

Malatya Centrality based Algorithm for Positive Influence Dominating Set: Malatya Positive Influence Algorithm

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Abstract – Social networks are used by individuals not so much for communication as for influencing each other. Social networks are modelled as graphs, and a special Dominating Set is obtained from the resulting graph. A new algorithm has been proposed for the Dominating set, where at least half of the neighbours of each node are elements of the Dominating Set. This algorithm is called as Malatya Positive Influence Algorithm, and its results for some graphs are given in this study.

Keywords – Social Networks, Dominating Set, Positive Influence Dominating Set

1. Introduction

Recently, social networks have become platforms used constantly throughout the day by many people. Everyone can be influenced by the messages (written or audio) that are posted, whether positive or negative in the social networks. People spend a significant amount of time each day trying to influence each other on social networks, and consequently, they strive to influence others. People generally more influenced by people with whom they have relative connections, or by people with whom they have social connections, rather than by those on social networks. As a result, the approach has been adopted that half or more of a person's neighbours have a positive influence on them.

The social networks are represented as graphs, where nodes represent individuals and edges represent the relationships between individuals. The problem of finding individuals who influence other individuals in a social network is called the Positive Influence Dominating Set (PIDS) problem, and it is a special case of the dominating set problem, where the resulting set is expected to have a minimum number of elements.

The solution to the PIDS problem can be used to construct a connected dominating set (DS) in wireless networks, thereby reducing the volume of repetitive information used in communication and decreasing energy consumption. Furthermore, the solution to the PIDS problem can be used in connected facility location selection and the analysis of adaptive networks.

In order to determine the dominating set, there are many improved methods to solve the dominating set problems (the dominating set problem is an NP-hard problem). In order to focus on how to locate the nodes that minimally positively impact the entire network, this situation where PIDS is connected and minimal, constructs a more effectively connected virtual backbone. Many studies have been conducted on solutions to DS and PIDS problems. First, after defining these problems, some of the studies on methods developed for this problem are presented.

Definition 1: Assume that $G=(V,E)$ is an undirected and simple graph. If at least one of the neighbours of the nodes in the set V is $D \subseteq V$, then D is called the Dominating Set (DS).

Definition 2: Assume that $G=(V,E)$ is a graph. If the node $S \subseteq V$ and $\forall v_i \in V$ have at least $\left\lceil \frac{v(v_i)}{2} \right\rceil$ neighbours that are elements of the set S , then the set S is called Positive Influence Dominating Set (PIDS) where $d(v_i)=|N(v_i)|$.

The DS problem is an NP-Complete problem, and Okumuş and Karci made a significant contribution to its solution in their work published in 2024 (Okumuş and Karci, 2024). The PIDS problem is an APX-hard problem, and therefore, the Greedy Algorithm method is used to solve this problem. Wang his/her friends (Wang et al, 2011) shown that the PIDS problem is APX-hard and presented a Greedy Algorithm for solving this problem. A method with Greedy Algorithm property was proposed for the connected PIDS solution on cubic graphs (Yao et al, 2020). They also shown that their work is valid for k-regular graphs.

The PIDS problem is a variant of the DS problem, and a fast Greedy Choice algorithm has been proposed for its solution (Pan and Bu, 2019), and experimental studies have been conducted on real networks. The aim was to demonstrate that the proposed method provides good results in terms of time. The minimum PIDs (MPIDS) problem in social networks is a significant problem, and heuristic, greedy choice, machine learning, and exact methods are used to solve it. A method with local selection capability has been proposed in the study (Sun et al, 2023).

The PIDS problem is a variant of the minimum DS problem, and attempts have been made to solve it using Integer Linear Programming (ILP) (Lin et al, 2018). This study also proposes a method combining ILP with a memetic algorithm. The aim is to identify a small number of influential individuals to spread positive influence in social networks (Bouaamama and Blum, 2021), and a greedy choice based heuristic algorithm is proposed for the MPIDS problem.

Since social networks began to take center stage in advertising, information dissemination, and similar areas, there has been considerable research into solving the problems associated with online social networks. One such study (sham et al, 2022) focuses on solving the MPIDS problem of online social networks, constructing an initial solution using greedy choice logic and then attempting to improve this solution in subsequent steps. An approach method and a proposed method for solving the MPIDS problem are presented in the study (Shong et al, 2025).

