline tuning associated with gain scheduling and adapts controller behavior in real time. State dependent switching laws for linear systems establish conditions for stable switching based on measurable state variables [11]. Switching PID control applied to nonlinear systems demonstrates improved transient performance under changing dynamics and external disturbances [12]. These results motivate the extension of variable structure PID concepts to multivariable systems with explicit coupling.

Despite these advances, applications of variable structure PID control to coupled helicopter dynamics remain limited. In

pitch yaw systems, coupling terms vary in magnitude and sign during transients and act as structured disturbances rather than constant offsets. Applying variable structure PID to such systems requires explicit modeling of cross-axis dynamics and integration of coupling effects within the control architecture itself.

This paper applies variable structure PID control to a helicopter pitch yaw helicopter model formulated as a two input two output system. The plant consists of linearized second order rotational dynamics with explicitly modeled pitch yaw coupling. Variable structure PID controllers are implemented on each axis using gain sets and state dependent switching logic consistent with previously established formulations [10], [11]. Coupling effects are selectively incorporated within the control structure to evaluate their influence on tracking performance and control effort. Simulation results compare variable structure PID control with fixed gain PID under identical plant dynamics, reference commands, and actuator constraints. The study demonstrates the extension of variable structure PID methods from single axis formulations to a physically meaningful multivariable helicopter system with strong coupling effects.

To examine the practical value of this gain switching in coupled rotary wing dynamics, this study focuses on its behavior under explicit cross-axis interaction. The main contributions of this work are overall summarized as follows:

- Extension of variable structure PID control to a coupled MIMO Aero2 attitude system with explicitly modeled cross-axis torque interaction.
- Development of a state-dependent gain switching strategy applied directly to a multivariable system exhibiting asymmetric coupling dynamics.
- Evaluation of switching behavior under transient conditions with varying coupling influence, with emphasis on disturbance amplification, convergence, and cumulative tracking deviation.
- Comparative analysis between fixed-gain and variable-structure PID control under identical plant dynamics and actuator constraints to isolate the effect of gain modulation.
- Identification of axis-dependent performance characteristics arising from the structure and asymmetry of cross-axis coupling.

The remainder of this paper is organized as follows. Section 2 presents the system description. Section 3 introduces the variable-structure PID design. The subsequent sections provide the simulation results, followed by the discussion in Section 5 and the conclusion.

2. SYSTEM DESCRIPTION

2.1. Symbolic Model

The system considered in this study represents a two degree of freedom rotary wing attitude platform with coupled pitch and yaw dynamics. The physical configuration of the helicopter platform, including the orientation of the pitch and yaw axes

and the placement of the front and rear rotors, is illustrated in Fig. 1. The model is also able to capture essential characteristics of eVTOL attitude subsystems like reaction torque effects and aerodynamic cross-axis interactions. It is formulated as a linear time-invariant, two-input two-output system derived from rigid-body rotational dynamics and linearized about a nominal operating point [14].

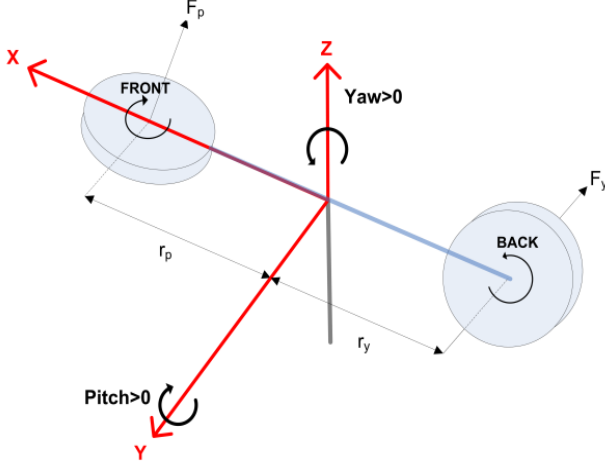


Fig. 1. Helicopter [13].

The control inputs are the rotor voltages V_p for pitch rotor voltage and V_y for yaw rotor voltage. Additionally, the outputs are the angular positions: θ as pitch angle and ψ as yaw angle. Given the helicopter model, the rotational dynamics are given by [13]:

$$J_\theta \ddot{\theta} + D_\theta \dot{\theta} + K_\theta \theta = \tau_\theta \quad (1)$$

$$J_\psi \ddot{\psi} + D_\psi \dot{\psi} = \tau_\psi \quad (2)$$

Here, J_θ and J_ψ are moments of inertia, D_θ and D_ψ are viscous damping coefficients, K_θ is the pitch restoring stiffness, and τ_θ , τ_ψ are the applied torques.

The actuator torques are modeled as linear functions of the rotor voltages with explicit cross-axis coupling [13],

$$\tau_\theta = a_p V_p + a_{\theta y} V_y \quad (3)$$

$$\tau_\psi = a_{\psi p} V_p + a_y V_y \quad (4)$$

where a_p and a_y are the primary control effectiveness coefficients, and $a_{\theta y}$ and $a_{\psi p}$ represent cross-axis coupling gains.

Substituting (3) and (4) into (1) and (2) yields:

$$J_\theta \ddot{\theta} + D_\theta \dot{\theta} + K_\theta \theta = a_p V_p + a_{\theta y} V_y \quad (5)$$

$$J_\psi \ddot{\psi} + D_\psi \dot{\psi} = a_{\psi p} V_p + a_y V_y \quad (6)$$

Taking the Laplace transform of (5) and (6) with null initial conditions gives [13]:

$$\frac{\Theta(s)}{V_p(s)} = \frac{a_p}{J_\theta s^2 + D_\theta s + K_\theta} + \frac{a_{\theta y}}{J_\theta s^2 + D_\theta s + K_\theta} \quad (7)$$

$$\frac{\Psi(s)}{V_p(s)} = \frac{a_{\psi p}}{J_\psi s^2 + D_\psi s} + \frac{a_y}{J_\psi s^2 + D_\psi s} \quad (8)$$

Defining the individual transfer functions,

$$G_{\theta p}(s) = \frac{a_p}{J_\theta s^2 + D_\theta s + K_\theta} \quad (9)$$

$$G_{\theta y}(s) = \frac{a_{\theta y}}{J_\theta s^2 + D_\theta s + K_\theta} \quad (10)$$

$$G_{\psi p}(s) = \frac{a_{\psi p}}{J_\psi s^2 + D_\psi s} \quad (11)$$

$$G_{\psi y}(s) = \frac{a_y}{J_\psi s^2 + D_\psi s} \quad (12)$$

Here, $G_{\theta p}(s)$, $G_{\theta y}(s)$, $G_{\psi p}(s)$, and $G_{\psi y}(s)$ are the pitch dynamics due to pitch actuation, the pitch response due to yaw actuation, the yaw response due to pitch actuation, and the yaw dynamics due to yaw actuation, respectively. The complete multivariable representation is therefore [13]:

$$\begin{bmatrix} \Theta(s) \\ \Psi(s) \end{bmatrix} = \begin{bmatrix} G_{\theta p}(s) & G_{\theta y}(s) \\ G_{\psi p}(s) & G_{\psi y}(s) \end{bmatrix} \begin{bmatrix} V_p(s) \\ V_y(s) \end{bmatrix} \quad (13)$$

Each denominator reflects the rotational dynamics of the affected axis. The pitch channel includes a restoring stiffness term associated with gravitational and aerodynamic moments while the yaw channel is more so dominated by inertia and damping. The coupling terms share the same denominator as the axis they influence, indicating that cross-axis inputs excite the same rotational modes as primary actuation.

2.2. Numerical Helicopter Model

Using the identified helicopter parameters obtained from system specifications and experimental characterization, the symbolic model is instantiated into a numerical linear time-invariant representation. Substituting the values given in Table I into (9)-(12) yields the following transfer functions [13]:

$$G_{\theta p}(s) = \frac{0.0281456}{s^2 + 0.0689655 s + 0.3793103} \quad (14)$$

$$G_{\theta y}(s) = \frac{0.0115469}{s^2 + 0.0689655 s + 0.3793103} \quad (15)$$

$$G_{\psi p}(s) = \frac{-0.0133663}{s^2 + 0.1386555 s} \quad (16)$$

$$G_{\psi y}(s) = \frac{0.0520581}{s^2 + 0.1386555 s} \quad (17)$$

The negative sign in $G_{\psi p}(s)$ reflects the opposing yaw torque induced by pitch actuation. This overall numerical

representation defines the exact plant used for the describing function analysis of the variable-structure PID controller presented in the subsequent sections.