The studies have been conducted on signed networks under time limits to maximize the effect in social networks with the development of electronic environments (He et al, 2023). Since information dissemination is time-dependent, the probability of impact for information dissemination has been calculated and an information dissemination index has been constructed to maximize the effect. At this aim, two heuristic algorithms are combined to determine the core nodes. Two effects (the consideration of elements with negative effects by few researchers, and the selection of Monte Carlo simulation for information dissemination) negatively affect information dissemination in social networks. Sheng and his/her friends (Sheng et al, 2020) focused on maximizing the positive effect by considering these two elements. They attempted to solve the MPIDS problem with a general greedy choice approach algorithm (Chen et al, 2022). A method combining genetic machine learning with particle swarm optimization has been proposed for studies on positive impact in social networks (Kundu and Choudhury, 2021).

Motivation: The problem of finding the positive influence dominating set is an NP-Hard problem, and the method proposed in this paper uses Malatya Centrality Algorithm to determine PIDS by robust and polynomial algorithms.

2. Malatya Positive Influence Algorithm: New Algorithm

We proposed an algorithm using Malatya Centrality algorithm to solve PIDS and MPIDS problems in social networks. Firstly, Malatya Centrality algorithm will be briefly explained.

Malatya Centrality was developed to calculate the relative influence of nodes in a graph, where $G=(V,E)$, and this centrality has been implemented (Karci et al, 2022; Yakut et al, 2023).

Definition 3: (Karci et al, 2022): (First Malatya Centrality) Assume that $G = (V, E)$ is a graph, where V is a nodes set and E is an edges set. The 1st Malatya Centrality of a node is given in Eq.1

$$\Psi_1(u) = \left(\sum_{\forall v_j \in N(u)} \frac{d(u)}{d(v_j)} \right) = \frac{1}{d(u)} \sum_{\forall v_j \in N(u)} \frac{d(u)}{d(v_j)} = \sum_{\forall v_j \in N(u)} \frac{1}{d(v_j)} \quad (1)$$

where $\forall u \in V$, $N(u)$ is the set of neighbours of node u except itself. The alternative definition for the 1st Malatya Centrality is given in Eq.2

$$\Psi_1(u) = \left(\sum_{\forall v_j \in N(u)} \frac{d(u)}{d(v_j)} \right) \frac{d_a(u)}{d(u)} = \frac{d_a(u)}{d(u)} \sum_{\forall v_j \in N(u)} \frac{d(u)}{d(v_j)} = d_a(u) \sum_{\forall v_j \in N(u)} \frac{1}{d(v_j)} \quad (2)$$

Malatya Centrality was developed to calculate the power / influence of a node within a graph and its relationship to its neighbours. By this way, the influence of the corresponding node in the region is revealed, and thus Malatya Centrality ensures which node is subject to which process.

Algorithm 1 Version-1 and Algorithm 1 Version 2 are two versions of the 1st Malatya Centrality. Assume that $G=(V,E)$ is an undirected simple graph where V is the set of nodes and E is the set of edges. $\forall v_i \in V$, $d(v_i)=|N(v_i)|$. Verion 2 of the 1st Malatya Centrality algorithm is used when it is to be used in the DS or PIDS. In these algorithms, $d_a(v_i)$ represents the active degree and the number of white neighbours of a node. Initially, all nodes in the given graph are considered as white, and when a node is selected for the PIDS set, its colour becomes black, and its neighbours are represented in gray colour.

Algorithm 1: 1stMalatyaCentrality(A, Ψ₁)-Version 1 (Eq.1)

```
Output: Ψ1
Input: A, A=Adjacency matrix, Ψ1=the first Malatya
Centrality values
i←1, ..., |V|
  Ψ1(vi)←0
i←1...n
  j←1, ..., n
    if vj∈N(vi) (A(i, j)==1)
      Ψ1(vi)← Ψ1(vi) +1/D(vj)
```

Algorithm 1: 1stMalatyaCentrality(A, Ψ₁)-Version 2 (Eq.2)

```
Output: Ψ1
Input: A, D, DA (A=Adjacency matrix, Ψ1=the first Malatya
Centrality values
i←1, ..., |V|
  Ψ1(vi)←0
i←1...n
  j←1, ..., n
    if vj∈N(vi) (A(i, j)==1)
      Ψ1(vi)← Ψ1(vi) +1/D(vj)
  Ψ1(vi) ← Ψ1(vi) *da(vi)
```

After calculating the 1st Malatya Centrality values, the next step is taken in care. The 1st Malatya Centrality takes in care a one-step (1-hop) distance. However, in order to determine the DS and PIDS, the two-step (2-hop) distance should be taken in care, and therefore the 2nd Malatya Centrality value is calculated (Eq.3).