Table I. Identified helicopter model parameters.

Parameter	Description
$J_\theta = 0.0232$	Pitch axis moment of inertia
$J_\psi = 0.0238$	Yaw axis moment of inertia
$D_\theta = 0.0016$	Pitch viscous damping coefficient
$D_\psi = 0.0033$	Yaw viscous damping coefficient
$K_\theta = 0.0088$	Pitch stiffness coefficient
$a_p = 0.000653$	Pitch motor gain constant
$a_{\theta y} = 0.000268$	Yaw-on-pitch coupling coefficient
$a_{\psi p} = -0.000318$	Pitch-on-yaw coupling coefficient
$a_y = 0.001239$	Yaw motor gain constant

3. VARIABLE-STRUCTURE PID DESIGN

The control objective is regulation of the pitch and yaw angles of the coupled helicopter system. The controller must ensure fast convergence while limiting overshoot and control effort under cross-axis interaction. The tracking errors are defined simply as

$$e_\theta(t) = \theta_d(t) - \theta(t) \quad (18)$$

$$e_\psi(t) = \psi_d(t) - \psi(t) \quad (19)$$

A conventional PID control law for either axis is expressed as

$$u(t) = K_p e(t) + K_i \int e(t) dt + K_d \dot{e}(t) \quad (20)$$

Here, K_p , K_i , K_d are the proportional, the integral and the derivative gains. Fixed PID gains impose a tradeoff between response speed and damping. Large gains reduce rise time but increase overshoot and amplify cross-axis coupling. Small gains improve damping but weaken disturbance rejection.

The VS-PID formulation is adopted following the phase plane switching strategy introduced in [10]. The switching signal is defined as

$$\sigma(t) = e(t)\dot{e}(t) \quad (21)$$

The sign of $\sigma(t)$ determines the gain set so that if

$$\sigma(t) > 0 \quad (22)$$

the error magnitude increases. An aggressive gain set is applied to increase corrective action if

$$\sigma(t) < 0 \quad (23)$$

In this case, the error decreases toward equilibrium. A damping oriented gain set is applied to reduce oscillation. The VS-PID gains are defined as piecewise constants,

$$K_p = \begin{cases} K_p^{(1)}, & \sigma(t) > 0 \\ K_p^{(2)}, & \sigma(t) < 0 \end{cases} \quad (24)$$

$$K_i = \begin{cases} K_i^{(1)}, & \sigma(t) > 0 \\ K_i^{(2)}, & \sigma(t) < 0 \end{cases} \quad (25)$$

$$K_d = \begin{cases} K_d^{(1)}, & \sigma(t) > 0 \\ K_d^{(2)}, & \sigma(t) < 0 \end{cases} \quad (26)$$

The resulting control input becomes

$$u(t) = K_p(\sigma)e(t) + K_i(\sigma)\int e(t) dt + K_d(\sigma)\dot{e}(t) \quad (27)$$

The final control structure is shown in Fig. 2. For the pitch and yaw subsystems, the control structure uses two VS-PID controllers. Coupling models are used to describe the interaction dynamics because of the intrinsic cross-coupling among the two axes. This ensures robust and precise tracking accuracy in the coupled helicopter system by enabling each controller to correct for cross-axis disturbances.

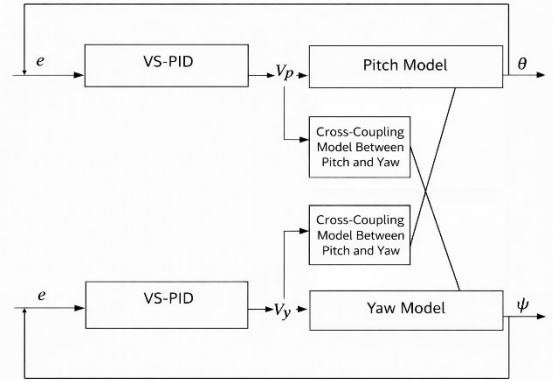


Fig. 2. Block diagram of the VS-PID controlled coupled system.

The gain values used in the simulations are defined as follows. For the pitch axis, the aggressive gain set is:

$$\begin{aligned} K_{p\theta}^{(1)} &= 500 \\ K_{i\theta}^{(1)} &= 30 \\ K_{d\theta}^{(1)} &= 70 \end{aligned}$$

The damping gain set is:

$$\begin{aligned} K_{p\theta}^{(2)} &= 40 \\ K_{i\theta}^{(2)} &= 10 \\ K_{d\theta}^{(2)} &= 40 \end{aligned}$$

For the yaw axis, the aggressive gain set is:

$$K_{p\psi}^{(1)} = 800$$

$$K_{i\psi}^{(1)} = 0.1$$

$$K_{d\psi}^{(1)} = 200$$

The damping gain set is:

$$K_{p\psi}^{(2)} = 200$$

$$K_{i\psi}^{(2)} = 0.1$$

$$K_{d\psi}^{(2)} = 100$$

For the fixed PID configuration, the damping gain set was selected for both pitch and yaw in order to avoid overly aggressive tuning and to ensure a fair comparison with the variable structure controller. The PID gain values used in this study were selected to provide aggressive and damping control behaviors consistent with the objectives of the variable structure setup. The gain sets were obtained through preliminary tuning and refinement of the closed-loop response of the linearized Aero2 model thus providing stable operation and reasonable transient performance under cross-axis coupling. The damping gain set was naturally chosen to yield a well-behaved response with limited overshoot and oscillation when applied as a fixed controller. The aggressive gain set was therefore defined by increasing proportional and derivative action relative to this baseline in order to enhance corrective response during points where the tracking error grows.

This structure is implemented independently for pitch and yaw axes. The plant remains linear as defined in (13). The closed loop becomes piecewise linear due to state dependent gain switching. The form follows the variable structure PID framework in [10] and forms the basis for the simulation.

4. SIMULATION RESULTS

The closed-loop performance of the coupled helicopter system was evaluated under cross-axis interaction using standard time-domain step response metrics. Performance measures include percent overshoot, Peak – steady-state deviation, settling time within a 2 percent tolerance band, and integral of absolute error (IAE).

Pitch Angle Fixed vs VS PID Comparison (w/ Coupling Effects)

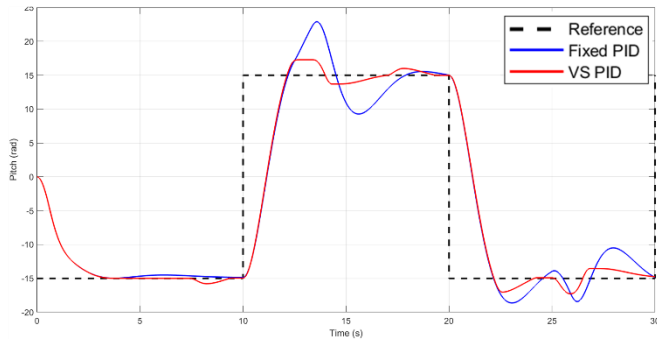


Fig. 3. Pitch angle tracking response for fixed PID and variable structure PID controllers with cross-axis coupling enabled.

Fig. 3 shows the pitch tracking response for the fixed PID and VS-PID controllers over the evaluation interval with cross-axis coupling enabled in the plant model. The coupling configuration represents the most demanding scenario for the controller due to cross-axis disturbance propagation.

The selected performance metrics evaluate transient amplification, convergence, and cumulative tracking accuracy under cross-axis coupling. Percent overshoot and Peak minus steady state deviation quantify transient excursion and damping quality which are directly affected by coupling induced disturbance. Settling time within a 2 percent band measures practical stabilization and convergence speed. IAE captures total tracking deviation over the evaluation interval and reflects overall regulation quality. These metrics are widely adopted in time domain controller assessment because they provide complementary measures of speed, stability, and accuracy in closed loop systems [15].

Table II. Performance metrics comparison of fixed PID and variable structure PID controllers for pitch angle.