Definition 4: (Okumuş and Karci, 2023: 2nd Malatya Centrality) $G=(V, E)$ is a graph, where $\forall u \in V$, $\Psi_2(u)$ is the 2nd Malatya Centrality value of node u . $\Psi_2(u)$ is computed as Eq.3 by summation of 1st Malatya centrality value of u over its neighbors' node the 1st Malatya centrality values.

$$\Psi_2(v_i) = \left(\sum_{\forall v_j \in N(v_i)} \frac{\Psi_1(v_i)}{\Psi_1(v_j)} \right) \frac{1}{d(v_i)} \frac{d_{active}(v_i)}{d(v_i)} = \frac{1}{d(v_i)} \frac{d_{active}(v_i)}{d(v_i)} \sum_{\forall v_j \in N(v_i)} \frac{\Psi_1(v_i)}{\Psi_1(v_j)} \quad (3)$$

After calculating the 2nd Malatya centrality values, DS of the given graph is determined. Once DS is determined, it is ensured that at least half of each node's neighbours are in the DS. As a result, care is taken to minimize the number of nodes in DS. Malatya Dominating Set Algorithm (MDSA) for DS solution developed by Okumuş and Karci (Okumuş and Karci, 2023) is shown in Algorithm 2.

Algorithm 2: 2ndMalatyaCentrality(A, Ψ₁, Ψ₂)

```
i←1, ..., |V|
  Ψ2(vi)←0
i←1...n
  j←1, ..., n
    if vj∈N(vi) (A(i, j)==1)
      Ψ2(vi)← Ψ2(vi) + Ψ1(vi) / Ψ1(vj)
  Ψ2(vi) ← Ψ2(vi)  $\frac{d_a(v_i)}{d^2(v_i)}$ 
```

After DS is determined, PIDS determination algorithm is run. PIDS algorithm guarantees that at least half of each node's neighbours are in DS, and its fundamental principle is the use of Malatya Centrality values. The PIDS algorithm is given in Algorithm 3.

Algorithm 3: MDSA(A, Ψ_2 , DS)

```

DS ← ∅
SelectedIndex ← MaxDugum(A,  $\Psi_2$ , NodeColour)
if SelectedIndex > 0
    DS ← DS ∪ {vSelectedIndex}
    NodeColour(vSelectedNode) ← Black
i ← 1, 2, ..., |V|
    if vi ∈ N(vSelectedNode)
        NodeColour(vi) ← Gray
RemoveGrayNode(A, D, DA, NodeColour)

```

Algorithm 4: MaxDugum(A, Ψ_2 , NodeColour)

```

GrayMax ← Max( $\Psi_2$ ) //max of gray nodes
WhiteMax ← Max( $\Psi_2$ ) //max of white nodes
if WhiteNeighbours(vGrayMax) > WhiteNeighbours(vWhiteMax) + 1
    SelectedIndex ← GrayMax
else SelectedIndex ← WhiteMax
Return(SelectedIndex)

```

Algorithm 5: RemoveGrayNode(A, D, DA, NodeColour)

```

i ← 1, 2, ..., |V|
    if NodeColour(vi) is Gray
        DA(vi) ← 0;
        j ← 1, 2, ..., |V|
            if vj ∈ N(vi) and NodeColour(vj) is White
                DA(vi) ← DA(vi) + 1;
        if (DA(vi) < 2)
            D(vi) ← 0 and DA(vi) ← 0
            NodeColour(vi) ← White
        k ← 1, 2, ..., |V|
            if vk ∈ N(vi)
                N(vi) ← N(vi) - {vk}

```

The algorithm proposed in this study, which ensures that at least half of each node's neighbours are in the dominating set after the dominating set has been determined, is shown in Algorithm 6. The developed algorithm has been called as Malatya Positive Influence Algorithm (MPIA).

Algorithm 6: MalatyaPositiveInfluence(A, D, InfluenceState, NodeColour)

```

while not(Termination)
    i ← 1, 2, ..., |V|
        j ← 1, 2, ..., |V|
            if ((vi, vj) ∈ E) && (NodeColour(vj) = Black)
                InfluencerNeighbour(vi) ← InfluencerNeighbour(vi) + 1
    i ← 1, 2, ..., |V|
        if InfluencerNeighbour(vi) ≥ ceil(D(i)/2)
            Influencerstate(vi) = 1;
    else
        k ← 1, 2, ..., |V|
            if ((vi, vk) ∈ E) and (NodeColour(vk) = Black)
                GrayNeighbour = GrayNeighbour + 1;