Metric	Fixed PID	VS-PID
Overshoot (%)	12.206	7.664
Peak – SS (units)	6.097766	2.973817
Settling Time 2% (s)	19.736	19.074
IAE ($\int e dt$)	108.529938	83.820334

Table II summarizes the corresponding quantitative performance metrics under coupling. For the pitch response, percent overshoot decreases from 12.206 percent in the fixed case to 7.664 percent under variable structure control. This corresponds to a reduction of approximately 37 percent in overshoot magnitude.

Peak – SS is computed as the difference between the maximum transient peak and the final steady-state value. The Peak – SS metric decreases from 6.0978 units to 2.9738 units, indicating a substantial reduction in maximum transient deviation under the variable structure formulation. This represents a reduction of approximately 51 percent in peak excursion amplitude.

The 2 percent settling time improves slightly from 19.736 s in the fixed PID case to 19.074 s under the variable structure controller. Although the improvement in convergence speed is modest, the switching formulation achieves comparable stabilization while significantly reducing transient amplification.

The integral of absolute error decreases from 108.5299 to 83.8203, corresponding to an approximate 23 percent reduction in cumulative tracking error. This confirms improved overall regulation quality under cross-axis interaction.

These results show that under cross-axis interaction, the variable structure PID reduces transient deviation and cumulative error relative to the fixed gain configuration and

significantly so.

Following the analysis of the pitch response, the yaw axis is evaluated under the same cross-axis coupling conditions. While the pitch dynamics exhibit pronounced sensitivity to coupling, the yaw axis shows comparatively smaller disturbance amplification. This distinction allows assessment of how the variable structure mechanism performs across axes with different coupling sensitivity.

In the helicopter configuration, the dominant cross torque term affects the pitch dynamics more strongly than the yaw dynamics. As a result, yaw disturbances induced by pitch commands remain limited in magnitude. Evaluating the yaw response under identical controller settings therefore provides a consistency check. If the switching mechanism introduces instability or excessive oscillation, it would be reflected in the yaw response despite its lower coupling sensitivity.

Yaw Angle Fixed vs VS PID Comparison (w/ Coupling Effects)

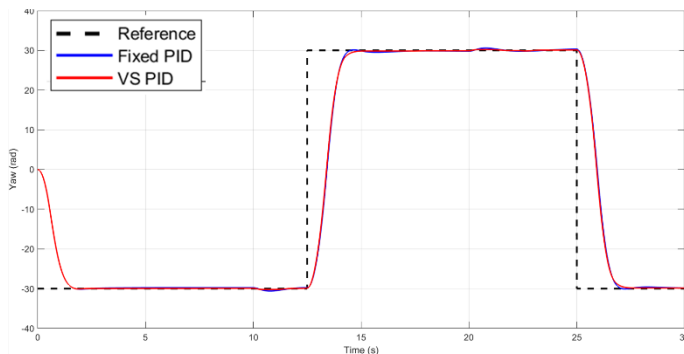


Fig. 4. Yaw angle tracking response for fixed PID and variable structure PID controllers with cross-axis coupling.

The influence of cross-axis coupling on the yaw dynamics is illustrated in Fig. 4.

Table III. Performance metrics comparison of fixed PID and variable structure PID controllers for yaw angle.

Metric	Fixed PID	VS-PID
Overshoot (%)	0.119	-
Peak – SS (units)	0.342441	0.014531
Settling Time 2% (s)	14.302	14.370
IAE ($\int e dt$)	115.681107	113.124118

Table III summarizes the quantitative performance metrics. For the yaw response, percent overshoot decreases from 0.119 percent in the fixed case to almost zero percent under the variable structure controller. Although the relative reduction is complete, the absolute magnitude of overshoot is already very small due to weaker cross-axis excitation in yaw.

The Peak – SS metric decreases from 0.3424 units to 0.0145 units, indicating a pronounced reduction in transient spike amplitude. This confirms that gain switching moderates transient amplification even in the less affected axis.

The 2 percent settling time remains nearly unchanged, with the fixed PID settling at 14.302 s and the variable structure PID

at 14.370 s. The minimal difference suggests that yaw convergence speed is not strongly influenced by gain switching under the present coupling configuration.

The integral of absolute error decreases from 115.6811 to 113.1241, corresponding to an approximate 2 percent reduction in cumulative tracking error. While smaller than the improvement observed in pitch, this reduction confirms that the switching mechanism does not degrade yaw tracking performance.

Compared to the pitch axis, the yaw axis demonstrates smaller absolute improvements. This behavior is consistent with the structure of the plant in (13) where pitch is more directly influenced by cross torque interaction. The variable structure mechanism therefore provides its largest benefit in the axis experiencing stronger coupling induced disturbance.

5. DISCUSSION

Evidently, the VS-PID improves transient behavior because the switching law adjusts loop aggressiveness according to the direction of the error trajectory. When cross-axis torque injects energy into the regulated axis, the error and its derivative share the same sign. The controller selects the higher gain set and increases corrective torque. This action counters disturbance energy during the growth phase of the transient. When the state moves toward equilibrium, the controller switches to the lower gain set and increases effective damping. The improvement associated with this mechanism appears more clearly in pitch because, as mentioned, the plant exhibits asymmetric coupling. The cross-torque term influencing pitch has greater relative effect on its dynamics than the reciprocal term influencing yaw. Cross-axis interaction therefore injects more transient energy into the pitch channel. Gain switching engages more frequently and with greater impact in this axis. In yaw, weaker coupling produces smaller disturbance amplification. The switching law remains active but operates over a narrower error range which explains the reduced magnitude of observable change.

The stability of the variable structure PID controller under switching is supported by the structure of the switching law and its consistency with established formulations in the literature. The switching signal defined as $\sigma(t) = e(t)\dot{e}(t)$ partitions the phase plane according to the direction of the error trajectory. When the error and its derivative share the same sign, the system state moves away from equilibrium, and the controller applies the higher gain set to increase corrective action. When the error decreases toward equilibrium, the lower gain set is selected to introduce effective damping. This mechanism aligns with previously reported variable structure PID strategies [7] and state dependent switching control approaches [8] where the stability is maintained by ensuring that switching occurs between locally stabilizing controller configurations. In the present implementation, both gain sets are selected to yield stable closed loop behavior for the linearized plant, and the switching logic does not introduce persistent excitation or discontinuous divergence in the system trajectories. The resulting closed loop system remains bounded and exhibits convergent behavior in all simulated cases, with no evidence of chattering or instability across either axis. These observations

are consistent with prior studies of switching PID control in nonlinear systems [9] in that appropriately designed switching laws preserve stability while improving transient response.

The performance of the proposed variable structure PID controller under non-ideal conditions such as parameter uncertainty, measurement noise, and time delay has not been explicitly evaluated in the present study and represents an important consideration for practical implementation. Modeling inaccuracies and parameter variations may affect the relative effectiveness of the predefined gain sets most significantly in the presence of strong cross-axis coupling. Time delays in sensing or actuation may further influence the timing of gain selection and reduce the effectiveness of the switching logic during fast transients. Nevertheless, the closed-loop responses obtained under nominal conditions remain smooth and bounded with no evidence of excessive oscillation or chattering which suggests stability of the switching formulation in the face of moderate disturbances. Despite this, a detailed stability analysis incorporating uncertainty, noise, and delay effects is left for future work to fully assess the applicability of the approach in real-world flight control systems. Also, It's also crucial to note that this study investigated the coupling effects and found that the controller stayed stable. This shows how well the controller performs in conditions that are not ideal

The controller suppresses peak growth and limits oscillatory amplitude while preserving overall convergence rate. The structure shapes the transient envelope rather than altering steady state characteristics. These observations have robustness implications under structured cross-axis disturbance. A fixed gain PID must balance aggressiveness and damping for all operating phases. Under coupling, this compromise leads to either excess amplification or slow correction. State dependent switching separates these roles across the transient. High gains act during disturbance growth. Lower gains act during decay. This separation reduces sensitivity to interaction without increasing controller order [16].

The approach retains the simplicity of classical PID implementation. The controller structure remains proportional, integral, and derivative action with two predefined gain sets. No additional states or model based observers are introduced. The nonlinear behavior arises from gain selection logic driven by measurable signals. This property supports implementation in embedded flight controllers with limited computational overhead. Moreover, the method has practical implications for multivariable attitude stabilization in rotary wing and eVTOL platforms. Cross-axis torque coupling acts as a structured disturbance during coordinated maneuvers. State dependent gain modulation offers a low complexity means to limit transient amplification in the most sensitive axis while maintaining stable regulation in the other. The architecture aligns with decentralized PID practice and requires only local error signals.