```

```

Remaining ← ceil(D/2) - GrayNeighbour
k ← 1;
while (Remaining > 0) && (k < n)
  if ((vi, vk) ∈ E) and (NodeColour(vk) = Gray)
    InfluencerState = InfluencerState ∪ {vk}
    DugumRengi(k) = 2;
    Remaining ← Remaining - 1
  k ← k + 1

```

3.Experimental Results

The proposed MPIA method has been applied to some graphs, and the results obtained are presented in the figures in Fig.1. In each graph, the nodes shown in red are elements of PIDs. In the experimental results shown in Fig.1, it is clearly seen that the obtained results are not MPIDS but PIDS.

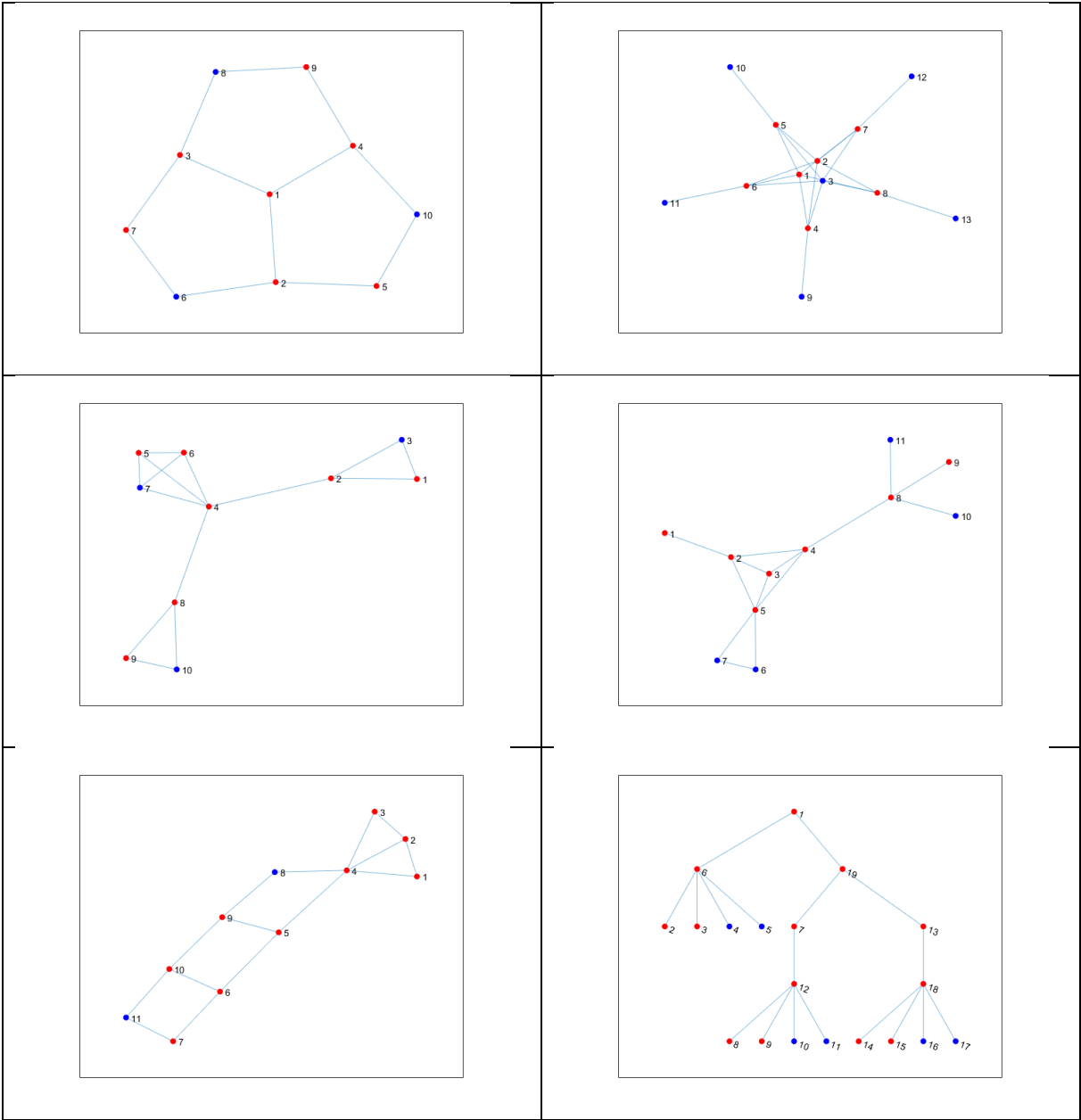


Figure 1. The applications of MPIA and its results

4. Conclusions

In this study, a new algorithm was developed for solving the problem of positive influence dominating set. This algorithm has a polynomial time complexity and is robust. It also consistently provides the same solution for the same graph.