The effectiveness of the strategy depends on coupling strength, disturbance magnitude, and gain set separation. If cross-axis interaction remains weak, switching produces limited benefit. If gain differences are excessive, switching may

induce chattering or undesired high frequency activity. The switching boundary defined by the error trajectory also influences responsiveness. Careful selection of gain values and switching logic remains necessary to ensure bounded and smooth behavior across operating conditions [17].

6. CONCLUSION

This study investigated state dependent gain switching in a coupled two degree of freedom helicopter model with explicit cross-axis torque interaction. The objective was to evaluate whether VS-PID control improves transient regulation under multivariable coupling when compared with fixed gain tuning. The analysis shows that gain switching enhances transient behavior in the axis most influenced by interaction while maintaining stable closed loop operation across both channels. The suggested controller improved the pitch overshoot by 37.22%, from 12.206% to 7.664%. The overshoot for yaw decreased from 0.119% to 0.000%. This suggests that the new controller can utilize less energy.

These findings indicate that nonlinear gain modulation within a classical PID structure provides a practical means of addressing structured cross-axis disturbance in multivariable attitude systems. The approach retains proportional, integral, and derivative action with predefined gain sets and requires no additional controller states. This property supports deployment in embedded flight control architectures where computational resources and certification constraints favor low order designs. The results suggest that structured switching offers an intermediate solution between fixed decentralized tuning and full model based multivariable synthesis for coupled MIMO plants [2].

Future work will focus on experimental validation on the physical platform to confirm performance under hardware constraints and potential sensor noise. Further investigation is indeed required to quantify sensitivity to switching threshold selection and gain separation. Continuous gain modulation strategies may be examined as an alternative to discrete switching to reduce potential discontinuities. Integration with adaptive or observer-based schemes could extend applicability to uncertain or time varying dynamics. Extension to higher order and fully nonlinear flight models will determine the scalability of the approach to broader multi-axis aerial systems [18].

ACKNOWLEDGEMENTS

The authors acknowledge the support provided by the Department of Aerospace Engineering and the Interdisciplinary Research Center for Aviation and Space Exploration at King Fahd University of Petroleum and Minerals. This research was funded by the Undergraduate Research Office (URO), KFUPM, Saudi Arabia. Yaqine Saada is an undergraduate student under the mentorship of Dr. Fadi Alyoussef as part of the independent research course (RES-200).

REFERENCES

- [1] E. Kurniawan *et al.*, "Attitude tracking of a multivariable 3-DoF helicopter via decentralized repetitive control," *J. Franklin Inst.*, vol. 362, no. 10, p. 107737, 2025, doi:

- [10.1016/j.jfranklin.2025.107737](https://doi.org/10.1016/j.jfranklin.2025.107737).
- [2] F. Gopmandal and A. Ghosh, “LQR-based MIMO PID control of a 2-DOF helicopter system with uncertain cross-coupled gain,” *IFAC-PapersOnLine*, vol. 55, no. 22, pp. 183–188, 2022, doi: [10.1016/j.ifacol.2023.03.031](https://doi.org/10.1016/j.ifacol.2023.03.031).
- [3] S. Mate, P. Pal, A. Jaiswal, and S. Bhartiya, “Simultaneous tuning of multiple PID controllers for multivariable systems using deep reinforcement learning,” *Digit. Chem. Eng.*, vol. 9, no. August, p. 100131, 2023, doi: [10.1016/j.dche.2023.100131](https://doi.org/10.1016/j.dche.2023.100131).
- [4] F. Alyoussef and I. Kaya, “Simple PI-PD tuning rules based on the centroid of the stability region for controlling unstable and integrating processes,” *ISA Trans.*, vol. 134, pp. 238–255, 2023, doi: [10.1016/j.isatra.2022.08.007](https://doi.org/10.1016/j.isatra.2022.08.007).
- [5] F. Alyoussef, A. Abdallah, and A. Farahat, “Robust optimal fractional-order proportional-integral and proportional-derivative controller design for integrating systems with time delays: Real-time application to quadrotors,” *ISA Trans.*, 2025, doi: [10.1016/j.isatra.2025.10.026](https://doi.org/10.1016/j.isatra.2025.10.026).
- [6] F. Alyoussef and I. Kaya, “Proportional–integral and proportional–derivative controller design based on analytically computed centroid point for controlling integrating processes,” *Proc. Inst. Mech. Eng. Part I J. Syst. Control Eng.*, 2023, doi: [10.1177/09596518221143815](https://doi.org/10.1177/09596518221143815).
- [7] F. Alyoussef and M. I. Alam, “Double-Integral Synergetic Control with Integral Backstepping for Robust PMSM Performance,” in *2025 29th International Conference on System Theory, Control and Computing (ICSTCC)*, IEEE, 2025, pp. 907–912. doi: [10.1109/ICSTCC66753.2025.11240458](https://doi.org/10.1109/ICSTCC66753.2025.11240458).
- [8] F. Alyoussef and I. Kaya, “Improved Adaptive Dynamic Non-singular Terminal Sliding Mode Controller with,” *Inf. Sci. (Ny)*, p. 119110, 2023, doi: [10.1016/j.ins.2023.119110](https://doi.org/10.1016/j.ins.2023.119110).
- [9] F. Alyoussef and I. Kaya, “A New Dynamic Sliding Mode Controller with Disturbance Observer for Controlling Integrating Processes with Time Delay,” *Int. J. Control*, vol. 97, no. 5, pp. 1136–1156, 2023, doi: [10.1080/00207179.2023.2201649](https://doi.org/10.1080/00207179.2023.2201649).
- [10] I. Boiko, “Variable-structure PID controller for level process,” *Control Eng. Pract.*, vol. 21, no. 5, pp. 700–707, 2013, doi: [10.1016/j.conengprac.2012.04.004](https://doi.org/10.1016/j.conengprac.2012.04.004).
- [11] D. Chen, “A state-dependent switching controller design approach for linear,” no. July, pp. 1719–1730, 2022, doi: [10.1049/cth2.12337](https://doi.org/10.1049/cth2.12337).
- [12] M. H. Kim and S.-U. Lee, “PID with a switching action controller for nonlinear systems of second-order controller canonical form,” *Int. J. Control Autom. Syst.*, vol. 19, no. 7, pp. 2343–2356, 2021, doi: [10.1007/s12555-020-0346-4](https://doi.org/10.1007/s12555-020-0346-4).
- [13] Quanser Inc., *Quanser AERO Laboratory Guide*. Markham, ON, Canada: Quanser Inc., 2016.
- [14] R. Fellag and M. Belhocine, “Comparative analysis of PID, fuzzy PID, and ANFIS controllers for 2-DOF helicopter trajectory tracking: simulation and hardware implementation,” *Arch. Mech. Eng.*, vol. 71, no. 3, pp. 323–349, 2024, doi: [10.24425/ame.2024.151331](https://doi.org/10.24425/ame.2024.151331).
- [15] S. Gulgonul, “IAE optimized PID tuning via second order step response target matching,” *arXiv preprint arXiv:2505.17268*, 2025, doi: [10.48550/arXiv.2505.17268](https://doi.org/10.48550/arXiv.2505.17268).
- [16] C. Wang, J. Tang, B. Jiang, and Z. Wu, “Sliding-mode variable structure control for complex automatic systems: a survey,” *Math. Biosci. Eng.*, vol. 19, no. 3, pp. 2616–2640, 2022, doi: [10.3934/MBE.2022120](https://doi.org/10.3934/MBE.2022120).
- [17] H. R. Ahmadzadeh and M. Shafiee, “Event-Triggered Variable Structure Control of Nonlinear Switched Descriptor Systems Subjected to Matched/Mismatched Disturbances,” *Iran. J. Sci. Technol. Trans. Electr. Eng.*, vol. 48, no. 4, pp. 1711–1727, 2024, doi: [10.1007/s40998-024-00744-8](https://doi.org/10.1007/s40998-024-00744-8).
- [18] S. Nekrouf, M. Bouhamida, and Z. Bellahcene, “Robust control of twin rotor MIMO system,” *Int. Rev. Autom. Control*, vol. 7, pp. 98–106, Jan. 2014, doi: [10.15866/ireaco.v7i1.1298](https://doi.org/10.15866/ireaco.v7i1.1298).