References

- Bouamama, S., Blum, C., (2021) "An Improved Greedy Heuristic for the Minimum Positive Influence Dominating Set Problem in Social Networks", Algorithms, <https://doi.org/10.3390/a14030079>.
- Chen, W., Zhong, H., Wu, L., Du, D.-Z., (2022) "A general Greedy Approximation Algorithm for finding Minimum Positive Influence Dominating sets in Social Networks", Journal of Combinatorial Optimization , <https://doi.org/10.1007/s10878-021-00812-3>.
- He, Q., Du, H., Liang, Z., (2023) "Positive Influence Maximization in Signed Networks within a Limited Time", IEEE Transactions on Computational Social Systems, <https://doi.org/10.1109/TCSS.2022.3192410>.
- Karci, A., Yakut, S., Öztemiz, F., (2022) "A New Approach Based on Centrality Value in Solving the Minimum Vertex Cover Problem: Malatya Centrality Algorithm", Journal of Computer Science, Vol:7, pp:81-88.
- Kundu, G., Choudhury, S., (2021) " A Discrete Genetic Learning enabled PSO for Targeted Positive Influence Maximization in Consumer Review Networks", Innovations in Systems and Software Engineering, <https://doi.org/10.1007/s11334-021-00396-5>.
- Lin, G., Guan, J., Feng, H., (2018) "An ILP based memetic algorithm for finding minimum positive influence dominating sets in social networks", Physic A, <https://doi.org/10.1016/j.physa.2018.02.119>.
- Pan, J., Bu, T.-M.,(2019) "A Fast Greedy Algorithm for Finding Minimum Positive Influence Dominating Sets in Social Networks", 2019 IEEE INFOCOM WJSHPS: CAOS-2019: Communication and Networking Aspects of Online Social Networks, pp:360-364.
- Okumuş, F., and Karci,Ş., (2023) "MDSA: A Dynamic and Greedy Approach to Solve the Minimum Dominating Set Problem," *Applied Sciences* 2024, Vol. 14, Page 9251, vol. 14, no. 20, p. 9251, Oct. 2024, doi: 10.3390/app14209251.
- Okumuş, F., Karci, Ş. (2024) "MDSA: A Dynamic and Greedy Approach to Solve the Minimum Dominating Set Problem", Applied Sciences, <https://doi.org/10.3390/app14209251>.
- Shan, Y., Kang, Q., Xiao, R., Chen, Y., Kang, Y., (2022) "An Iterated Carousel Greedy Algorithm for Finding Minimum Positive Influence Dominating Sets in Social Networks", IEEE Transactions On Computational Social Systems, Vol:9, pp:830-838.
- Sheng, J., Chen, L., Cehn, Y., Liu, W., (2020) "Positive Influence Maximization in Signed Social Networks under Independent Cascade model", Soft Computing, <https://doi.org/10.1007/s00500-020-05195-x>.
- Sun, R., Wu, J., Jin, C., Wang, Y., Zhou, W., Yin, M., (2023) "An Efficient Local Search Algorithm for Minimum Positive Influence Dominating Set problem", Computers and Operations Research, Vol:154, <https://doi.org/10.1016/j.cor.2023.106197>.
- Wang, F., Du, H., Camacho, E., Xu, K., Lee, W., Shi, Y., Shan, S., (2011), "On positive influence dominating sets in social networks", Theoretical Computer Science, Vol:412, pp:265-269.
- Yakut,S., Öztemiz,F., Karci,A.,(2023) "A New Approach Based on Centrality Value in Solving the Maximum Independent Set Problem: Malatya Centrality Algorithm", Journal of Computer Science, Vol:8, pp:16-23.
- Yao, X., Huang, H., Du, H., (2020) "Connected positive influence dominating set in k-regular graph", Discrete Applied Mathematics, <https://doi.org/10.1016/j.dam.2020.08.010>.
- Zhong, H., Li, W., Zhang, Q., Lin, R., Tang, Y., (2025) "Efficient Approximation Algorithms for Several Positive Influence Dominating Set Problems in Social Networks", IEEE Transactions On Computational Social Systems, <https://doi.org/10.1109/TCSS.2025.3541230>.